Future developments of Light-Cone Sum Rules

Prospects for global determinations of $|V_{ub}|$

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Disclaimer

What this talk is:

- expressing my impression of where LCSRs can be headed
- relying on the state of the literature
- presenting lower limits on accuracies

What this talk is not:

- not presenting new results
- not providing usable results (all numbers / plots shown herein are simulations only)

Principle of QCD sum rules in a nut shell

- contrary to what some colleagues say, sum rules are not a form of “black magic”
- infers knowledge of more exclusive hadronic matrix element f
  - construct artificial correlation functions $\mathcal{F}$

$$\mathcal{F}(k^2) \sim \int d^4x\, e^{ik\cdot x} \langle 0|\mathcal{T}\{J_1(x), J_2(0)\}|\overline{B}\rangle$$
$$\sim \int d\omega \sum_{n,t} \left(\frac{\alpha_s}{4\pi}\right)^n \mathrm{tr}\{T_{n,t}\mathcal{M}_t(\omega)\}$$

  - phase space (k) chosen to ensure light-cone dominance of T-product
  - less exclusive hadronic matrix elements $\langle 0|\overline{q}(x)\Gamma b_v(0)|\overline{B}\rangle$
  - perturbatively calculable quantities $T_{n,t}$
  - relate $\mathcal{F}$ to the quantitiy of interest within a dispersion relation

$$\mathcal{F}(k^2) \sim \frac{f}{m^2 - k^2} + \int ds \frac{\rho^{\text{cont.}}(s)}{s - k^2}$$

  - spectral information taken from from experimental data

Motivation

- in order to extract $|V_{ub}|$ from present and future experimental data we need information on the relevant hadronic matrix elements
  - genuine non-perturbative quantities
  - for this talk: infer information from Light-Cone Sum Rules (LCSRs)
  - for the lattice perspective, wait for Andreas' talk hereafter
- Light-Cone Sum Rules (LCSRs) currently provide complementary information to lattice QCD
  - probe different region of phase space, where final state meson is energetic in the B rest frame
  - status quo of $B \to \pi$, $B \to K$ form factors: complementarity expected to stay for some time
- what are the prospects for developments of LCSRs in the future?

What we should aim for

- global analysis of exclusive $b \to u\ell\nu$ transitions:
  - $B \to \pi\ell\nu$
  - $B \to \pi\pi\ell\nu$
  - $B \to \gamma\ell\nu$
  - $B \to \tau\nu$

- infer hadronic matrix elements exclusively from LCSRs (and f_B from two-point sum rules)
  - $\Rightarrow$ only use data in LCSR-accessible phase space
  - benefit over lattice: semileptonic decays have larger partial rates in LCSR-accessible phase space
  - fully complementary to the lattice analyses
  - extrapolation to lattice (e.g. via z-expansion) only a-posteriori

De-Motivation

- light-meson LCSRs have a large proliferation of nuisance parameters
- for $B \to \pi$ form factor study [e.g. Imsong/Khodjamirian/Mannel/DvD 1409.7816]
  - 7 parameters for π-LCDAs
  - 1 threshhold
- for $B \to V$, in order to achieve the same level of sophistication
  - twice as many LCDAs $\Rightarrow$ twice as many LCDA parameters
  - 3 threshholds: one per form factor (4 if semitauonic decays are considered)
- analysis involving $B \to \tau\nu$ and $B \to \{\pi, \rho, \omega\}\ell\nu$ would require > 50 parameters
  - hard but not impossible to do in a global analysis
- however: no benefit from global analysis
  - little to no correlations among nuisance parameters!

for this talk: consider global analysis using B-LCSRs only

What we should worry about

- LCSRs rely on information on B-meson Light-Cone Distribution Amplitudes (LCDAs)
  - expectation: we can infer LCDAs from $\mathcal{B}(B^- \to \gamma \ell^- \bar{\nu})$
  - Q **how sensitive are we to the leading-twist B-meson LCDA?**
  - Q **are there ways to improve the present sensitivity?**

- LCSRs rely on modelling of the continuum contributions
  - usually simple “pole + step function” models
  - Q **are there ways to improve this modelling?**

Determination of LCDA parameter(s) from $B^- \to \gamma \ell^- \overline{\nu}$

2-particle LCDAs: inverse (logarithmic) moments

Leading-twist 2-particle LCDA ϕ_+ defined previously during this workshop

[see A. Rusov's and V. Braun's talks]

Hadronic matrix elements for $B \to \gamma\ell\nu$ depend to leading-twist and NLL accuracy on only three moments of the ϕ_+ distribution amplitude [Beneke/Rohrwild 1110.3228]

$$\frac{1}{\lambda_{B,+}(\mu)} \equiv \int_0^\omega \frac{\mathrm{d}\omega}{\omega}\phi_+(\omega;\mu)$$

$$\sigma_{B,+}^{(n)}(\mu) \equiv \lambda_{B,+}(\mu,\mu_0)\int_0^\omega \frac{\mathrm{d}\omega}{\omega}\ln^n\frac{\mu_0}{\omega}\phi_+(\omega;\mu)$$

prospect to extract $\lambda_{B,+}$ from measurements of the rate [e.g. Beneke/Rohrwild 1103.228]

caveat: potentially large soft-contributions if $\lambda_{B,+} < 0.3\,$GeV [Wang 1609.09813]

Higher twist and 3-particle LCDAs

- work by V. Braun and collaborators is a game changer:

$$
\begin{aligned}
\phi_+(\omega) &= \omega\, f(\omega) \\
\phi_3(\omega_1,\omega_2) &= -\frac{1}{2}\varkappa(\lambda_E^2 - \lambda_H^2)\,\omega_1\omega_2^2\,\partial_{\omega_2} f(\omega_1+\omega_2) \\
\psi_4(\omega_1,\omega_2) &= \varkappa\lambda_E^2\,\omega_1\omega_2\, f(\omega_1+\omega_2) \\
\widetilde{\psi}_4(\omega_1,\omega_2) &= \varkappa\lambda_H^2\,\omega_1\omega_2\, f(\omega_1+\omega_2) \\
2\omega_1\,\phi_4(\omega_1,\omega_2) &= \omega_2\left[\psi_4(\omega_1,\omega_2) + \widetilde{\psi}_4(\omega_1,\omega_2)\right]
\end{aligned}
$$

[V. Braun's talk yesterday, p. 21]

- make any choice of $f(\omega)$ (which introduces model-dependency!)
- due to the EOM our knowledge of $\lambda_{B,+}$ furthers our knowledge of all LCDAs up to and including twist 4.
- caveat: only holds at $O\left(\alpha_s^0\right)$ / large N_C limit

Q: how sensitive are we to ϕ_+ / its inverse moments?

$\mathrm{d}\mathcal{B}(B \to \gamma\ell\nu)/\mathrm{d}E_\gamma$ only probes $|V_{ub}|f_B/\lambda_{B,+}$!
Answer: Branching ratio alone is not sensitive at all!

Rescue comes at hand of normalisation to $\mathcal{B}(B \to \tau\nu)$ [Braun/Khodjamirian 1210.4454]

- in absence of experimental results on this ratio a "global" analysis is required
- fit for $|V_{ub}|f_B$ and $\lambda_{B,+}$ simultaneously
- results on the ratio will likely benefit from cancellation of some of the experimental uncertainties

Combining $B \to \gamma \ell \nu$ with $B \to \tau \nu$

[Ball/Braun 1210.4453]

schematically:

$$\frac{\mathcal{B}(B \to \gamma \ell \nu)}{\mathcal{B}(B \to \tau \nu)} \propto \frac{1}{\lambda_{B,+}} + \text{soft contributions}$$

theory

$B \to \tau \nu$

- only used to remove product $|V_{ub}|\, f_B$

$B \to \gamma \ell \nu$

- hard contrib. known to NLL precision [Beneke/Rohrwild 1110.3228]
- soft contrib. known in disp. approach [Braun/Khodjamirian 1210.4454]
  3-particle contr. [Wang 1609.09813]

experiment

- expected uncertainty on $B \to \tau \nu$ for Belle II: $\mathbf{3}\%$ [B2TIP]
- expected uncertainty on $B \to \mu \nu$ for Belle II: $\mathbf{7}\%$ [B2TIP]
- no expected uncertainty on $B \to \gamma \ell \nu$ for Belle II yet
  - photons not a problem for Belle II!
  - $\mathcal{B}(B \to \gamma \ell \nu)\big|_{E_\gamma > 1.7\,\text{GeV}} \sim \mathcal{B}(B \to \mu \nu)$
  - not unreasonable to assume uncertainty of $\sim \mathbf{10}\%$ (assuming $\lambda_{B,+} \simeq 0.35\,$GeV)

fit yields lower limit on the uncertainty

$$\sigma(\lambda_{B,+}) = 0.03\,\text{GeV} \qquad [\text{assuming } \lambda_{B,+} = 0.35\,\text{GeV}]$$

Q: are there ways to improve the sensitivity?

Specifically:

- how to maximize amount of information inferred from $B \to \gamma \ell \nu$?

- two suggestions come to mind
  - moments of the photon energy

$$M_k \equiv \int_{E_{\gamma,\text{min}}}^{M_B/2} \mathrm{d}E_\gamma \, \frac{\mathrm{d}\Gamma(B \to \gamma \ell \nu)}{\mathrm{d}E_\gamma} \left(\frac{2E_\gamma}{M_B}\right)^k \tag{1}$$

  - angular analysis of the decay

$$\frac{\mathrm{d}^2\Gamma}{\mathrm{d}E_\gamma \, \mathrm{d}E_\ell} \to \frac{\mathrm{d}^2\Gamma}{\mathrm{d}E_\gamma \mathrm{d}\cos\theta_\ell}$$

Moments of photon energy spectrum

- in rate-normalised observables, any sensitivity to $\lambda_{B,+}$ stems from interference of non-$\lambda_{B,+}$-dep. terms with $\lambda_{B,+}$-dep. terms
- decay rate depends on combination $|F_A|^2 + |F_V|^2$
- interferences cancel to larged extent in this combination

Angular distribution of $B \to \gamma \ell \nu$

- why not look into the angular distribution of this $1 \to 3$ decay? Maximises exploitation of data in semi-leptonic decays!
- can be easily included in global analysis

$$\frac{\mathrm{d}^2\Gamma}{\mathrm{d}x_\gamma\,\mathrm{d}x_\ell} \propto (1-x_\gamma)\left[(1-x_\nu)^2\,(F_A+F_V)^2+(1-x_\ell)^2\,(F_A-F_V)^2\right]$$

where $x_i \equiv 2E_i/M_B$, $x_\nu = 2 - x_\gamma - x_\ell$

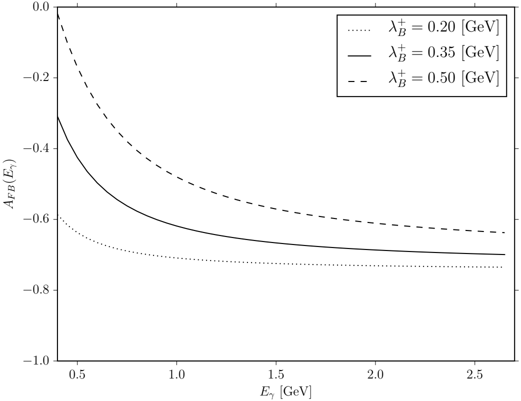
- using $1 - x_{\ell(\nu)} = x_\gamma(1 \pm \cos\theta_\ell)/2$ obtain angular distribution

$$\frac{\mathrm{d}^2\Gamma}{\mathrm{d}x_\gamma\,\mathrm{d}\cos\theta_\ell} \propto (1-x_\gamma)\,x_\gamma^3\,\left[(1+\cos^2\theta_\ell)\left(|F_A|^2+|F_V|^2\right)-4\cos\theta_\ell\,\mathrm{Re}\,F_A^*F_V\right]$$

- beside rate, only further observables is forward-backward asymmetry A_{FB}

$$A_{\mathsf{FB}}(E_\gamma) \equiv -\frac{3}{2}\,\frac{\mathrm{Re}\,F_A^*F_V}{|F_A|^2+|F_V|^2}$$

Angular distribution of $B \to \gamma \ell \nu$



- use hypothetical measurements of binned A_{FB}
  - $[1.7\,\text{GeV}, 2.0\,\text{GeV}]$
  - $[2.0\,\text{GeV}, 2.3\,\text{GeV}]$
  - $[2.3\,\text{GeV}, M_B/2]$
- assume a lower limit of $\mathbf{5\%}$ on the uncertainty in each bin

fit yields a lower limit on the precision:

$$\sigma(\lambda_{B,+}) = 0.05\,\text{GeV} \qquad [\text{assuming } \lambda_{B,+} = 0.35\,\text{GeV}]$$

Modelling the continuum spectrum

$B \to \pi$ sum rule as an example

only 2-particle contributions and for $q^2 = 0$ [Khodjamirian/Mannel/Offen hep-ph/0611193]

$$f_+(0) \propto \int_0^{s_0^\pi} \exp(-s/M^2)\phi_-(s/M_B) + \text{3-particle contr.}$$

- threshhold parameter s_0^π is a “reparametrization of our ignorance”
- information on s_0 crucial for determination of the form factor

Daughter sum rule for s_0^π determination

only 2-particle contributions and for $q^2 = 0$ [Khodjamirian/Mannel/Offen hep-ph/0611193]

– define s moments of a sum rule (again $B \to \pi$ as an example)

$$\langle s^k \rangle \equiv \int_0^{s_0^\pi} s^k \exp(-s/M^2)\phi_-(s/M_B) + \text{3-particle contr.}$$

– Quark Hadron Duality (QHD) motivates equality first s moment of OPE result and hadronic model

– technically can be done using derivative with respect to Borel parameter

– first moment can be used witin a Bayesian framework to reduce uncertainties
[demonstrated for π-LCSRs in Imsong/Khodjamirian/Mannel/DvD 1409.7816]

– naive result on normalised first moment: m^2, where m is the mass of the interpolated state

– proof of principle for incorporation within a statistical framework
  - B-meson interpolation
  - central value taken from experimental results on $M_B \pm 1\%$
  - uncertainty inflated by factor 350, due to lack of information on spectrum of interpolating current

Daughter sum rules for B-LCSRs

- $B \to \pi$ literature uses threshhold $s_0^\pi = 0.7\,\text{GeV}^2 \simeq [0.83\,\text{GeV}]^2$
  [Khodjamirian/Mannel/Offen hep-ph/0611193]
- reaches beyond 3π-threshhold, which starts at $\sim 0.18\,\text{GeV}^2 \simeq [0.42\,\text{GeV}]^2$
- suggests that first moment of the sum rule should be larger than M_π^2
- additional contributions stem from $B \to 3\pi$ and even $B \to 5\pi$ form factors
- spectral information from $\tau \to [3\pi, 5\pi]\nu_\tau$ can help pinning down first moment of the sum rule

Benefits within global analysis in $B \to V\ell\nu$

- full angular distribution will allow to constrain ratios of the form factors
  [e.g. Faller/Feldmann/Khodjamirian/Mannel/DvD 1310.6660]
- for a given LCDA model, ratios of the form factors strongly depend on the threshhold parameters
- very useful as cross check of the inputs / validation of the LCDA model

Summary

my personal opinion on future developments and prospects for LCSRs

– light-meson LCSRs will not benefit from combination in global analyses
  – should be used individually, and $|V_{ub}|$ averaged a-posteriori

– B-LCSRs will benefit from global analyses
  – however, benefits will not overcome inherently larger theory uncertainties with respect to B-LCDA inputs

– input(s) for B-LCSRs based on data will keep being rather uncertain
  – minimal uncertainty of $30 \ldots 50$ MeV
  – more realistically: $50 \ldots 75$ MeV