RARE KAON DECAYS ON THE LATTICE

LmC, Siegen, 18-20.09.2017

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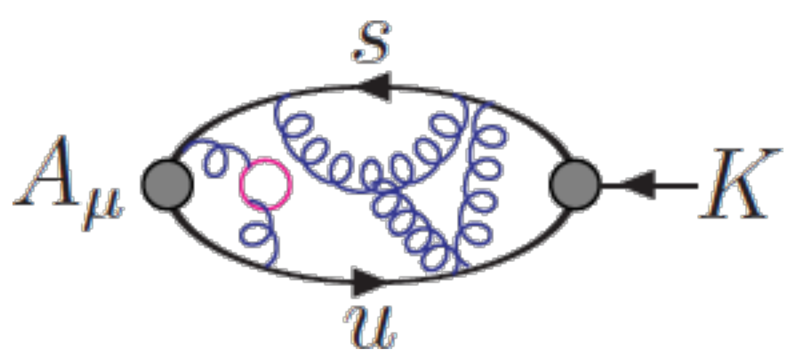
NON-RARE KAON DECAYS

- *leptonic kaon decys*
- *semileptonic kaon decays*

LEPTONIC KAON DECAYS

Leptonic kaon decay:



$$\langle 0|\bar{s}/\bar{d}\gamma_\mu\gamma_5 u|K^+/\pi^+(p)\rangle = i f_{K^+/\pi^+} p_\mu$$

ratio of decay constants → ratio of CKM MEs:

$$\frac{\Gamma(K^+ \to l^+\nu_l(\gamma))}{\Gamma(\pi^+ \to l^+\nu_l(\gamma))} = \left(\frac{|V_{us}|}{|V_{ud}|}\frac{f_{K^+}}{f_{\pi^+}}\right)^2 \frac{m_K\left(1 - m_l^2/m_K^2\right)^2}{m_\pi\left(1 - m_l^2/m_\pi^2\right)^2} \underbrace{\left(1 + \delta_{\rm EM}^{\rm ChPT}\right)}_{0.9930(35)}$$

Marciano, PRL. 93 (2004) 231803
hep-ph/0402299

experimental status:

$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_{K^+}}{f_{\pi^+}} = 0.2758(5)$$

FLAVIA Kaon WG EPJ C 69, 399-424 (2010)
arXiv:1005.2323 KTeV, Istra, KLOE

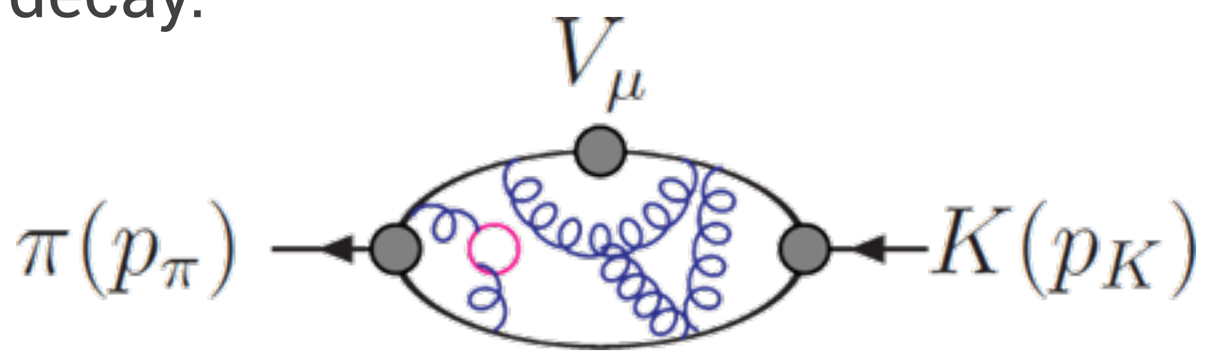
update e.g. Moulson

$$\left|\frac{V_{\rm us}}{V_{\rm ud}}\right|\frac{f_{K^+}}{f_{\pi^+}} = 0.2760(4)$$

arXiv:1411.5252

SEMILEPTONIC KAON DECAYS

semi-leptonic kaon decay:



matrix element and form factors:

$$\langle \pi(p_\pi)|V_\mu(0)|K(p_K)\rangle = f_+^{K\pi}(q^2)(p_K+p_\pi)_\mu + f_-^{K\pi}(q^2)(p_K-p_\pi)_\mu$$

$$\Gamma(K\to\pi l\nu) = C_K^2 \frac{G_F^2 m_K^5}{192\pi^2} S_{\mathrm{EW}}(1+\delta_{SU(2)}^{\mathrm{ChPT}}+\delta_{\mathrm{EM}}^{\mathrm{ChPT}})^2\, I\, \left(f_+^{K^0\pi^-}(0)|V_{us}|\right)^2$$

charged and neutral kaon decays

experimental status:

$$|V_{us}|f_+^{K^0\pi^-}(0) = 0.2163(5)$$

FLAVIA Kaon WG EPJ C 69, 399-424 (2010)
arXiv:1005.2323

update e.g. Moulson

$|V_{us}|f_+^{K^0\pi^-}(0) = 0.2165(4)$

arXiv:1411.5252

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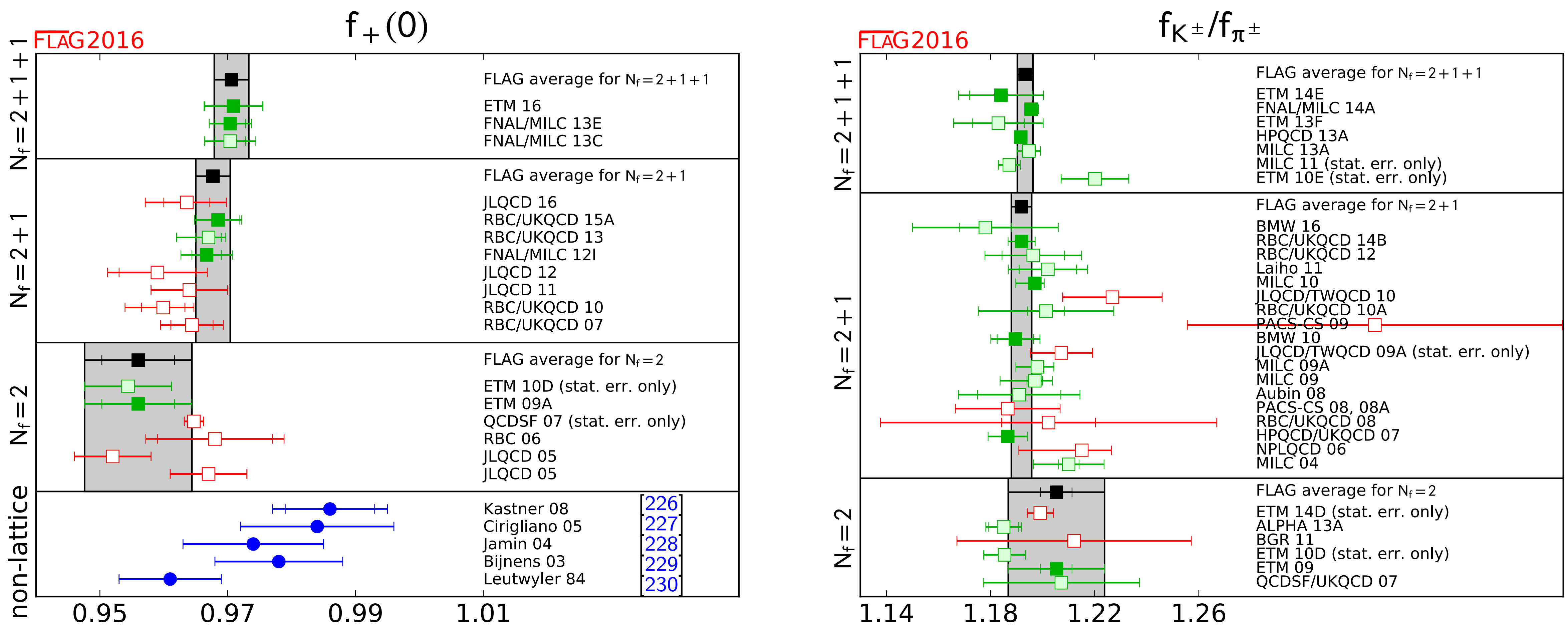
BEYOND PRECISION?

kaon phenomenology on the lattice has become a precision tool



FLAG Eur. Phys. J. C (2017) 77:112

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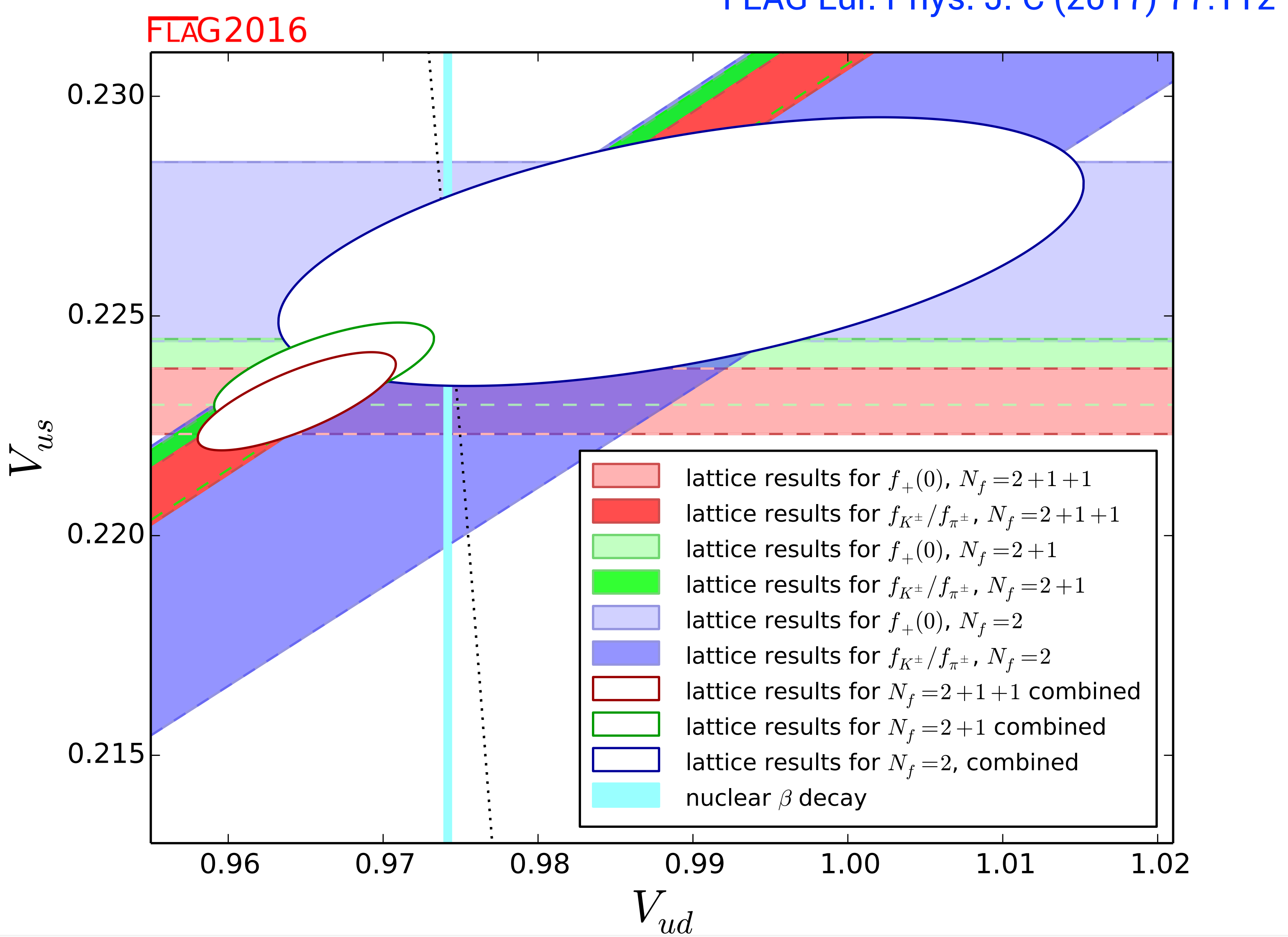
KAON PHYSICS

$$|V_{us}|f_+^{K^0\pi^-}(0) = 0.2165(4) \qquad \frac{f_{K^+}}{f_{\pi^+}}\frac{|V_{us}|}{|V_{ud}|} = 0.2760(4)$$ Moulson arXiv:1411.5252

FLAG Eur. Phys. J. C (2017) 77:112



high precision test of SM unitarity - no worrisome tension at sub-percent-level precision

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KAON PHYSICS

transition matrix element

$$\langle \pi(p_\pi)|V_\mu(0)|K(p_K)\rangle = f_+^{K\pi}(q^2)(p_K + p_\pi)_\mu + f_-^{K\pi}(q^2)(p_K - p_\pi)_\mu$$

quite standard now and can be done with high precision

sub-percent precision has been reached, further progress requires inclusion of QED effects ($\alpha_{EM} \approx 1\%,\ (m_d - m_u)/\Lambda_{\mathrm{QCD}} \approx 1\%$*)*

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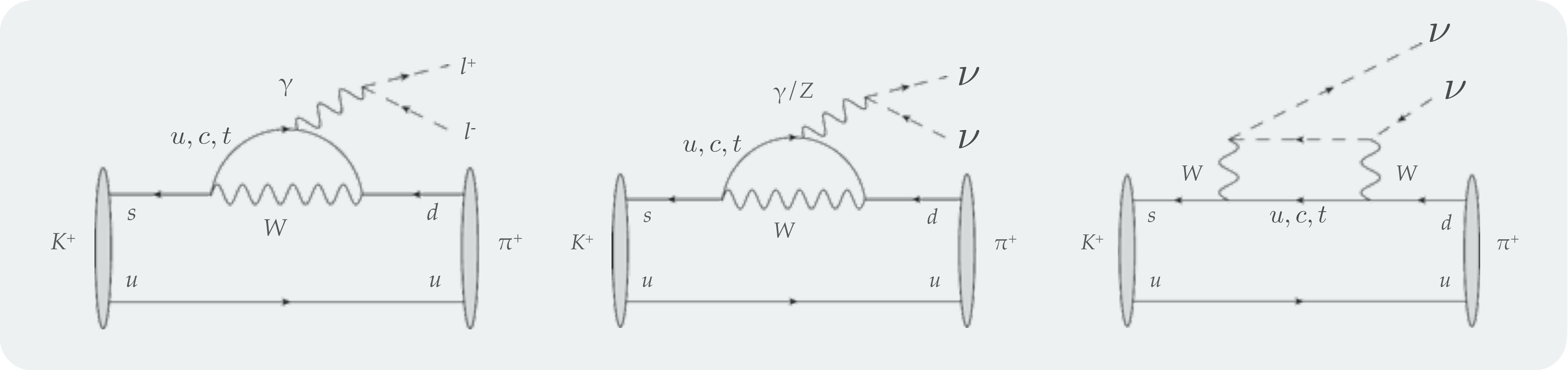
RARE KAON DECAYS

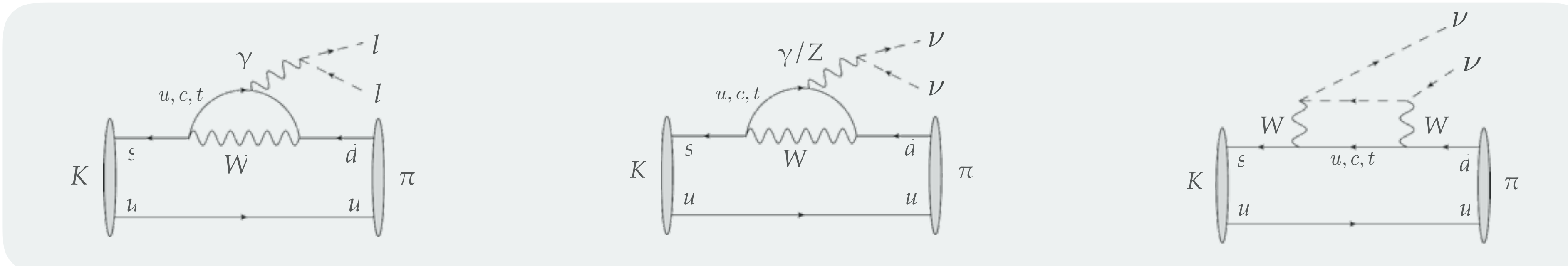
blindly increasing precision is pointless, so what should we do?

- *think about QED corrections to the semileptonic decay* ✓
- *think about how to do rare Kaon decays on the lattice* ✓



  - *FCNC (W-W or γ/Z-exchange diagrams)*
  - *deep probe into flavour mixing and SM/BSM due to suppression in the SM (2nd order weak)*
  - *can determine V_{td}, V_{ts} and test SM*

RARE KAON DECAYS – EXPERIMENTS



$K_L \to \pi^0 \nu \bar{\nu}$

- *KOTO (J-PARC)*
- *direct CP violation*
- *exp. BR* $\leq 2.6 \times 10^{-8}$
  theory BR $3.0(3) \times 10^{-11}$
- *GIM → top dominated and charm suppressed, pure SD*

$K^+ \to \pi^+ \nu \bar{\nu}$

- NA62 (CERN)
- CP conserving
- exp. BR $1.73(^{+1.15}_{-1.05}) \times 10^{-10}$
  theory BR $0.911(72) \times 10^{-10}$
- small **LD contribution**, candidate for lattice

$K^+ \to \pi^+ l^+ l^-$

$K_S \to \pi^0 l^+ l^-$

- *both exp. observed*
- *1-photon exchange **LD dom.***
- *indirect contribution to CP rare K_L decay*
- *SM prediction mainly ChPT*
- *lattice can predict ME and LECs*
- *well suited for experiment*

compute in lattice QCD

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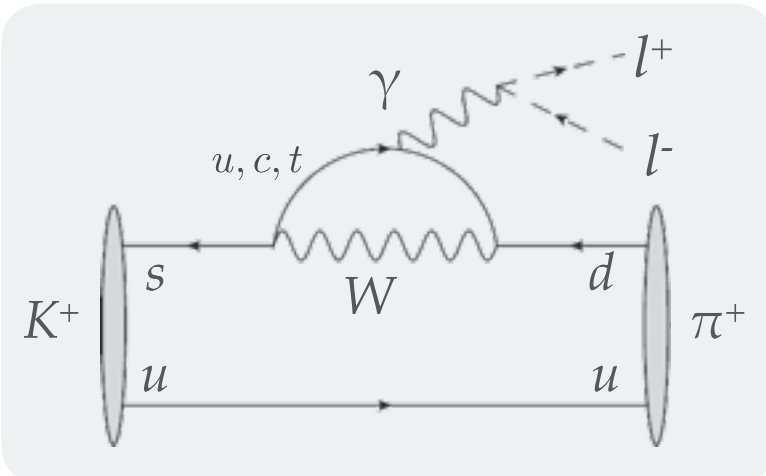
RARE KAON DECAYS – FORM FACTOR



Concentrate on $K^+ \to \pi^+ l^+ l^-$
dominant 1-photon contribution

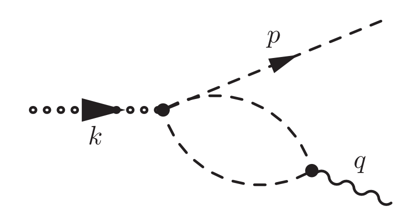
$$\mathcal{A}_\mu = (q^2) \int d^4x \langle \pi(p) | T \left[J_\mu(0) H_W(x) \right] | K(k) \rangle$$

Decay amplitude in terms of elm. transition form factor:

D'Ambrosio et al., JHEP 9808, 004 (1998)

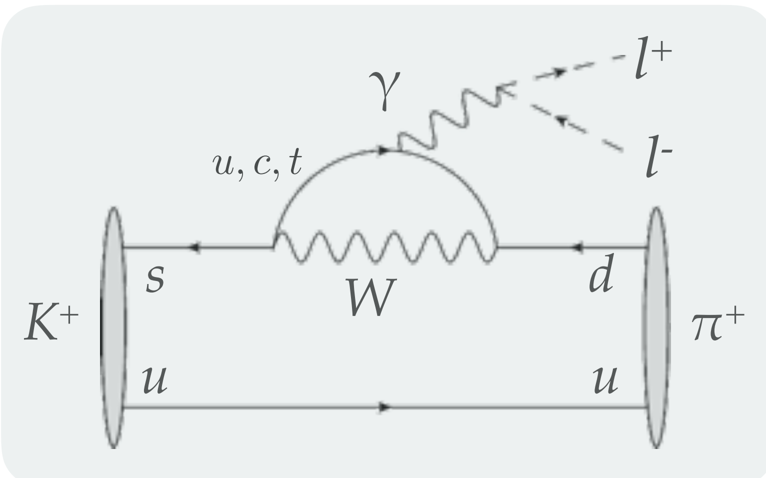
$$\mathcal{A}^c_\mu(q^2) = -i \frac{G_F}{4\pi}^2 \left[q^2(k+p)_\mu - (M_K^2 - M_\pi^2) q_\mu \right] V_c(q^2/M_K^2)$$

$$V_c(q^2/M_K^2) = a_c + b_c q^2/M_K^2 + V_c^{\pi\pi}(q^2/M_K^2)$$



- *the $|a_S|$ and $|a_+|$ can be extracted from branching ratios*
- *a_S parameterises also the CP-violating contribution to the K_L BR*
- *sign of a_s unknown - could be predicted by lattice — plays crucial role in BR prediction for $K_L \to \pi^0 e^+ e^- / \mu^+ \mu^-$*

RARE KAON DECAYS – RENORMALISATION



4-flavour

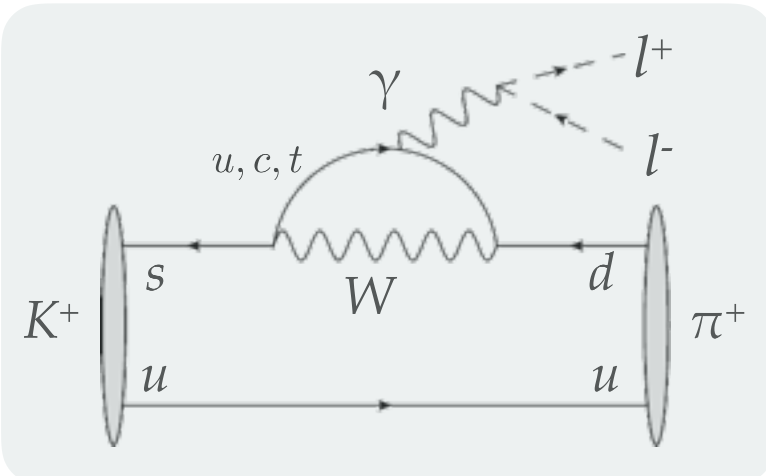
$$\mathcal{A}_\mu = (q^2) \int d^4x \langle \pi(p) | T \left[J_\mu(0) H_W(x) \right] | K(k) \rangle$$

GIM

$$H_W(x) = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \left[C_1 (Q_1^u - Q_1^c) + C_2 (Q_2^u - Q_2^c) \right]$$

$$Q_1^q = \left(\bar{s}_i \gamma_\mu^L d_i \right) \left(\bar{q}_j \gamma_\mu^L q_j \right), \qquad Q_2^q = \left(\bar{s}_i \gamma_\mu^L q_i \right) \left(\bar{q}_j \gamma_\mu^L d_j \right)$$

RENORMALISATION – RENORMALISATION

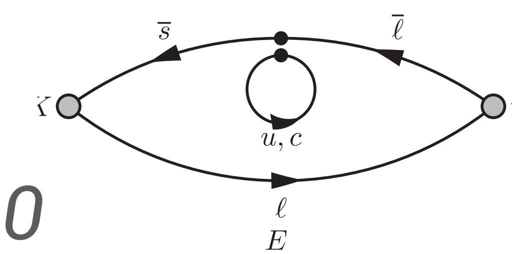


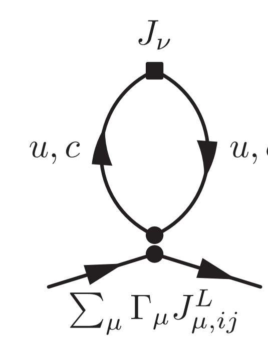
4-flavour

$$\mathcal{A}_\mu = (q^2) \int d^4x \langle \pi(p)| T\left[J_\mu(0) H_W(x)\right] |K(k)\rangle$$

$$H_W(x) = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \left[C_1(Q_1^u - Q_1^c) + C_2(Q_2^u - Q_2^c)\right]$$

- *Q_1 and Q_2 in H_W renormalise multiplicatively (chiral fermions)*
- *J_μ conserved*
- *divergences:*
  - *quadratic divergence can appear as x→0 but gauge invariance reduces it to a logarithmic one*
  - *remaining logarithmic divergence cancelled via GIM (→ need charm quark in lattice simulation)*





SPECTRAL REPRESENTATION – MINKOWSKI

$$\mathcal{A}^c_\mu(q^2) = \int d^4x \langle \pi^c(p)| T\left[J_\mu(0) H_W(x)\right] |K^c(k)\rangle$$

non-strange intermediate states

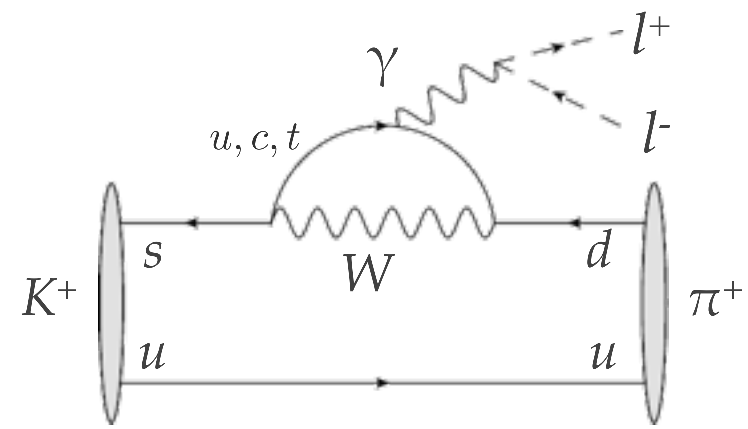
$$\begin{aligned}\mathcal{A}^c_\mu(q^2) &= i\int_0^\infty dE\,\frac{\rho(E)}{2E}\,\frac{\langle \pi^c(\mathbf{p})|J_\mu(0)|E,\mathbf{k}\rangle\langle E,\mathbf{k}|H_W(0)|K^c(\mathbf{k})\rangle}{E_K(\mathbf{k})-E+i\epsilon} \\ &- i\int_0^\infty dE\,\frac{\rho_S(E)}{2E}\,\frac{\langle \pi^c(\mathbf{p})|H_W(0)|E,\mathbf{p}\rangle\langle E,\mathbf{p}|J_\mu(0)|K^c(k)\rangle}{E-E_\pi(\mathbf{p})+i\epsilon}\end{aligned}$$

strange intermediate states

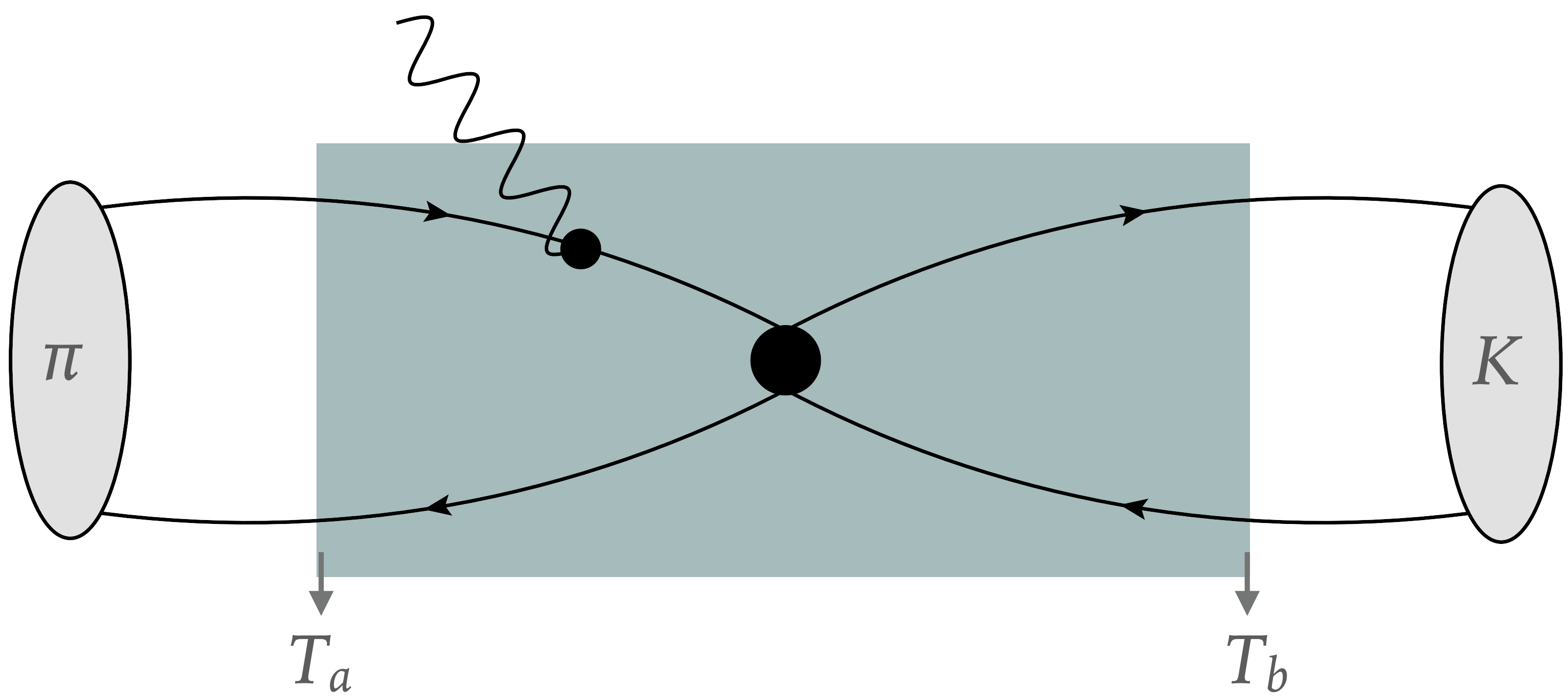
dealing with intermediate (multi) hadron states constitutes a considerable technical challenge when J_μ and H_W separated by hadronic length scales (LD)

SPECTRAL REPRESENTATION – EUCLIDEAN

RBC/UKQCD, PRD94 (2016) no.11, 114516, PRD92 (2015) no.9, 094512



$$\mathcal{A}_\mu = (q^2) \int d^4x \langle \pi(p) | T \left[J_\mu(0) H_W(x) \right] | K(k) \rangle$$



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SPECTRAL REPRESENTATION - EUCLIDEAN

RBC/UKQCD, PRD94 (2016) no.11, 114516, PRD92 (2015) no.9, 094512

$$
\begin{aligned}
I_\mu(T_a,T_b,q^2) &= -i\int_0^\infty dE\,\frac{\rho(E)}{2E}\,\frac{\langle\pi^c(\mathbf{p})|J_\mu(0)|E,\mathbf{k}\rangle\langle E,\mathbf{k}|H_W(0)|K^c(\mathbf{k})\rangle}{E_K(\mathbf{k})-E}\left(1-e^{(E_K(\mathbf{k})-E)T_a}\right)\\
&+\int_0^\infty dE\,\frac{\rho_S(E)}{2E}\,\frac{\langle\pi^c(\mathbf{p})|H_W(0)|E,\mathbf{p}\rangle\langle E,\mathbf{p}|J_\mu(0)|K^c(k)\rangle}{E-E_\pi(\mathbf{p})}\left(1-e^{-(E-E_\pi(\mathbf{k}))T_b}\right)
\end{aligned}
$$

$$
\mathcal{A}^c_\mu(q^2) = -i\frac{G_F}{\sqrt{2}}V^*_{us}V_{ud}\lim_{T_a,T_b\to\infty} I_\mu^{\mathrm{subtracted}}(T_a,T_b,q^2)
$$

exponential in first terms on r.h.s.

- *1st line:*
  - *$E>E_K$: exponential term vanishes as $T_a\to\infty$*
  - *$E<E_K$: exponential term grows as $T_a\to\infty$, must be removed (possible intermediate states π, ππ, πππ)*
- *2nd line: no problem, all intermediate states E larger E_π*

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SPECTRAL REPRESENTATION - EUCLIDEAN

RBC/UKQCD, PRD94 (2016) no.11, 114516, PRD92 (2015) no.9, 094512

$$I_\mu(T_a,T_b,q^2) = -i\int_0^\infty dE\,\frac{\rho(E)}{2E}\,\frac{\langle\pi^c(\mathbf{p})|J_\mu(0)|E,\mathbf{k}\rangle\langle E,\mathbf{k}|H_W(0)|K^c(\mathbf{k})\rangle}{E_K(\mathbf{k})-E}\left(1-e^{(E_K(\mathbf{k})-E)T_a}\right)$$

$$+\int_0^\infty dE\,\frac{\rho_S(E)}{2E}\,\frac{\langle\pi^c(\mathbf{p})|H_W(0)|E,\mathbf{p}\rangle\langle E,\mathbf{p}|J_\mu(0)|K^c(k)\rangle}{E-E_\pi(\mathbf{p})}\left(1-e^{-(E-E_\pi(\mathbf{k}))T_b}\right)$$

subtraction of exponentially increasing states:

➤ *π: either get amplitudes from 2pt and 3pt functions and subtract* ***or*** *replace*

$$H_W(x)\to H'_W(x)=H_W(x)+c_S(\mathbf{k})\bar{s}(x)d(x)$$

where c_S such that $\langle\pi^c(\mathbf{k})|H'_W(0,\mathbf{k})|K^c(\mathbf{k})\rangle=0$ kills the unwanted divergent contribution and does not contribute to the amplitude itself

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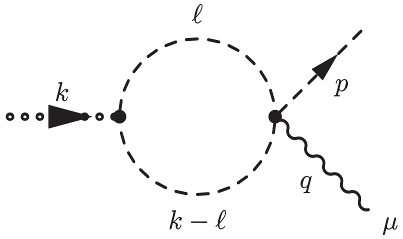
SPECTRAL REPRESENTATION - EUCLIDEAN

RBC/UKQCD, PRD94 (2016) no.11, 114516, PRD92 (2015) no.9, 094512

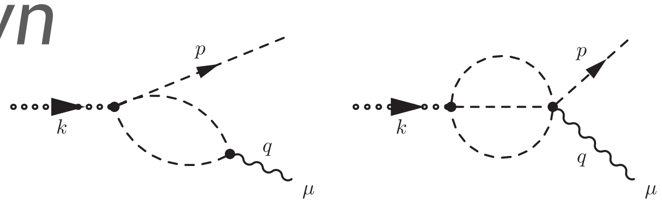
$$
\begin{aligned}
I_\mu(T_a, T_b, q^2) \quad &= \quad -i\int_0^\infty dE\,\frac{\rho(E)}{2E}\,\frac{\langle \pi^c(\mathbf{p})|J_\mu(0)|E,\mathbf{k}\rangle\langle E,\mathbf{k}|H_W(0)|K^c(\mathbf{k})\rangle}{E_K(\mathbf{k})-E}\left(1-e^{(E_K(\mathbf{k})-E)T_a}\right)\\
&+ \quad \int_0^\infty dE\,\frac{\rho_S(E)}{2E}\,\frac{\langle \pi^c(\mathbf{p})|H_W(0)|E,\mathbf{p}\rangle\langle E,\mathbf{p}|J_\mu(0)|K^c(k)\rangle}{E-E_\pi(\mathbf{p})}\left(1-e^{-(E-E_\pi(\mathbf{k}))T_b}\right)
\end{aligned}
$$

subtraction of exponentially increasing states:

- *ππ: disallowed by O(4) invariance but can be present as discretisation effect — needs to be monitored*



- *πππ: comparison of experimental width (PDG) suggests*
  - *πππ to be highly suppressed wt. respect to ππ*
  - *techniques similar as for ππ possible but it's own research topic (K→πππ)*



(a) (b)

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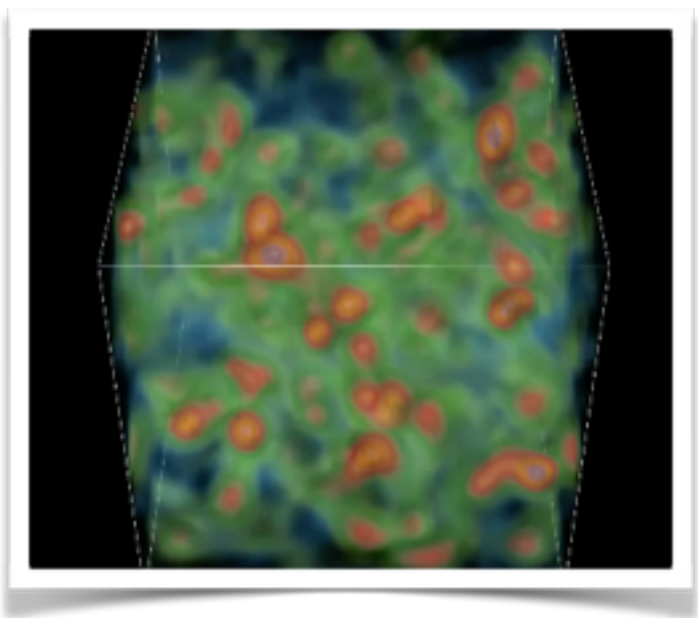
EXPLORATORY STUDY - LATTICE SETUP

RBC/UKQCD, PRD94 (2016) no.11, 114516, PRD92 (2015) no.9, 094512

RBC/UKQCD ***exploratory*** *study — unphysical m_π (because it's cheap)*



- *domain wall fermions*
- *m_π~430MeV, m_K~625MeV*
  $E_K(\mathbf{k})<2M_\pi$ → only one intermediate state
- *unphysically light charm quark mass*
  m_c~533MeV
- *no disconnected diagrams*
- *kaon at rest*

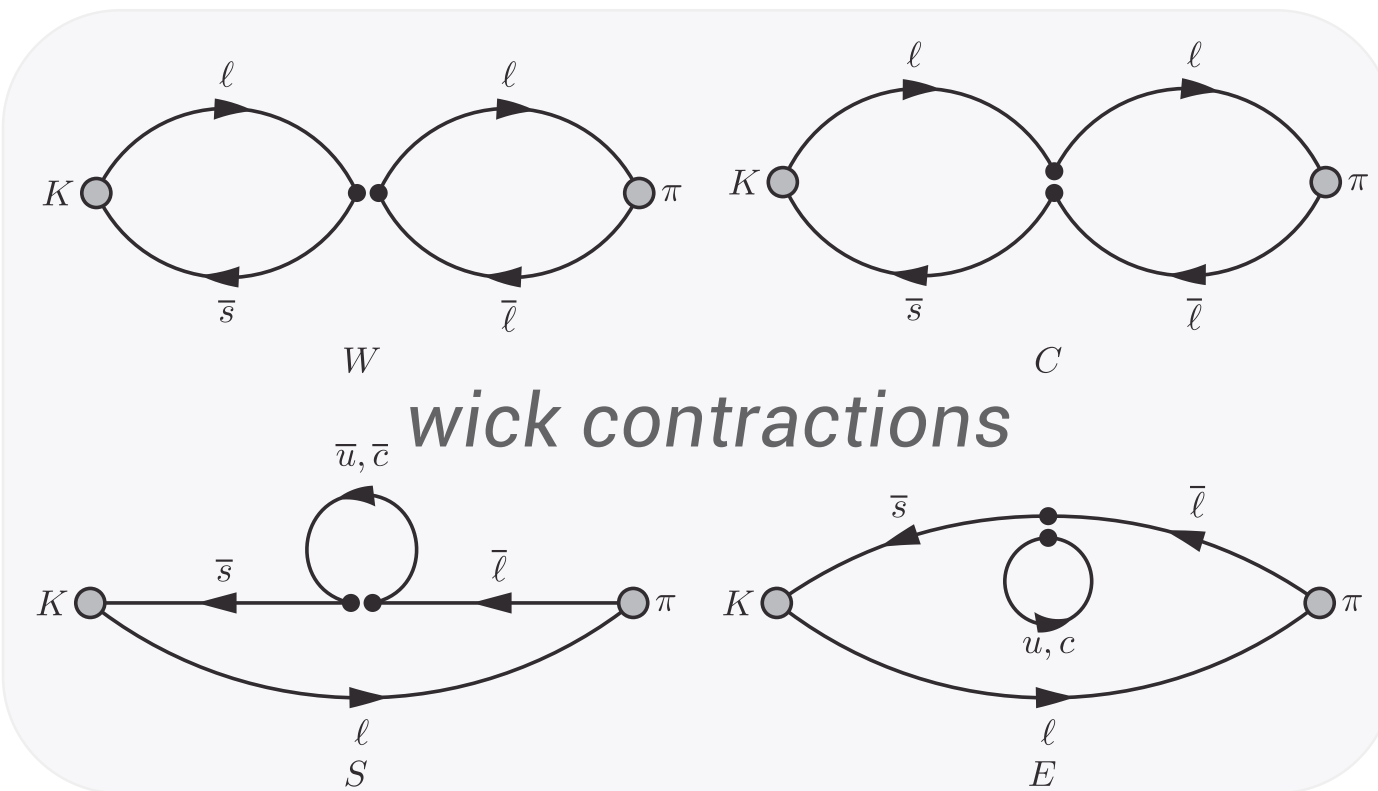
EUCLIDEAN CORRELATION FUNCTIONS

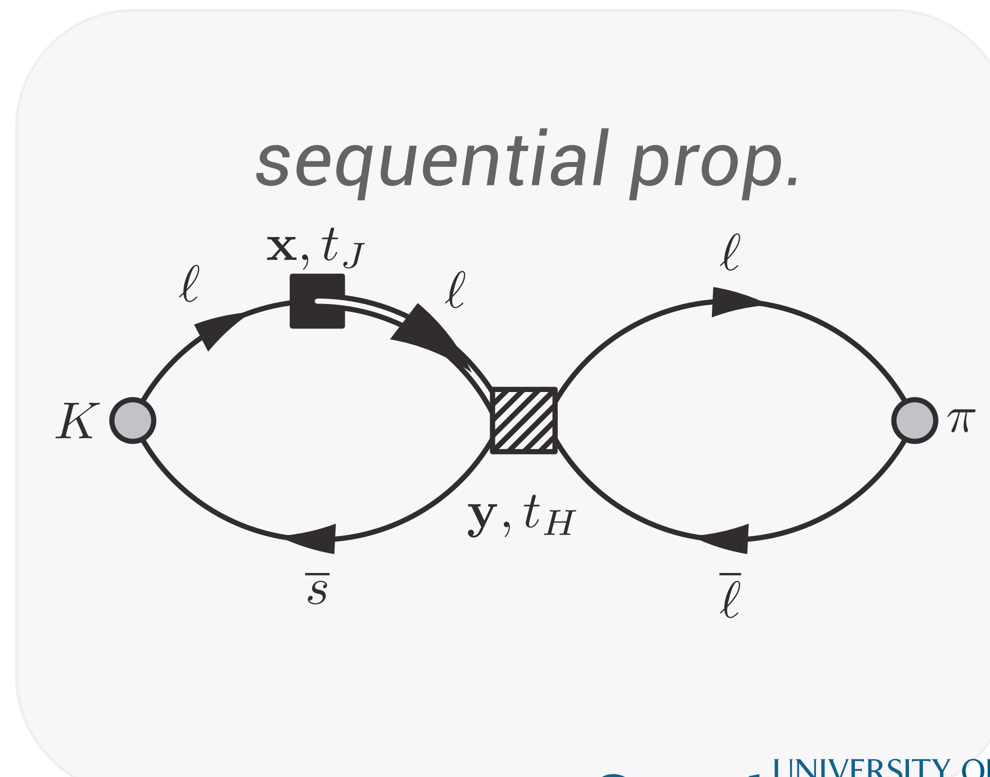
RBC/UKQCD, PRD94 (2016) no.11, 114516, PRD92 (2015) no.9, 094512

$$\Gamma_{\mu}^{(4),\,c}(x,\mathbf{k},\mathbf{p}) = \langle \phi_{\pi^c}(t_\pi,\mathbf{p}) T\left[J_\mu(0) H_W(x)\right] \phi_{K^c}(t_K,\mathbf{k})^\dagger \rangle$$

$$\Gamma_{\mu}^{(4)\,c}(t_H,t_J,\mathbf{k},\mathbf{p}) = \int d^3\mathbf{x} \int d^3\mathbf{y} e^{-i\mathbf{q}\cdot\mathbf{x}} \langle \phi_{\pi^c}(t_\pi,\mathbf{p}) T\left[J_\mu(t_j,\mathbf{x}) H_W(t_H,\mathbf{y})\right] \phi^\dagger_{K^c}(0,\mathbf{k}) \rangle$$

$$\Gamma_{\mu}^{(4),\,c}(x,\mathbf{k},\mathbf{p}) = \frac{Z_\pi Z_K^\dagger e^{-E_\pi(\mathbf{p})t_\pi} e^{E_K(\mathbf{k})t_K}}{4E_\pi(\mathbf{p})E_K(\mathbf{k})} \langle \pi^c(\mathbf{p})|T\left[J_\mu(0) H_W(x)\right]|K^c(\mathbf{k})\rangle$$





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RESULTS - DOMINANT CONTRIBUTIONS AND GIM SUBTRACTION

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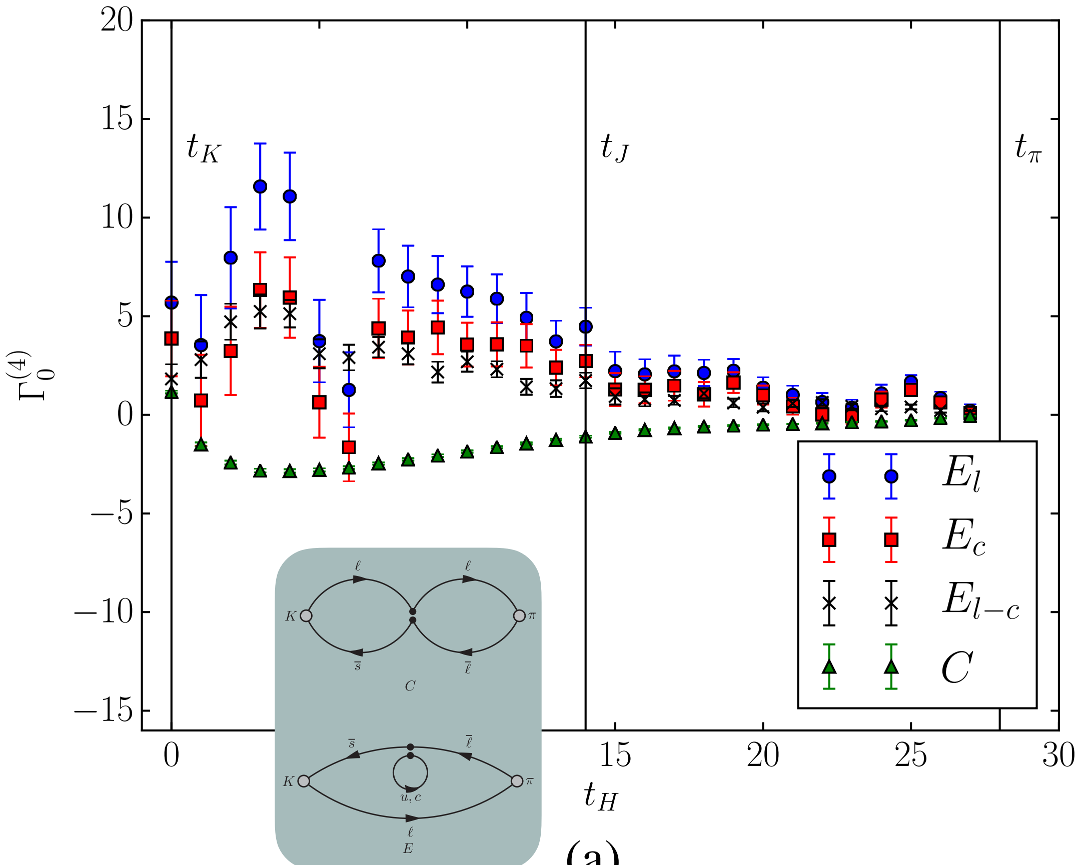
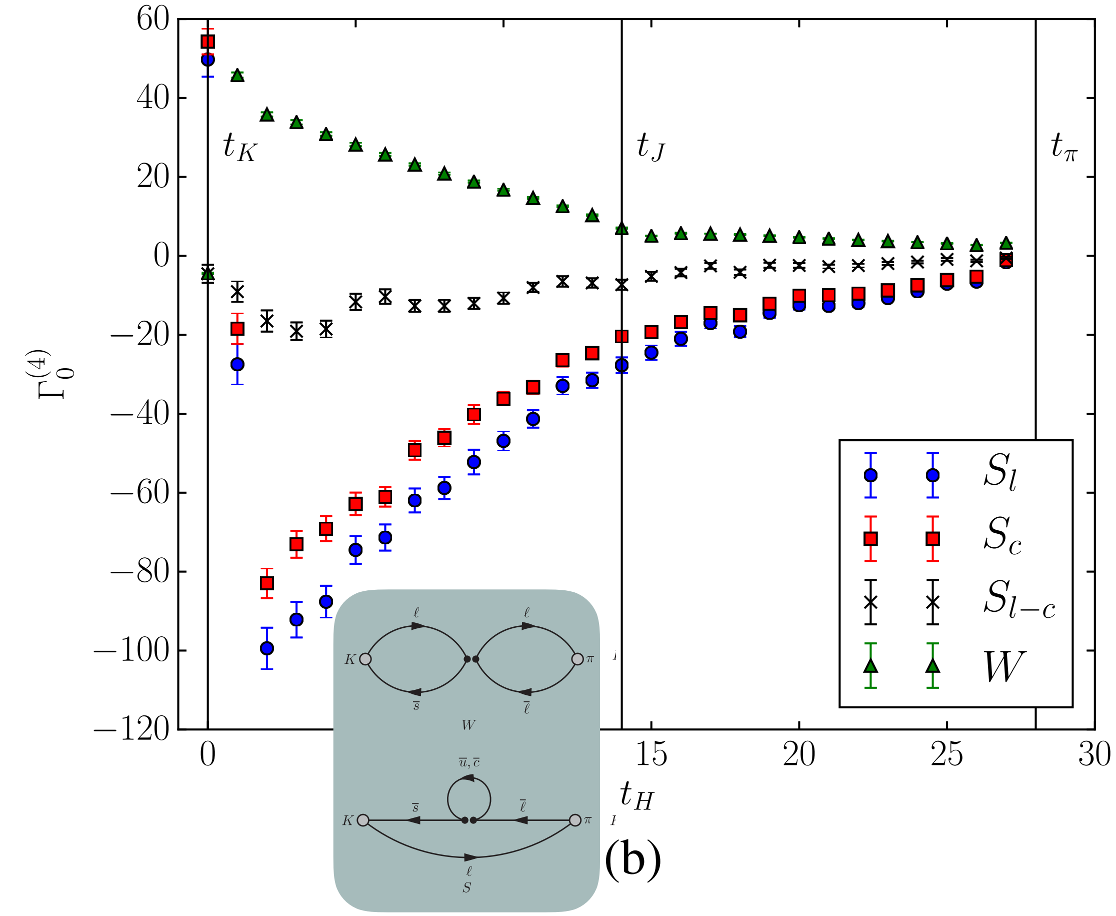


(a)



(b)

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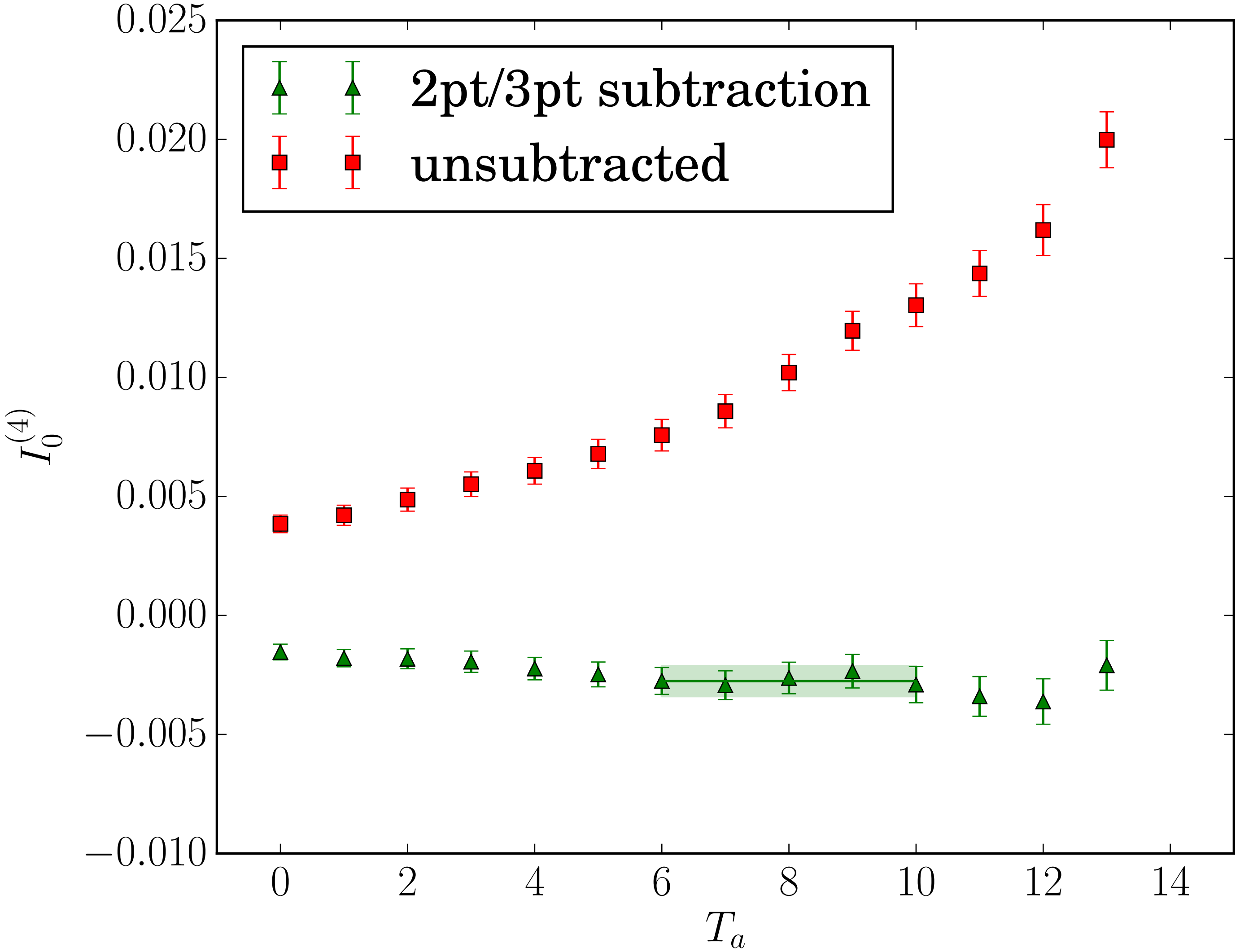
REMOVING THE EXPONENTIALLY RISING TERMS

RBC/UKQCD, PRD94 (2016) no.11, 114516, PRD92 (2015) no.9, 094512

$$I_\mu(T_a,T_b,q^2) = -i\int_0^\infty dE\,\frac{\rho(E)}{2E}\,\frac{\langle\pi^c(\mathbf{p})|J_\mu(0)|E,\mathbf{k}\rangle\langle E,\mathbf{k}|H_W(0)|K^c(\mathbf{k})\rangle}{E_K(\mathbf{k})-E}\left(1-e^{(E_K(\mathbf{k})-E)T_a}\right)$$

$$+\int_0^\infty dE\,\frac{\rho_S(E)}{2E}\,\frac{\langle\pi^c(\mathbf{p})|H_W(0)|E,\mathbf{p}\rangle\langle E,\mathbf{p}|J_\mu(0)|K^c(k)\rangle}{E-E_\pi(\mathbf{p})}\left(1-e^{-(E-E_\pi(\mathbf{k}))T_b}\right)$$



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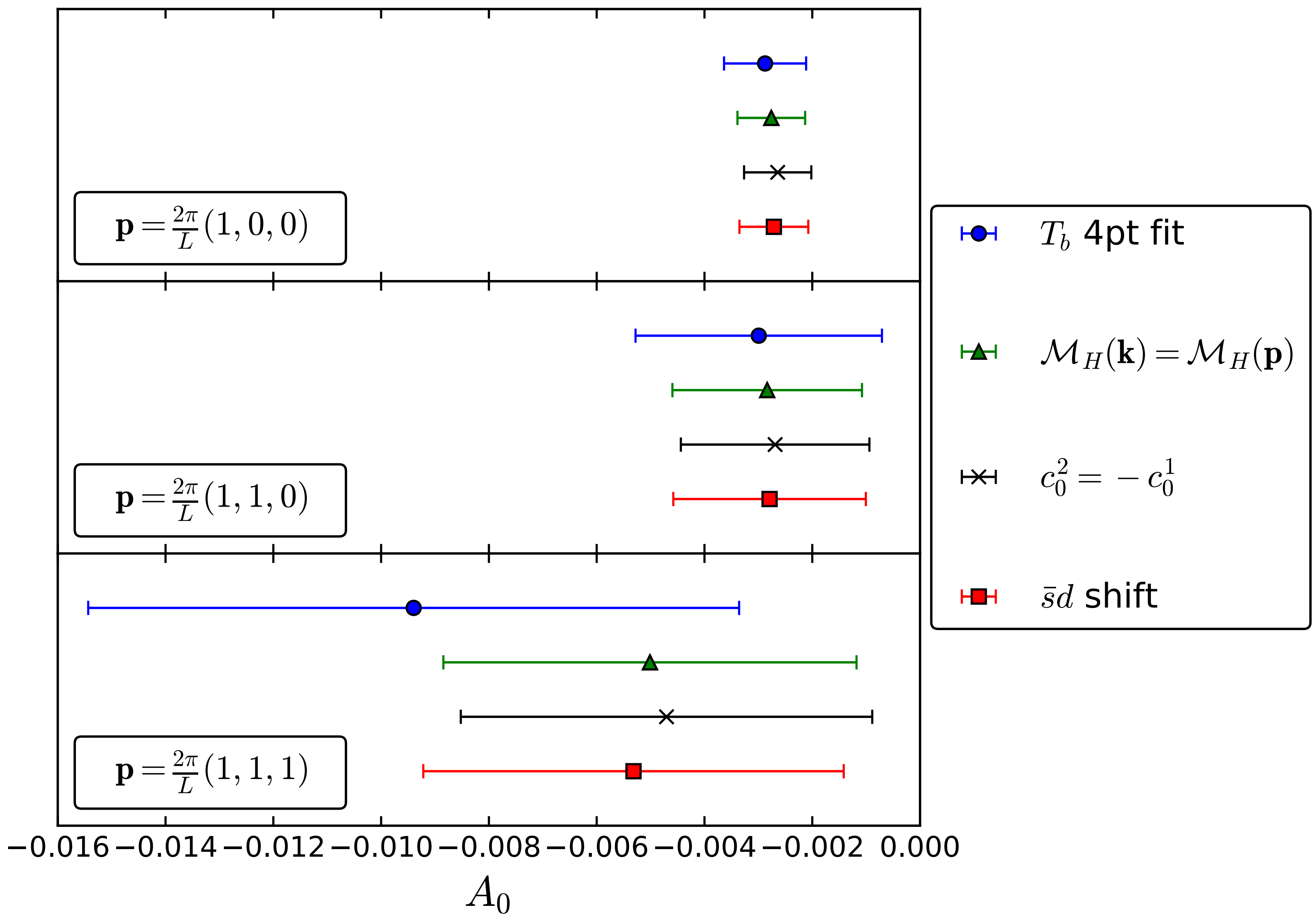
REMOVING THE EXPONENTIALLY RISING TERMS – COMPARISON OF METHODS

RBC/UKQCD, PRD94 (2016) no.11, 114516, PRD92 (2015) no.9, 094512



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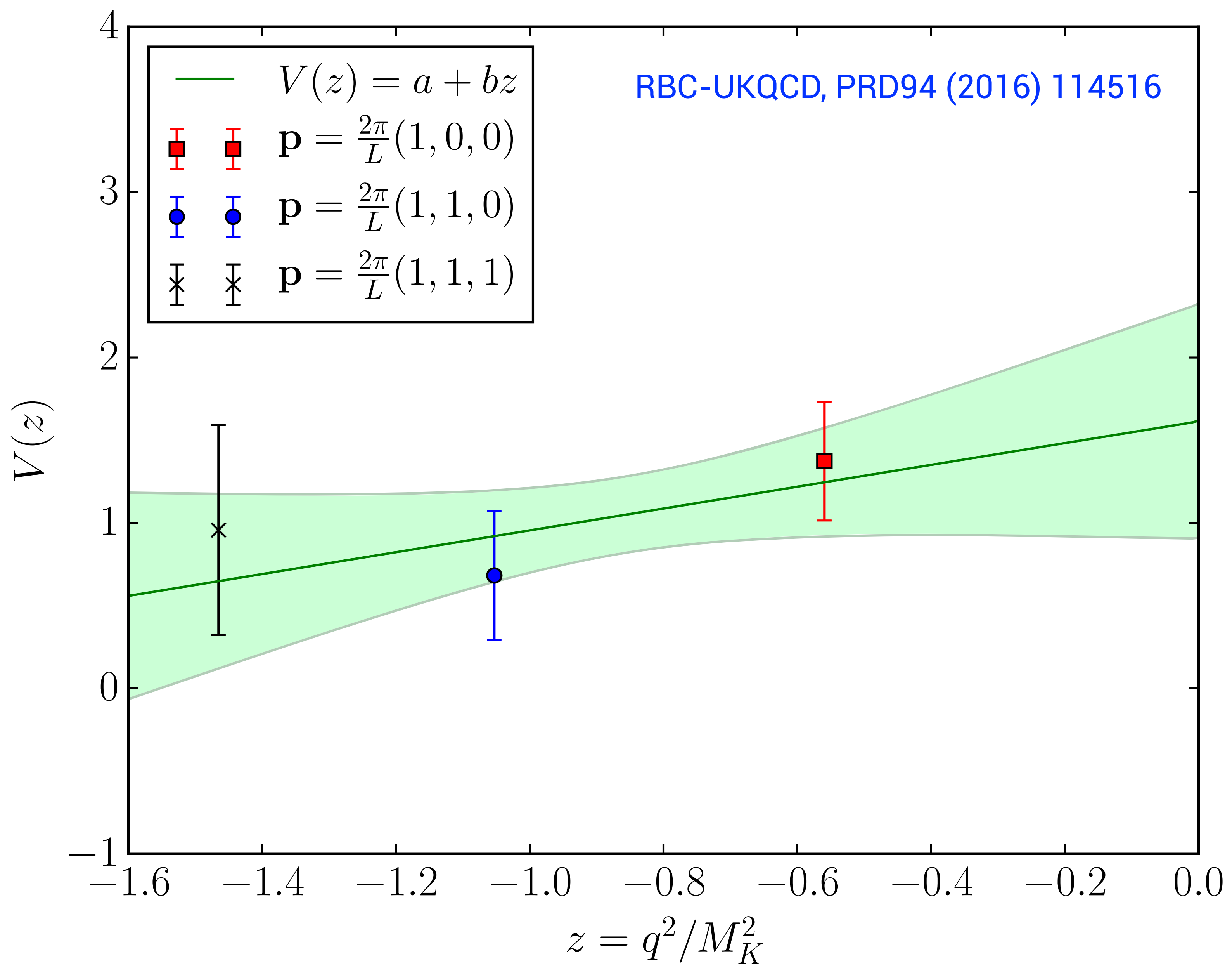
$K^+ \to \pi^+ l^+ l^-$ FORM FACTOR – RESULT EXPLORATORY STUDY

RBC/UKQCD, PRD94 (2016) no.11, 114516, PRD92 (2015) no.9, 094512



$V_+(z)=a_+ + b_+ q^2/m_K^2$ our result: $a_+=1.6(7)$, $b_+=0.7(8)$

pheno fit to exp. data: $a_+=-0.58(2)$, $b_+=0.78(7)$

Cirigliano, et. al., Rev. Mod. Phys. 84 (2012) 399

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$K^+ \to \pi^+ l^+ l^-$ FORM FACTOR – RESULT EXPLORATORY STUDY

- *first lattice evaluation of this form factor*
- *we get a consistent signal*
- *this study shows that it is feasible to reliably rare kaon decay ff*
- *we are working on more 'physical' simulations*
    - *need to reduce m_π on large volume lattices*
    - *$\pi\pi\pi$ state will be kinematically allowed*
    - *m_c needs to be physical as well — discretisation effects are a concern*

EXPENSIVE

- *alternatively consider N_f=2+1 H_W — treat charm perturbatively absence of GIM leads to log divergence which needs to be dealt with*

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$K^+ \to \pi^+ \nu\bar{\nu}$ DECAY

~29% SD
~3% LD

$$\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})_{\mathrm{SM}} = \kappa_+ (1 + \Delta_{\mathrm{EM}}) \cdot \left[\left(\frac{\mathrm{Im}\lambda_t}{\lambda^5} X_t(x_t) \right)^2 + \left(\frac{\mathrm{Re}\lambda_c}{\lambda} \boxed{P_c} + \frac{\mathrm{Re}\lambda_t}{\lambda^5} X_t(x_t) \right)^2 \right]$$

$$= (9.11 \pm 0.72) \times 10^{-11}$$ Buras et al. 2015

~68%

$$P_c = P_c^{SD} + \delta P_{c,u}$$

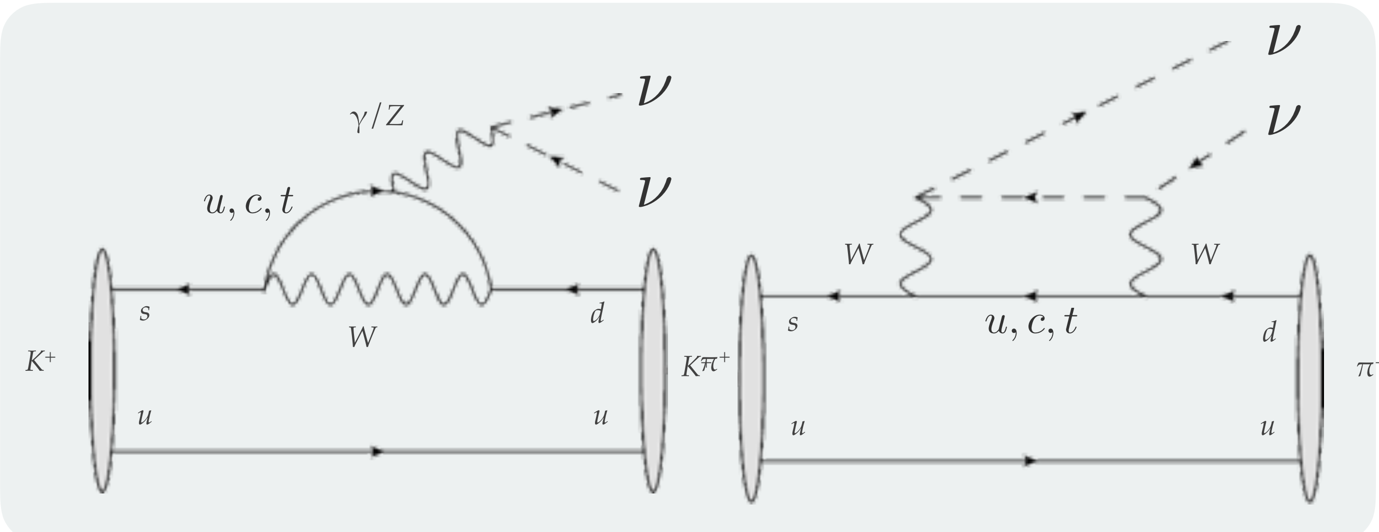
Energy >m_c *up- and charm-quark loops ~3%?*

Buras, Gorbahn, Haisch, Nierste JHEP 0611 (2006) 002
Isidori, Mescia, Smith Nucl.Phys. B718 (2005) 319-338

compute P_c on the lattice in 4-flavour theory thus avoiding PT at around the charm scale

$K^+ \to \pi^+ \nu\bar{\nu}$ DECAY

RBC/UKQCD PRD93 (2016) no.11, 114517, PRL 118 (2017) no.25, 252001

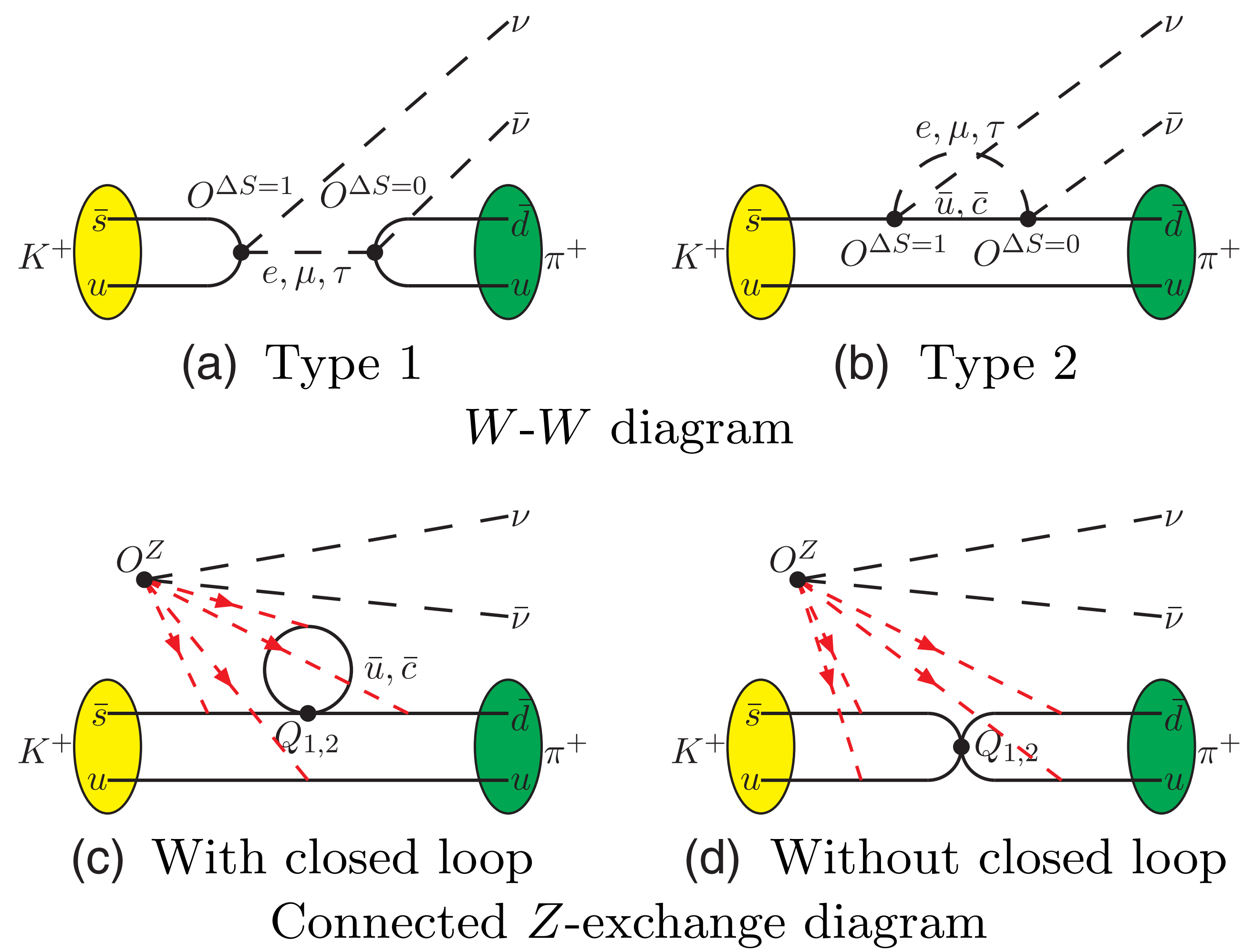


involves two genuinely weak operators with V-A structure

V part renormalises similarly to $K^+ \to \pi^+ l^+ l^-$

A-part causes log-div which needs to be subtracted

$$\mathcal{O}(y) = \sum_{A,B} \int d^4x T[C_A Q_A(x) C_B Q_B(y)] + C_0 Q_0(y).$$



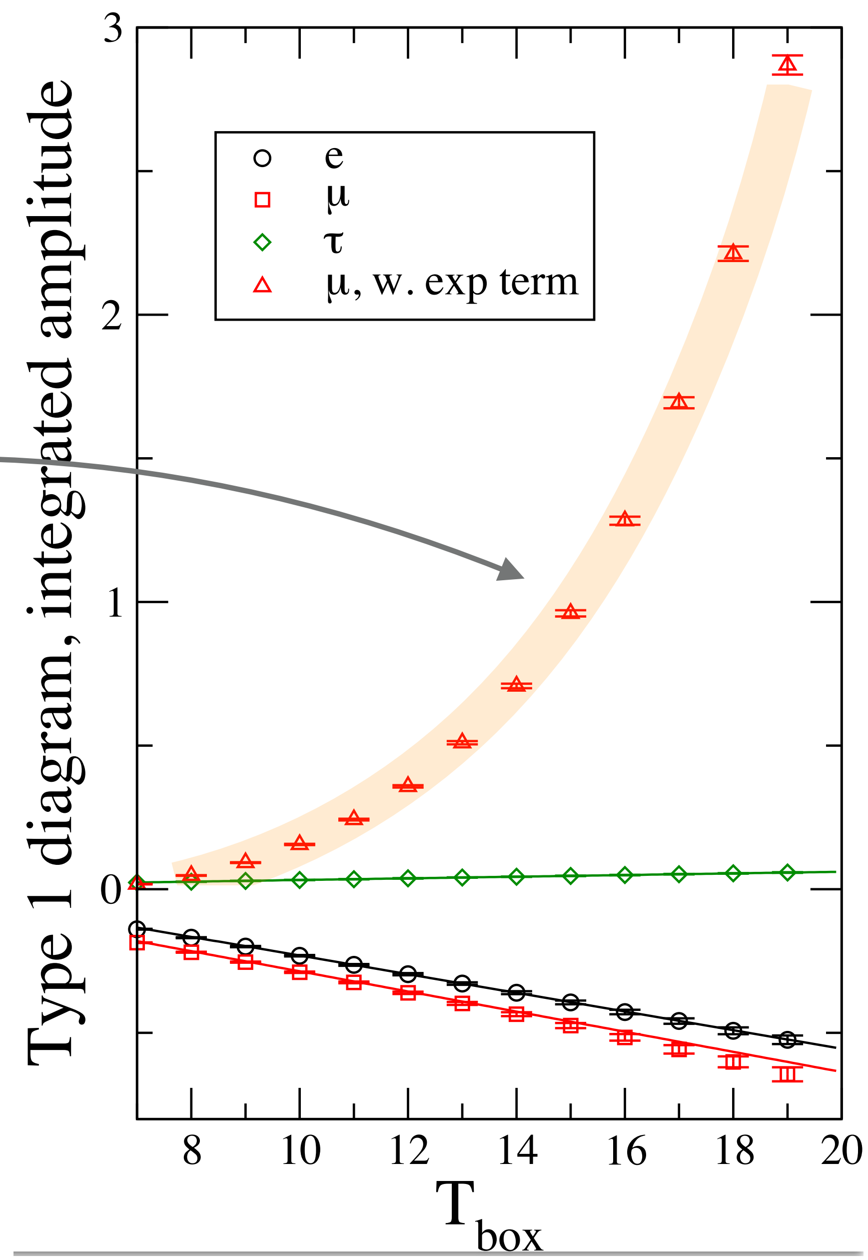
DEFINITIONS

RBC/UKQCD PRD93 (2016) no.11, 114517, PRL 118 (2017) no.25, 252001

$$\int_{-T_a}^{T_b} dx_0 \langle \pi^+\nu\bar{\nu}|T\{H_A(x_0)H_B(0)\}|K^+\rangle = \sum_n \left\{ \frac{\langle \pi^+\nu\bar{\nu}|H_A|n\rangle\langle n|H_B|K^+\rangle}{E_n - E_K}\left(1 - e^{(E_K - E_n)T_b}\right) + \frac{\langle \pi^+\nu\bar{\nu}|H_B|n\rangle\langle n|H_A|K^+\rangle}{E_n - E_K}\left(1 - e^{(E_K - E_n)T_a}\right)\right\}$$

intermediate states:

$$|n\rangle = |l^+\nu\rangle,\ |\pi^0 l^+\nu\rangle,\ |(\pi^+\pi^0)^{I=2}\rangle$$



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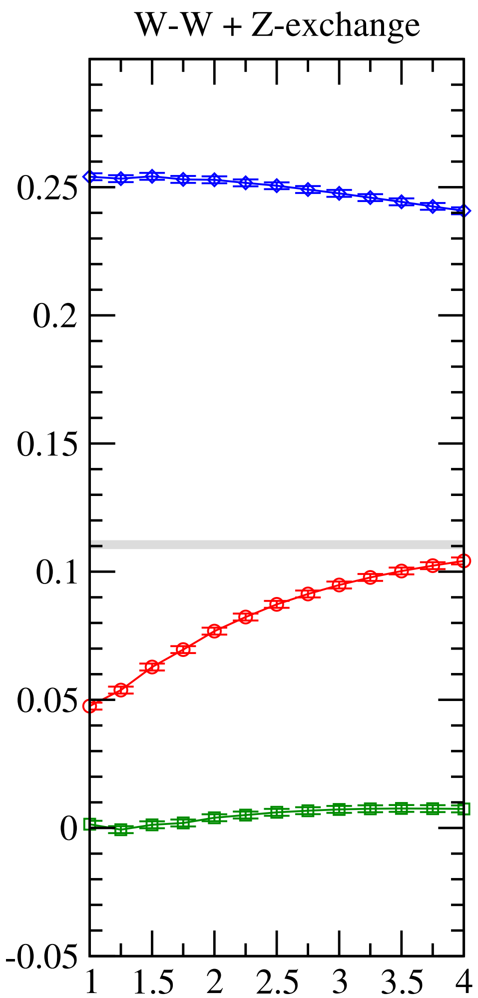
$K^+ \to \pi^+ \nu\bar{\nu}$ DECAY

RBC/UKQCD PRD93 (2016) no.11, 114517, PRL 118 (2017) no.25, 252001

lattice result for m_π=420MeV, m_c=860MeV



3) after subtr. of divergence

1) bare P_c

2) bilocal contrib.

4) diff. wt. resp. to PT

lattice result for m_π=420MeV, m_c=860MeV

$$P_c = 0.2529(\pm 13)_{stat}(\pm 32)_{scale}(-45)_{FV}$$

$$P_c - P_c^{SD} = 0.0040(\pm 13)_{stat}(\pm 32)_{scale}(-45)_{FV}$$

- *unphysical simulation*
- *residual scale dependence small*
- *$P_c - P_c^{SD}$ small due to cancellation between W-W and Z*
  will this persist in more physical simulation?

SUMMARY AND OUTLOOK

- *kaon tree decays have become a precision game with QCD+QED the new challenge*
- *kaon rare decays constitute a new theoretical and technical challenge worthwhile to pursue in view of experimental efforts*
- *techniques also applicable to other LD effects ΔM_K, ε_K*
- *intermediate state subtraction and renormalisation are main challenges*
- *physical point simulation starting for $K^+ \to \pi^+ l^+ l^-$*
- *the experiments running, we are looking forward to their results in particular prospect of $K^+ \to \pi^+ l^+ l^-$ @ NA62*

THANK YOU!

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NEUTRAL KAON MASS DIFFERENCE

$$\boxed{\Delta M_K = m_{K_S} - m_{K_L} = \boxed{2\mathrm{Re}M_{0\bar{0}}}} \qquad M_{\bar{0}0} = \mathcal{P}\sum_{\lambda} \frac{\langle \overline{K^0}|H_W|\lambda\rangle\langle\lambda|H_W|K^0\rangle}{m_K - E_\lambda}$$

- experimentally $\Delta M_K = 3.483(6)\times 10^{-12}$MeV (PDG)
- 2nd order EW, suppressed by 14 orders of magnitude with respect to QCD
  → poses strong BSM constraints (e.g. $(1/\Lambda)^2\,\bar{s}d\bar{s}d$ BSM contribution) knowing
  ΔM_K at 10%-level → **$\Lambda \geq 10^4$TeV**
- SD about 70% of experimental value - rest LD?

Brod, Gorbahn PRL 108 121801 (2012) **arXiv:1108.2036**

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NEUTRAL KAON MASS DIFFERENCE

N. Christ et al. PRD 88 (2013) 014508 [arXiv:1212.5931]
Bai et al. PRL 113 (2014) 112003 [arXiv:1406.0916]

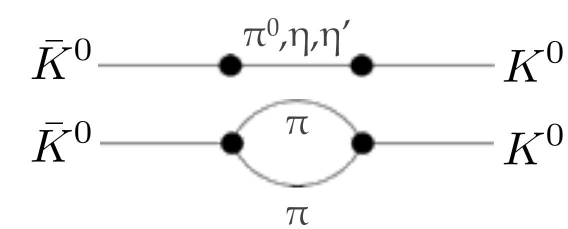
$$\mathcal{A} = N_K^2 e^{-M_K(t_f - t_i)} \sum_n \frac{\langle \bar{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{M_K - E_n} \left(-T - \frac{1}{M_K - E_n} + \frac{e^{(M_K - E_n)T}}{M_K - E_n} \right)$$

amplitude Δm_K^{FV}

irrelevant constant

exponential term needs to be subtracted

- multiple hadrons in intermediate states causing difficulties and need to be subtracted



- finite volume corrections from two-particle intermediate state can be sizeable extension of Lellouch-Lüscher correction to 2nd order weak MEs N. Christ et al. PRD91 (2015) 114510 [arXiv:1504.01170]
also: Briceno, Hansen [arXiv:1502.04314]

- what happens when the two H_W approach each other (GIM in action)?
→ to take advantage of GIM cancellation it needs to be implemented on the lattice and it has been shown to work!

$$\mathrm{Br}(K_L \to \pi^0 e^+ e^-)_{\mathrm{CPV}} = 10^{-12} \times \left[15.7|a_S|^2 \pm 6.2|a_S| \left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}}\right) + 2.4 \left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}}\right)^2\right]$$

$$\mathrm{Br}(K_L \to \pi^0 \mu^+ \mu^-)_{\mathrm{CPV}} = 10^{-12} \times \left[3.7|a_S|^2 \pm 1.6|a_S| \left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}}\right) + 1.0 \left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}}\right)^2\right] \ ,$$