Cosmology with negative mass

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Outline

1. Motivations: Antimatter and gravity
   - The Dirac-Milne Universe
2. Newtonian gravity with negative mass
   - Antigravity, Bondi masses, Dirac-Milne scenario
   - Models, linear analysis
3. Cosmological structure formation
   - Comoving coordinates
   - Expanding Dirac-Milne universe and structure formation
4. Conclusions

Antimatter and gravity — open questions

- **Why matter−antimatter imbalance?**
  - Standard model predicts same amount in the early universe
  - Where has all the antimatter gone?

- **Gravitational behavior of antimatter**
  - Equivalence principle never tested directly for antimatter
    - GBAR, ALPHA, AEGIS, ASACUSA, ATRAP
  - Fundamental questions for our understanding of gravity
    - Acceleration of the expanding universe → Dark energy
    - Matter content of the universe → Dark matter
    - Alternative theories such as MOND (Modifed Newtonian Dynamics)

- ***Our understanding of gravity, even at the Newtonian level, may still be very incomplete***

Antimatter and gravity

- **Gravitational behavior of antimatter**
  - Same as matter (attraction)
  - Slightly different (attraction, but different coupling)
  - Matter-antimatter repulsion

- **Dirac-Milne universe**
  - *A. Benoit-Levy and G. Chardin, Astron. Astroph. (2012)*
  - Matter-antimatter symmetric universe
  - Matter and antimatter repel each other
  - Linear expansion factor, $a(t) \sim t$ (Milne)
  - Solves horizon problem (no inflation)
  - No need for dark matter/energy.

Mass in Newtonian mechanics

- Active gravitational mass m_a: $\Delta\phi = 4\pi G\rho = 4\pi G m_a n$
- Passive gravitational mass m_p: $\boldsymbol{F} = -m_p \nabla\phi$
- Inertial mass m_i: $\boldsymbol{p} = m_i \dot{\boldsymbol{r}}$
- Equation of motion: $\ddot{\boldsymbol{r}} = -(m_p/m_i)\nabla\phi.$

		Active grav. mass	Passive grav. mass	Inertial mass
matter	A (standard)	+	+	+
antimatter	B (antiplasma)	−	−	+
	C (Bondi)	−	+	+
	D (antiinertia)	+	−	+

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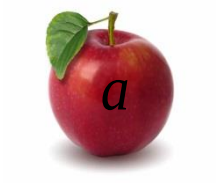
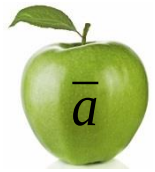
Antiplasma





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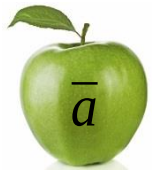

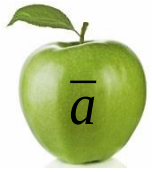

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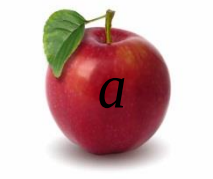
Bondi



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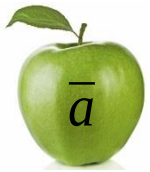

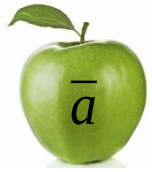

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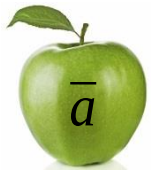
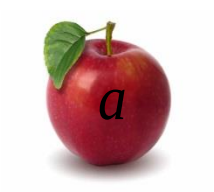
Bondi: runaway acceleration





Dirac-Milne scenario

- However, the above scenarios are not suited to model the Dirac-Milne universe
- **Antiplasma:**
  - Does not respect the EP
  - Allows formation of negative mass structures
- **Bondi:**
  - Requires negative inertial mass to ensure energy conservation
  - Exotic features such as runaway acceleration

- We need a **generalization of Newtonian gravity** for two particles species

Type of matter	Type of matter	Interaction
$+$	$+$	Attraction
$-$	$-$	Repulsion
$-$	$+$	Repulsion
$+$	$-$	Repulsion

- Cannot be realized with a single Poisson's equation

$$\begin{aligned} \Delta\phi_+ &= 4\pi G m(+n_+ - n_-), \\ \Delta\phi_- &= 4\pi G m(-n_+ - n_-) \end{aligned}$$

- **Antimatter spreads uniformly**
- **Matter coalesces in structures**

General matrix formalism

$$\Delta\Phi = 4\pi G m \, \widehat{\mathsf{M}} \, \mathsf{n},$$

Matrix Poisson's equation

$$\Phi = \begin{pmatrix} \phi_+ \\ \phi_- \end{pmatrix}, \quad \mathsf{n} = \begin{pmatrix} n_+ \\ n_- \end{pmatrix}, \quad \widehat{\mathsf{M}} = \begin{pmatrix} M_{++} & M_{+-} \\ M_{-+} & M_{--} \end{pmatrix} \qquad M_{ij} = \pm 1$$

$$\mathcal{L}(\phi_+, \phi_-) = \frac{\nabla\Phi^T \cdot \nabla\Phi}{8\pi G} + \Phi^T \, \widehat{\mathsf{M}} \, \Phi$$

(can be obtained from Lagrangian)

$$\widehat{\mathsf{M}}_{\mathrm{plasma}} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}, \quad \widehat{\mathsf{M}}_{\mathrm{Bondi}} = \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} \qquad \widehat{\mathsf{M}}_{\mathrm{DM}} = \begin{pmatrix} 1 & -1 \\ -1 & -1 \end{pmatrix}$$

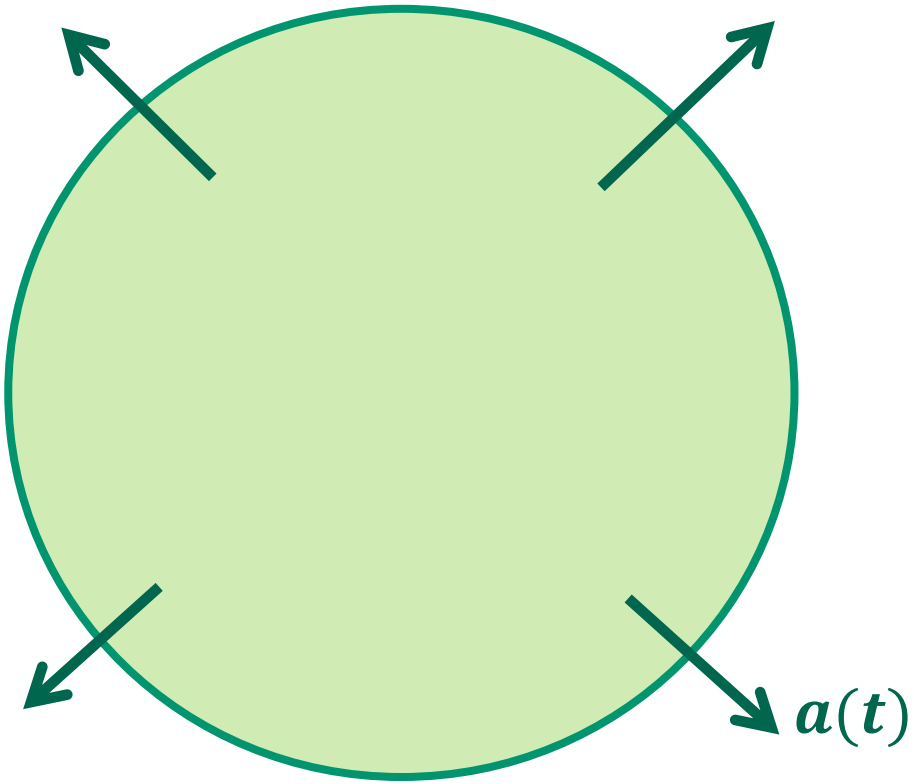
Expanding universe – Comoving coordinates

Equation of motion

$$\frac{d^2 r}{dt^2} = E_r(r,t),$$

Scaling factor

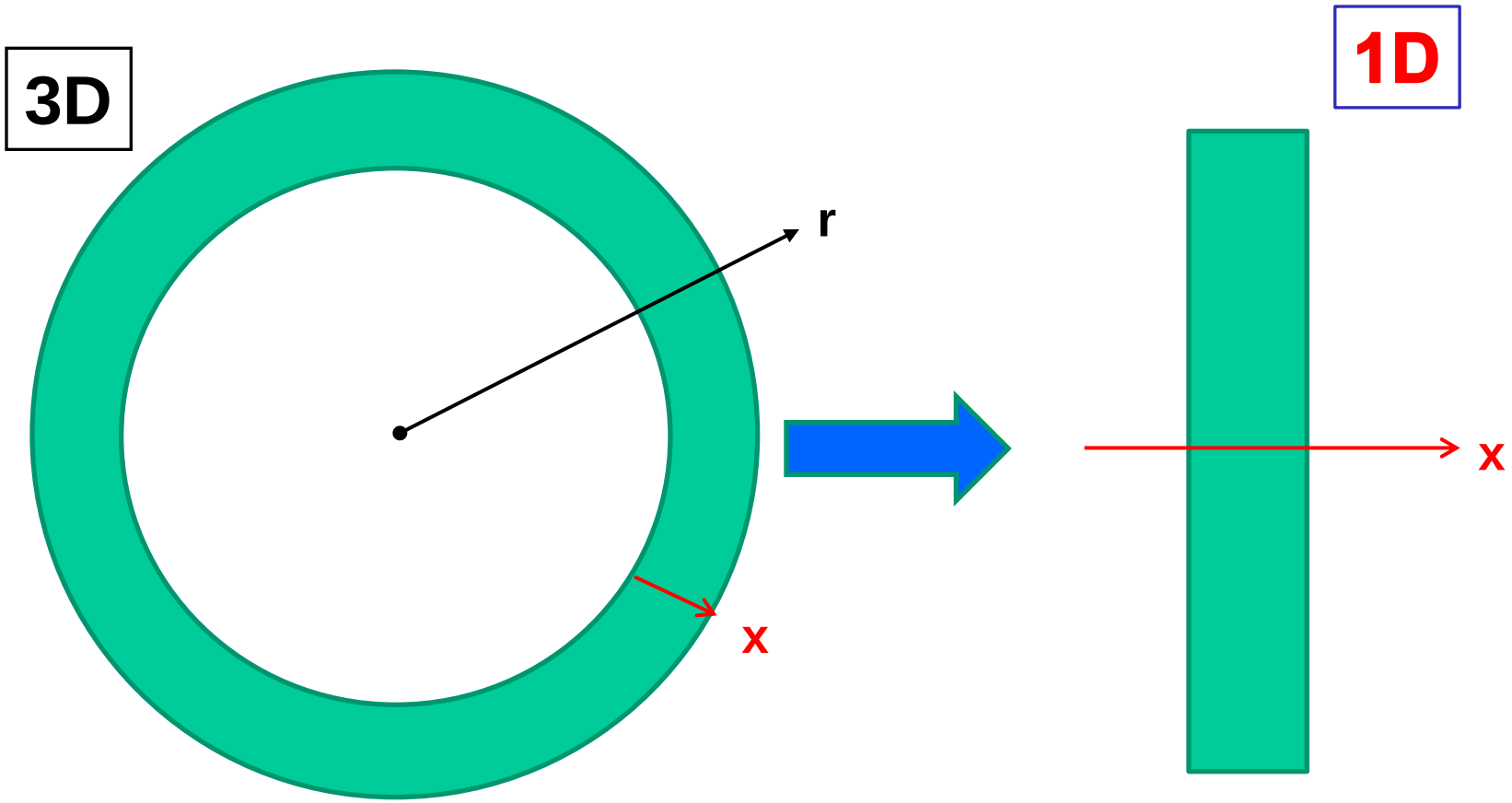
$$r = a(t)\hat{r},$$



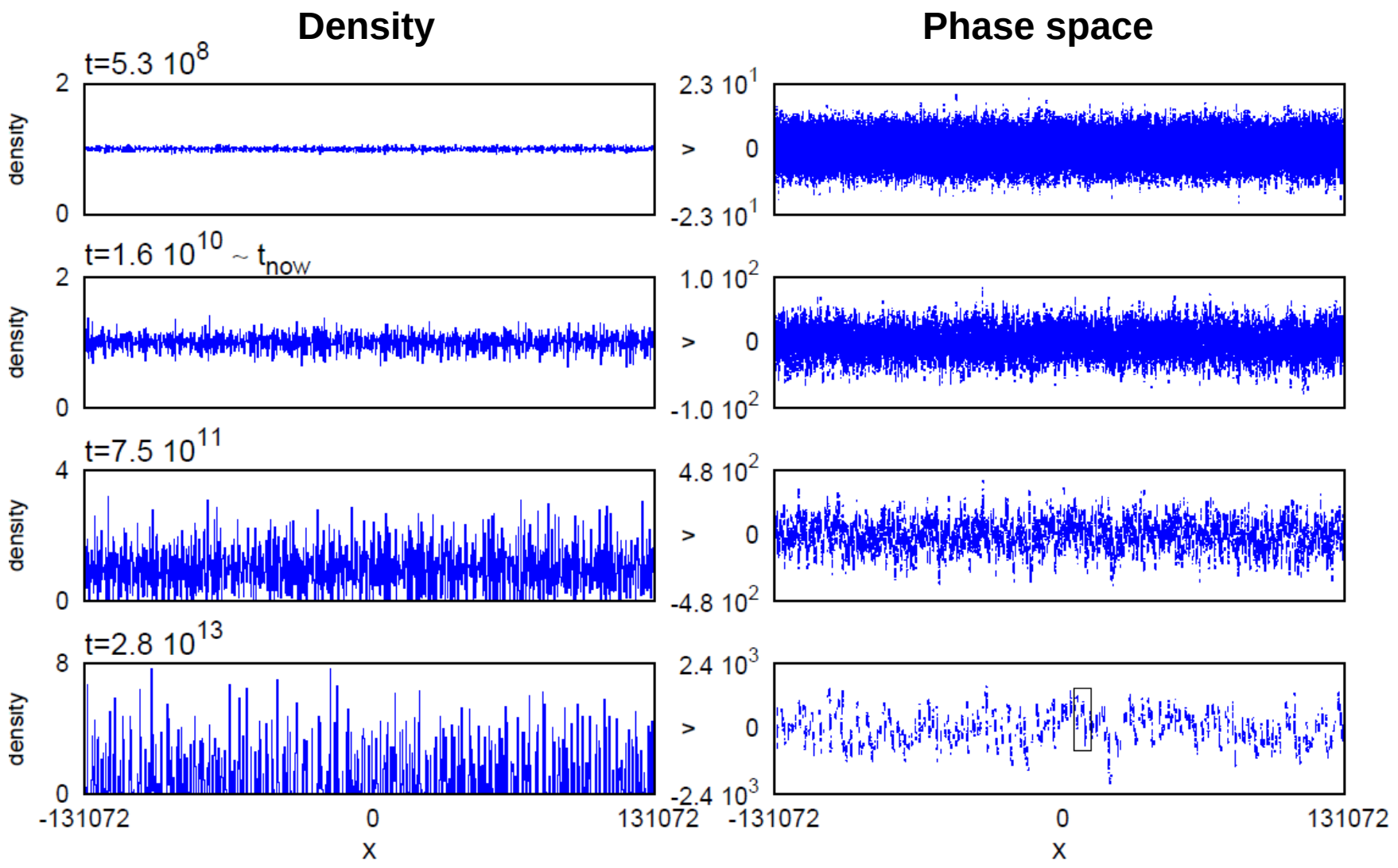
$a(t) \sim t^{2/3}$ Einstein – de Sitter universe (matter dominated)

$a(t) \sim t$ Dirac-Milne universe

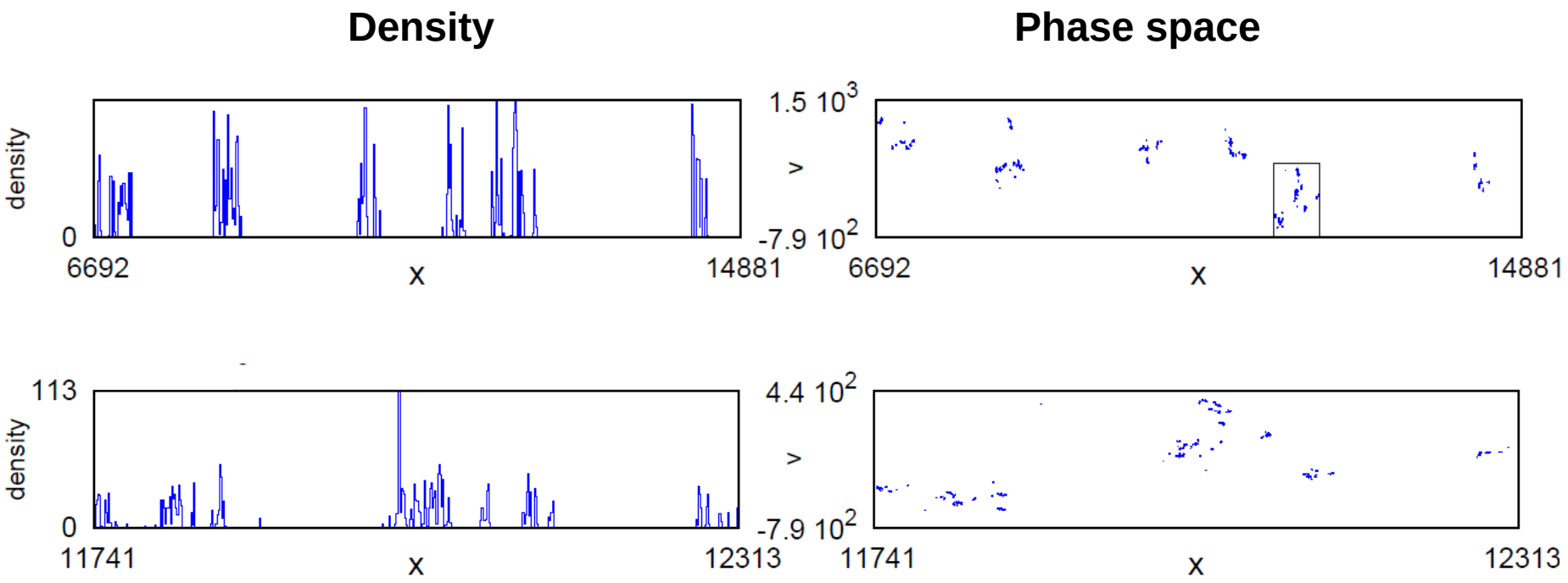
One-dimensional geometry



Einstein-de Sitter universe



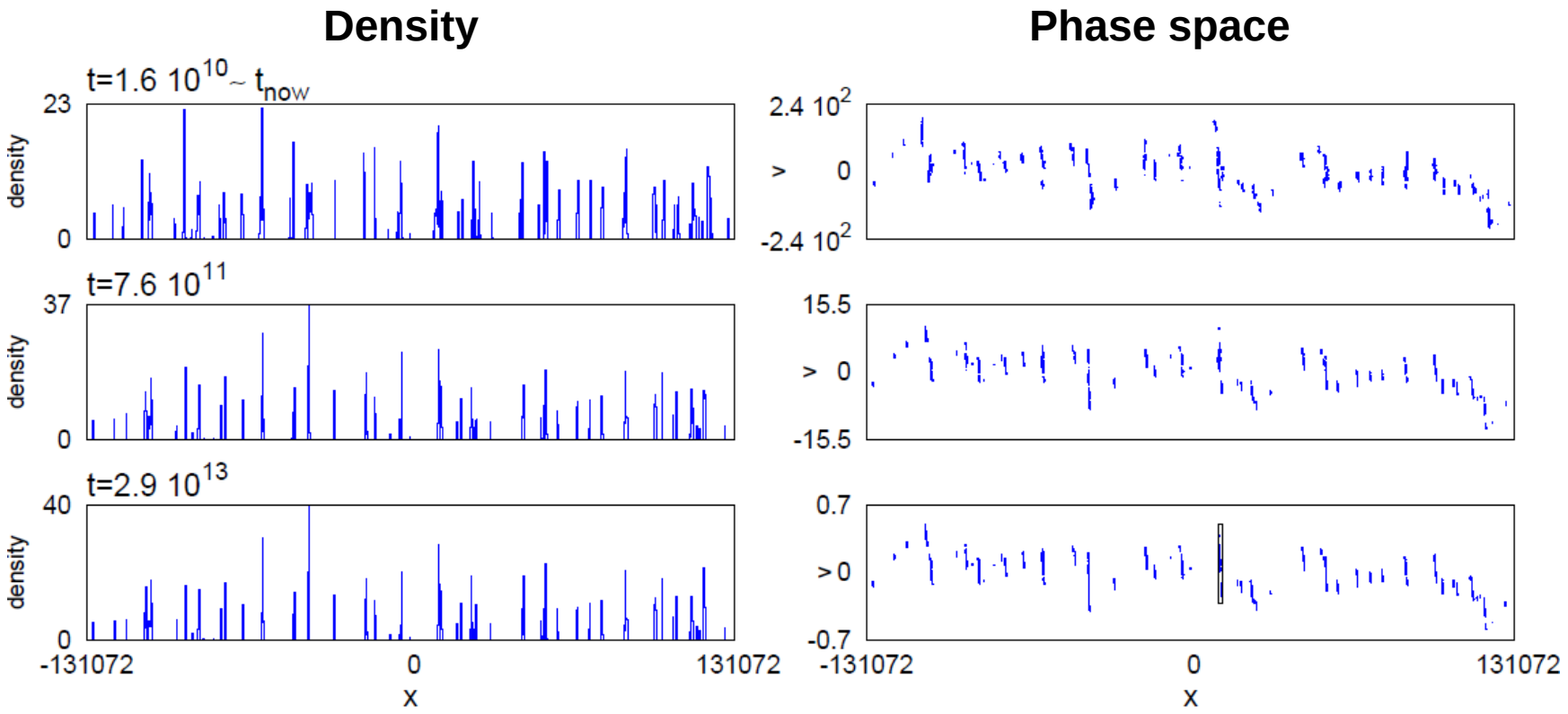
Einstein-de Sitter universe: zooms



- **Continuous generation of gravitational clusters and sub-clusters**
- **Self-similar structures**
- **Fractal geometry**

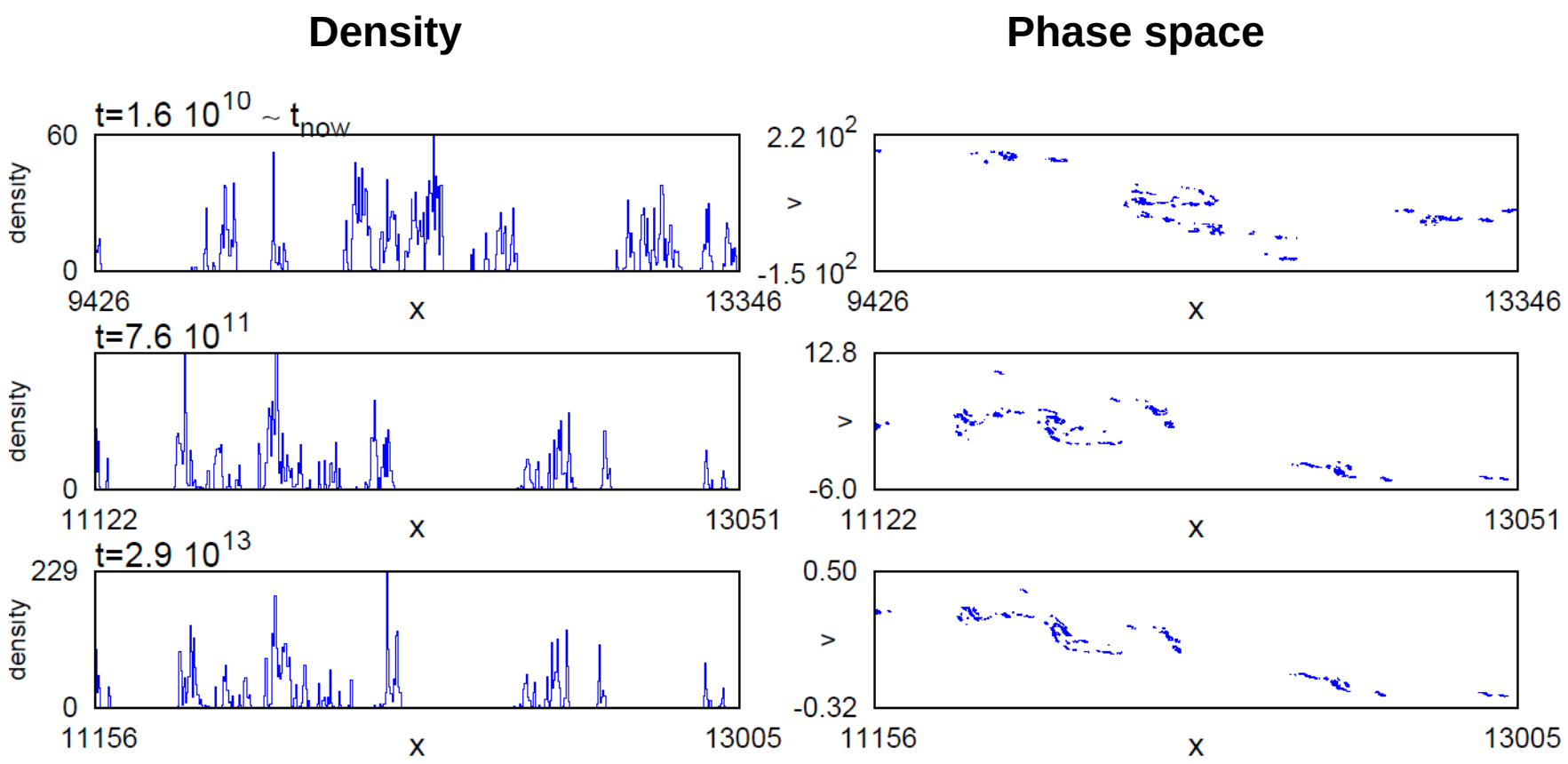
Bruce N. Miller and Jean-Louis Rouet, J. Stat. Mech. P12028 (2010).

Dirac-Milne universe

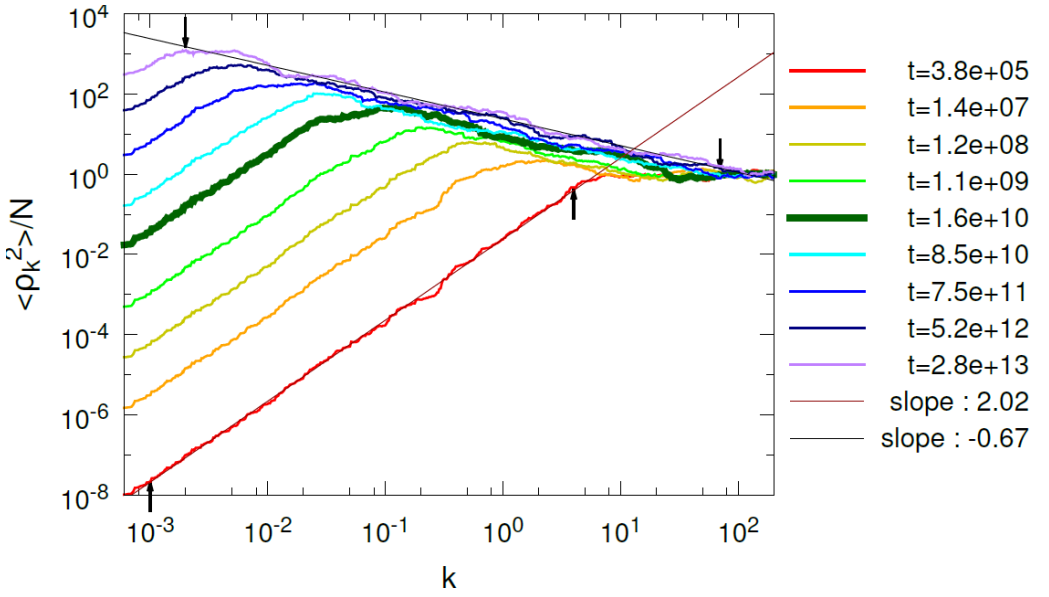


- We show only matter (m>0); antimatter constitutes a low-density repulsive background
- **Structure formation stops before the present epoch (t_{now})**
- Note: no cosmological constant needed

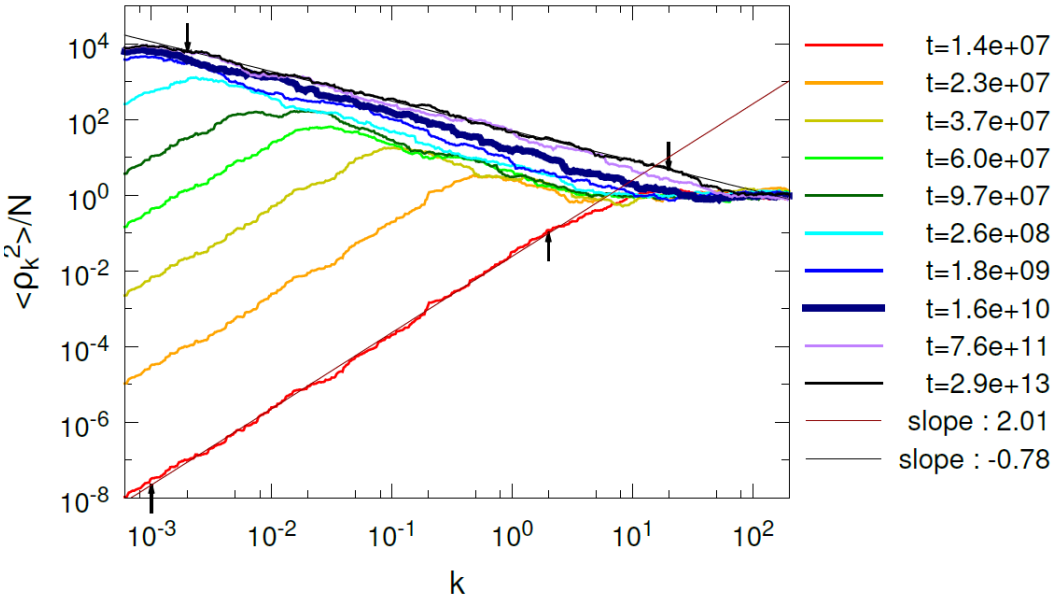
Dirac-Milne universe: zoom



Matter-density power spectrum

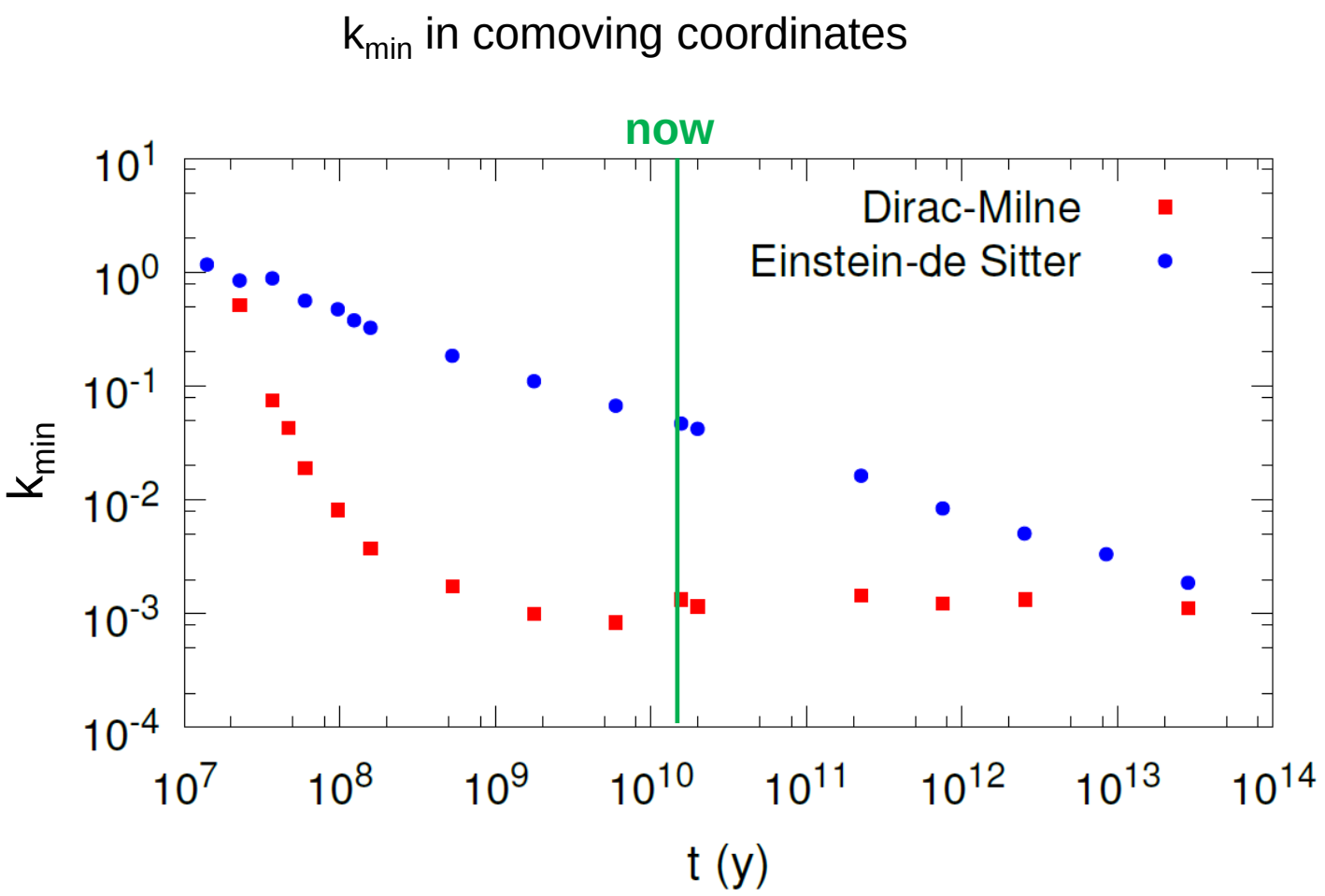


Einstein – de Sitter



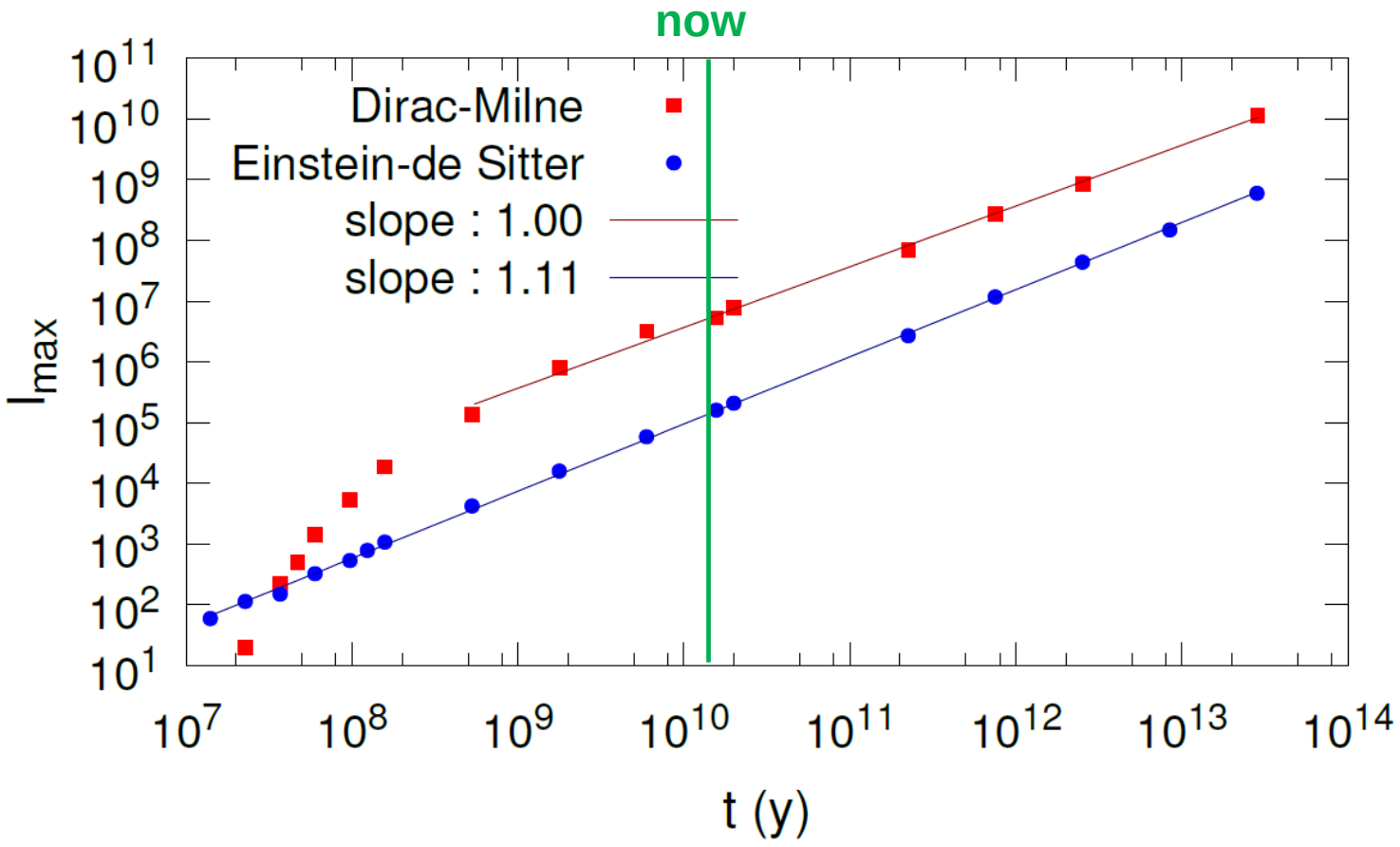
Dirac-Milne

Evolution of power spectrum peak



Typical cluster size

$l_{max} = \frac{2\pi a(t)}{k_{min}}$ in fixed (non comoving) coordinates



Conclusions

- **Newtonian gravity with negative mass**
  - Standard cases with various choices of m_i, m_a, m_p (Bondi, antiplasma,…)
  - Alternative “bimetric” theories → Dirac-Milne
- **Cosmological structure formation with negative mass**
  - Comparison between Einstein-de Sitter and Dirac-Milne
  - In the Dirac-Milne universe, structure formation stops before 10^{10} Gy
  - Similar to ΛCDM