Heavy Flavor Correlations under Thermal Stochastic Influence

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CP & Critical Phenomena

$$\underbrace{\textit{Collider} \qquad\qquad \textbf{\textit{Fixed target}}}_{\textit{strong interacting matter @ high } T \,\&\, \mu_B}$$

In the proximity of *CP*:

- **Matter becomes weakly coupled**
- **Color is no more confined**
- **Chiral symmetry is restored**

➢ **Phase transition is associated with breaking of symmetry**
What is the symmetry?

Instructive: ***CP*** **clarified through** $(\mu_B - T)$ **plane**

scanning of $(\mu_B - T)$ **phase diagram**

scanning of $(QCD)_\beta$ **observables**

CP & Critical Phenomena

A few questions arise:

- ➢ ***CP* meaning?**
- ➢ **Basic observables to be measured when *CP* achieved?**
- ➢ **New knowledge if *CP* approached?**

Answer: in terms of $\mathcal{QCD}_\beta$ @ large distances

N/Perturbative phenomena: $\chi\mathcal{SB}$ & Confinement of color

$\downarrow$ *?relations?* $\downarrow$

Phase transition of $\chi\mathcal{S}$ Restoration Deconfinement

$\downarrow$ **correlations** $\downarrow$

important issue

NO correct solution (massless quarks in the theory)

Effective models, e.g., with topological defects

Phase transitions ⇔ Topological defects (TD's)

TD's exist only in phase with SSB where $\langle \phi \rangle_{vacuum}$ emerges

Non-broken symmetry phase: ***no solutions relevant to TD's***

Minimal model: TD's (strings) arise in Abelian Higgs-like model

(Nielsen, Olesen, 1973)

$$SU(N) \xrightarrow{reduction} \left[U(1)\right]^{N-1}$$ **dual scalar thery**

gauge symmetry breaking ↓ Higgs-like mechanism

- **MA Gauge suggests special properties of QCD vacuum**
  Abelian dominance
- **Condensation of scalar d.o.f.** **(Ezawa, Iwasaki, 1982)**
  which provides
- **Dual superconductor picture of QCD vacuum** **('t Hooft 1981)**

Effective model

CP **• Fluctuation measure • Observables**

may be visible **↓ through**

Fluctuations of characteristic length ξ of chiral end mode

Model: effective dual approach to QCD.

Fluctuations based on the order parameter $m \sim \xi^{-1}$

- Deal with gauge-invariant quantities, TPCF as a function of $C_\mu(x)$

- Dual color string: $U_C(x,y) \sim exp\left[ig\int_y^x dz^\mu C_\mu(z)\right]$**,** C_μ^a *dual to* A_μ^a

- ***Particles:*** **Bound states in terms of flux tubes**

Flux tubes

Excitations above vacuum: narrow flux tubes, $r_s \sim \xi \sim m^{-1}$

(in the center, $r_s \to 0$, scalar condensate vanishes)

Ensemble of a single flux tube system, $N(R)$ configurations of f.t.'s

$$Z_{flux} = \sum_{\beta} \sum_{R} N(R) \exp\left[-\beta \, E(m,R)\right] D\left(|\vec{x}|, \beta; M\right)$$

effective energy: $E(m,R) \sim m^2 R\left[a + b \ln(\tilde{\mu} R)\right]$ ***GK, 2010***

Dual gauge field C_μ - critical end mode!

$$m^2(\beta) \sim g^2(\beta)\delta^{(2)}(0)$$

$$\downarrow$$

$$c/\left(\pi r_s^2\right), \quad c \sim O(1)$$

TPCF

At large distances for any correlator (observables)

$$\langle O(\tau,\vec{x})O(\tau,0)\rangle \sim A|\vec{x}|^c D(|\vec{x}|,\beta;M) \text{ as } |\vec{x}|\to\infty$$

$$D(|\vec{x}|,\beta;M)=\exp[-M(\beta)|\vec{x}|],\ \ D(|\vec{x}|,\beta;M)\neq 0 \text{ even at } \beta=\beta_c$$

$M^{-1}(\beta)$ is the measure of screening effect of color electric field

$$SU(N=2,3)\ \ M(\beta)=M^{LO}(\beta)+N\alpha T\ln\left(\frac{M^{LO}(\beta)}{4\pi\alpha T}\right)+4\pi\ \alpha\ T\ y_{n/p}(N)+O(\alpha^2 T)$$

$$M^{LO}(\beta)=\sqrt{4\pi\alpha\left(\frac{N}{3}+\frac{N_f}{6}\right)}\ T$$

Kajante et al. 1997

$$\langle O(\tau,\vec{x})O(\tau,0)\rangle \sim L_W^{-4}-\frac{T}{V}\sigma_0(\beta)\xi^2\left[\frac{1}{\xi}\sqrt{\frac{8\pi}{\sigma_0}}-N\ln\left(\xi\sqrt{2\pi\sigma_0}\right)+\ldots\right]!$$

GK, 2014

- ***TPCF disappears more sharply for heavier flavors @ small $|\vec{x}|$***
- ***Large fluctuations ξ, - TPCF's disappear (CP does approach)***

Result:

effective theory in terms of non-perturbative TPCF describes the fluctuations at distances $g\xi/\sqrt{\pi} < |\vec{x}| < M^{-1}$ ***up to CP***

String tension $\sigma_0(\beta) \sim m^2(\beta)\alpha(\beta)$ *GK 2010*

Flux-tube scheme:

- $\xi \sim m^{-1}$ the penetration length of color-electric field
- $\xi \sim r_s$ "string"-like radius
- $l \sim m_\phi^{-1}$ coherent length of scalar (dilaton) condensate
- $\tau = \sqrt{4/(3\alpha)}\xi$ formation time of flux tube ($\to \infty$ @ CP)

Flux tube formation time becomes shorter for *heavier flavors*

Dual QCD vacuum.

In $SU(3)$ gluodynamics vacuum is characterized

$$k_{GL} = \frac{\xi}{l} \sim \frac{m_\phi}{m} \begin{array}{l} <1 \quad (type\ I\ vacuum,\ \text{flux tubes}\ attracted) \\ >1 \quad (type\ II\ vacuum,\ \text{flux tubes}\ repel) \end{array}$$

Scalar fields, dilatons ϕ (condensate) remain massive up to the CP (1^{st} order PT)

$k_{GL} \to \infty$ **Deconfinement!**

If $k_{GL} = 1$ parallel strings (carry the same flux) do not interact each other.

Singularity of $Z_{flux} \Rightarrow$ $$k_{GL} \geq \frac{3}{4}\frac{\alpha(\beta)}{\xi m_{q\bar{q}}}\left[1 + \frac{4}{3}\frac{\xi^2}{\alpha(\beta)\beta} M(\beta)\frac{L_W}{R}\right]$$ GK 2014

$\downarrow$ **Type-I vac** $\quad$ $\downarrow$ **Type-II vac** $\quad \to \infty \ \ as \ \ \xi \to \infty$

Observation of correlations between two bound states (strings) is rather useful & instructive to check the *CP* is approached!

Field theory $\Rightarrow$ RG $\Rightarrow$ Critical Behavior

Phase transitions $\Leftarrow$ presence and the properties of ***fixed points***

IR attracted Fixed Points $\left\{ \begin{array}{c} \mathit{Phase\ \ Transitions\ \ of\ \ 2nd\ \ kind} \\ \mathit{Critical\ \ scaling} \end{array} \right.$

RG Fluxes (solutions) may leave physical domain containing **IRFP** (even to ∞)

$\Downarrow$

Phase transition of the 1st kind

From theory to phenomenology: *Bose-Einstein Correlations @ finite T*

Def.:

BEC's are the quantum effect which enhances the probability that multiple bosons be found in the same state, same position, same momentum

Sample with production

$AA\ (pp) \rightarrow high\ \ T\ \ quark - gluon\ \ \ bubble \rightarrow hadronization \rightarrow$

$\rightarrow chaotic\ \ \mathbf{hadron's}\ \ production\ \ \ with\ \ \ different\ \ \ directions,\ \ momenta,$
$angles$

What's happened once the critical *T is approached and above*

- **Shape of correlation behavior?**
- **Correlation radius size?**
- **Other characteristics to be measured?**

Size of the particle source

- Possible approach to ***CP*** study through spatial correlations of final state particles
- Size effect of space composed of "hot" particles $\Rightarrow$ derive theoretical formulas for $2-,\dots,N-$ particle ***distribution-correlation functions*** (stochastic, chaotic behavior)

Stochastic scale (size) in C's Bose-Einstein GK (2008-2010)

$$C_2(q,\lambda) \approx \eta(N)\left[1+\lambda(\nu)e^{-\Delta_{qL}}\right], \quad \Delta_{qL} = q^{\mu}\Re_{\mu\nu}q^{\nu}, \quad \eta(N) = \frac{\langle N(N-1)\rangle}{\langle N\rangle^2}$$

event-to-event fluctuations

Chaoticity function:

$$\lambda(\nu) = 1/(1+\nu)^2, \; 0<\nu<\infty, \; N \sim V\int d\omega\, m^2 \frac{1}{e^{(\omega-\mu)\beta}-1}$$

$$b(x) = a(x) + R(x), \; \tilde{R}(p_\mu) = \sqrt{\nu\, \Xi(p,p)}, \; \Xi = \langle a^+(p)a(p)\rangle$$ GK'98-02

Strength of BE correlations $\tilde{\lambda}(k_T,\beta)$

(almost immediate emission of the particle's pairs from a source,)

$$C_2(q,\beta) \approx \eta(N)\left\{1+\tilde{\lambda}(\beta)e^{-q^2L_{st}^2}\left[1+\lambda_1(\beta)e^{+q^2L_{st}^2/2}\right]\right\}(1+\delta\cdot q+\ldots)$$

$$\tilde{\lambda}(\beta)=\frac{\gamma(\omega,\beta)}{\left[1+\nu(N)\right]^2},\ \lambda_1 \approx 2\nu,\ \nu\sim\frac{1}{n}\frac{1}{k_{GL}^2},\ \text{as}\ T\to T_c,\ \ \gamma(\omega,\beta)\sim O(1)$$

Proposal: *GK 2009-2014*

✓ $\tilde{\lambda}(k_T,\beta)\to 1$ **as** ***CP*** **approached,** $k_{GL}\to\infty$ ***DECONFINEMENT***

Origin: infinite fluctuation length $\xi\to\infty$

- When ***CP*** approached:
  - **NO** signal of enhancement of pairs of same-sign charge particles is observed!

BEC C_2 ***- function does not deviate from 1.***

OBSERVABLES?

The scaling form C_2 is useful to predict behavior of observables @ **CP**

$L_{st} \to \infty$ *as* $T \to T_c$, $\mu \to \mu_c$ indicate the vicinity of **CP**

- Observable, e.g., $k_T^2 = \dfrac{1}{\nu(N)\; T^3\; L_{st}^5}$, $\quad k_T = \left|\vec{p}_{T_1} + \vec{p}_{T_2}\right|$ GK(2011)

Exp: L3/CMS(2011)/ATLAS(2015)/ALICE L decreasing (smooth) with k_T

- Chaoticity λ measured. *(Most important theor. study)*

$$0 \;<\lambda\left[\nu(N)\right]\leq\; 1 \qquad\qquad \nu(N) \approx \frac{1}{n\,k_{GL}^2}\,O\!\left(\frac{m_\phi^2}{m^2}\right)$$

$\Downarrow$ fully coherent phase

$\Downarrow$ chaotic *(critical behavior from BM to AM)*

Chiral restoration & Particle emission size

Theory: $L_{st} = L_{st}\left(\beta, k_T, m, \nu(N)!\right) \sim \dfrac{1}{\nu^{1/5}(N)\, m_h^{\alpha} T^{\gamma}}$ ***GK, 2009-2010***

$$\nu(N) = \frac{2 - \tilde{C}_2(0) + \sqrt{2 - \tilde{C}_2(0)}}{\tilde{C}_2(0) - 1}, \qquad \tilde{C}_2(0) = \frac{C_2(q=0)}{\dfrac{\langle N^2 \rangle}{\langle N \rangle^2} - \dfrac{1}{\langle N \rangle}}$$

$$\langle N \rangle \geq 1 + C_2(0)/2, \quad C_2(0) \leq 2$$

CMS (2011): $\sqrt{s}$ =0.9 TeV; 7 TeV,- L_{st} increases with $\langle N \rangle$

ATLAS (2015): $\sqrt{s}$ =0.9 TeV; 7 TeV,- L_{st} increases with $\langle N \rangle$ as well

Small distortion ν (*vicinity of CP*):

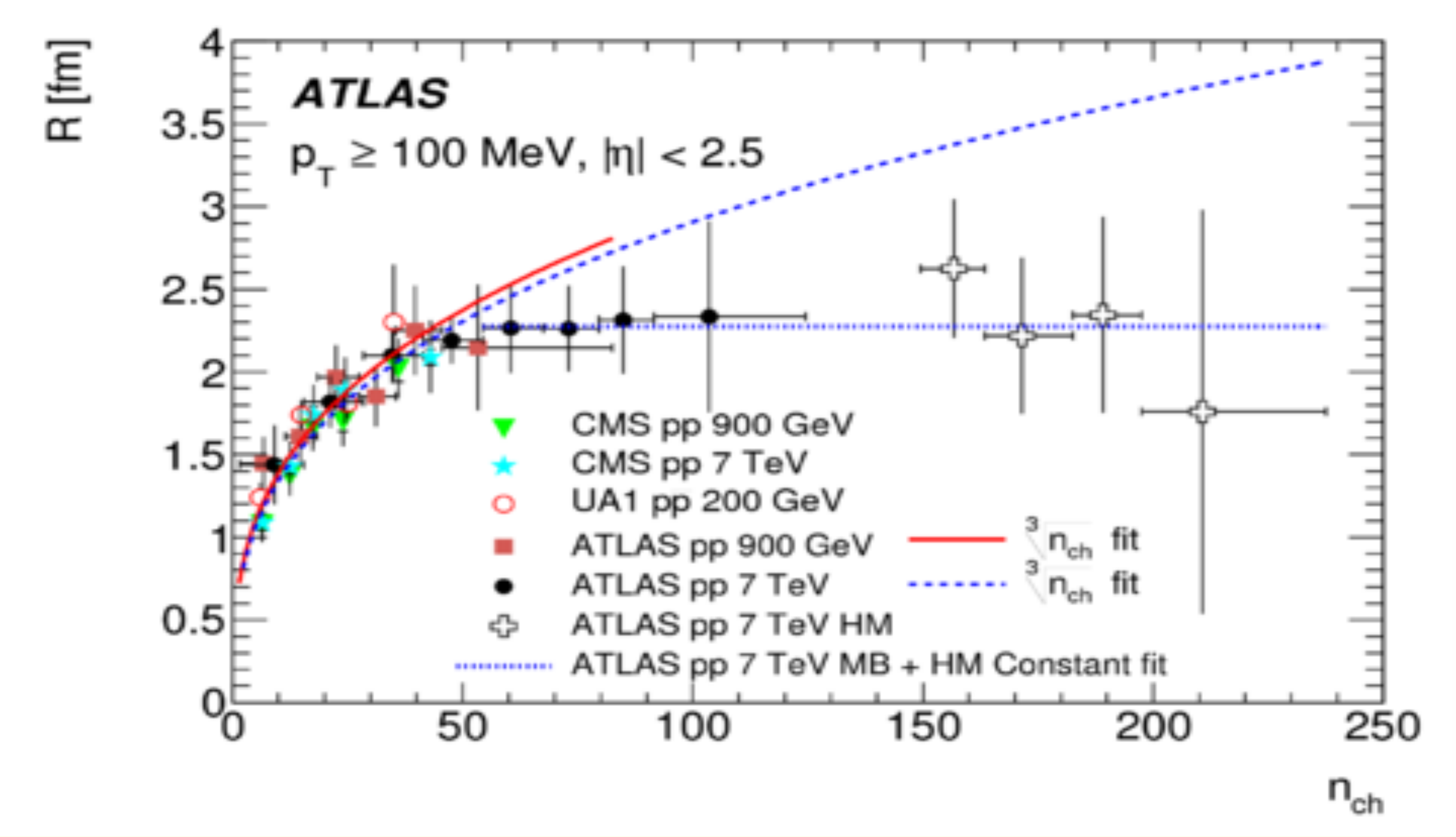
combined $\bar{\lambda} = \lambda \cdot \lambda_1 \sim \nu$ decreases with ***heavy flavor mass***

Charactersitic size of the Correlation source

In terms of Ginzburg-Landau criterium k_{GL}

«Radius» increases with $n \sim n_{ch}$ $R \sim L_{st} \sim \left(\frac{n\ k_{GL}^2}{k_T^2\ T^3} \right)^{1/5}$ *GK (2009)*

ATLAS Coll., Eur. Phys. J.C75 (2015) 466



- **Expansion of particle emission size**

$$L_{st}(\beta) \sim \left[\nu(N) k_T^2 \; T^3\right]^{-1/5} \to \infty \;\; \boldsymbol{as}\; \nu(N) \to 0 \text{ at } T \to T_c$$ ***KG 2010***

The temperature at which the signal of two-particles correlations disappears is the critical temperature at *CP* : $C_2(q, T_c) = 1$

! Too rapid phase transition can include the ***explosion*** of a "hadronic fireball" just after a phase transition

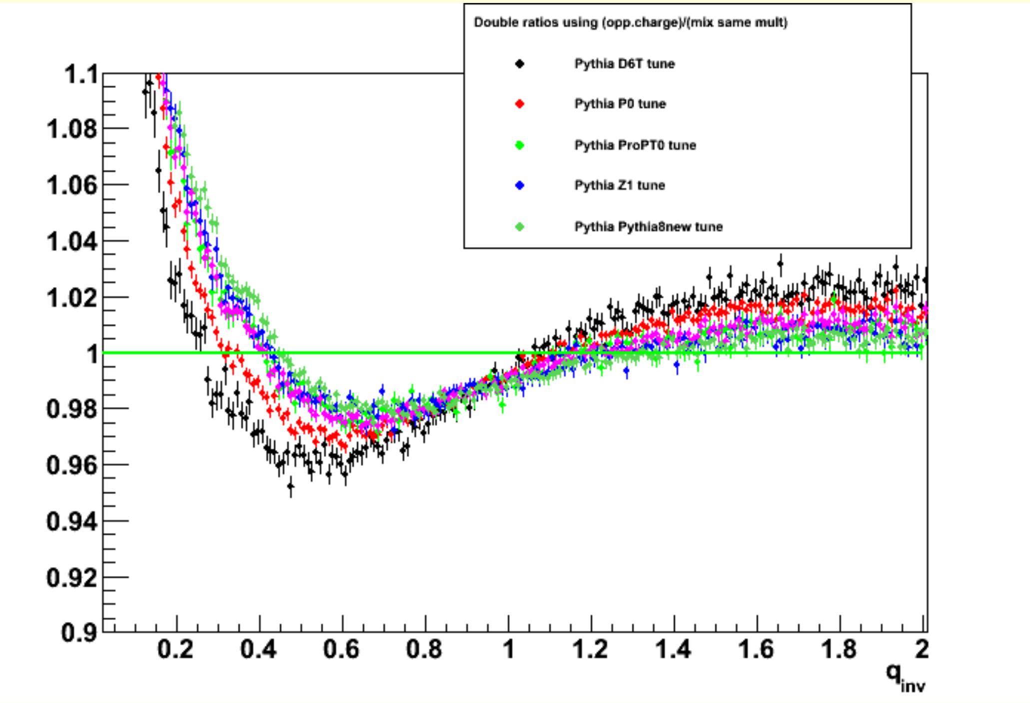
Dip-effect

The effect of anti-correlations (the dip-effect) is predicted at low charged-particle multiplicity in the event: $C_2(q,N)<1$?! ***KG 2010***

The depth of the dip in the anti-correlation region decreases as N increases.

Observed by CMS at LHC [CMS Coll., JHEP 5 (2011) 029]

Proposal: dip-effect disappears at *CP*



Random Fluctuation Walk *(BM ⇒ AM)*

Random stochastic (chaotic) walk with respect to quantum correlations of identical particles. Cross-over walk.

Model 1D x-oriented $(-\infty < x < \infty)$

$$P\left(x;\bar{\lambda},\mu_c\right) = p\sum_{j=0}^{\infty}\bar{\lambda}^j\frac{1}{2}\sqrt{\frac{\pi}{t}}\left[e^{-y_-^{2j}/4t} + e^{-y_+^{2j}/4t}\right]$$

$$y_{\pm}^j = x\mu_c \pm a^j,\ a = \left(\mu/\mu_c\right) > 1,\ t = l\mu_c \text{ (lattice spacing)}$$

$$\underbrace{\lim}_{\substack{l\to 0\\ \mu_c\neq 0}} P\left(x;\bar{\lambda},\mu_c\right) = p\left(\bar{\lambda}\right)\sum_{j=0}^{\infty}\bar{\lambda}^j\pi\left[\delta\left(x\mu_c - a^j\right) + \delta\left(x\mu_c + a^j\right)\right]$$

$$p\left(\bar{\lambda}\right) = \frac{1}{2\pi}\left(1-\bar{\lambda}\right),\ 0 < \bar{\lambda} \le 1,$$ NC: $$2\left(p + \bar{\lambda}p + \ldots + \bar{\lambda}^j p + \ldots\right) = 1$$

The limit $\bar{\lambda} \to 1 \Rightarrow$ broad behavior of P: *vicinity of* ***CP*** *is approached*

$\bar{\lambda} \to 0 \Rightarrow P\left(x;\bar{\lambda}\to 0,\mu_c\right) \to 1/(2\pi)$ trivial

Analyticity of probability $P(x;\lambda,\mu_c)$

Large x (sharp increasing of L_{st}) / or $k \to 0$ (IR analogue)

To smooth the particularity (speciality) of $P(x;\lambda,\mu_c)$

$$P(x;\lambda,\mu_c) \to G(k;\lambda,\mu_c) = p(\lambda)\sum_{j=0}^{\infty}\lambda^j \cos\left(\frac{k}{\mu_c}a^j\right), \quad G(0;\lambda) = \frac{1}{2\pi}$$

Fluctuation length through the even moments of the order $2s$:

$$\xi^2_{(2s)}(\lambda) \sim m_{(2s)}(\lambda) = \frac{\partial^{2s}G(k;\lambda,\mu_c)}{\partial k^{2s}}\Big|_{k=0}, \quad \bar{\lambda} \to \lambda(\nu) = \left[1+\nu(N)\right]^{-2}$$

- Finite ξ will provide analytical form of G, however large $\xi \to$ non-analytical behavior of G @ $k \to 0$

The dual QCD vacuum will influence *(through k_{GL})* ξ up to cross-over: unified process of phase transition between BM and AM

Analyticity of probability $P(x;\lambda,\mu_c)$ cont'd I

$G(k;\lambda,\mu_c)$ in terms of CBE function $C_2(q,\lambda)$

$$G(k;\lambda,\mu_c)=\frac{2-\bar{C}_2(0)}{2\pi}\sum_{j=0}^{\infty}\left[\bar{C}_2(0)-1\right]^j\cos\left(\frac{k}{\mu_c}a^j\right),\quad \bar{C}_2(q;\lambda)\equiv\frac{C_2(q;\lambda)}{\eta(N)}$$

CP: $\underbrace{\lim}_{\bar{C}_2(0)\to 2}\ r.h.s.\to\infty$ at high multiplicities N, $\bar{C}_2(q=0,\lambda)=\bar{C}_2^{\exp}(0)$

Fluctuation length result: $\left|\xi_{(2s)}^2(\lambda)\right|=p(\lambda)\sum_{j=0}^{\infty}\left(\frac{a^j}{\mu_c}\right)^{2s}\lambda^j,\ a=\frac{\mu}{\mu_c}>1$

Converged @ $(a/\mu_c)^{2s}\lambda<1,\ \left|\xi_{(2s)}^2(\lambda)\right|\approx p(\lambda)\mu_c^{-2s}(1+a^{2s}\lambda)$ finite

If $\mu\approx\mu_c$ and $\lambda\to 1,\ \xi\to\infty$ divergence $/\xi\Leftrightarrow$ **CP/**

Analyticity of probability $P(x;\lambda,\mu_c)$ cont'd II

- Infinite # of divergent (singular) terms in $G(k;\lambda,\mu_c)$

  Why?

Because wide range of $\lambda,$ μ; singularity @ $k << \mu_c$ $(k \to 0)$

- To find non-analytical part @ $k \approx 0$

$G(k;\lambda,\mu_c) = G_{BM}(k;\lambda,\mu_c) + G_{AM}(ak;\lambda,\mu_c)$ linear non-homog. eq.

BM $G_{BM}(k;\lambda,\mu_c) = p(\lambda)\boldsymbol{cos}(k/\mu_c)$ regular if $k \approx 0$, for all λ

AM $G_{AM}(k;\lambda,\mu_c) = \lambda G(ak;\lambda,\mu_c)$, for $a^{-2} < \lambda < 1$

$G_{AM}(k;\lambda,\mu_c) \to 0,$ *if* $\lambda \to 0$ The phase with **BM** does exist only

While $G(ak;\lambda \approx 0,\mu_c) \approx (1/2\pi)\boldsymbol{cos}(k\mu/\mu_c^2)$ Finite

Solution for AM phase

$$G_{AM}(k;\lambda,\mu_c) \sim \frac{i^{\alpha-2}}{\Gamma(\alpha-1)} \xi^2_{(\alpha-2)}(\lambda) \left|\frac{k}{\mu_c}\right|^{\alpha} \frac{1}{\log a} \sum_{m=0}^{\infty} b_m \cos\left[2\pi m \frac{\log(|k|)}{\log a} + \vartheta_m\right]$$

Divergent if a) $\lambda \to 1$ because of $\xi(\lambda)$

b) $a = \dfrac{\mu}{\mu_c} \to 1$ from above

For **AM** upper limit of μ_c**:** $\mu_c \leq \mu\left[1 + \frac{1}{n\, k_{GL}^2} O\left(\frac{m_\phi^2}{m^2}\right)\right]^{-1}$

CP (cross-over) $\mu = \mu_c$ at $k_{GL} \to \infty$

Conclusions

1. RFW solution: **BM** (regular) + **AM** (singular).

2. Cross-over between **BM** and **AM**

Influence on heavy flavor production

2. Main points: $\lambda(\nu),\ k_{GL},\ \xi,\ (\mu/\mu_c)>1.$

3. Heavy flavor production source:

Source size L_{st} *increases (smoothly) with* N *at low* T.

✓ L_{st} blows up as $T \to T_c$ $/\nu(N)\to 0,\ m_h \to 0/$

✓ L_{st} singular @ CP.