STUDY ON D-D-SIGMA AND D*-D*-SIGMA COUPLING CONSTANTS

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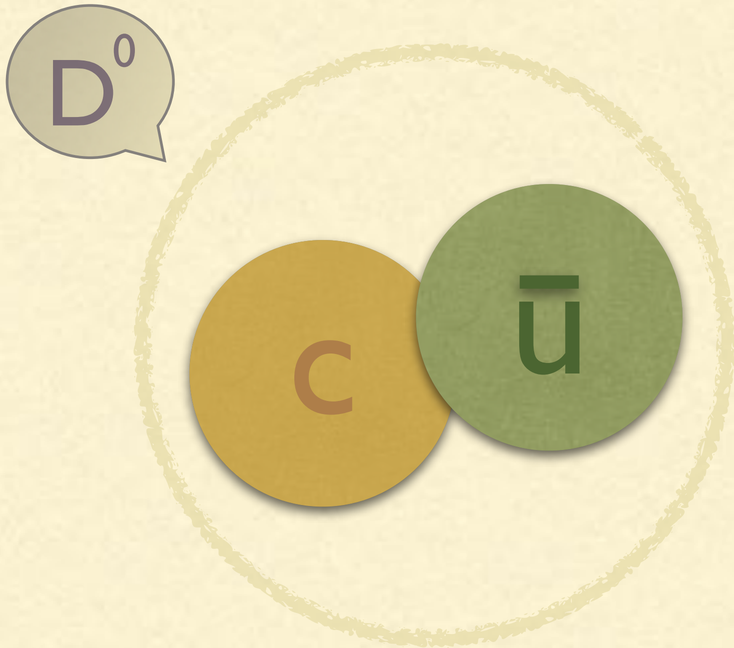
CONTENTS

- MOTIVATION
- EFFECTIVE LAGRANGIAN
- FEYNMAN AMPLITUDES
- CROSSING SYMMETRY
- RESCATTERING EQUATION
- SPECTRAL FUNCTION
- COUPLING CONSTANTS
- SUMMARY

MOTIVATION

- D(D*) meson



- Charmed meson
- Pseudoscalar (Vector) particle
- Isospin : I=1/2

MOTIVATION

- X(3872) are near in the DD* threshold (~3871)
    - The importance of D mesons arises
- Couplings with mesons (pion, rho, etc)

EFFECTIVE LAGRANGIAN

EFFECTIVE LAGRANGIAN

- Effective Lagrangians

$$\mathcal{L}_{DD^*\pi} = g_{DD^*\pi} D^*_\mu \vec{\tau} \cdot \partial^\mu \vec{\pi} D + \text{h.c.}$$

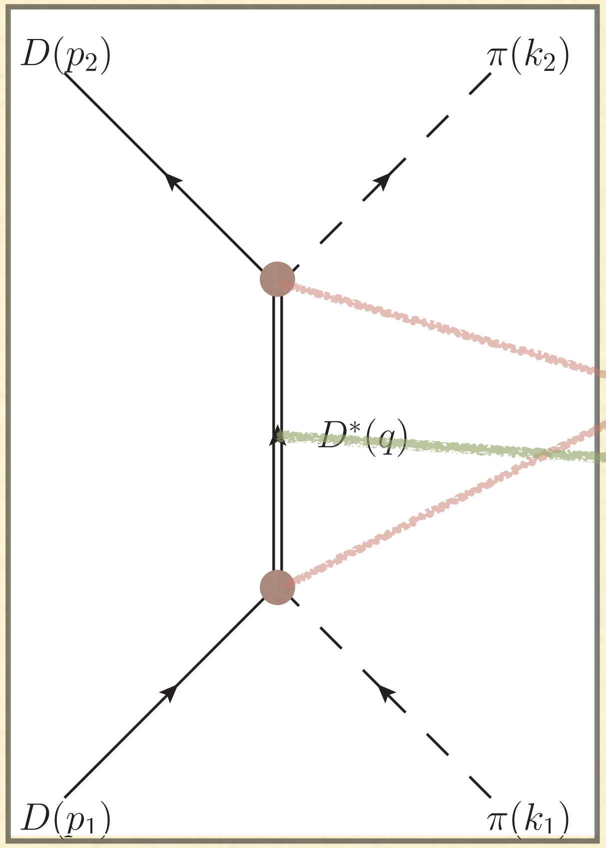
$$\mathcal{L}_{D^*D^*\pi} = g_{D^*D^*\pi} \varepsilon^{\alpha\beta\mu\nu} D^*_\alpha \vec{\tau} \cdot \partial_\mu \vec{\pi} \partial_\nu D^*_\beta + \text{h.c.}$$

$C.\ H.\ Zhang, L.\ Ding, Z.\ W.\ Yan, K.\ WuandW.\ Z.\ Zhao, Commun.\ Theor.\ Phys.\ \mathbf{67}, no.6, 648(2017).$

FEYNMAN AMPLITUDES

PI-D ⟶ PI-D SCATTERING

- The s-channel Feynman Amplitude



$$i\mathcal{M}^s_{\alpha\beta} = (ig_{DD^*\pi})\tau_\alpha(-ik_1^\mu)$$
$$\times \frac{i\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{M_{D^*}^2}\right)}{q^2 - M_{D^*}^2}$$
$$\times (ig_{DD^*\pi})\tau_\beta(ik_2^\nu)$$

PI-D ⟶ PI-D SCATTERING

- s-channel Feynman Amplitude

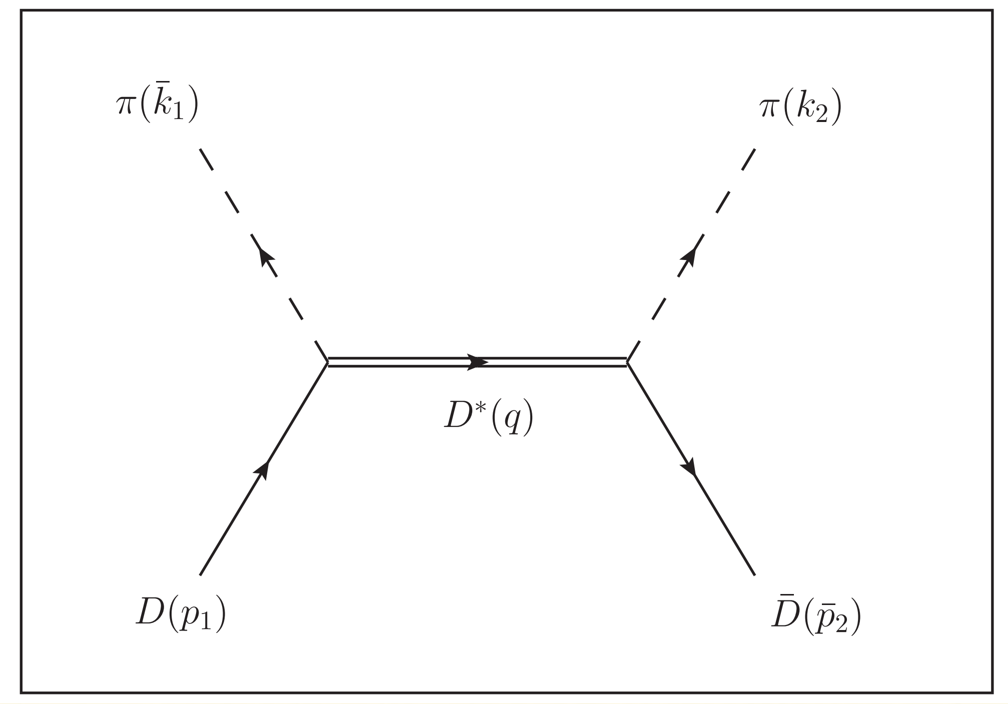
$$i\mathcal{M}^{s}_{\alpha\beta} = i g^2_{DD^*\pi} \frac{m_\pi^2 - (t/2) - (s - M_D^2 + m_\pi^2)^2/4M_{D^*}^2}{s - M_{D^*}^2} \tau_\alpha \tau_\beta$$

- u-channel Feynman Amplitude

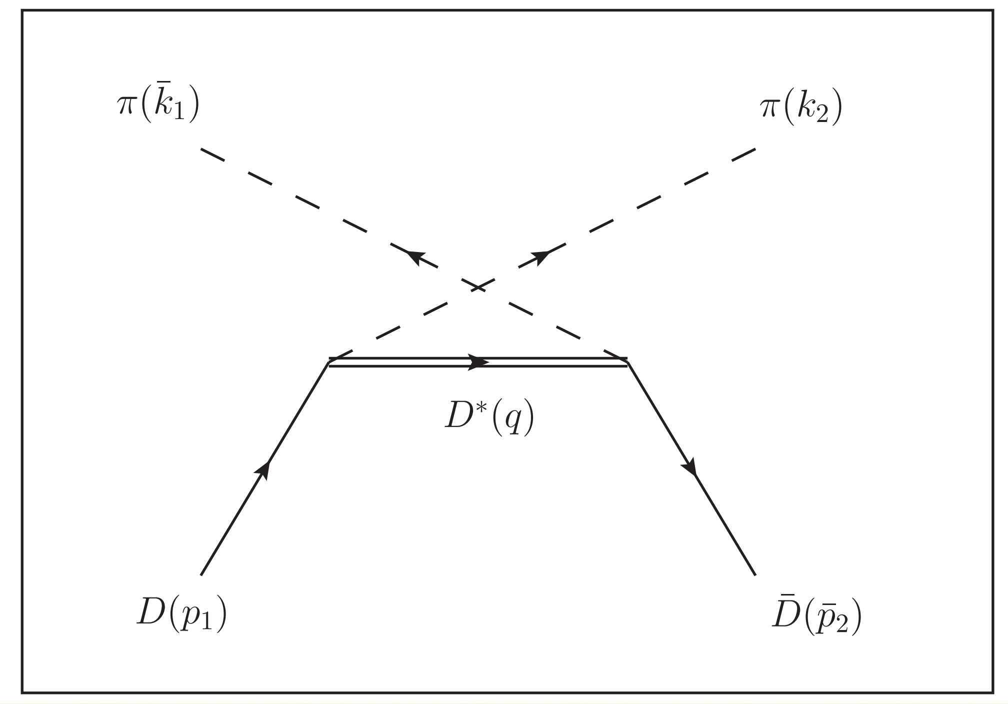
$$i\mathcal{M}^{u}_{\alpha\beta} = i g^2_{DD^*\pi} \frac{m_\pi^2 - (t/2) - (u - M_D^2 + m_\pi^2)^2/4M_{D^*}^2}{u - M_{D^*}^2} \tau_\beta \tau_\alpha$$

$D\bar{D} \longrightarrow$ PIPI SCATTERING

- Feynman Diagrams



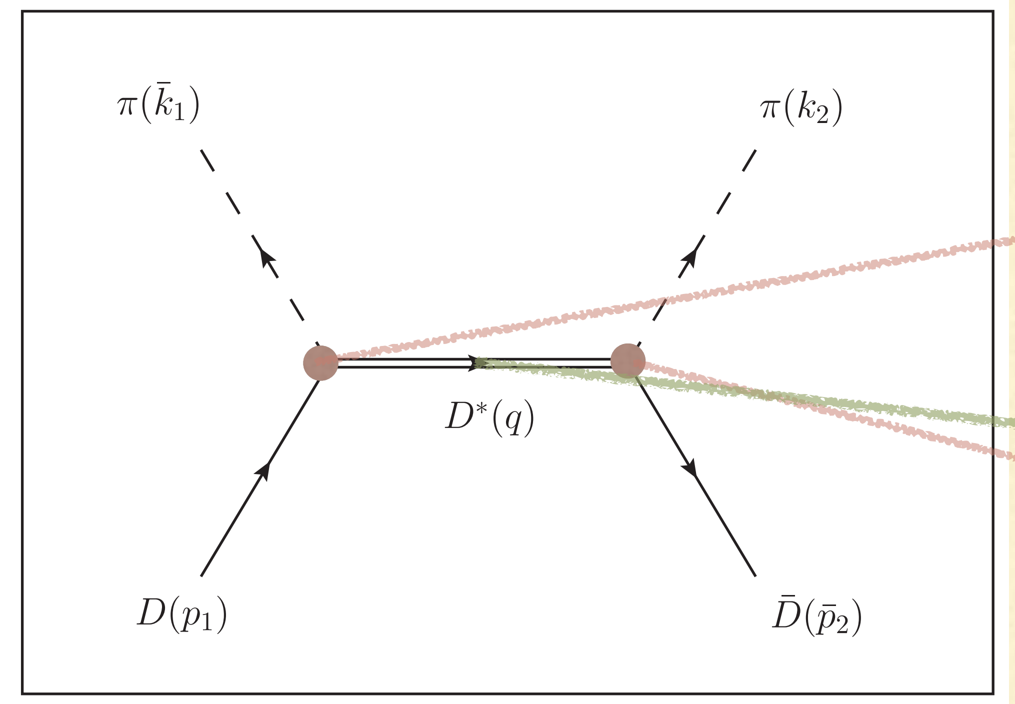
t-channel



u-channel

PI-D ⟶ PI-D SCATTERING

- The t-channel Feynman Amplitude



$$i\mathcal{M}^t_{\alpha\beta} = (ig_{DD^*\pi})\tau_\alpha(i\bar{k}_1^\mu)$$

$$\times \frac{i\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{M^2_{D^*}}\right)}{q^2 - M^2_{D^*}}$$

$$\times (ig_{\bar{D}D^*\pi})\tau_\beta(ik_2^\mu)$$

$D\bar{D} \longrightarrow$ PIPI SCATTERING

- s-channel Feynman Amplitude

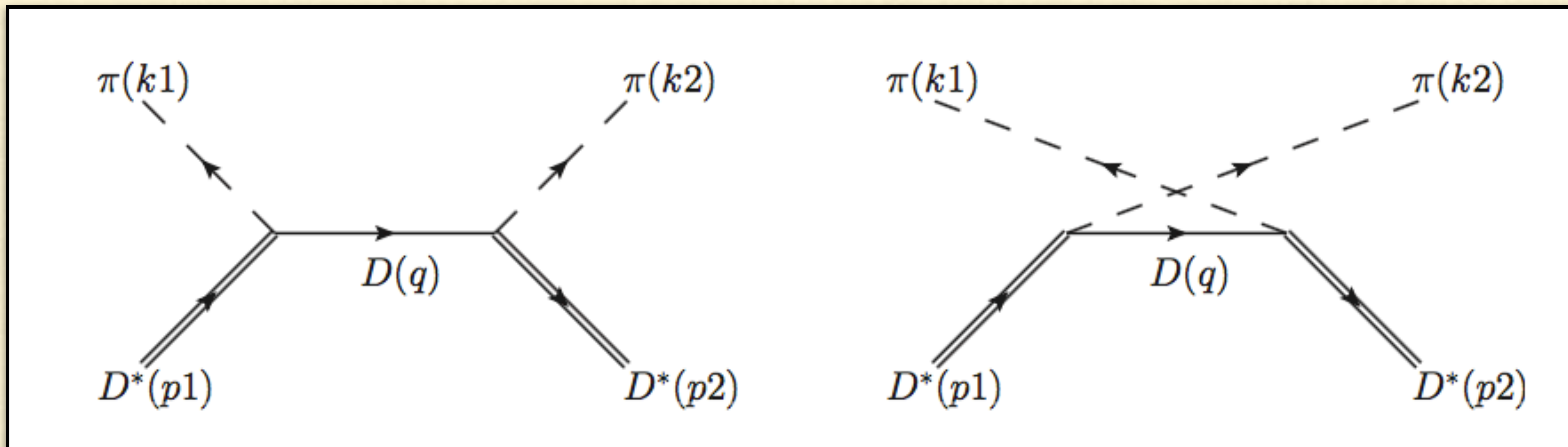
$$i\mathcal{M}^{s}_{\alpha\beta} = i g^2_{DD^*\pi} \frac{m_\pi^2 - (t/2) - (s - M_D^2 + m_\pi^2)^2/4M_{D^*}^2}{s - M_{D^*}^2} \tau_\alpha \tau_\beta$$

- u-channel Feynman Amplitude

$$i\mathcal{M}^{u}_{\alpha\beta} = i g^2_{DD^*\pi} \frac{m_\pi^2 - (t/2) - (u - M_D^2 + m_\pi^2)^2/4M_{D^*}^2}{u - M_{D^*}^2} \tau_\beta \tau_\alpha$$

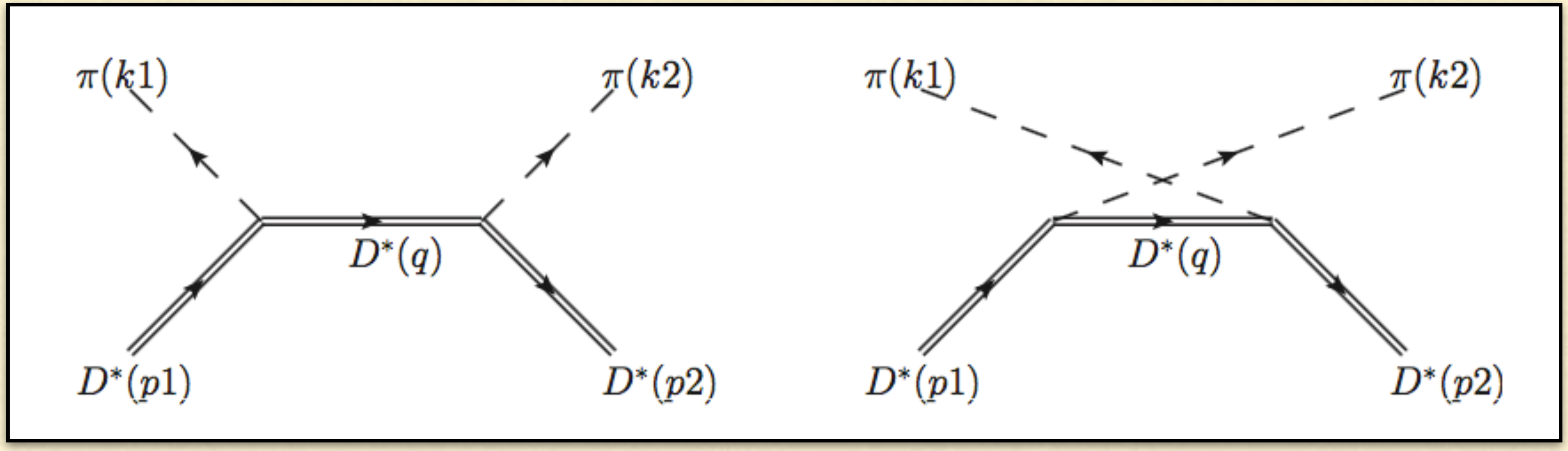
$D^*\bar{D}^* \longrightarrow$ PIPI SCATTERING

- Feynman Diagrams : D-exchange



$D^*\bar{D}^* \longrightarrow$ PIPI SCATTERING

- Feynman Diagrams : D*-exchange



$D^*\bar{D}^* \longrightarrow$ PIPI SCATTERING

- s-channel Feynman Amplitude

$$i\mathcal{M}^{D,s}_{\alpha\beta} = -ig_{DD^*\pi}g_{D\bar{D}^*\pi}\epsilon^{(\lambda)}_{\mu}(p_1)\epsilon^{(\lambda')}_{\nu}(p_2)\frac{k_1^{\mu}k_2^{\nu}}{s-M_D^2}\tau_\alpha\tau_\beta$$

$$\mathcal{M}^{\alpha\gamma}_{s} = 4g_{D^*D^*\pi}g_{D^*\bar{D}^*\pi}\varepsilon_{\alpha\beta\mu\nu}\varepsilon_{\gamma\delta\rho\sigma}g_{\beta\delta}p_1^{\nu}p_2^{\sigma}\frac{k_1^{\mu}k_2^{\rho}}{s-M_{D^*}^2}$$

$D^*\bar{D}^* \longrightarrow$ PIPI SCATTERING

- u-channel Feynman Amplitude

$$i\mathcal{M}_{\alpha\beta}^{D,u} = -ig_{DD^*\pi}g_{D\bar{D}^*\pi}\epsilon_\mu^{(\lambda)}(p_1)\epsilon_\nu^{(\lambda')}(p_2)\frac{k_2^\mu k_1^\nu}{u-M_D^2}\tau_\beta\tau_\alpha$$

$$\mathcal{M}_u^{\alpha\gamma} = 4g_{D^*D^*\pi}g_{D^*\bar{D}^*\pi}\varepsilon_{\alpha\beta\mu\nu}\varepsilon_{\gamma\delta\rho\sigma}g_{\beta\delta}p_1^\nu p_2^\sigma\frac{k_2^\mu k_1^\rho}{u-M_{D^*}^2}$$

D*D̄* ⟶ PIPI SCATTERING

- u-channel Feynman Amplitude

$$i\mathcal{M}_{\alpha\beta}^{D,u} = -ig_{DD^*\pi}g_{D\bar{D}^*\pi}\epsilon_\mu^{(\lambda)}(p_1)\epsilon_\nu^{(\lambda')}(p_2)\frac{k_2^\mu k_1^\nu}{u - M_D^2}\tau_\beta\tau_\alpha$$

$$\mathcal{M}_u^{\alpha\gamma} = 4g_{D^*D^*\pi}g_{D^*\bar{D}^*\pi}\varepsilon_{\alpha\beta\mu\nu}\varepsilon_{\gamma\delta\rho\sigma}g_{\beta\delta}p_1^\nu p_2^\sigma\frac{k_2^\mu k_1^\rho}{u - M_{D^*}^2}$$

D*D̄* → PIPI SCATTERING

$$
\begin{aligned}
i\mathcal{M}_{ij}^{D^*} &= 4ig_{D^*D^*\pi}g_{D^*\bar{D}^*\pi}\varepsilon_{\alpha\beta\mu\nu}\varepsilon_{\gamma\delta\rho\sigma}g_{\beta\delta}p_1^\nu p_2^\sigma \\
&\qquad \times \epsilon_\alpha^{(\lambda)}(p_1)\epsilon_\gamma^{(\lambda')}(p_2)\left[\frac{k_1^\mu k_2^\rho}{s-M_{D^*}^2}\tau_i\tau_j + \frac{k_2^\mu k_1^\rho}{u-M_{D^*}^2}\tau_j\tau_i\right] \\
&= i\mathcal{M}_{\lambda\lambda'}^{(+)}\delta_{ij} + i\mathcal{M}_{\lambda\lambda'}^{(-)}\frac{1}{2}[\tau_i,\tau_j]
\end{aligned}
$$

$$
\mathcal{M}_{\lambda\lambda'}^{(+)} = 4g_{D^*D^*\pi}g_{D^*\bar{D}^*\pi}\varepsilon_{\alpha\beta\mu\nu}\varepsilon_{\gamma\delta\rho\sigma}g_{\beta\delta}p_1^\nu p_2^\sigma\epsilon_\alpha^{(\lambda)}(p_1)\epsilon_\gamma^{(\lambda')}(p_2)\left[\frac{k_1^\mu k_2^\rho}{s-M_{D^*}^2} + \frac{k_2^\mu k_1^\rho}{u-M_{D^*}^2}\right]
$$

$$
\mathcal{M}_{\lambda\lambda'}^{(-)} = 4g_{D^*D^*\pi}g_{D^*\bar{D}^*\pi}\varepsilon_{\alpha\beta\mu\nu}\varepsilon_{\gamma\delta\rho\sigma}g_{\beta\delta}p_1^\nu p_2^\sigma\epsilon_\alpha^{(\lambda)}(p_1)\epsilon_\gamma^{(\lambda')}(p_2)\left[\frac{k_1^\mu k_2^\rho}{s-M_{D^*}^2} - \frac{k_2^\mu k_1^\rho}{u-M_{D^*}^2}\right]
$$

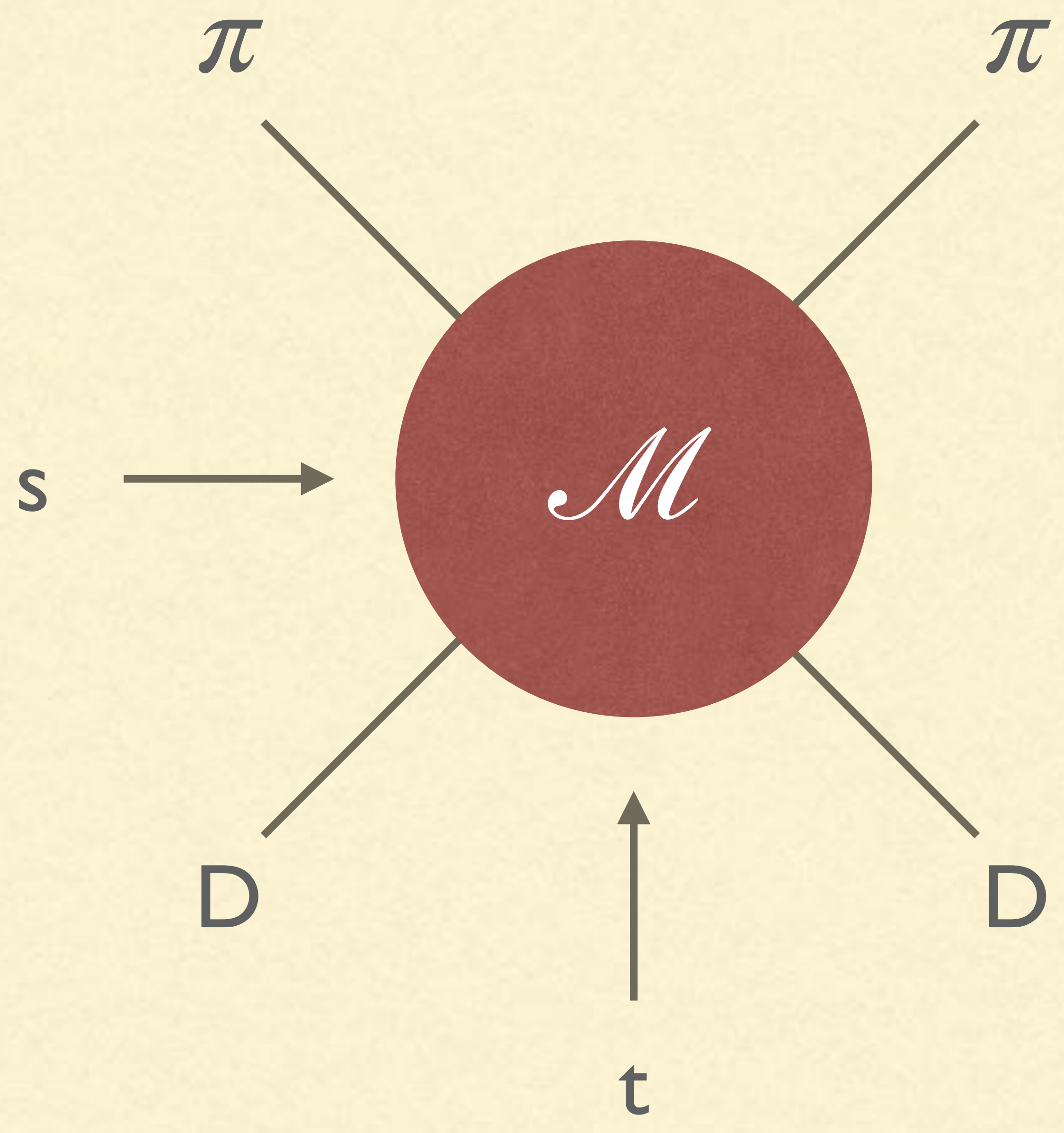
$D^*\bar{D}^* \longrightarrow$ PIPI SCATTERING

- Feynman amplitude

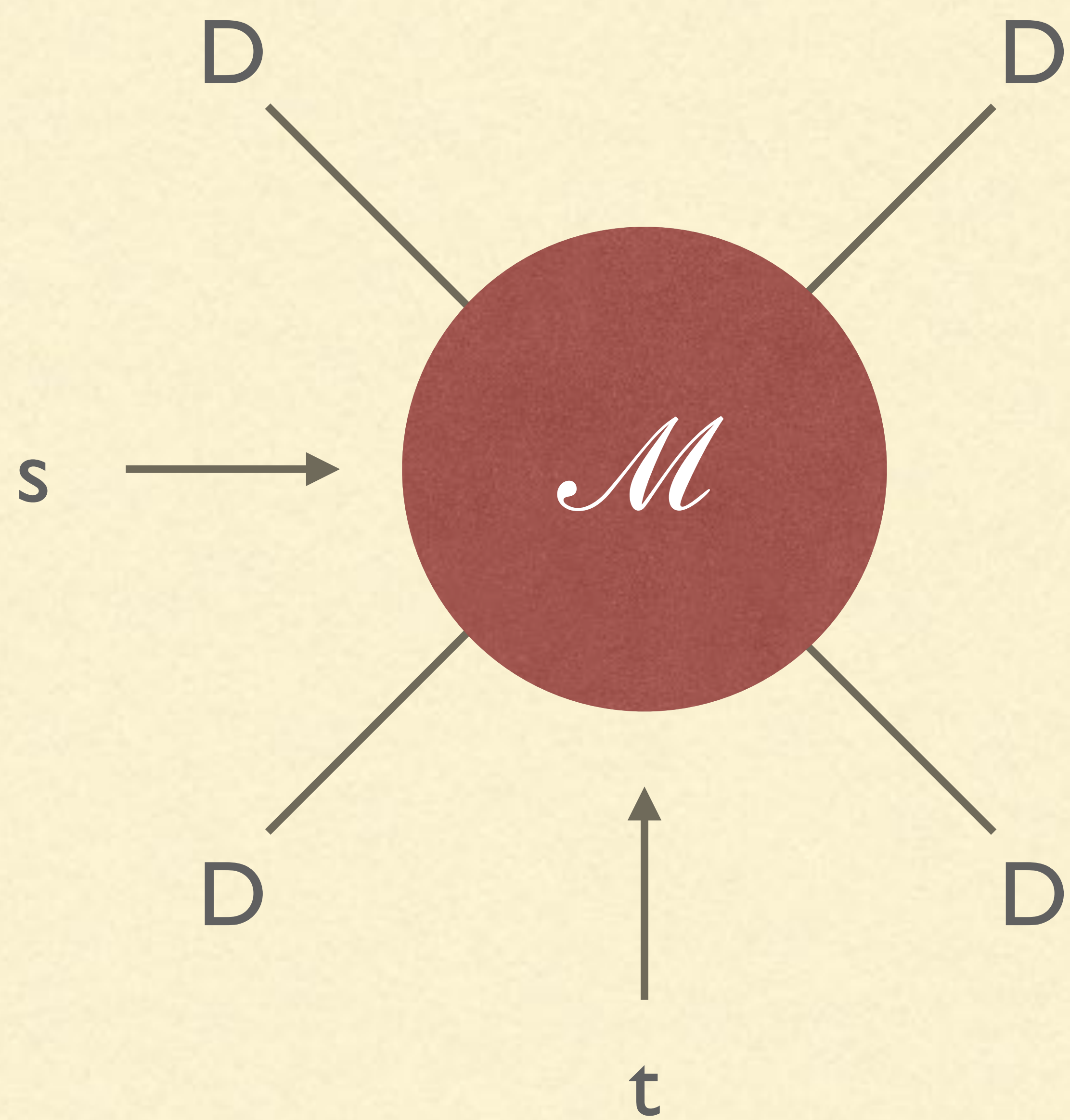
$$\mathcal{M}^{(+),\lambda_1\lambda_2}_{D^*\bar{D}^*\to\pi\pi} = -\,g^2_{DD^*\pi}\left[\frac{(\epsilon_{\lambda_1}\cdot q_1)(\epsilon_{\lambda_2}\cdot q_2)}{s-M_D^2} + \frac{(\epsilon_{\lambda_1}\cdot q_2)(\epsilon_{\lambda_2}\cdot q_1)}{u-M_D^2}\right]$$
$$+\,4g^2_{D^*D^*\pi}\varepsilon_{\mu\mu'\alpha\beta}\varepsilon_{\nu\nu'\gamma\delta}\epsilon^{\mu}_{\lambda_1}\epsilon^{\nu}_{\lambda_2}g^{\mu'\nu'}p_1^{\beta}p_2^{\delta}\left[\frac{q_1^{\alpha}q_2^{\gamma}}{s-M^2_{D^*}} + \frac{q_2^{\alpha}q_1^{\gamma}}{u-M^2_{D^*}}\right]$$

CROSSING SYMMETRY

CROSSING SYMMETRY



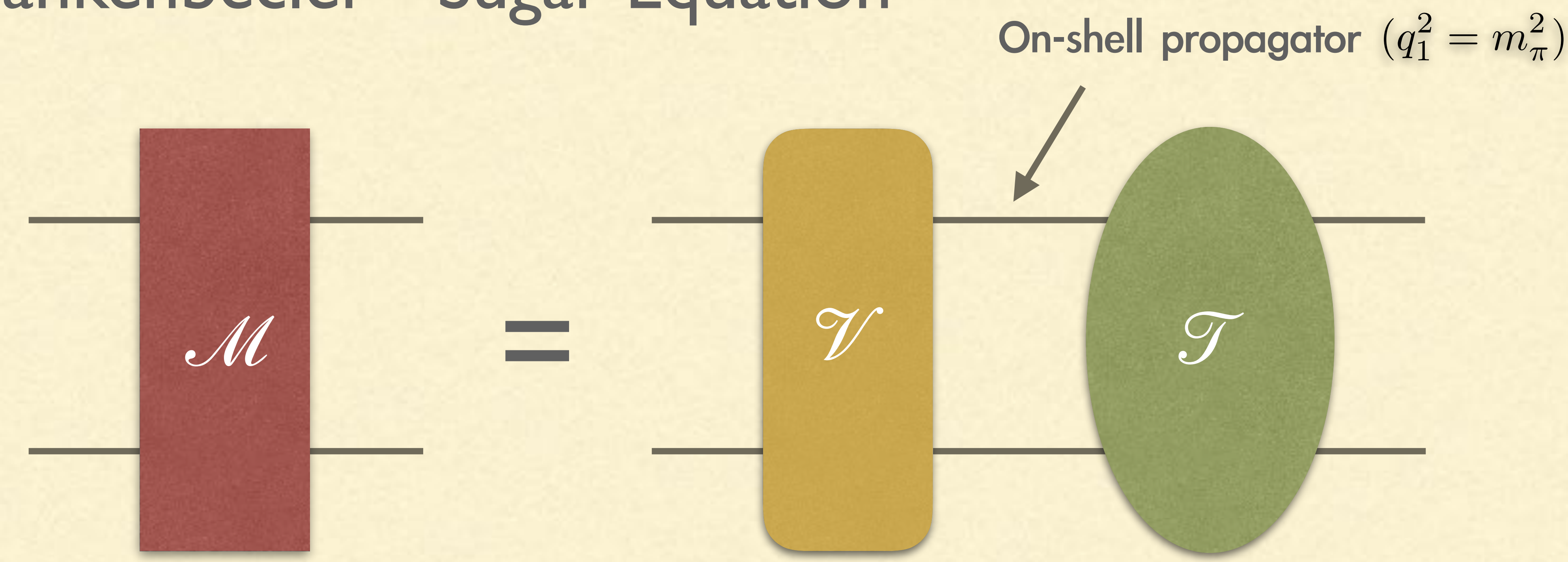
CROSSING SYMMETRY



RESCATTERING EQUATION

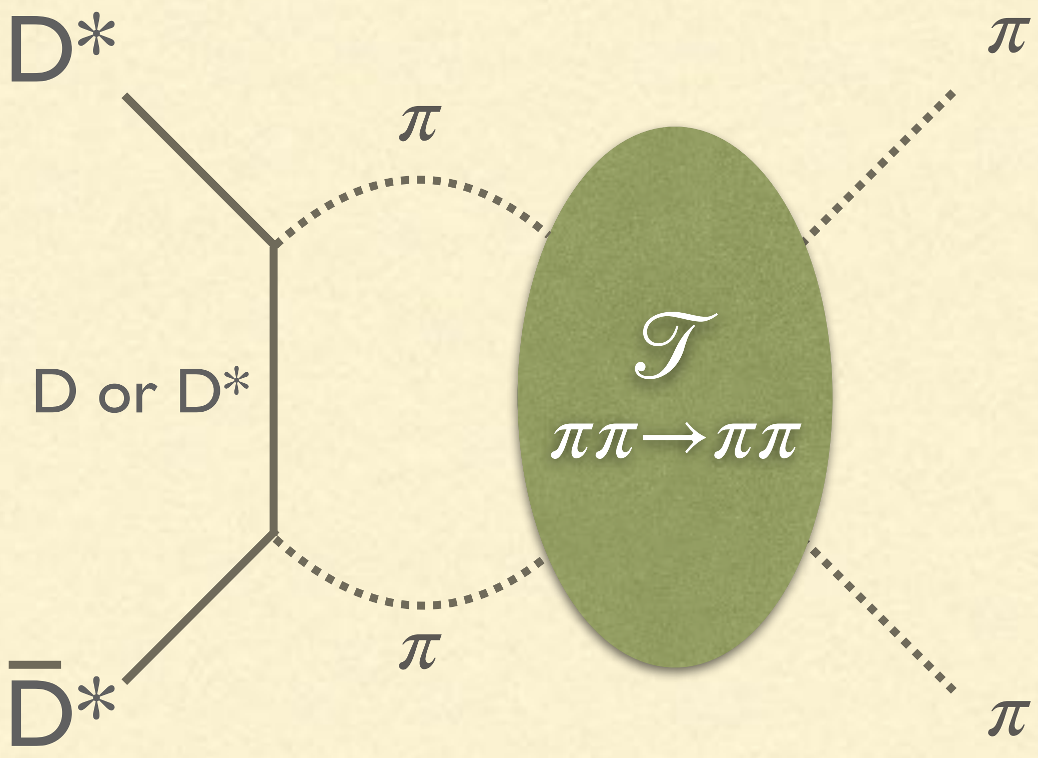
RESCATTERING EQUATION

- Blankenbecler - Sugar Equation



$$\mathcal{M}(p,p';s) = \mathcal{V}(p,p';s) + \int d^3q \, \frac{1}{(2\pi)^3} \frac{1}{2\omega_q} \frac{\mathcal{M}(p,q;s)\mathcal{T}(q,p';s))}{s - 4\omega_q^2 + i\varepsilon}$$

RESCATTERING EQUATION



- We are interested only in the scalar channel → Partial wave expansion
- Not only decomposing equation but also having simpler form of equation

$$\mathcal{M}^{J=0,\lambda_1\lambda_2}_{D^*\bar{D}^*\to\pi\pi} = \mathcal{M}^{\mathrm{born},J=0;\lambda_1\lambda_2}_{D^*\bar{D}^*\to\pi\pi} + \int dq\, q^2 \frac{1}{(2\pi)^3 2\omega_q (s - 4\omega_q^2 + i\varepsilon)} \mathcal{M}^{J=0,\lambda_1\lambda_2}_{D^*\bar{D}^*\to\pi\pi} \mathcal{T}^{J=0}_{\pi\pi\to\pi\pi}$$

SPECTRAL FUNCTION

TWO-BODY UNITARITY

The S-matrix is unitary : $S_{fn}S^{\dagger}_{ni} = \delta_{fi}$

It may be expressed in terms of invariant amplitude :

$$S = 1 + iT = 1 + i(2\pi)^4\delta^{(4)}(\sum P)\mathcal{M}$$

⬇

If both of the initial and final states are D*D* , we have

$$-i\left(\mathcal{M}_{D^*\bar{D}^*} - \mathcal{M}^{\dagger}_{D^*\bar{D}^*}\right) = \sum_n (2\pi)^4\delta^{(4)}(P_f - P_n)|\mathcal{M}_{D^*\bar{D}^*\to n}|^2$$

TWO-BODY UNITARITY

Since we now consider two pion exchange, we may write

$$2\,\mathrm{Im}\mathcal{M}_{D^*\bar{D}^*} = \frac{1}{2}\int (2\pi)^4\delta^{(4)}(p_1+p_2-q_1-q_2)|\mathcal{M}_{D^*\bar{D}^*\to\pi\pi}|^2 d\Pi_2$$

Due to the exchange of the two identical particles, we multiplied 1/2.

After some algebra, in the center of mass frame, we get

$$\mathrm{Im}\mathcal{M}_{D^*\bar{D}^*} = \frac{1}{128\pi^2}\sqrt{\frac{t-4m_\pi^2}{t}}\int d\Omega\,|\mathcal{M}_{D^*\bar{D}^*\to\pi\pi}|^2$$

PARTIAL WAVE EXPANSION

- Connection formula

$$|JM\lambda_1\lambda_2\rangle = \sqrt{\frac{2J+1}{4\pi}} \int d\Omega\, D^{J*}_{M\lambda}(\phi,\theta,0))|\phi\,\theta\lambda_1\lambda_2\rangle$$

$$\begin{aligned}
\langle\phi\,\theta\lambda_1'\lambda_2'|JM\lambda_1\lambda_2\rangle &= \sqrt{\frac{2J+1}{4\pi}} \int d\Omega'\, D^{J*}_{M\lambda}(\phi',\theta',0)\langle\phi\,\theta\lambda_1'\lambda_2'|\phi'\,\theta'\lambda_1\lambda_2\rangle \\
&= \sqrt{\frac{2J+1}{4\pi}} \int d\Omega'\, D^{J*}_{M\lambda}(\phi',\theta',0)\delta(\cos\theta-\cos\theta')\delta(\phi-\phi')\delta_{\lambda_1\lambda_1'}\delta_{\lambda_2\lambda_2'} \\
&= \sqrt{\frac{2J+1}{4\pi}} D^{J*}_{M\lambda}(\phi,\theta,0)
\end{aligned}$$

PARTIAL WAVE EXPANSION

- Partial wave expansion

$$\mathcal{M}^{\lambda_1\lambda_2,\lambda_3\lambda_4}_{D^*\bar{D}^*} = \sum_J \frac{2J+1}{4\pi} D^{J*}_{\lambda\lambda'}(\phi_{p'},\theta_{p'},0)\mathcal{M}^{J,\lambda_1\lambda_2,\lambda_3\lambda_4}_{D^*\bar{D}^*}$$

$$\mathcal{M}^{\lambda_3\lambda_4}_{D^*\bar{D}^*\to\pi\pi} = \sum_{J'} \frac{2J'+1}{4\pi} D^{J'*}_{0\lambda'}(\bar{\phi},\bar{\theta},0)\mathcal{M}^{J',\lambda_3\lambda_4}_{D^*\bar{D}^*\to\pi\pi}$$

$$\mathcal{M}^{\lambda_1\lambda_2\dagger}_{D^*\bar{D}^*\to\pi\pi} = \sum_J \frac{2J+1}{4\pi} D^{J*}_{\lambda 0}(\phi_q,\theta_q,0)\mathcal{M}^{J,\lambda_1\lambda_2}_{D^*\bar{D}^*\to\pi\pi}$$

SPECTRAL FUNCTION

We can write equation again neglecting common factors

$$\mathrm{Im}\mathcal{M}^{J,\lambda_1\lambda_2,\lambda_3\lambda_4}_{D^*\bar{D}^*} = \frac{1}{128\pi^2}\sqrt{\frac{t-4m_\pi^2}{t}} \times \mathcal{M}^{J,\lambda_1\lambda_2}_{D^*\bar{D}^*\to\pi\pi}\mathcal{M}^{J,\lambda_3\lambda_4\dagger}_{D^*\bar{D}^*\to\pi\pi}$$

Note that J=0 implies that λ and λ' are 0 :

$$D^{J=0}_{\lambda\lambda'}\ ; \quad \lambda = \lambda_1 - \lambda_2 = 0 \text{ and } \lambda' = \lambda_3 - \lambda_4 = 0$$

SPECTRAL FUNCTION

Let the spectral function ρ(t) be unpolarized amplitude :

$$\rho(t) \equiv \mathrm{Im}\mathcal{M}^{J=0}_{D^*\bar{D}^*} = \frac{1}{128\pi^2}\sqrt{\frac{t-4m_\pi^2}{t}}\sum \mathcal{M}^{J=0\dagger}_{D^*\bar{D}^*\to\pi\pi}\mathcal{M}^{J=0}_{D^*\bar{D}^*\to\pi\pi}$$

To find out possible D*D* $\to\pi\pi$ amplitudes, we need to know the selection rules for transitions. It is rather helpful to use LSJ basis.

SPECTRAL FUNCTION

- Two-body unitarity in the LSJ basis

$$\mathrm{Im}\langle JML'S'|\mathcal{M}^J|JMLS\rangle = \frac{1}{128\pi^2}\sqrt{\frac{t-4m_\pi^2}{t}} \times \sum_{L_{\pi\pi}} \langle JML'S'|\mathcal{M}^J|JML_{\pi\pi}S_{\pi\pi}\rangle\langle JML_{\pi\pi}S_{\pi\pi}|\mathcal{M}^J|JMLS\rangle$$

I would like to define partial spectral function for convenience as follow

$$\rho(L'S';LS) \equiv \mathrm{Im}\langle JML'S'|\mathcal{M}^J|JMLS\rangle$$

SPECTRAL FUNCTION

From parity and G-parity conservation, we can construct the selection rules :

$$(-1)^L = \eta_\pi \eta_\pi (-1)^{L_{\pi\pi}} = (-1)^{L_{\pi\pi}}$$

$$(-1)^{J+I} = G_\pi G_\pi = 1$$

We are now dealing with the scalar and isoscalar channel.

	J^P	I^G	$L_{\pi\pi}$	$S_{\pi\pi}$	L	S
σ	0^-	0^+	0	0	0	0
			0	0	2	2

TABLE I. Allowed transitions of σ-channel

SPECTRAL FUNCTION

- Connections between LSJ and helicity descriptions

$$|0000\rangle_{LSJ} = \sum_{\lambda_1\lambda_2} (000\lambda|0\lambda)(1\lambda_1 1{-}\lambda_2|0\lambda)|00\lambda_1\lambda_2\rangle$$

$$= \frac{1}{\sqrt{3}}\left(|11\rangle - |00\rangle| - 1{-}1\rangle\right)$$

$$|0022\rangle_{LSJ} = \sum_{\lambda_1\lambda_2} (202\lambda|0\lambda)(1\lambda_1 1{-}\lambda_2|2\lambda)|00\lambda_1\lambda_2\rangle$$

$$= \frac{1}{\sqrt{6}}\left(|11\rangle + 2|00\rangle + | - 1{-}1\rangle\right)$$

SPECTRAL FUNCTION

It is convenient that use of this notation :

$$\langle L'S'|\mathcal{M}^J|LS\rangle_{\mathrm{LSJ}} \equiv \langle JML'S'|\mathcal{M}^J|JMLS\rangle$$

According to the selection rule, four LSJ amplitudes are possible :

$$\langle 00|\mathcal{M}^{J=0}|00\rangle_{LSJ},\ \langle 00|\mathcal{M}^{J=0}|22\rangle_{LSJ},\ \langle 22|\mathcal{M}^{J=0}|00\rangle_{LSJ},\ \langle 22|\mathcal{M}^{J=0}|22\rangle_{LSJ}.$$

SPECTRAL FUNCTION

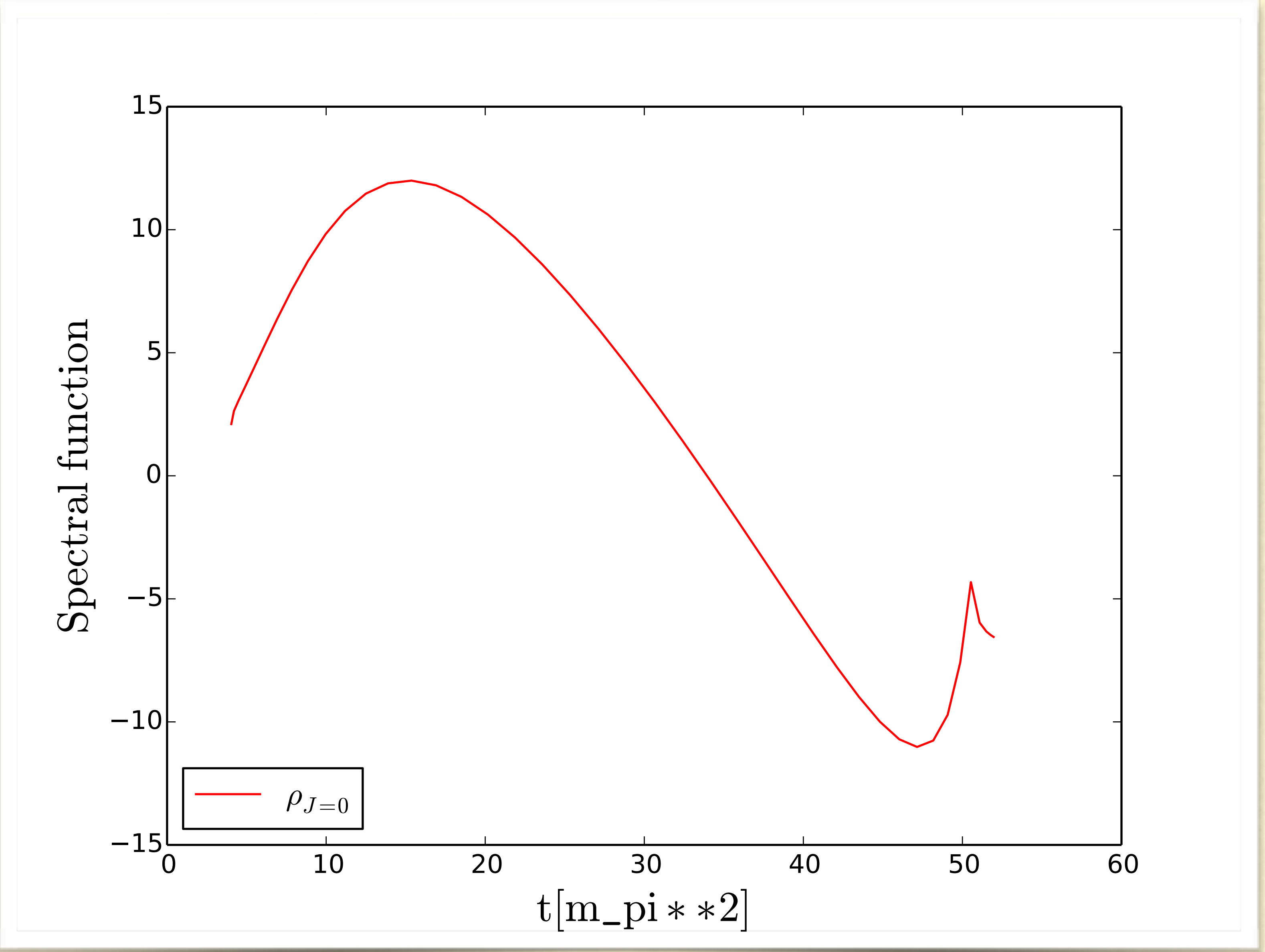
The partial LSJ amplitudes is expressed in terms of the helicity amplitudes.

$$\mathrm{Im}\langle 00|\mathcal{M}|00\rangle_{LSJ} = \frac{1}{128\pi^2}\sqrt{\frac{t-4m_\pi^2}{t}}\frac{1}{3} \times \left(\mathcal{M}^{00\dagger}_{D^*\bar{D}^*\to\pi\pi}\mathcal{M}^{00}_{D^*\bar{D}^*\to\pi\pi} - 4\mathcal{M}^{00\dagger}_{D^*\bar{D}^*\to\pi\pi}\mathcal{M}^{11}_{D^*\bar{D}^*\to\pi\pi} + 4\mathcal{M}^{11\dagger}_{D^*\bar{D}^*\to\pi\pi}\mathcal{M}^{11}_{D^*\bar{D}^*\to\pi\pi}\right)$$

$$\mathrm{Im}\langle 00|\mathcal{M}|22\rangle_{LSJ} = \frac{1}{128\pi^2}\sqrt{\frac{t-4m_\pi^2}{t}}\frac{1}{3\sqrt{2}}\left(-2\mathcal{M}^{00\dagger}_{D^*\bar{D}^*\to\pi\pi}\mathcal{M}^{00}_{D^*\bar{D}^*\to\pi\pi} + 2\mathcal{M}^{00\dagger}_{D^*\bar{D}^*\to\pi\pi}\mathcal{M}^{11}_{D^*\bar{D}^*\to\pi\pi} + 4\mathcal{M}^{11\dagger}_{D^*\bar{D}^*\to\pi\pi}\mathcal{M}^{11}_{D^*\bar{D}^*\to\pi\pi}\right)$$

$$\mathrm{Im}\langle 22|\mathcal{M}|22\rangle_{LSJ} = \frac{1}{128\pi^2}\sqrt{\frac{t-4m_\pi^2}{t}}\frac{2}{3}\left(\mathcal{M}^{00\dagger}_{D^*\bar{D}^*\to\pi\pi}\mathcal{M}^{00}_{D^*\bar{D}^*\to\pi\pi} + 2\mathcal{M}^{00\dagger}_{D^*\bar{D}^*\to\pi\pi}\mathcal{M}^{11}_{D^*\bar{D}^*\to\pi\pi} + \mathcal{M}^{11\dagger}_{D^*\bar{D}^*\to\pi\pi}\mathcal{M}^{11}_{D^*\bar{D}^*\to\pi\pi}\right)$$

SPECTRAL FUNCTION



SUMMARY

SUMMARY

- D meson is crucial for study exotic mesons
- I am calculating spectral function of D*D* amplitude via two-body unitarity and the Blankenbecler - Sugar equation
- I am computing the spectral function analytically
- I am going to compute D*D*σ coupling constant

THANK YOU VERY MUCH!