

PUSAN WORKSHOP

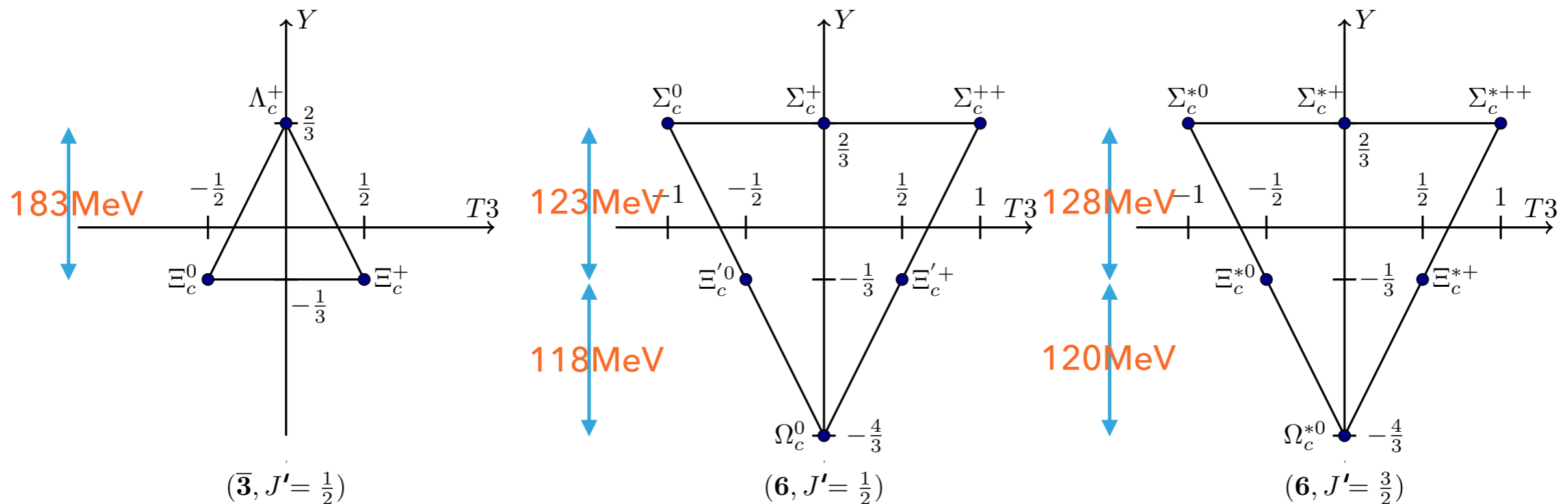
**Mass of singly and doubly heavy baryons
within the $SU(3)$ chiral quark-soliton model**

CONTENTS

- ▶ Motivation
- ▶ Chiral quark-soliton model
- ▶ Mass splitting of the singly heavy baryons
- ▶ Masse of the singly heavy baryons
- ▶ Masse of the doubly heavy baryons

MOTIVATION

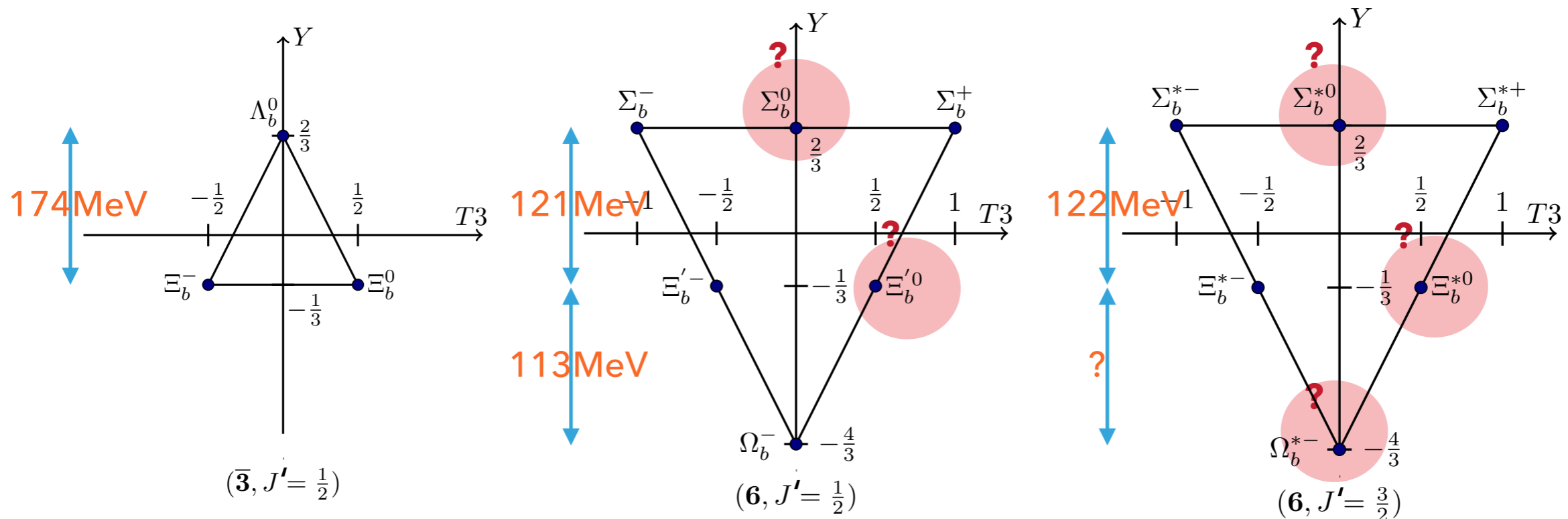
- ▶ The mass of light baryons are well known in the ground state and this model extremely well explain the mass splittings.
- ▶ We want to take account of the heavy baryon sector.



- ▶ Heavy quark spin is conserved. it indicates that the soliton spin is conserved. $m_Q \rightarrow \infty$
- ▶ The soliton and heavy quark is decoupled and the heavy quark play a role of the static color source.

MOTIVATION

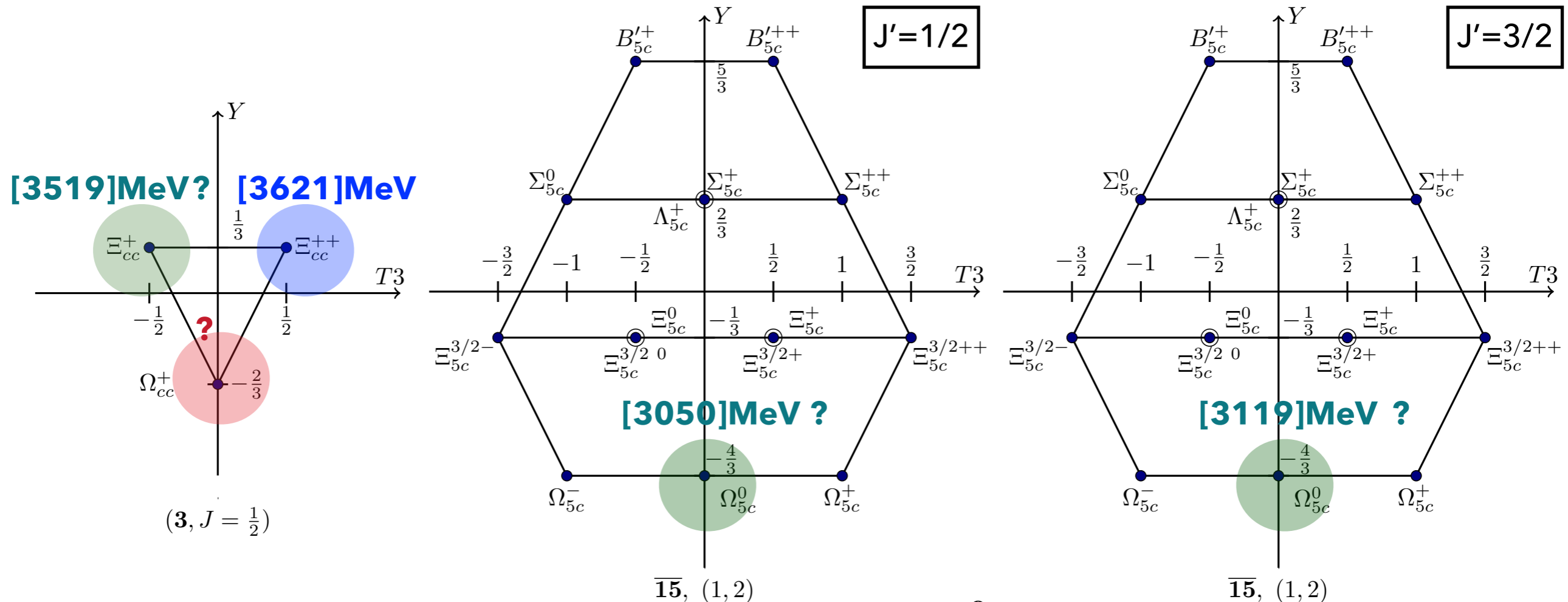
- ▶ The masses of charmed baryons are well known. Thus we will check the validity of the present approach.



- ▶ There are unknown baryon masses in the bottom baryon sector.
- ▶ We can predict the mass of the Ω_b^{*-} without isospin symmetry breaking.

MOTIVATION

- ▶ Mass of the Ω_{cc}^+ is unknown. Recently, mass of the Ξ_{cc}^+ and Ξ_{cc}^{++} are founded [1] therefore we can predict mass of Ω_{cc}^+ .



- ▶ Moreover, there are observed particles Ω_c^0 in LHCb. The particle is classified within chiral-quark soliton model by using model independent calculation[2].

[1] M. Mattson et al [SELEX Collaboration], Phys. Rev. Lett. **89**, 112001 (2002).

[2] H. C. Kim, M. V. Polyakov and M. Praszalowicz, Phys. Rev. **D96**, no. 1, 014009 (2017)

▶ Effective chiral action

$$S_{\text{eff}} = -N_c \text{TrLn}(i\gamma_\mu \partial^\mu + i\hat{m} + iMU\gamma_5)$$

$$U\gamma_5 = e^{i\pi^a \tau^a \gamma_5} \quad \hat{m} = \text{diag}(m_u, m_d, m_s)$$

▶ Classical soliton mass for light baryon

$$\underline{M_{cl} \approx 1300\text{MeV}}$$

▶ Collective Hamiltonian

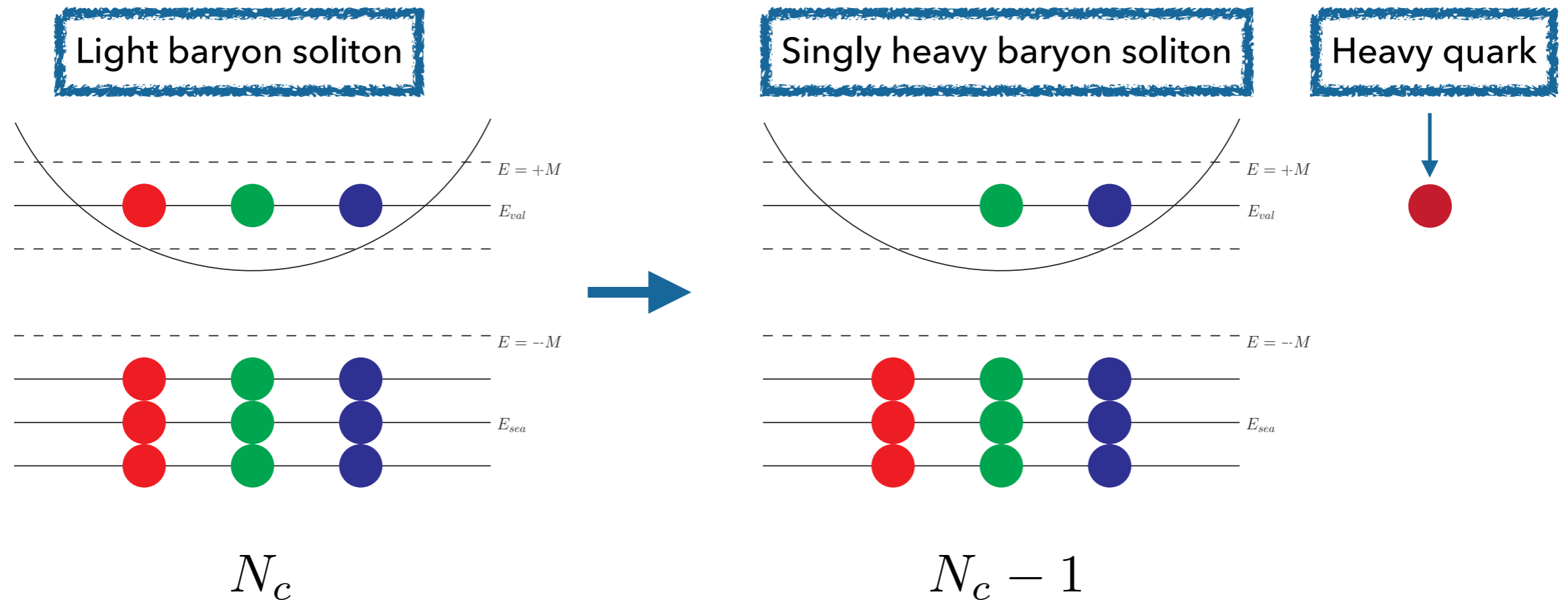
$$H = H_{\text{sym}} + H_{\text{sb}}^{(1)} + H_{\text{hf}},$$

$$H_{\text{sym}} = M_{cl} + \frac{1}{2I_1} \sum_{i=1}^3 \hat{J}_i^2 + \frac{1}{2I_2} \sum_{a=4}^7 \hat{J}_a^2,$$

$$H_{\text{hf}} = \frac{2}{3} \frac{\kappa}{m_Q M_{\text{sol}}} \vec{J} \cdot \vec{J}_Q,$$

$$H_{\text{sb}}^{(1)} = \alpha D_{88}^{(8)} + \beta \hat{Y} + \frac{\gamma}{\sqrt{3}} \sum_{i=1}^3 D_{8i}^{(8)} \hat{J}_i,$$

Change of the pion mean fields for Singly heavy baryon



- ▶ The mass of the heavy baryon soliton changed under the pion mean field change.

$$M_{sol} = (N_c - 1)E_{val} + E_{sea} \approx 1100\text{MeV}.$$

Change of the pion mean fields for Singly heavy baryon

- ▶ The moment of inertias and dynamical parameters are also changed under the pion mean fields change. One can take account of the I_1 .

$$I_1 = (N_c - 1)I_1^{\text{val}} + I_1^{\text{sea}}$$

Light baryon	This work	Ref [14]	Ref [32]	Singly heavy baryon	This work
\tilde{I}_1 [fm]	1.108	1.230 ± 0.002	1.06	I_1 [fm]	0.844
\tilde{I}_2 [fm]	0.529	0.420 ± 0.006	0.48	I_2 [fm]	0.404
\tilde{K}_1 [fm]	0.428	-	0.42	K_1 [fm]	0.286
\tilde{K}_2 [fm]	0.272	-	0.26	K_2 [fm]	0.181
\tilde{N}_0 [fm]	0.457	-	-	N_0 [fm]	0.499
\tilde{N}_1 [fm]	0.410	-	-	N_1 [fm]	0.380
\tilde{N}_2 [fm]	0.323	-	-	N_2 [fm]	0.286
$\tilde{\Sigma}_{\pi N}$ [MeV]	43.7	36.4 ± 3.9	41	$\Sigma_{\pi N}$ [MeV]	40.0
$\tilde{\alpha}$ [MeV]	-392.0	-262.9 ± 5.9	-197.0	α [MeV]	-326.3
$\tilde{\beta}$ [MeV]	-99.3	-144.3 ± 3.2	-93.7	β [MeV]	-77.8
$\tilde{\gamma}$ [MeV]	-49.4	-104.2 ± 2.4	-52.6	γ [MeV]	-37.8
\tilde{M}_{sol} [MeV]	1296.1	-	-	M_{sol} [MeV]	1099.9

$$m_s = 180 \text{ MeV}$$

[14] Gh. S. Yang, H.-Ch. Kim, M. V. Polyakov and M. Praszalowicz, Phys. Rev. **D94**, 071502 (2016)

[32] T. Ledwig, A. Silva and H.-Ch. Kim, Phys. Rev. **D82**, 034022 (2010)

ROTATIONAL ENERGY

$$\overline{M}_{\mathcal{R},Q} = M_{\text{cl}} + \frac{1}{2I_1} J(J+1) + \frac{1}{2I_2} [C_2(p, q) - J(J+1)] - \frac{3}{8I_2} \overline{Y}^2.$$

$$\begin{cases} M_{\text{cl}} = M_{\text{sol}} & \text{Light baryon} \\ M_{\text{cl}} = M_{\text{sol}} + m_Q & \text{Singly heavy baryon} \end{cases}$$

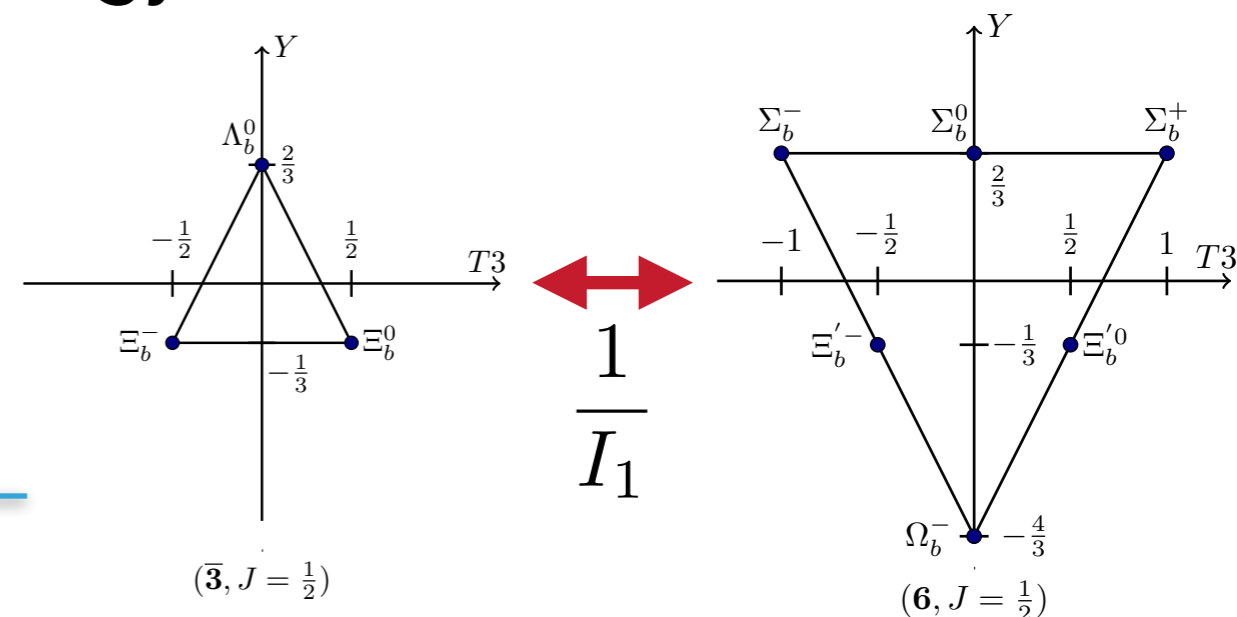
$$\begin{cases} \overline{Y} = N_c/3 & \text{Light baryon} \\ \overline{Y} = (N_c - 1)/3 & \text{Singly heavy baryon} \end{cases}$$

$\overline{\mathbf{3}} \rightarrow (p, q) = (0, 1) \rightarrow J = 0$ antitriplet

$\mathbf{6} \rightarrow (p, q) = (2, 0) \rightarrow J = 1$ sextet.

- ▶ The rotational energy yields the energy difference between the $\overline{\mathbf{3}}$ and the $\mathbf{6}$.

$$\overline{M}_{\mathbf{6},c} - \overline{M}_{\overline{\mathbf{3}},c} = \frac{1}{I_1}.$$



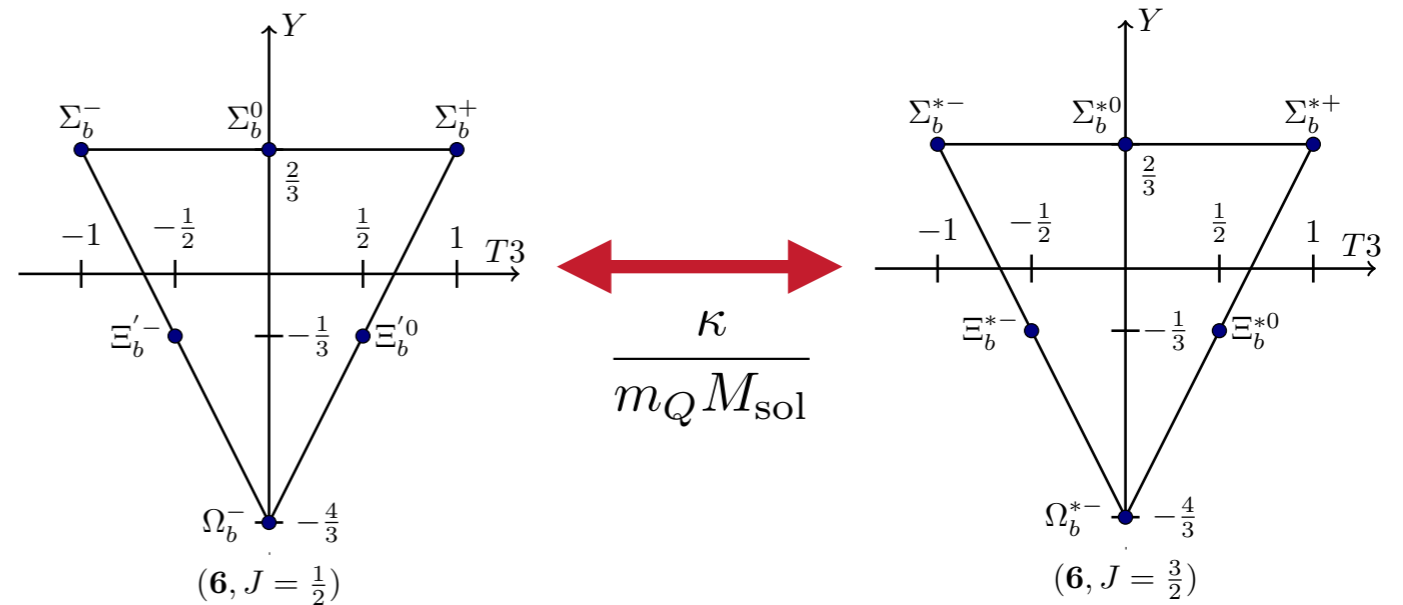
HYPERFINE MASS SPLITTING

- ▶ The mass difference between baryon spin 1/2 and 3/2 arises from the spin-spin interaction.

$$H_{\text{hf}} = \frac{2}{3} \frac{\kappa}{m_Q M_{\text{sol}}} \vec{J} \cdot \vec{J}_Q,$$

$$M_{\mathbf{6}_{1/2}, Q}^{(\text{hf})} = -\frac{2\kappa}{3m_Q M_{\text{sol}}},$$

$$M_{\mathbf{6}_{3/2}, Q}^{(\text{hf})} = \frac{\kappa}{3m_Q M_{\text{sol}}},$$



- ▶ The hyperfine mass splitting is determined by using experimental data[1]

$$\frac{\kappa}{m_c M_{\text{sol}}} = 68.1 \pm 1.1 \text{ MeV}$$

$$\frac{\kappa}{m_b M_{\text{sol}}} = 20.3 \pm 1.0 \text{ MeV}$$

SU(3) SYMMETRY BREAKING

$$H_{\text{sb}}^{(1)} = +\alpha D_{88}^{(8)} + \beta \hat{Y} + \frac{\gamma}{\sqrt{3}} \sum_{i=1}^3 D_{8i}^{(8)} \hat{J}_i,$$

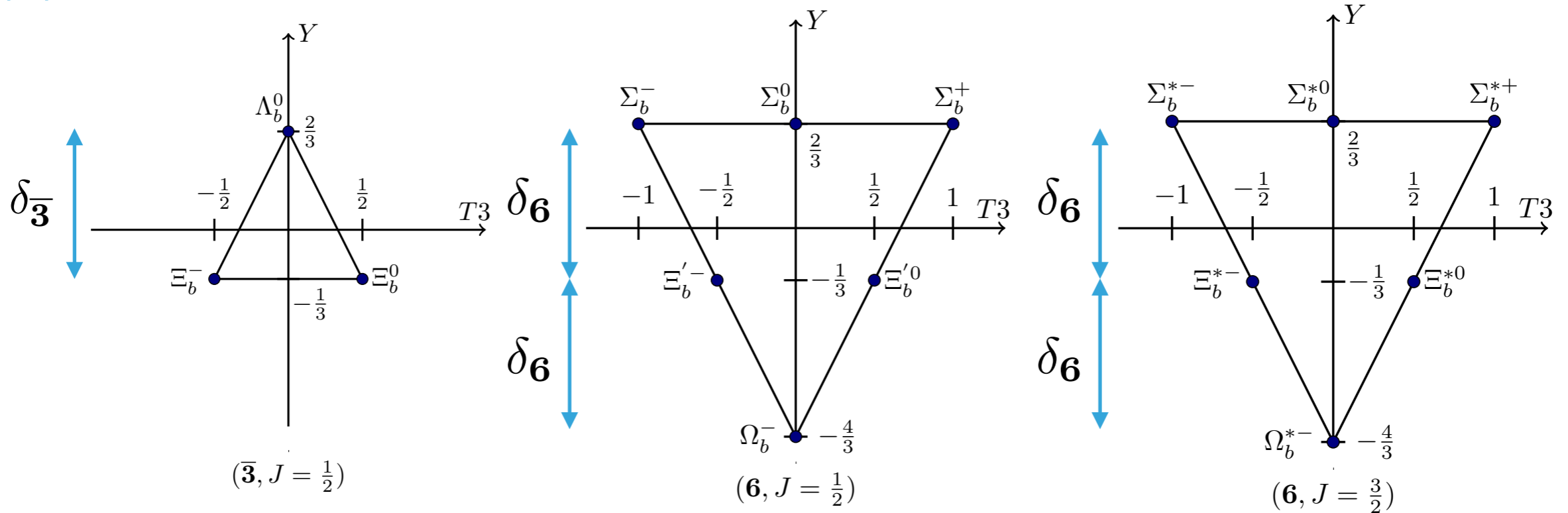
$$\alpha = \left(-\frac{\Sigma_{\pi N}}{3m_0} + \frac{K_2 \bar{Y}}{I_2} \right) m_s, \quad \beta = -\frac{K_2}{I_2} m_s, \quad \gamma = 2 \left(\frac{K_1}{I_1} - \frac{K_2}{I_2} \right) m_s.$$

- ▶ Mass correction in the perturbation theory

$$M_{B,\mathcal{R}}^{(1)} = \langle B, \mathcal{R} | H_{\text{sb}}^{(1)} | B, \mathcal{R} \rangle = Y \delta_{\mathcal{R}}$$

$$\delta_{\bar{\mathbf{3}}} = \frac{3}{8} \alpha + \beta, \quad \delta_{\mathbf{6}} = \frac{3}{20} \alpha + \beta - \frac{3}{10} \gamma.$$

SU(3) SYMMETRY BREAKING



model calculation

$$\delta_{\bar{\mathbf{3}}} = -203.6\text{MeV}, \quad \delta_{\mathbf{6}} = -117.4\text{MeV},$$

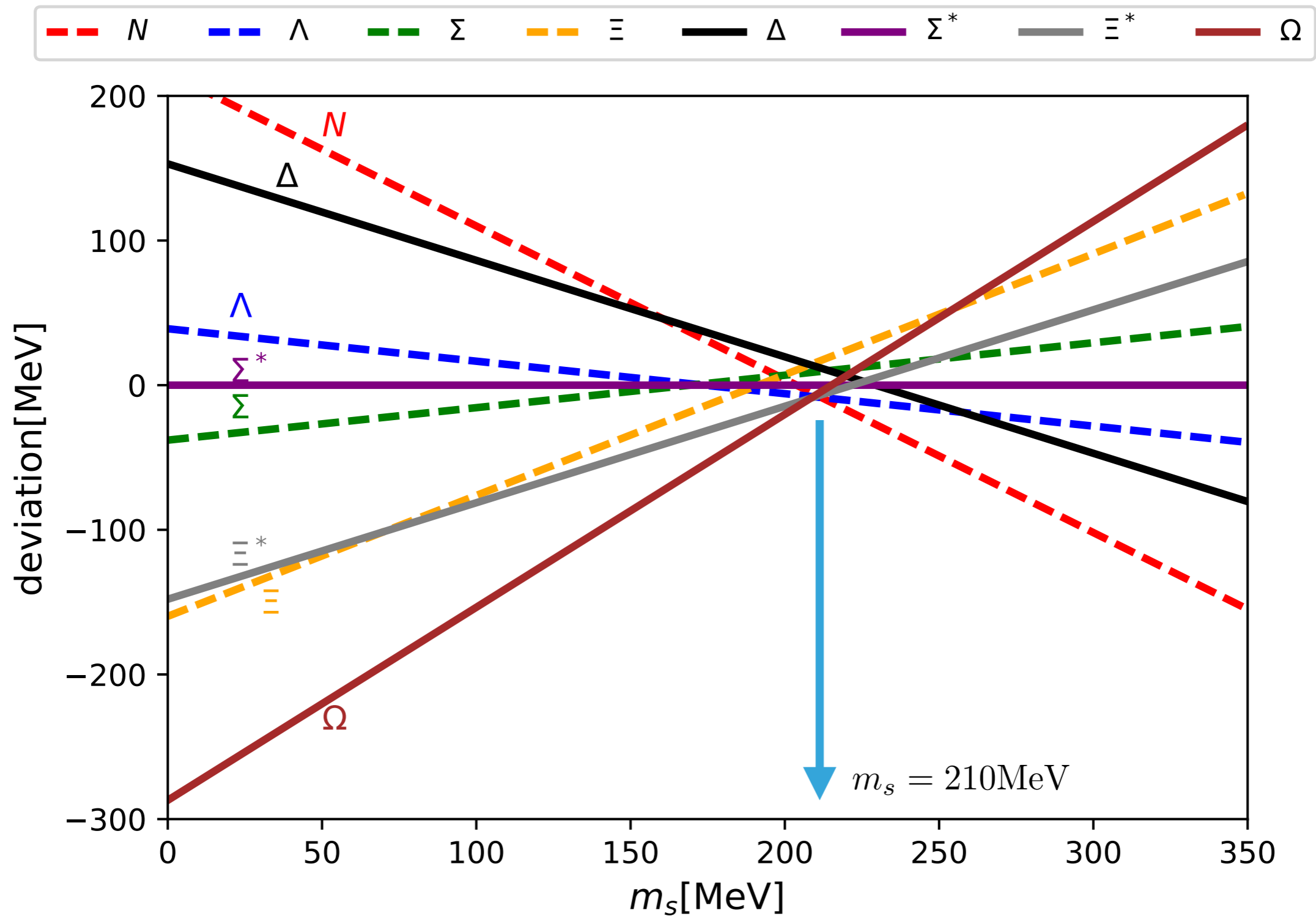
model independent value

$$\delta_{\bar{\mathbf{3}}}^{\text{exp}} = -178.5\text{MeV}, \quad \delta_{\mathbf{6}}^{\text{exp}} = -120.7\text{MeV},$$

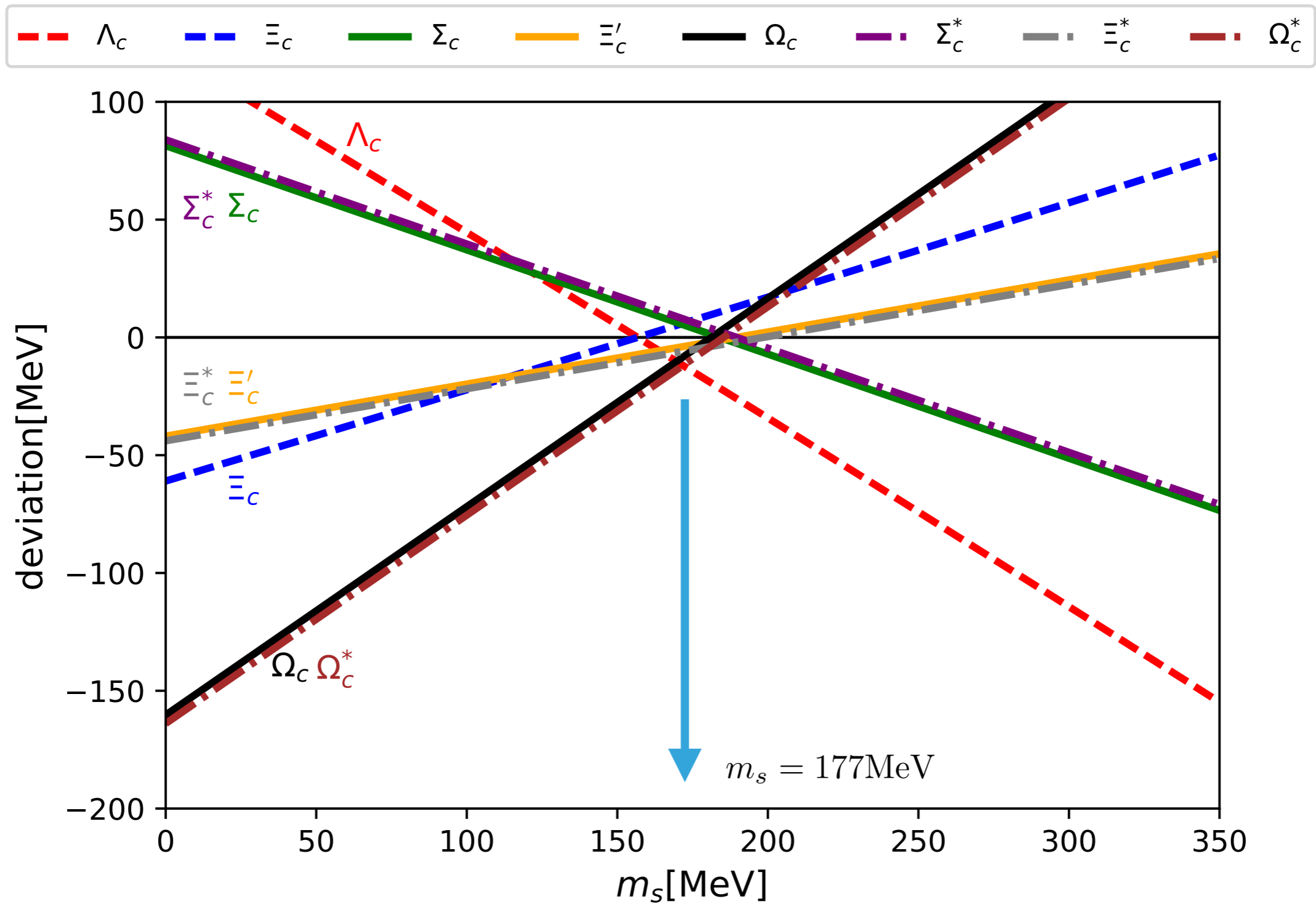
$$M_B^{(2)(\text{wf})} = \sum_{\mathcal{R} \neq \mathcal{R}'} \frac{|\langle B, \mathcal{R}' | H_{\text{sb}}^{(1)} | B, \mathcal{R} \rangle|^2}{\bar{M}_{\mathcal{R}, Q} - \bar{M}_{\mathcal{R}', Q}}.$$

$$M_B^{(2)(\text{op})} = \langle B, \mathcal{R} | H_{\text{sb}}^{(2)} | B, \mathcal{R} \rangle.$$

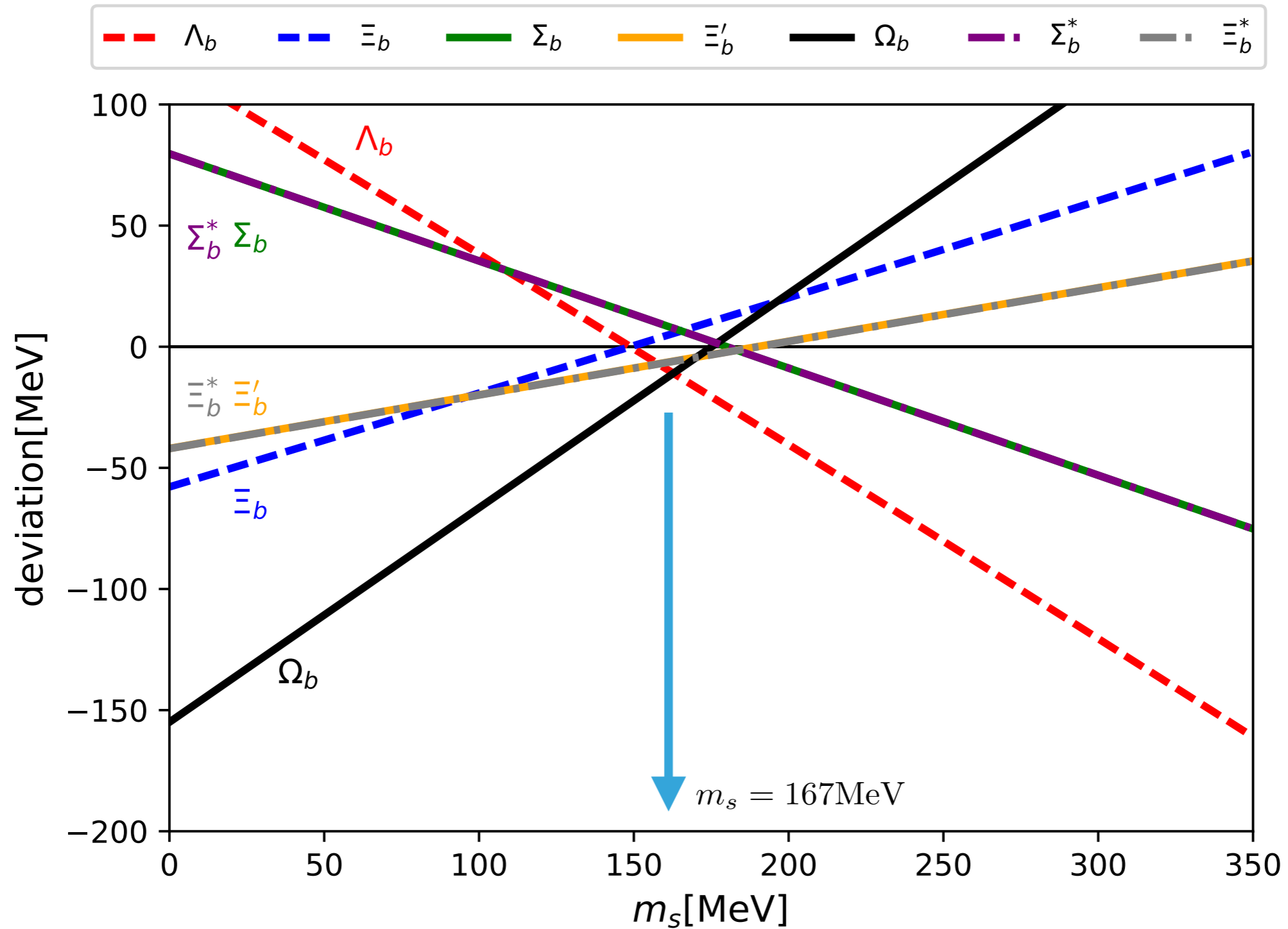
LIGHT BARYON



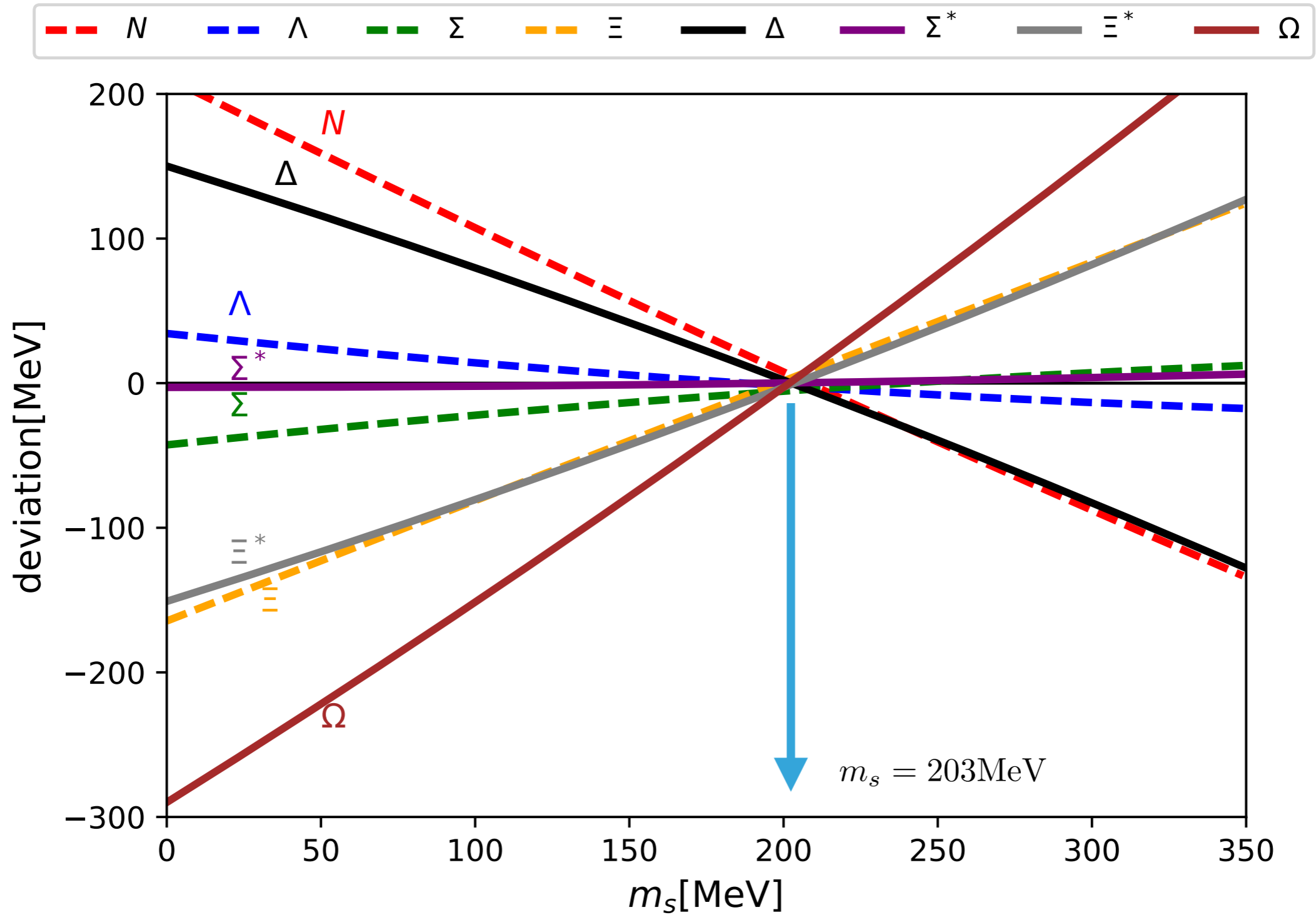
CHARMED BARYON



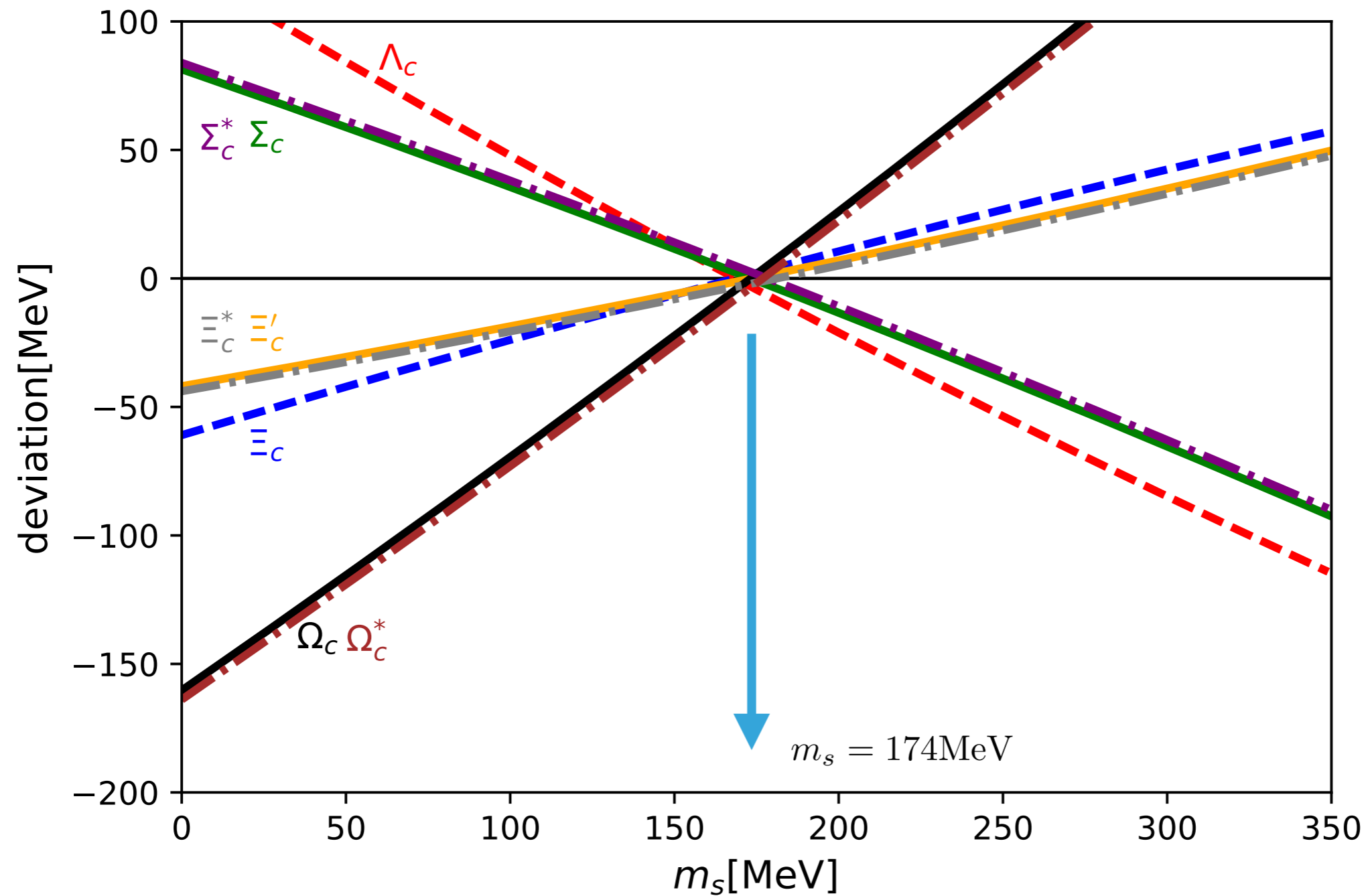
BOTTOM BARYON



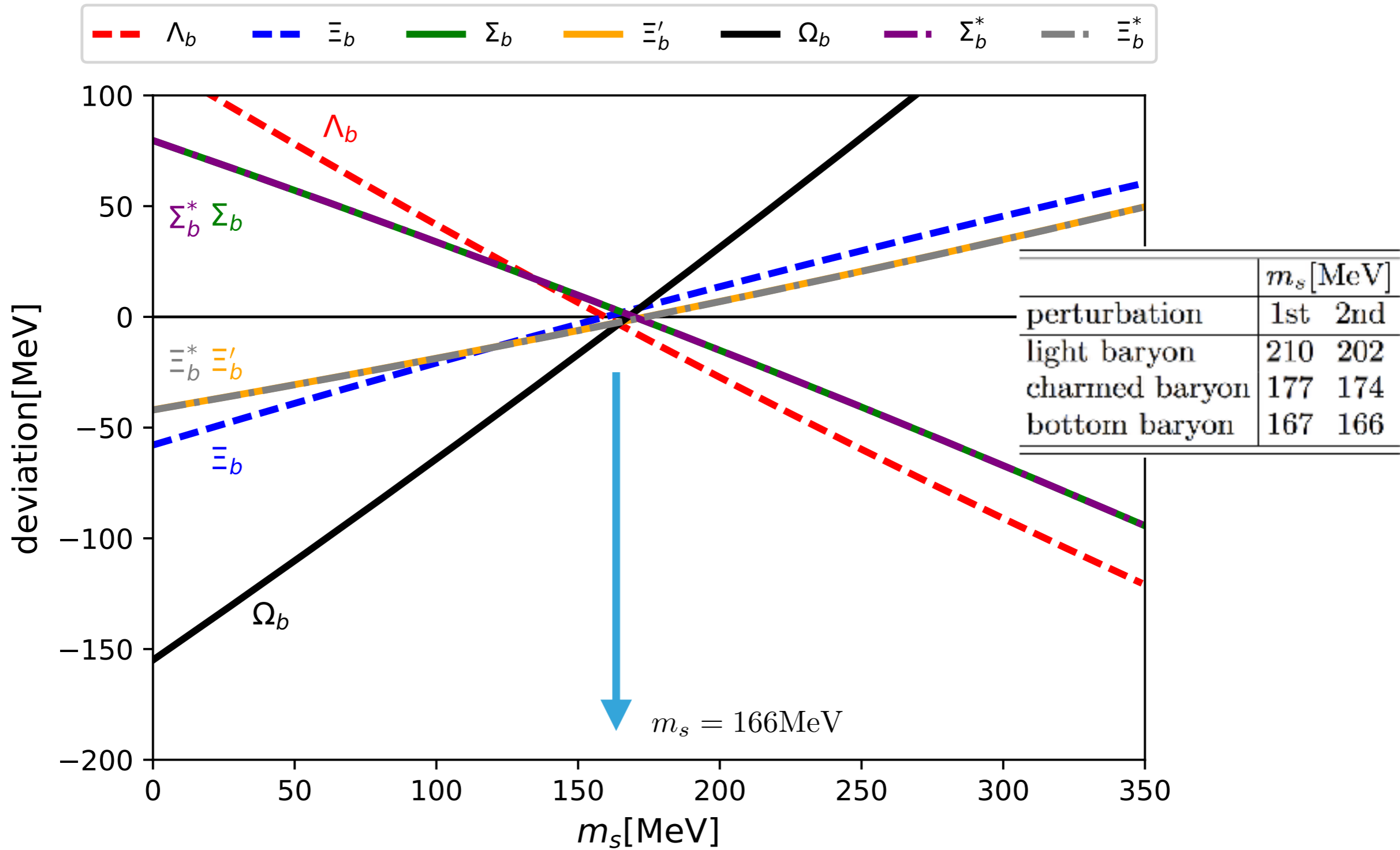
LIGHT BARYON



CHARMED BARYON



BOTTOM BARYON



CHARMED BARYON

- ▶ The heavy baryon masses can be calculated by using the determined strange current quark mass.

	m_s [MeV]	
perturbation	1st	2nd
light baryon	210	202
charmed baryon	177	174
bottom baryon	167	166

				Perturbative	
		Experiment [MeV]	ref[14] [MeV]	1st order	2nd order
\mathcal{R}_J^Q	B_c			$m_s = 177$ [MeV]	$m_s = 174$ [MeV]
$\bar{\mathbf{3}}_{1/2}^c$	Λ_c	2286.5 ± 0.1	2272.5 ± 2.3	2272.7	2282.8
$\bar{\mathbf{3}}_{1/2}^c$	Ξ_c	2469.4 ± 0.3	2476.3 ± 1.2	2476.3	2471.2
$\mathbf{6}_{1/2}^c$	Σ_c	2453.5 ± 0.1	2445.3 ± 2.5	2456.1	2453.2
$\mathbf{6}_{1/2}^c$	Ξ'_c	2576.8 ± 2.1	2580.5 ± 1.6	2573.6	2576.9
$\mathbf{6}_{1/2}^c$	Ω_c	2695.2 ± 1.7	2715.7 ± 4.5	2691.0	2695.9
$\mathbf{6}_{3/2}^c$	Σ_c^*	2518.1 ± 0.8	2513.4 ± 2.3	2525.0	2520.3
$\mathbf{6}_{3/2}^c$	Ξ_c^*	2645.9 ± 0.4	2648.6 ± 1.3	2642.4	2644.0
$\mathbf{6}_{3/2}^c$	Ω_c^*	2765.9 ± 2.0	2783.8 ± 4.5	2759.9	2763.1

BOTTOM BARYON

- Especially, the mass of Ω_b^* can be predicted in the chiral quark-soliton model.

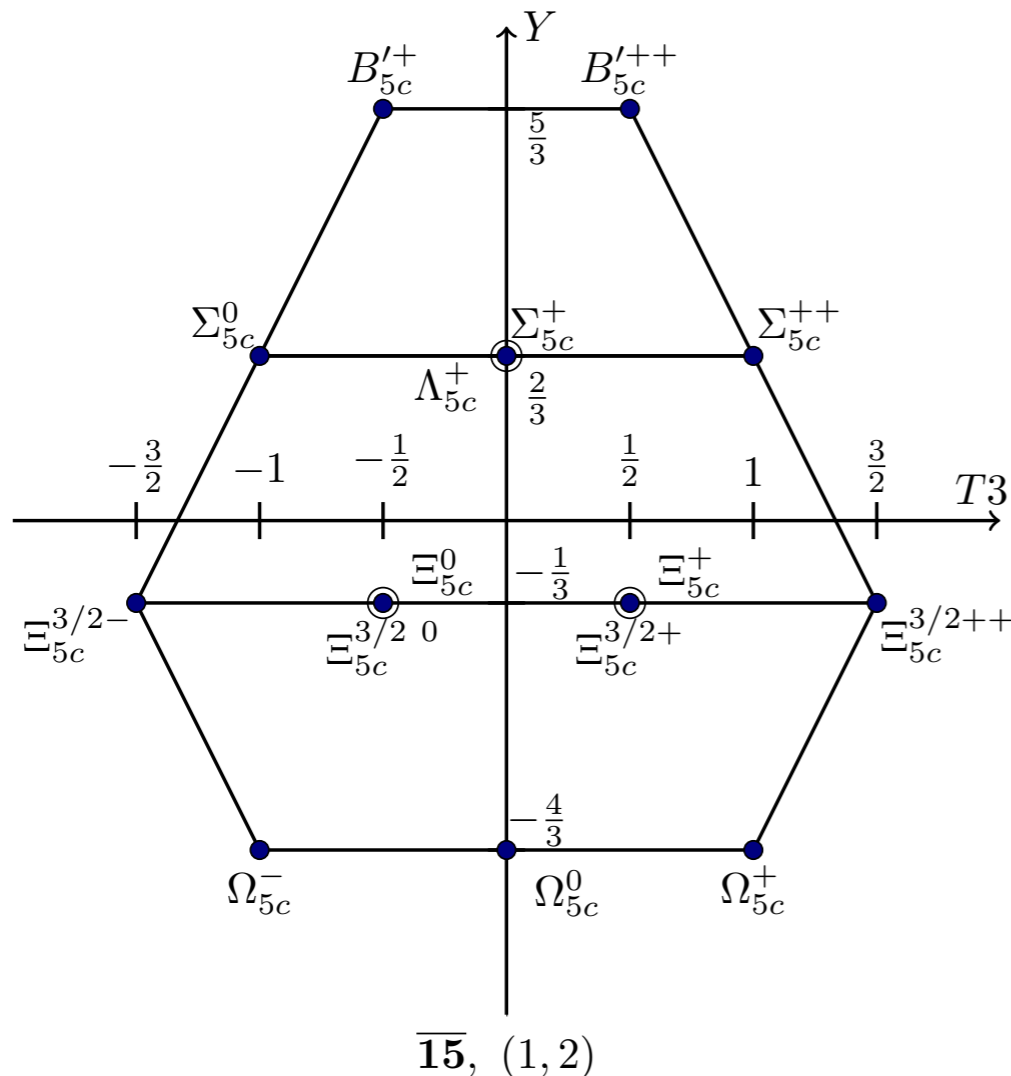
	m_s [MeV]	
perturbation	1st	2nd
light baryon	210	202
charmed baryon	177	174
bottom baryon	167	166

				Perturbative	
				1st order	2nd order
\mathcal{R}_J^Q	B_c	Experiment [MeV]	ref[14] [MeV]	$m_s = 167$ [MeV]	$m_s = 166$ [MeV]
$\bar{\mathbf{3}}_{1/2}^b$	Λ_b	5619.5 ± 0.2	5599.3 ± 2.4	5607.1	5615.1
$\bar{\mathbf{3}}_{1/2}^b$	Ξ_b	5793.1 ± 0.7	5803.1 ± 1.2	5799.3	5795.3
$\mathbf{6}_{1/2}^b$	Σ_b	5813.4 ± 1.3	5804.3 ± 2.4	5821.0	5815.3
$\mathbf{6}_{1/2}^b$	Ξ'_b	5935.0 ± 0.05	5939.5 ± 1.5	5931.8	5933.0
$\mathbf{6}_{1/2}^b$	Ω_b	6048.0 ± 1.9	6074.7 ± 4.5	6042.6	6046.4
$\mathbf{6}_{3/2}^b$	Σ_b^*	5833.6 ± 1.3	5824.6 ± 2.3	5840.9	5835.5
$\mathbf{6}_{3/2}^b$	Ξ_b^*	5955.3 ± 0.1	5959.8 ± 1.2	5951.7	5953.2
$\mathbf{6}_{3/2}^b$	Ω_b^*	-	6095.0 ± 4.4	6062.5	6066.6

$$M_{\Omega_b^*} = M_{\Omega_b} + \frac{\kappa}{m_Q m_{\text{sol}}} = 6068.0 \pm 2.1 \text{ MeV}$$

HEAVY PENTAQUARK

- ▶ The $\Omega_c^0[3050]$ and the $\Omega_c^0[3119]$ mass difference is 69MeV that would be hyperfine mass splitting for charmed baryon, which is the scenario of our model.



	$M_{B_{5c}}^{(1)}$
$\langle B'_{5c} H_{coll} B'_{5c} \rangle$	$\frac{1}{8}\alpha + \frac{5}{3}\beta - \frac{1}{4}\gamma$
$\langle \Sigma_{5c} H_{coll} \Sigma_{5c} \rangle$	$\frac{1}{12}\alpha + \frac{2}{3}\beta - \frac{1}{6}\gamma$
$\langle \Xi_{5c}^{3/2} H_{coll} \Xi_{5c}^{3/2} \rangle$	$\frac{1}{24}\alpha - \frac{1}{3}\beta - \frac{1}{12}\gamma$
$\langle \Omega_{5c} H_{coll} \Omega_{5c} \rangle$	$-\frac{1}{6}\alpha - \frac{4}{3}\beta + \frac{1}{3}\gamma$
$\langle \Lambda_{5c} H_{coll} \Lambda_{5c} \rangle$	$\frac{2}{3}\beta$
$\langle \Xi_{5c} H_{coll} \Xi_{5c} \rangle$	$-\frac{1}{12}\alpha - \frac{1}{3}\beta + \frac{1}{6}\gamma$

$$\frac{\kappa}{m_c M_{sol}} = 68.1 \pm 1.1 \text{MeV}$$

HEAVY PENTAQUARK

- ▶ One can calculate the whole heavy pentaquark masses by using masses of the Ω_c^0 particles.

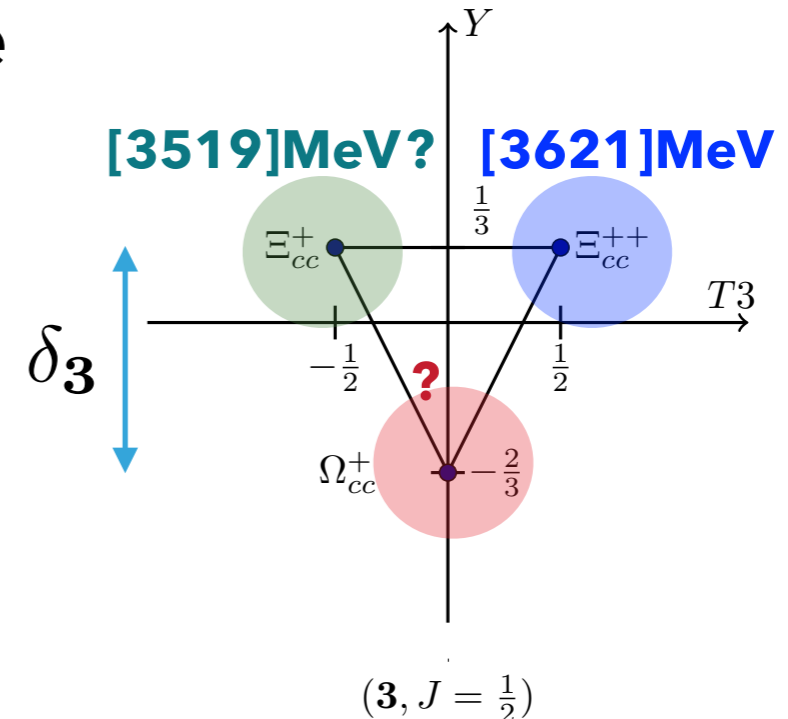
	Perturbative	
	1st order	2nd order
m_s	177	174
$M_{B_{5c}}$	2738.1	2751.7
$M_{\Sigma_{5c}}$	2827.9	2861.0
$M_{\Lambda_{5c}}$	2849.1	2859.9
$M_{\Xi_{5c}^{3/2}}$	2917.7	2923.3
$M_{\Xi_{5c}}$	2949.6	2964.3
M_{Ω_c}	[Input]	
$M_{B_{5c}}$	2808.1	2820.7
$M_{\Sigma_{5c}}$	2897.9	2930.0
$M_{\Lambda_{5c}}$	2919.1	2928.9
$M_{\Xi_{5c}^{3/2}}$	2987.7	2992.3
$M_{\Xi_{5c}}$	3019.6	3033.7
M_{Ω_c}	[Input]	

DOUBLY CHARMED BARYON

- ▶ Doubly charmed baryon consist of the two heavy quarks and $N_c - 2$ light quark.
- ▶ One can get the masses of the Ω_{cc}^+ by using masses of the Ξ_{cc}^+ and Ξ_{cc}^{++}

$$M_{B_{Q_1 Q_2}}^{(1)} = Y \delta_{\mathcal{R}}.$$

$$\delta_{\mathbf{3}} = \frac{3}{16} \alpha' + \beta' - \frac{9}{32} \gamma'.$$



DOUBLY CHARMED BARYON

- ▶ There is dependence of the strange current quark mass as a result of the SU(3) symmetry breaking
- ▶ The strange current quark mass cannot be determined for now but one can predict the mass of the Ω_{cc}^+ as a function of the quark mass.

	m_s [MeV]	$M_{\Omega_{cc}}$ [MeV]	
		1st order	2nd order
$M_{\Xi_{cc}}^{\text{exp}} = 3519.0 \text{ MeV [43]}$	150	3613.3	3614.5
	160	3619.6	3621.0
	170	3625.8	3627.4
	180	3632.1	3633.9
$M_{\Xi_{cc}}^{\text{exp}} = 3621.4 \text{ MeV [42]}$	150	3715.8	3717.0
	160	3722.1	3723.5
	170	3728.3	3729.9
	180	3734.6	3736.4

- ▶ Pion mean field play a important role in the heavy baryon sector.
- ▶ Unfortunately, mass difference of the representations undershoots by 25%.
- ▶ If the strange current quark mass determined in the doubly heavy baryon, one can predict the exact heavy baryon masses.
- ▶ Decay width of the heavy baryon can be calculated in the chiral quark-soliton model as well.

THANK YOU VERY MUCH!