Exclusive production of Υ production in proton-proton collisions

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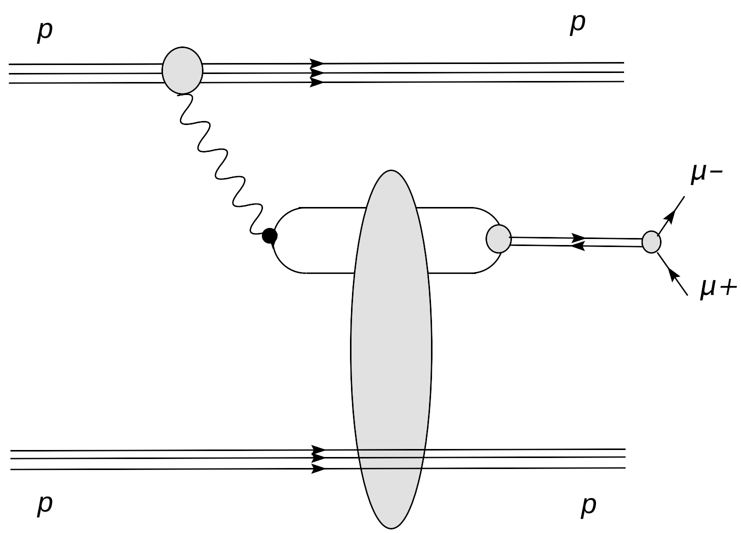
Outline

- Exclusive production of Υ production in $pp-$ collisions
  photoproduction cross section for $\gamma p \to \Upsilon p$ subprocess
- QCD factorization approach at NLO
  [L. Szymanowsi, G. Krasnikov, D.I. '04]
  suffers from huge NLO corrections at $x \to 0$
- High energy resummations
  - The method, analitical results
  - Numerics
- Conclusions

Exclusive Υ production in pp- collisions



Experiment: will be measured at LHC and Tevatron

Theory: main ingredients

- **Main production mechanism – photoproduction subprocess. Possible Odderon contribution (Leszek Motyka)**
- **gap survival, interesting physics**

HERA data: $\sigma^{\gamma p \to \Upsilon(1S) p}(W)$, 100 **GeV** $< W < 180$ **GeV**

Many theoretical calculations (dipole model, $k_\perp$ factorization):
Klein, Nystrand; Motyka, Watt; Rybarska, Schäfer, Szczurek; Goncalves, Machado; Forshow, Shaw ...
– my apologies to those not mentioned!

Personal opinion: Theoretical uncertainties of these methods remain to be poorly understood

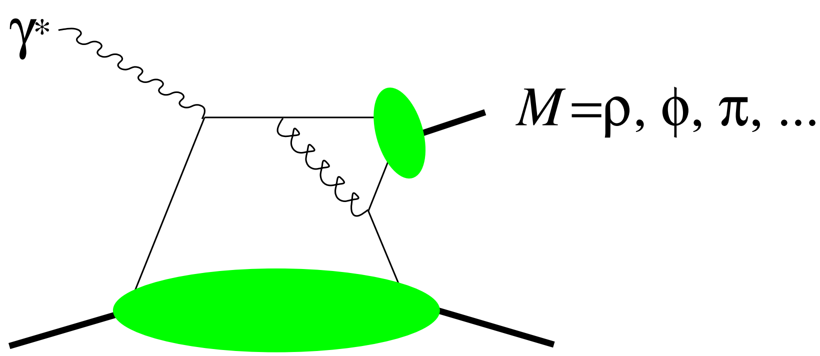
Our approach – Leading twist collinear factorization + NRQCD

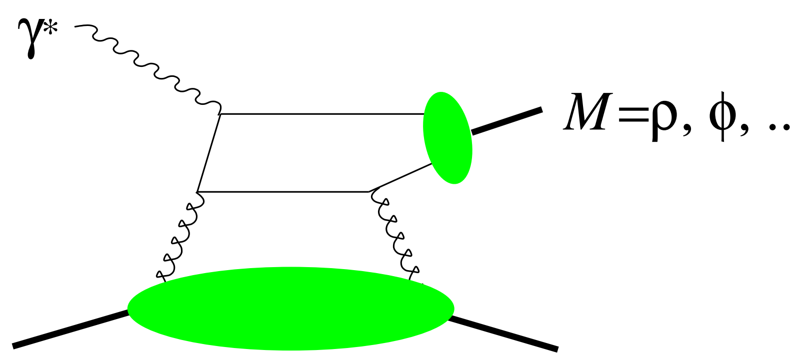
Υ photoproduction. Heavy quark mass provides a hard scale.
Kinematic scale: $Q^2 = M_\Upsilon^2 \sim 100$ **GeV**^{2},
for decay in the central ATLAS and CMS detectors, one could probe
$x = M_\Upsilon^2 / W^2$ down to $x \sim 10^{-4}$

We believe that DGLAP works in this kinematical range for F_2, F_L.

Why should not try it for exclusive Υ production?

Factorization theorem for exclusive processes **Collins, Frankfurt, Strikman '96**





$$A = \sum_{i=q,g} \int \int \; dx\; dz\; H^i(x,\xi,t;\mu_F)\; C^i(x,z,\mu_F)\; \phi(z;\mu_F)$$

$$\xi = x_B/(2-x_B)\;, \quad x_B = \frac{Q^2}{W^2+Q^2}$$

- **light meson production:** $\gamma^* + p \longrightarrow (\rho, \phi, \pi, 2\pi) + p$
- **valid in Bjerken limit:** $Q^2 \rightarrow \infty$ **and** x_B **fixed**
- **bad news : at small** x_B **an approach is unstable for** ρ^0, ϕ

- **M. Ryskin (1993) – LO prediction:** $M \sim x\, g(x, Q^2)$**,** $\sigma \sim (x\, g(x, Q^2))^2$
  later $x\, g(x, Q^2) \to$ **GPD** $H_g(x, \xi)$

- **No all order factorisation theorem for quarkonium photoproduction.**

- **Explicit NLO calculation:[Krasnikov, Schäfer, Szymanowski, D.I. '04]**
  – collinear factorization is consistent with the use of the nonrelativistic approximation to the NLO accuracy.

We believe it can be systematically extended to higher orders using the NRQCD expansion:

$$\mathcal{M} \quad = \quad \frac{4\pi\sqrt{4\pi\alpha}\, e_q (e_V^* e_\gamma)}{N_c\, \xi} \left(\frac{\langle O_1 \rangle_V}{m^3} \right)^{1/2} \int\limits_{-1}^{1} dx \left[\, T_g(x,\xi)\, F^g(x,\xi,t) + T_q(x,\xi) F^{q,S}(x,\xi,t) \right] ,$$

$$F^{q,S}(x,\xi,t) \; = \sum_{q=u,d,s} F^q(x,\xi,t) \, .$$

Here $F^{g(q)}(x,\xi,t;\mu_F^2)$ **– the gluon (quark) GPDs,**
$\xi = M^2/(2W^2 - M^2)$ **is the skewedness paramerer.**

NRQCD – all information about the quarkonium structure is encoded in the NRQCD matrix element $\langle O_1 \rangle_V$ **which enters the leptonic decay rate**

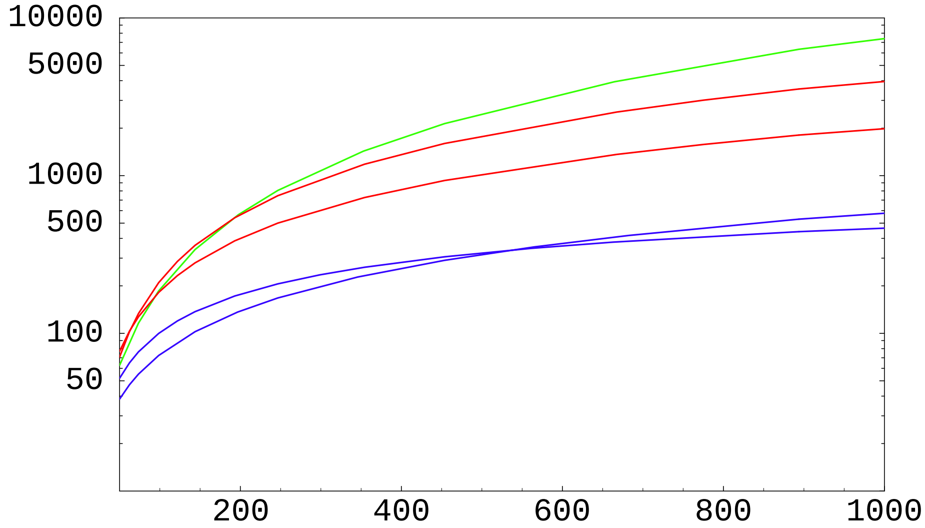
$$\Gamma[V \to l^+ l^-] = \frac{2 e_q^2 \pi \alpha^2}{3} \frac{\langle O_1 \rangle_V}{m^2} \left(1 - \frac{8\alpha_S}{3\pi} \right)^2 .$$

NLO corrections are large

$\sigma(W)$, **in** pb, **NLO vs LO**

$Q^2/4 \leq \mu_F^2 \leq Q^2$

- LO
- NLO
- **par. of HERA data(L. Motyka)**
  $\sigma(W) = 0.12$ **pb** $\times \left(\frac{W}{\textbf{GeV}}\right)^{1.6}$



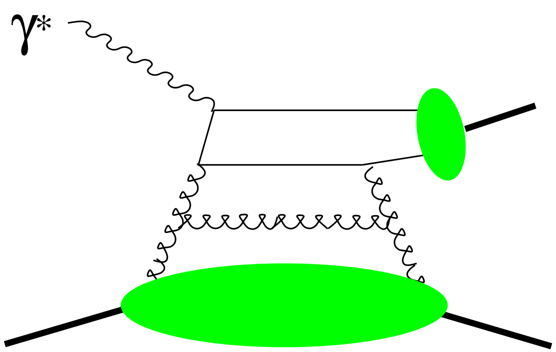
High-energy resummation

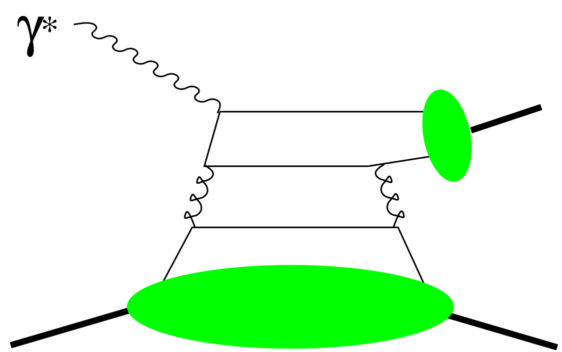
- **Why NLO corrections are large at small x_B?**
  large contribution comes from $\xi \ll x \ll 1$ ($Q = M$)

$$Im A^g \sim H^g(\xi,\xi) + \frac{3\alpha_s}{\pi}\left[\log\frac{Q^2}{\mu_F^2} - \log 4\right]\int_\xi^1 \frac{dx}{x} H^g(x,\xi)$$

$H^g(x,\xi) \sim xg(x) \sim const$, **therefore** $\int dx/x H^g(x,\xi) \sim \log(1/\xi) H^g(\xi,\xi)$

$x \sim \frac{s_{\gamma^* g}}{Q^2}$, **Born contribution for BFKL:**





At higher orders powers of energy log are generated

$$\mathcal{I}m A^g \sim H^g(\xi,\xi) + \int\limits_{\xi}^{1} \frac{dx}{x} H^g(x,\xi) \sum_{n=1} C_n(L) \frac{\bar{\alpha}_s^n}{(n-1)!} \log^{n-1}\frac{x}{\xi}$$

$C_n(L)$ - **polynomials of** $L = \log \frac{Q^2}{\mu_F^2}$**, maximum power is** L^n

- **Can one calculate $C_n(L)$ using BFKL equation?**
- **Consistently with collinear factorization, in terms of corrections to coeff. functions and anomalous dimensions; say, in $\overline{MS}$ scheme**
- **for DIS a technique suggested by Catani, Ciafaloni and Hautmann; [Catani, Hautmann '94]**
- **May the method used in DIS be generalized to exclusive, nonforward processes?**

In Mellin moment space high energy terms generate poles

$$\frac{1}{x}\frac{\bar{\alpha}_s^n}{(n-1)!}\log^{n-1}\frac{x}{\xi} \rightarrow \left(\frac{\bar{\alpha}_s}{N}\right)^n$$

For high energy resummation we need to the study $N \rightarrow 0$ limit.

Like in DIS:

$$\mathcal{I}m A(\xi,t) = \frac{1}{\xi}\int\limits_{\xi}^{1} dx \left[D^{(+)}\left(\frac{\xi}{x}\right)\mathcal{H}^{(+)}(x,\xi,t) + \frac{1}{\xi} D^{g}\left(\frac{\xi}{x}\right)\mathcal{H}^{g}(x,\xi,t)\right]$$

Mellin moments not factorize:

$$D_N(t) = \int\limits_0^1 d\xi\, \xi^N \mathcal{I}m A(\xi,t) =$$

$$\int\limits_0^1 \int\limits_0^1 du\, dx\, u^{N-1} x^N \left[D^{(+)}(u)\, \mathcal{H}^{(+)}(x,u\,x,t) + \frac{1}{u\,x} D^{g}(u)\, \mathcal{H}^{g}(x,u\,x,t)\right]$$

Polynomiality property:

$$\int_0^1 dx\, x^n \mathcal{H}^g(x,\eta,t) = \sum_{j=0,\text{even}}^{n} (2\eta)^j A^g_{n+2,j}(t) + (2\eta)^{n+2}\, C^g_{n+2}(t)$$

For the odd N we have

$$D_N(t) = \sum_{k=0}^{\infty} 2^k \left[D^{(+)}_{N+k-1} A^q_{N+k+1,k}(t) + D^g_{N+k-2} A^g_{N+k+1,k}(t) \right]$$

$N \to 0$ **limit. Singularities of coeff. functions are due to $k=0$ term only.**

$$D_N(t) = C^{(+)}_N q^{(+)}_N(t) + C^g_N g_N(t) + D^{\text{reg}}_N(t)$$

At $t \to 0$, $q^{(+)}_N(t)$ $g_N(t)$ reduce to the moments of usual PDFs

$$q^{(+)}_N(t) \to q^{(+)}_N = \int_0^1 dx\, x^N q^{(+)}(x)\,, \quad g_N(t) \to g_N = \int_0^1 dx\, x^N g(x)$$

method:

- Curci, Furmanski, Petronzio approach:
  Amplitudes on parton (quark, gluon) target.

$$\gamma^* g \to V g \ , \quad \gamma^* q \to V q$$

  Separation of leading twist.

- Dimensional regularization, $1/\epsilon^n$ poles. Collinear factorization. Poles are absorbed into definition of parton densities.

$$\Gamma_N = \exp\left(\frac{1}{\epsilon} \int\limits_0^{\alpha_S (\mu_F/\mu)^\epsilon S_\epsilon} \frac{d\alpha}{\alpha} \gamma_N(\alpha) \right) \ , \quad S_\epsilon = \exp\{-\epsilon(\psi(1) + \ln 4\pi)\}$$

- BFKL equation in $D = 4 + 2\epsilon$ is not scale invariant. Iterative solution. Collinear singularities – $1/\epsilon^n$ poles

The high energy terms are defined from the expression

$$C_N^g \sim h_V(\gamma) R \left(\frac{Q^2}{\mu_F^2}\right)^\gamma . \quad (1)$$

The gluon anomalous dimension is determined by $1 = (\bar{\alpha}_s/N)\chi(\gamma)$**, where** $\chi(\gamma)$ **is the BFKL eigenfunction, function** R **depends on** $\bar{\alpha}_s/N$**. Expanding** C_N^g **in the series of variable** $y = \bar{\alpha}_s/N$ **one can obtain analytical expressions for the polynomials** $C_n(L)$**.**

k_t **dependent amplitude of the gluon subprocess (**$Q = M$**)**

$$h_V(k_t^2) = \frac{Q^2}{4{k_t}^2 + Q^2} , \quad (2)$$

then we calculate its Mellin transform

$$h_V(\gamma) = \gamma \int\limits_0^\infty \frac{d{k_t}^2}{{k_t}^2} \left(\frac{{k_t}^2}{Q^2}\right)^\gamma h_V(k_t^2) = 4^{-\gamma}\Gamma[1+\gamma]\Gamma[1-\gamma] . \quad (3)$$

$$L = \log\left(\frac{Q^2}{\mu_F^2}\right) \quad , \qquad x = \frac{\bar{\alpha}_s}{N}$$

Υ**:**

$$1 + x(L - \log 4) + \frac{x^2}{6}\Big(\pi^2 + 3\log^2 4 + 3L(L - \log 16)\Big) + \ \ldots + \mathcal{O}(x^{10})$$

ρ^0**:**

$$1+x(L-2)+\frac{x^2}{2}\left(8 + L(L-4)\right)+\frac{x^3}{6}\left(28\zeta(3) - 48 + L\left(L(L-6) + 24\right)\right)+\ldots+\mathcal{O}(x^{10})$$

Coeff. functions at small x / their Mellin moments at $N \to 0$

F_L – [Catani, Hautmann '94] $x = \frac{3\alpha_s}{\pi}\frac{1}{N}$

$\mu_F^2 = Q^2$

F_L: $1 - \frac{1}{3}\,x + 2.13\,x^2 + 2.27\,x^3 + 0.434\,x^4 + \ldots$

ρ^0: $1 - 2\,x + 4\,x^2 - 2.39\,x^3 - 4.09\,x^4 + \ldots$

Υ: $1 - 1.39\,x + 2.61\,x^2 + 0.481\,x^3 - 4.96\,x^4 + \ldots$

$\mu_F^2 = Q^2/4$

F_L: $1 + 1.05\,x + 2.63\,x^2 + 5.35x^3 + 8.97\,x^4 + \ldots$

ρ^0: $1 - 0.614\,x + 2.19\,x^2 + 1.68\,x^3 - 0.958\,x^4 + \ldots$

J/Ψ: $1 + 0.\,x + 1.64\,x^2 + 3.21\,x^3 + 1.08\,x^4 + \ldots$

$$\mathcal{I}m A^g \sim H^g(\xi,\xi) + \int\limits_{\xi}^{1} \frac{dx}{x} H^g(x,\xi) \sum_{n=1} C_n(L) \frac{\bar{\alpha}_s^n}{(n-1)!} \log^{n-1}\frac{x}{\xi}$$

without loss of accuracy $H^g(x,\xi) \to x g(x)$, but LO – $H^g(\xi,\xi)$ should be kept.

$C_n(L)$ - L^n **polynomials we have calculated,** $L = \log\frac{Q^2}{\mu_F^2}$

in the numerics below we used:

- **– CTEQ6M PDFs**

- **for LO term: $x\, g(x) \to H^g(\xi,\xi)$ – Radyushkin DD model**
- **substitute the first term ($n = 1$) in the sum, by the exact NLO coefficient**

- **BFKL logs affect both the coeff. functions and the parton distributions themselves.**
- **The importance of such logarithms for the evolution of parton distributions is a subject of ongoing debate.**
  It seems that resulted effect at $Q^2 \sim 50$ GeV$^2, x \sim 10^{-2} \div 10^{-4}$ is not much different from the conventional NLO DGLAP evolution.
- **Whereas the BFKL corrections to parton distributions are universal, the corrections to hard-scattering amplitudes are process-dependent**
- **BFKL-related corrections to coeff. functions for Υ prod. are very large, factor several larger that the similar corrections to, say, the longitudinal structure function $F_L(x, Q^2)$.**
- **Because of this, the LO resummation of BFKL energy logarithms in the coeff. functions can make sense and be a valid approximation (at least in a limited energy range)**

BUT!

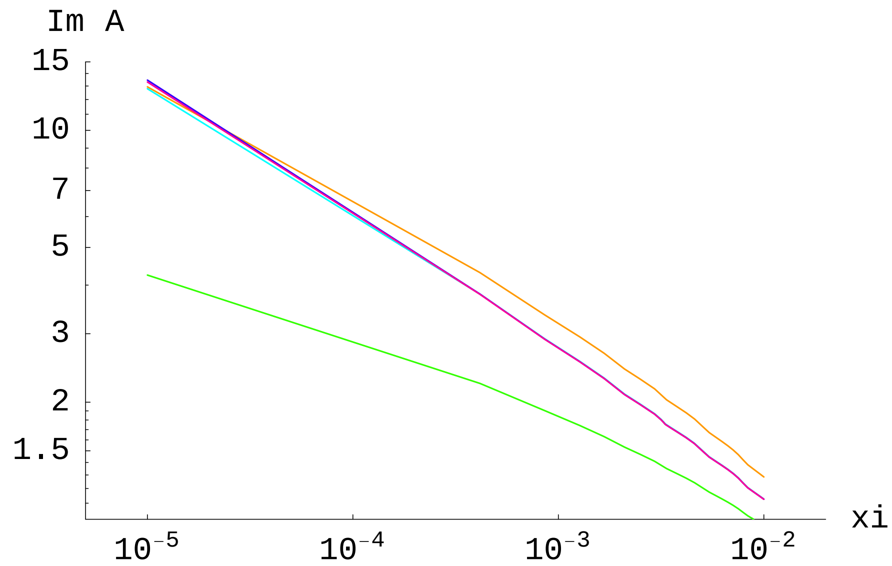
More realistic numerics still should include:

- **resummed anomalous dimensions approach for μ_F evolution for $g(x, \mu_F)$**

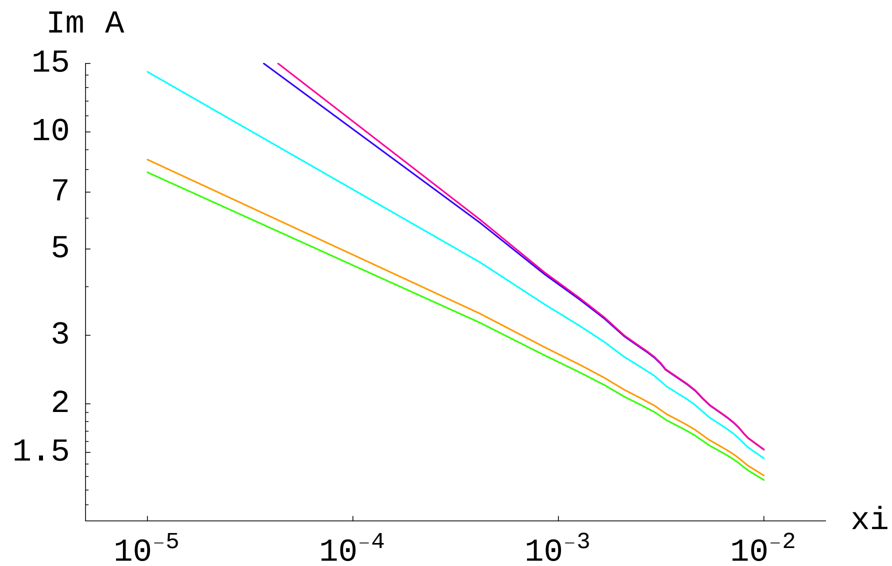
Convergence of resummation

$\mu_F^2 = 100$ **GeV**^{2}

- LO
- $LO + 1st$ **high en. term**
- $LO + 2$ **high en. terms**
- $LO + 5$ **high en. terms**
- $LO + 6$ **high en. terms**



$\mu_F^2 = 25$ **GeV**^{2}



Comparison with ZEUS (96-07) 468 pb^{-1} data:

$$\sigma^{\gamma p \to \Upsilon(1S) p} = 160 \pm 51^{+48}_{-21}\; pb \quad \textbf{for} \quad < W >= 100 \text{ GeV}$$

$$\sigma^{\gamma p \to \Upsilon(1S) p} = 321 \pm 88^{+45}_{-114}\; pb \quad \textbf{for} \quad < W >= 180 \text{ GeV}$$

Input for our calculation:

$$m_b = 4.9 \text{ GeV}\,, \quad b_\Upsilon = 4.5 \text{ GeV}^{-2}$$

Our results:

$Q^2/4 < \mu_F^2 < Q^2$

$$\sigma^{LO}(100) = 214 \div 185\; pb\,, \quad \sigma^{res}(100) = 159 \div 144\; pb$$

$$\sigma^{LO}(180) = 492 \div 358\; pb\,, \quad \sigma^{res}(180) = 427 \div 332\; pb$$

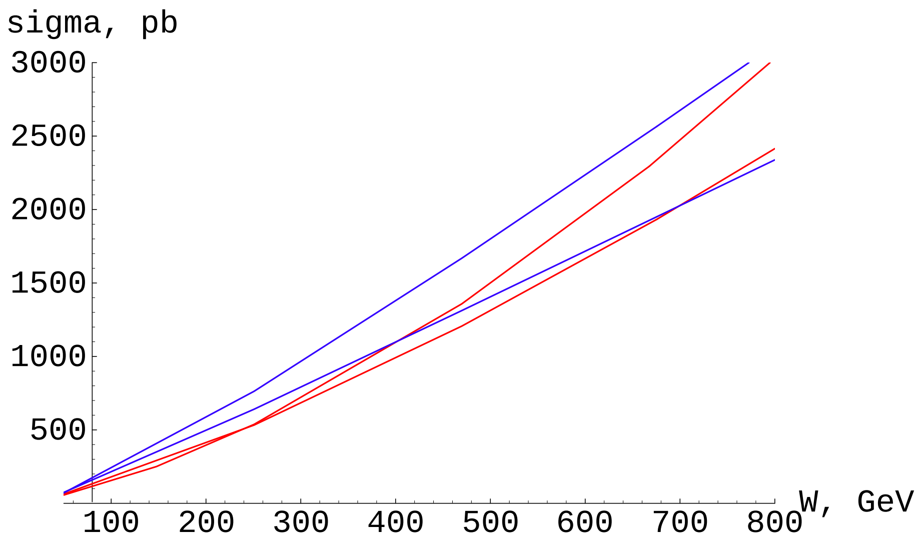
$Q^2/2 < \mu_F^2 < Q^2$

$$\sigma^{LO}(100) = 214 \div 202\; pb\,, \quad \sigma^{res}(100) = 159 \div 156\; pb$$

$$\sigma^{LO}(180) = 492 \div 430\; pb\,, \quad \sigma^{res}(180) = 345 \div 332\; pb$$

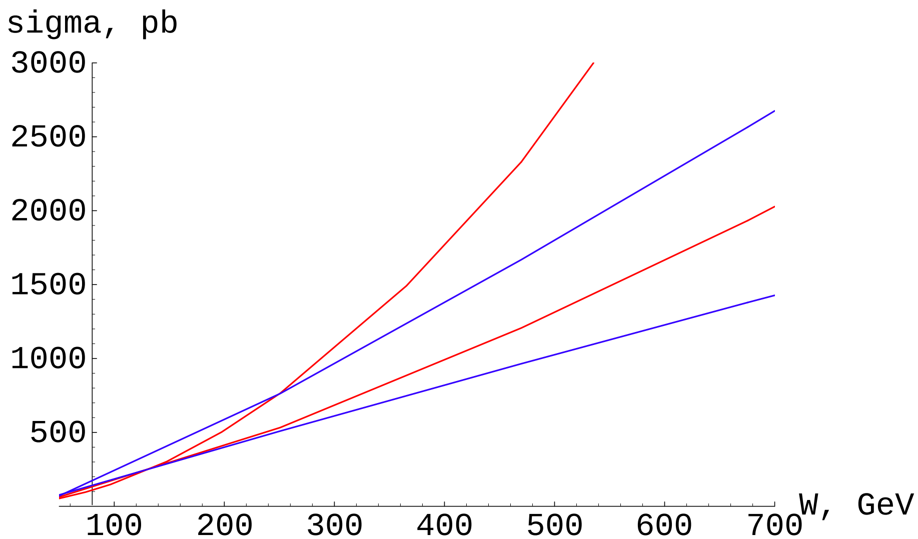
LO **vs Resummed cross section**

100 **GeV**^{2} $<$ μ_F^2 $<$ 50 **GeV**^{2}



- LO
- **Resummed**

100 **GeV**$^2 < \mu_F^2 < 25$ **GeV**^{2}



Summary

- **NRQCD – natural framework for Υ production. Expansion parameter: $\frac{v}{c} \sim \alpha_s$, first rel. correction $\sim \left(\frac{v}{c}\right)^2 \sim \alpha_s(m_b)^2 \sim 0.046$.**

- **Scale parameter: $Q^2 \sim m_\Upsilon^2 \sim 100$ GeV^2. Leading twist collinear QCD factorization should be an adequate tool.**

- **NLO correction are large at small x. Need for high-energy resummation.**

- **The approach used earlier for DIS may be generalized to hard exclusive reactions.**
  We obtain analitical results for resummed coefficient functions.
  Numerical estimates give encouraging results.

To be done:

- **Incorporate resummed parton PDFs in the analysis**
- **Consider other processes: F_L; ρ, J/Ψ – exclusive production**