# High energy resummation in direct photon production

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G.D., *Nucl. Phys.* B (2009), doi:10.1016/j.nuclphysb.2009.09.001

Low-x workshop, Ischia island
12 September 2009

# Outline

# A single-scale problem

- Collinear factorization

$$\sigma(x, Q^2) = C(x, Q^2/\mu^2, \alpha_s(\mu^2)) \otimes F(x, \mu^2)$$

$$x \equiv \frac{Q^2}{S}$$

- Resummation of collinear logs by a renormalization group approach $\Longrightarrow$ DGLAP PDFs evolution

$$\alpha_s^k \log^n \frac{Q^2}{\mu^2}$$

Accurate predictions in the region

$$\Lambda^2 \ll Q^2 \sim S$$

# A two-scale problem

In the TeV energy range (@LHC) hadronic collisions enter a two scale regime:

$$\Lambda^2 \ll Q^2 \ll S$$

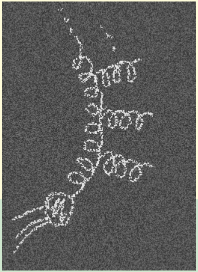
where QCD perturbation theory is affected by large

$$(\alpha_s log x)^n$$


**RESUMMATION!!**

# Ladders, Logs and $k_t$-factorization

The small-$x$ contributions to the cross section stem from strong ordered gluon emissions in the $t$-channel (BFKL ladders).



The $k_t$-factorization theorem[a]

$$\sigma(x, Q^2) = \int_0^{Q^2} \frac{d\mathbf{k}^2}{\mathbf{k}^2} \int_x^1 \frac{dz}{z} \hat{\sigma}(\mathbf{k}^2/Q^2, x/z) \mathcal{F}(\mathbf{k}^2, z)$$

provides a simple recipe to resum the LL$x$ enhancement of the coefficient function in terms of the partonic cross section $\hat{\sigma}$ with off-shell incoming gluons.

[a] S. Catani, M. Ciafaloni, F. Hautmann (1991)

# Conjugate variables

We introduce the dimensionless variables

$$\xi = \frac{\mathbf{k}^2}{Q^2}, \ x = \frac{Q^2}{S}$$

and the impact factor in Mellin space

$$h(N, M) = M \int_0^1 dx\, x^{N-1} \int_0^\infty d\xi\, \xi^{M-1} \hat{\sigma}(\xi, x)$$

$x$ vs $N$
$\xi$ vs $M$

High energy in $N$ space:

$$\log^k x \to \frac{1}{N^{k+1}}$$

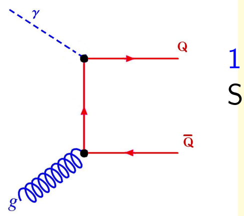
# High energy resummation HOWTO


The high energy resummation at LL$x$ can be performed by using the following recipe

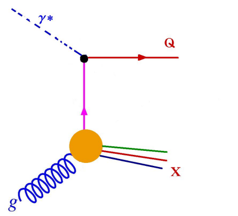
1. compute the cross section at leading order with off-shell incoming gluons:
   - Sudakov parametrization: $k^\mu = z_i p_i^\mu + \mathbf{k}^\mu \rightarrow k^2 = -\mathbf{k}^2$
   - Polarization sum (eikonal) $\sum_\lambda \epsilon_\mu^\lambda(k)\epsilon_\nu^\lambda(k) = \frac{\mathbf{k}_\mu \mathbf{k}_\nu}{\mathbf{k}^2}$
2. perform the Mellin integrations to obtain $h(N, M)$
3. identify $M$ as the sum of the leading singularities of the small-$x$ resummed anomalous dimension:

$$\sigma(N, \cdots) = h\left(N, M = \gamma_s\left(\frac{\alpha_s}{N}\right)\right)(1 + \mathcal{O}(\alpha_s))$$

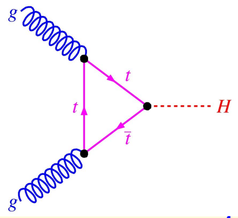
# Applications



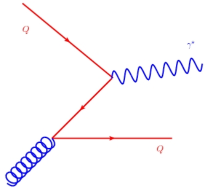

1. Heavy flavour photo- lepto- and hadro- production
S. Catani, M. Ciafaloni, F. Hautmann (1991)




2. DIS
S. Catani, F. Hautmann (1994),
G. Altarelli *et Al.* (2008)




3. Higgs production
S. Marzani *et Al.* (2009)




4. Drell-Yan
S. Marzani, R. D. Ball (2009)

# Prompt photon production

## Motivations

- PP is the most important reducible background for the $H \to \gamma\gamma$ signal.
- an useful tool to probe the gluon density.
- very low values of $x$! ($p_\perp^\gamma \gtrsim 20$ Gev $\to x \gtrsim 10^{-5}$)

## Known results

- NLO cross section ($\mathcal{O}(\alpha\alpha_s^2)$)
  P. Aurenche *et Al.* (1988)
  R. K. Ellis, D. A. Ross (1990)
  L. E. Gordon, W. Vogelsang (1993)
- NLL Sudakov resummation
  S. Catani *et Al.* (1999)

# Direct-photon production: $H_1(P_1) + H_2(P_2) \to \gamma(q) + X$

## Collinear Factorization

direct fragmentation

$$\sigma^{\gamma}(N) = \sum_{a,b} F_{a/H_1}(N) F_{b/H_2}(N) \left( \mathcal{C}^{\gamma}_{ab}(N) + \sum_{c} D_{c/\gamma}(N) \mathcal{C}^{c}_{ab}(N) \right)$$

## Direct component at LO

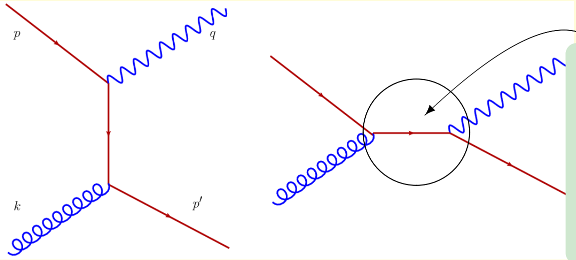
$$\begin{aligned} q(\bar{q})g &\to \gamma q(\bar{q}) \\ q\bar{q} &\to \gamma g \end{aligned}$$

$$\sigma^{\gamma}(N) = \int_0^1 dx\, x^{N-1} \mathbf{q}^3 \frac{d\sigma(x)}{d\mathbf{q}}$$

$\mathbf{q}$ = transverse momentum of $\gamma$

Scaling variable $\to x = \dfrac{4\mathbf{q}^2}{S}$

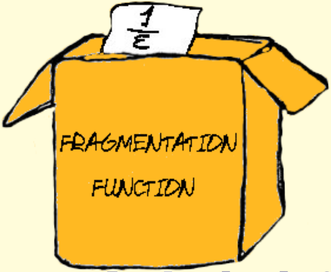
# Off-shell amplitude and collinear divergence




With an off-shell incoming gluon $\gamma$ and $q$ can be collinear!

$\Rightarrow$ Divergence!

- Phase space integrations in $4+2\epsilon$ dimensions
- The final state collinear divergence must be absorbed in the **fragmentation function** definition.



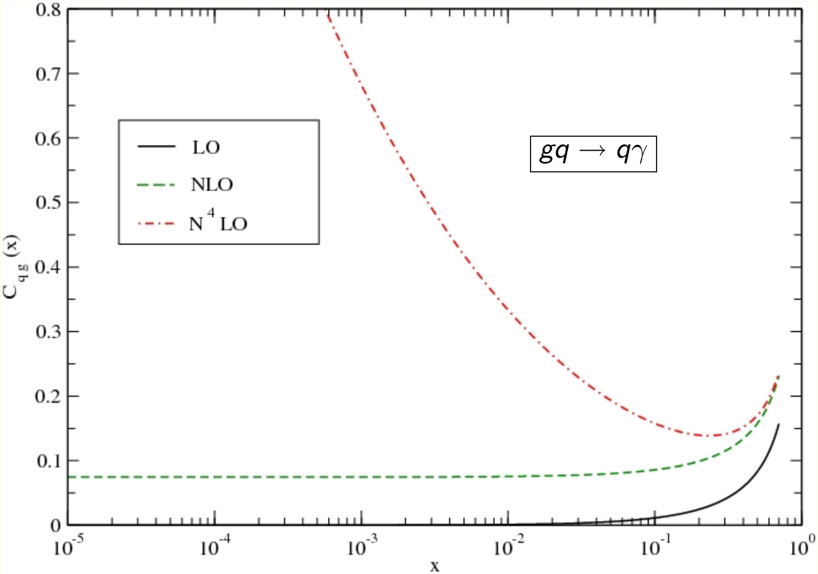
# Impact factor

$$h(N=0,M) = \frac{\alpha\alpha_s\pi}{C_A}\left\{\frac{(7-7M+2M^2)}{(M-1)(M-2)(2M-3)}\left(\pi\cot(M\pi)+ +2H_{M-2}+\frac{2}{M-1}\right)+\frac{1}{2-M}\right\}$$

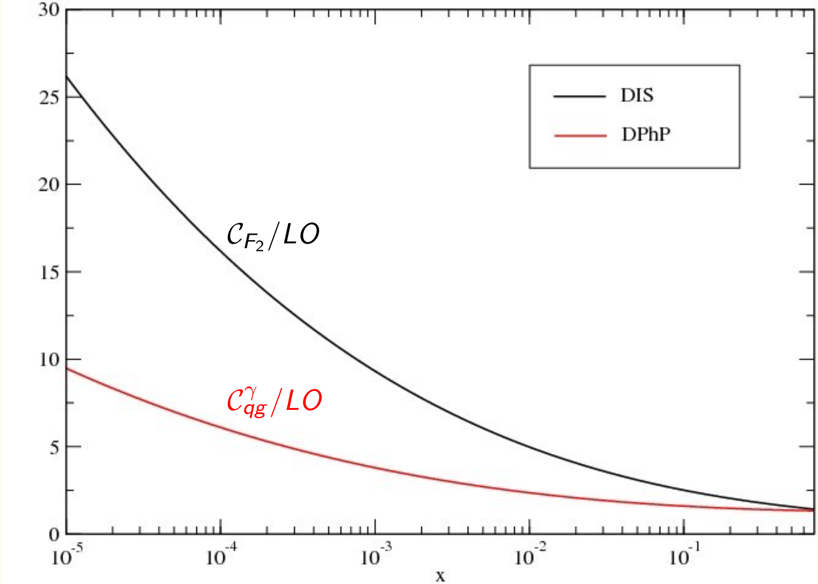
where poles in $N=0$ are produced by the identification:

$$M=\gamma_s\left(\frac{\alpha_s}{N}\right)=\frac{\alpha_s C_A}{\pi N}+\mathcal{O}\left(\frac{\alpha_s}{N}\right)^4$$

# Coefficient function



# Direct photon and DIS



# Conclusions and outlook

- Direct photon production at high energy: impact factor and coefficient function.
- Fragmentation component?
- Resummed coefficient functions + resummed evolution
- Phenomenology!