Heavy flavors at HERA - physics around one BFKL node

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Low x 2009

Two topics

to be discussed:

- Spectrum and solutions of the color dipole BFKL equation: nodes of eigenfunctions, intercepts and all that...
- Physics implications of nodal properties of the BFKL eigenfuctions: HERA data on F_L and heavy flavor production $F_2^b, \; F_2^c$

Asymptotic Freedom and BFKL

The BFKL equation for the color dipole cross section $\sigma(x,r)$ (Nikolaev, Zakharov and VRZ, 1994):

$$\frac{\partial \sigma(x,r)}{\partial \log(1/x)} = \mathcal{K} \otimes \sigma(x,r),$$

The kernel $\mathcal{K}$ is related to the flux of the Weizsäcker - Williams gluons

$$\mathcal{K} \propto |\vec{E}(\vec{\rho}) - \vec{E}(\vec{\rho} + \vec{r})|^2.$$

AF dictates : ch. el. field $\vec{E}(\vec{\rho})$ must be calculated with the running QCD charge $g_S(\rho)$

$$\vec{E}(\vec{\rho}) = g_S(\rho)\vec{\rho}/\rho^2 \times (\text{screening factor}).$$

(screening factor=infrared regularization)

!

The running coupling introduced in this way does not exhaust all NLO effects but correctly describes the enhancement of long distance effects and suppression of short distance effects by AF.

BFKL spectrum and solutions

The spectrum of the running BFKL equation is a series of moving poles in the complex j-plane (Fadin, Kuraev and Lipatov 1975). Hence, the BFKL-Regge expansion:

$$\sigma(x,r) = \sigma_0(r)x^{-\Delta_0} + \sigma_1(r)x^{-\Delta_1} + \sigma_2(r)x^{-\Delta_2} + ...$$

Eigenfunctions:

$$\sigma_n(r)$$

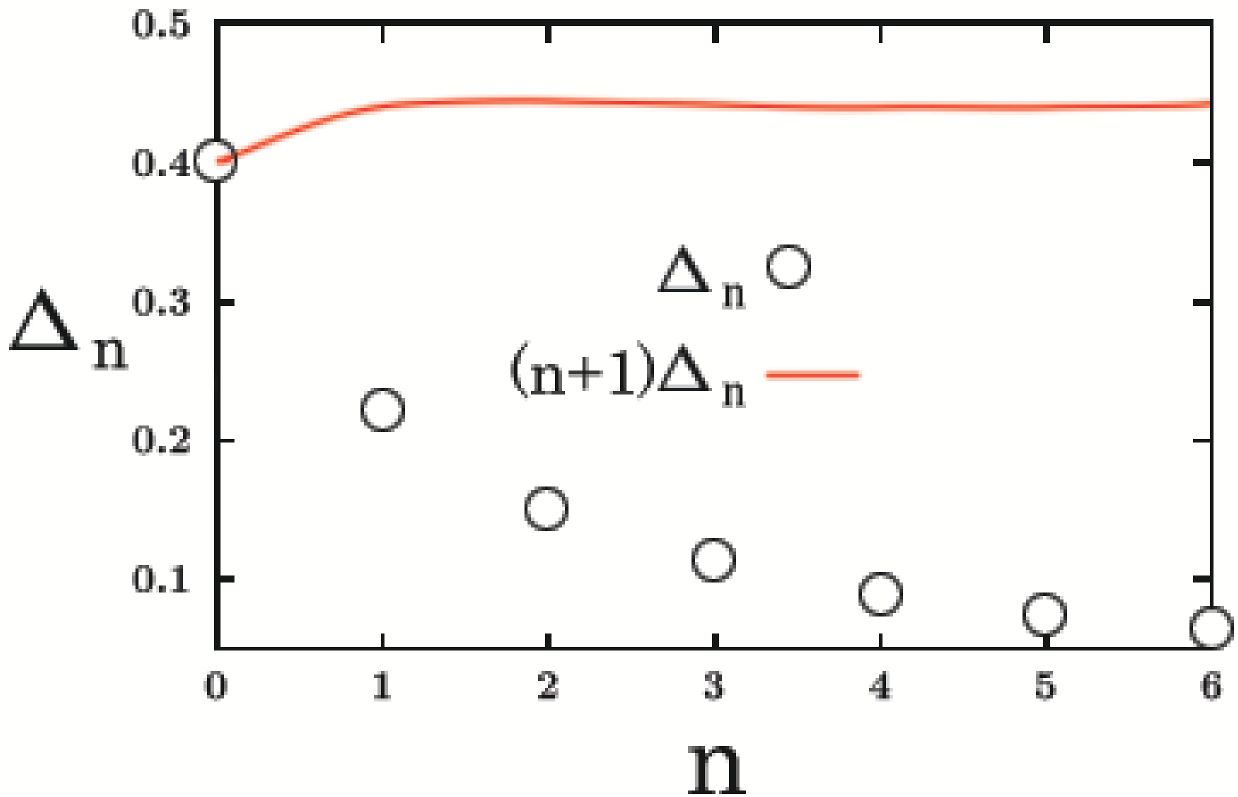
a solution of the eigenvalue problem

$$\mathcal{K} \otimes \sigma_n = \Delta_n \sigma_n(r).$$

with the pomeron intercept Δ_n as the eigenvalue

$$\Delta_n = j_n - 1$$

Δ_n vs. quasiclassical approx.



Quasiclassical approximation:

$$\Delta_n \approx \frac{\Delta_0}{(n+1)}, \;\; n \gg 1$$

Eigenvalues (intercepts)

- The eigenvalues Δ_n closely, to better than 10%, follow Lipatov's quasiclassical approx.

$$\Delta_n = \frac{\Delta_0}{(n+1)}$$

- The intercept of the leading pole trajectory, with our infrared regularization is

$$\Delta_0 = 0.4.$$

(Nikolaev, Zakharov and VRZ 1994)

1. eigenfunctions $\sigma_n(r)$

- subleading BFKL eigenfunctions $\sigma_n(r)$ are oscillating functions of the color dipole size r

$$r^{-1}\sigma_n(r) \propto cos[\phi(r)].$$

- Period of oscillations is rather large because the relevant variable is

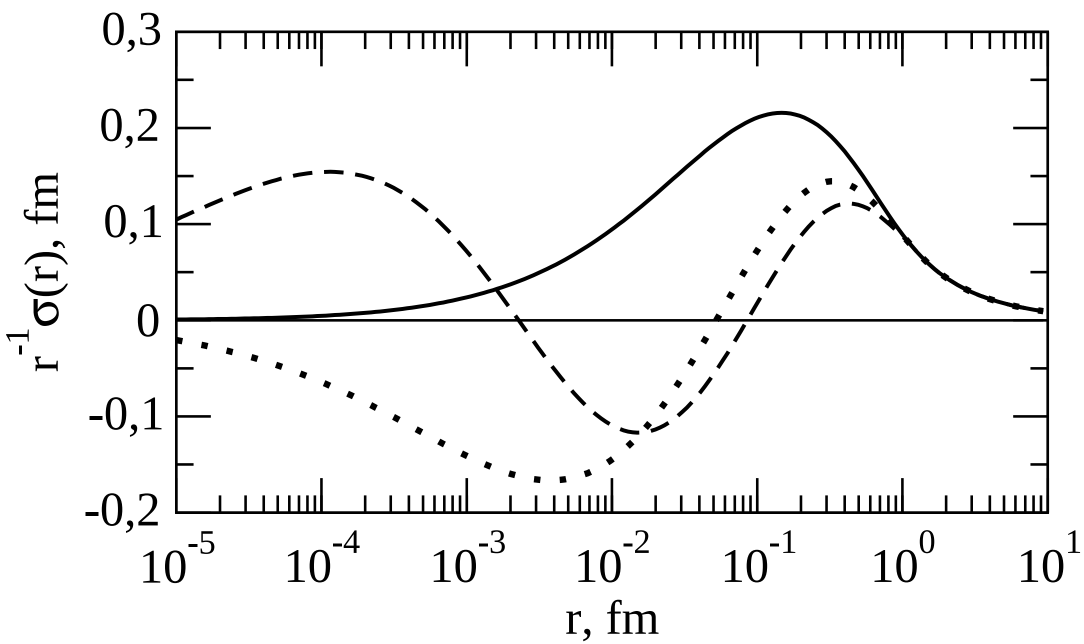
$$\phi \sim [1/\alpha_S(r)]^\gamma$$

- only one node located at

$$r \sim 0.1\,\mathrm{fermi}$$

  is within the reach of HERA experiments on charm and beauty production.

3. eigenfunctions $\sigma_n(r)$



- With our IR regularization the node of $\sigma_1(r)$ is at $r_1 \simeq 0.06 - 0.07\,\mathrm{fm}$
- for larger n its position r_1 moves to larger $r \sim 0.1\,\mathrm{fm}$.
- the first nodes for all n accumulate at $r \sim 0.1\,\mathrm{fm}$ - accumulation point

Charm: observables, scales.

In the color dipole representation

$$F_2^c(x,Q^2) = \frac{Q^2}{4\pi\alpha_{em}} \int_0^1 dz \int d^2\vec{r}|\Psi^{cc}(z,r)|^2 \sigma(x,r).$$

and the integral over r is dominated by

$$\frac{4}{Q^2+4m_c^2} \lesssim r^2 \lesssim \frac{1}{m_c^2}$$

$$r_c \sim 1/\sqrt{m_c^2 + Q^2/4} \lesssim 0.1\,\mathrm{fermi}.$$

phenomenon of decoupling

In open charm production for moderately large Q^2 the relevant r is close to the accumulation point - r_1.

$$r \sim r_1 \simeq 0.06 - 0.1\,\mathrm{fermi}\,.$$

Subleading pomerons decouple from charm structure function (Nikolaev and VRZ 1997)

$$F_2^c(x,Q^2) = f_0(Q^2)x^{-\Delta_0} + f_1(Q^2)x^{-\Delta_1} + f_2(Q^2)x^{-\Delta_2} + \ldots$$

$$\approx f_0(Q^2)x^{-\Delta_0}$$

Hence, $F_2^c(x,Q^2)$ gives a direct access to the intercept of the rightmost BFKL pole which is

$$\Delta_0 = 0.4$$

(Nikolaev, Zakharov and VRZ 1994)

charm at high Q^2

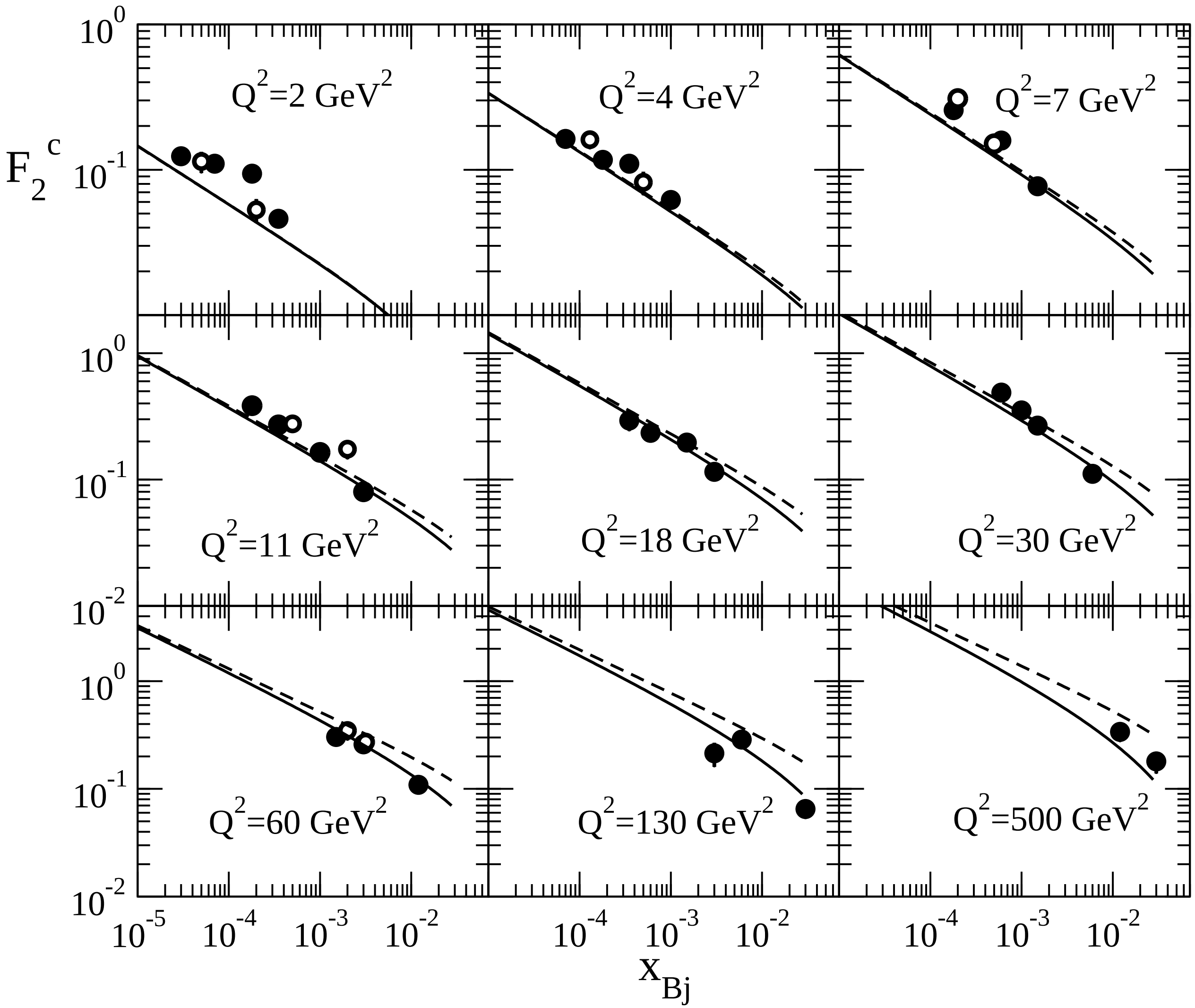
For large Q^2, far beyond the nodal region, the effect of cancellations disappears and

$$f_n^c(Q^2) \propto \left[\alpha_S(Q^2)\right]^{-\gamma_n}, \;\gamma_n = \frac{4}{3\Delta_n}$$

Remind, at $r \to 0$ the CD BFKL equation has an exact solution

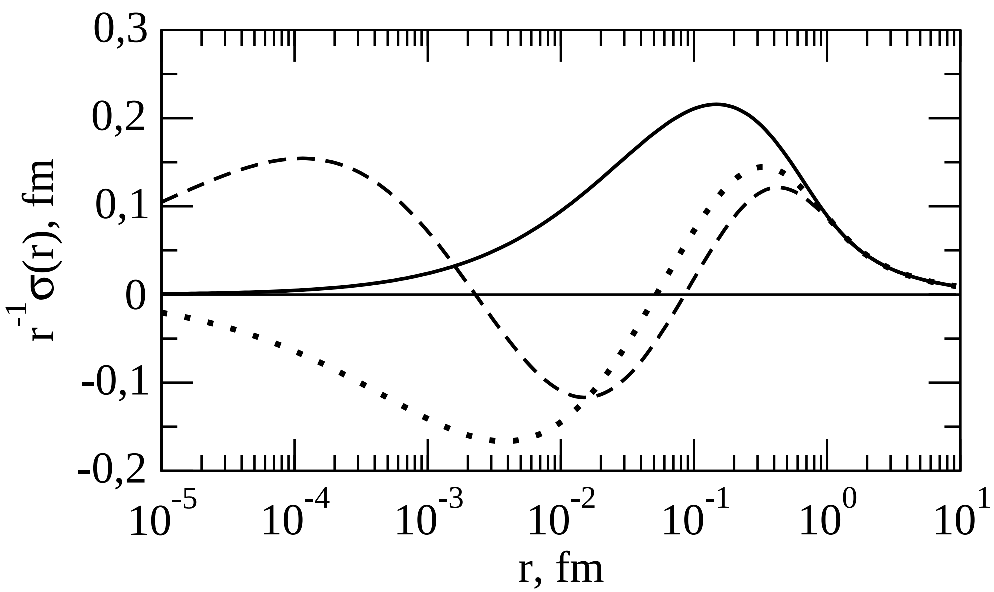
$$\sigma_n(r) = r^2 \left[\frac{1}{\alpha_S(r)}\right]^{\gamma_n - 1}, \;\gamma_n \Delta_n = 4/3\,.$$

F_2^c: th. vs. exp.



Data: H1 Collab. Eur.Phys.J. C45, 23 (2006)

Beauty production

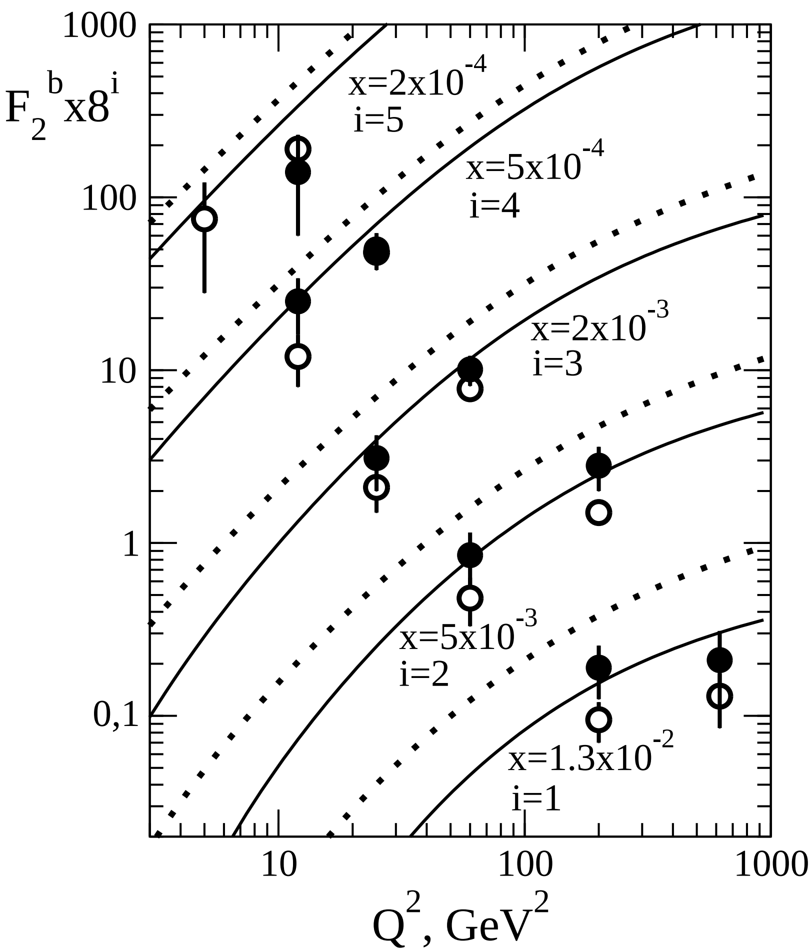


Relevant distances are smaller than the rightmost node location

$$r_b \sim 1/\sqrt{m_b^2 + Q^2/4} \ll r_1 \sim 0.1\,\mathrm{fermi}$$

There are large negative valued contributions to F_2^b, coming from subleading BFKL singularities.

1. F_2^b : th. vs. exp.



full circles - H1 Collab., 2006 (Eur.Phys.J. C45, 23 (2006))

open circles - H1 Collab., 2008 (preliminary)

2. F_2^b : th. vs. exp.

Our 1999 predictions for beauty SF agree well with the determination of F_2^b by the H1 published in 2006 but slightly overshoot preliminary H1 results on F_2^b reported in 2008.

to conclude

the open beauty production probes the vacuum exchange for color dipoles smaller than $r_1 \sim 0.1$ fermi and picks up significant subleading contribution to $F_2^b(x, Q^2)$.
However, the approximation

$$F_2^b = \mathbb{P}_0 + \mathbb{P}_1 + \mathbb{P}_2$$

remains numerically accurate at least for $Q^2 \lesssim 300$ GeV^2.
Digression: The approximation

$$F_2 = \mathbb{P}_0 + \mathbb{P}_1 + \mathbb{P}_2 + soft$$

also works well for the proton light flavor structure functions

Nikolaev, Zakharov and VRZ, (1997)
Ellis, Kowalski and Ross (2008)

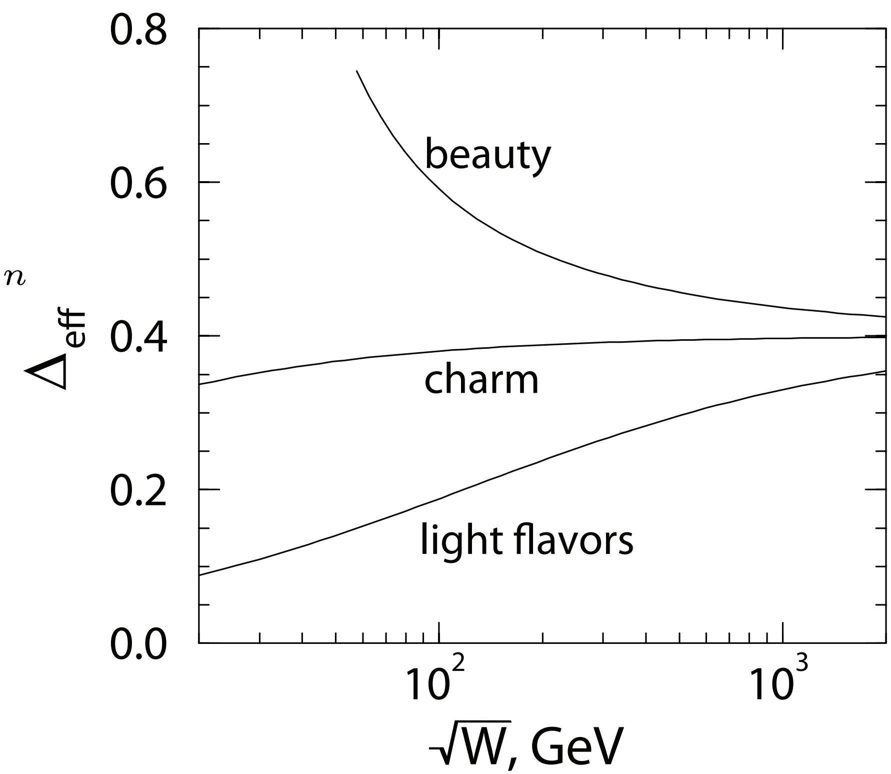
$Q\bar{Q}$ photoproduction

The hierarchy of pre-asymptotic intercepts:

$$\sigma^{Q\bar{Q}}(W) = \sum_{n=0} \sigma_n \cdot W^{\Delta_n}$$

$$\propto W^{\Delta_{\rm eff}}$$

$$r_n = \sigma_n/\sigma_0$$



$$\Delta_{\rm eff} = \Delta_0 \left[1 - \sum_{n=1} r_n (1 - \Delta_n/\Delta_0)\,(W_0/W)^{\Delta_0 - \Delta_n}\right]$$

Δ_0 from measurements of $F_L(x, Q^2)$

$F_L(x, Q^2)$ is known as local probe of the dipole cross section at

$$r^2 \simeq 10/Q^2.$$

(Nikolaev and Zakharov 1994)

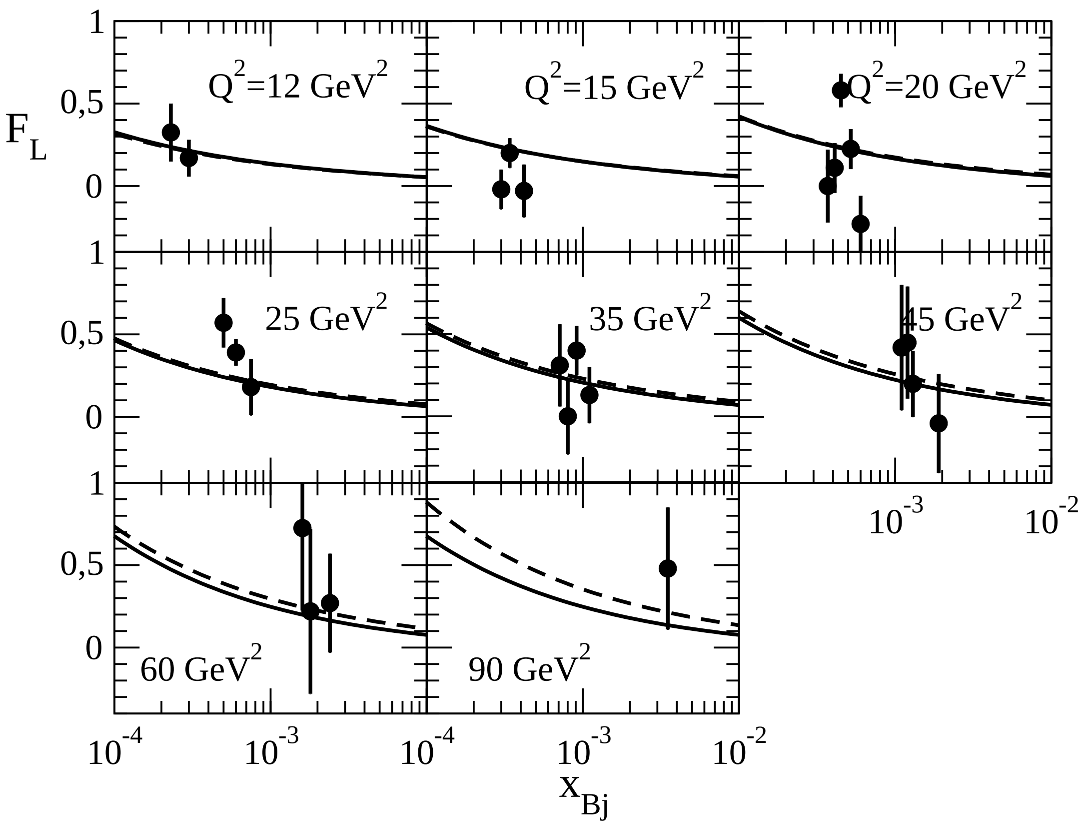
The subleading CD BFKL cross sections have their rightmost node at

$$r_1 \sim 0.1\,\mathrm{fermi}$$

Therefore, one can separate the leading BFKL pole contribution and measure the pomeron intercept Δ_0 from the x-dependence of $F_L(x, Q^2)$ at

$$Q^2 \sim 10 - 30\,\mathrm{GeV}^2$$

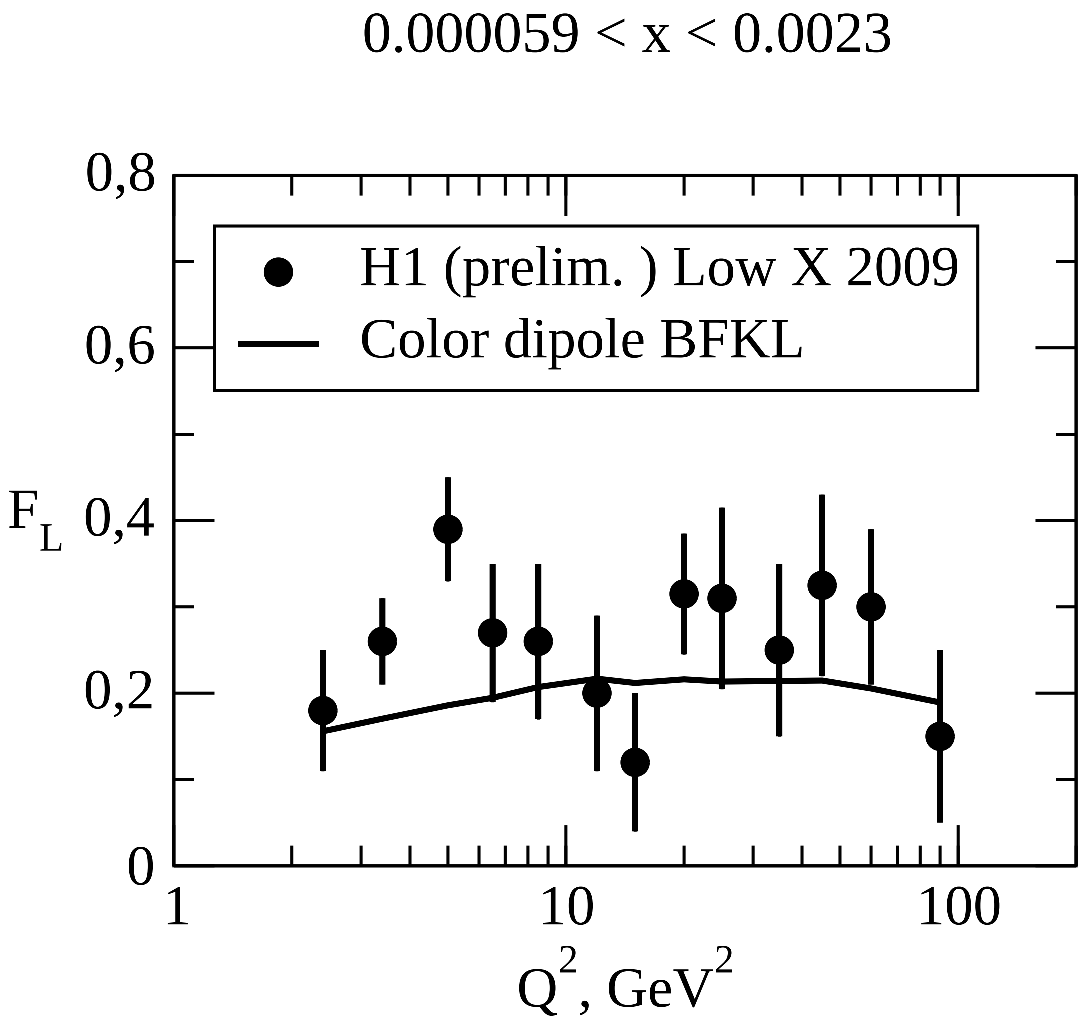
F_L: theory & experiment



Data: H1 Collab. Phys.Lett. B665, 139 (2008)

- Magnitude of F_L is reproduced correctly, but the data do not allow to draw conclusions on the x-dependence.

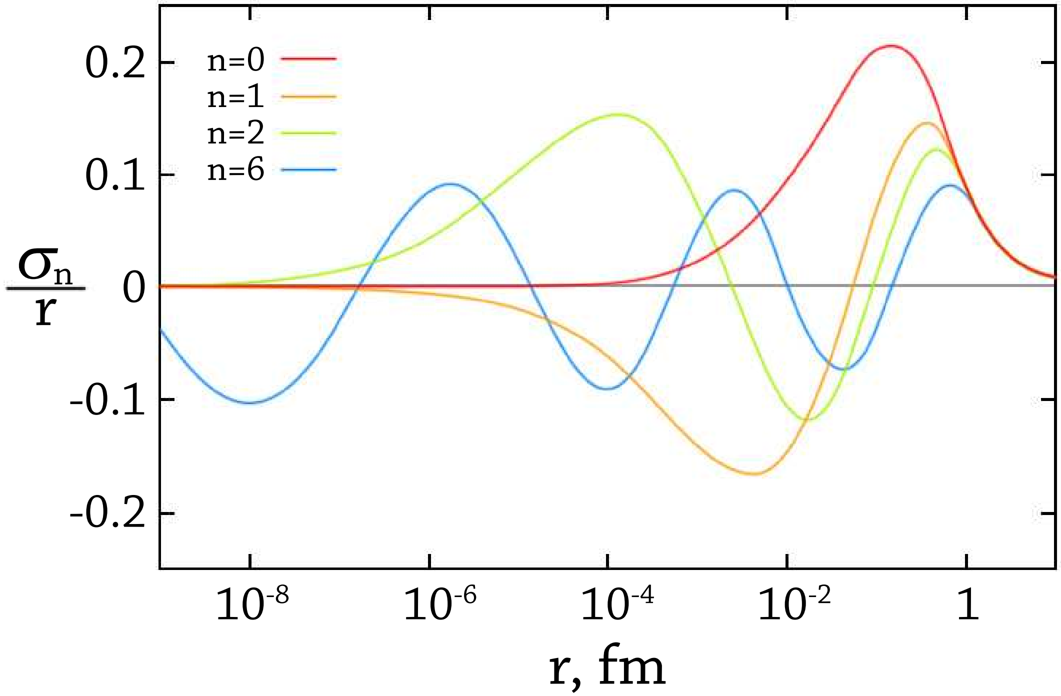
F_L from Mandy and CD BFKL



summary

The rightmost nodes of subleading BFKL eigenfunctions accumulate at dipole sizes $r \sim 0.1$ fermi, in the region probed by heavy flavor production. Due to the decoupling phenomenon this observation greatly simplifies analysis of observables and strengthens predictive power of the CD BFKL approach.

2. eigenfunctions $\sigma_n(r)$



- The leading eigenfunction $\sigma_0(r)$ ("ground state") is node free.
- The subleading $\sigma_n(r)$ ("excited states") has n nodes.
- At $r \to 0 \quad \sigma_n(r) = r^2 \left[\frac{1}{\alpha_S(r)}\right]^{\gamma_n - 1}, \ \gamma_n \Delta_n = 4/3\,.$