

Higher twists in DIS from the DGLAP improved saturation model

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Overview

Motivation

Higher twist evolution in QCD at small x

From QCD to saturation model

Estimates of higher twists in F_2 and F_L

Summary

Based on results obtained with J. Bartels and K. Golec-Biernat

Motivation

At fixed Q^2 importance of higher twist operators in QCD increases quickly with \sqrt{s}

Reason: rapid higher twist evolution at large Q^2 and small x

$$\text{Gluons: } \frac{\text{Twist 4}}{\text{Twist 2}} \sim \frac{1}{Q^2 R^2} \exp \left(\sqrt{b \log(Q^2) \log(1/x)} \right)$$

Higher twists \longrightarrow corrections to main HERA observables: F_2 and F_L

\longrightarrow provide input to precision determination of PDF

Goal 1: Explicit determination of higher twist corrections to F_2 and F_L at small x

Goal 2: QCD analysis of Saturation Model

\longrightarrow identify necessary approximations

\longrightarrow interpret ingredients in terms of QCD

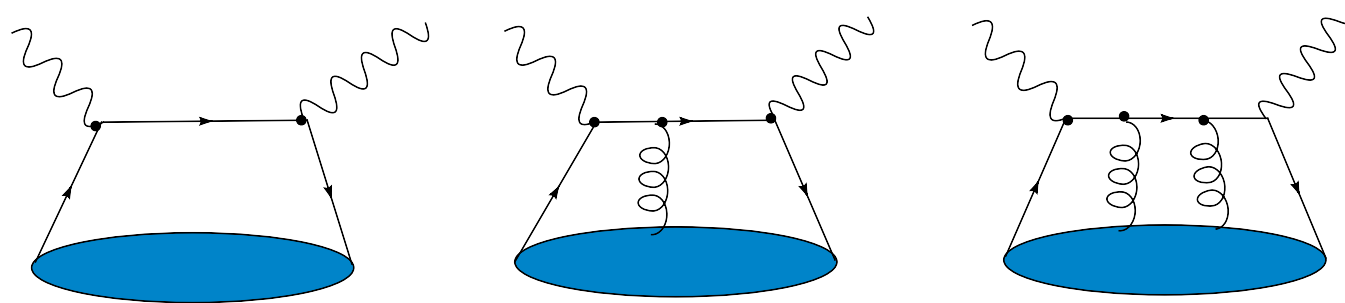
\longrightarrow exploit the information on higher twists

Extension of twist analysis of GBW model [Bartels, Golec-Biernat, Peters]

Higher twist evolution: Quasi Partonic Operators (BFKLs)

DIS: OPE for hadronic tensor:
$$W^{\mu\nu} = \sum_{\tau} \left(\frac{\Lambda}{Q}\right)^{\tau-2} \sum_i C_{\tau,i}^{\mu\nu} \otimes f_{\tau,i}(Q^2/\Lambda^2)$$

Complete twist 4 analysis of $q\bar{q}gg$ evolution [Ellis, Furmanski and Petronzio, 1983]



At small x gluon evolution expected to drive DIS cross-sections at all twists — complete analysis of twist 4 evolution for gluons does not exist

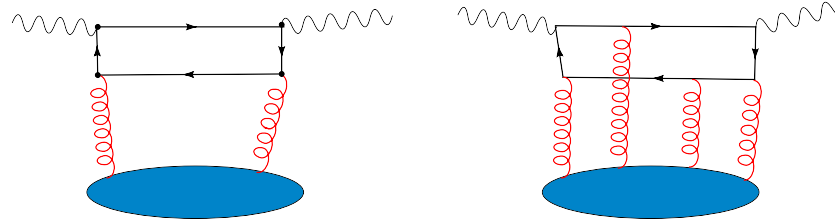
Known at all twists: evolution of Quasi-Partonic Operators [Bokhvestov, Kuraev, Lipatov, Frolov, 1985]

QPO hadronic matrix elements at twist $\tau \sim \tau$ free partons. Examples: $(\partial \cdot A_{\alpha}^{\perp})^2 (\partial \cdot A_{\beta}^{\perp})^2$, $\bar{\psi}\psi\bar{\psi}\psi$

Collinear evolution kernel for twist τ at LLA splits into disconnected pairwise parton interactions — non-forward (in x and p_T) DGLAP kernels

4-gluon evolution at twist 4

At small x the dominant contribution should come from diagrams of the type:

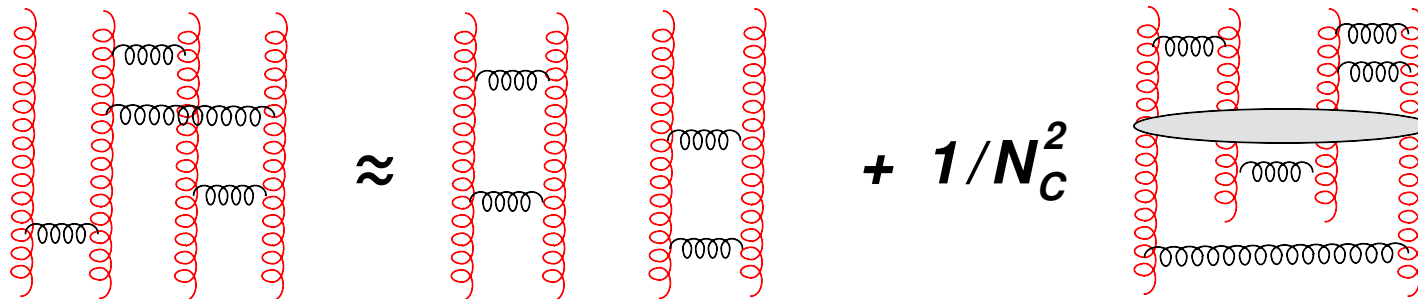


For twist-4, $N_c \rightarrow \infty$, in the leading $\alpha_s \log(Q^2) \log(1/x)$ approximation dominant singularity:

$$\gamma = \frac{4N_c \alpha_s}{\pi} \frac{1}{\omega}$$

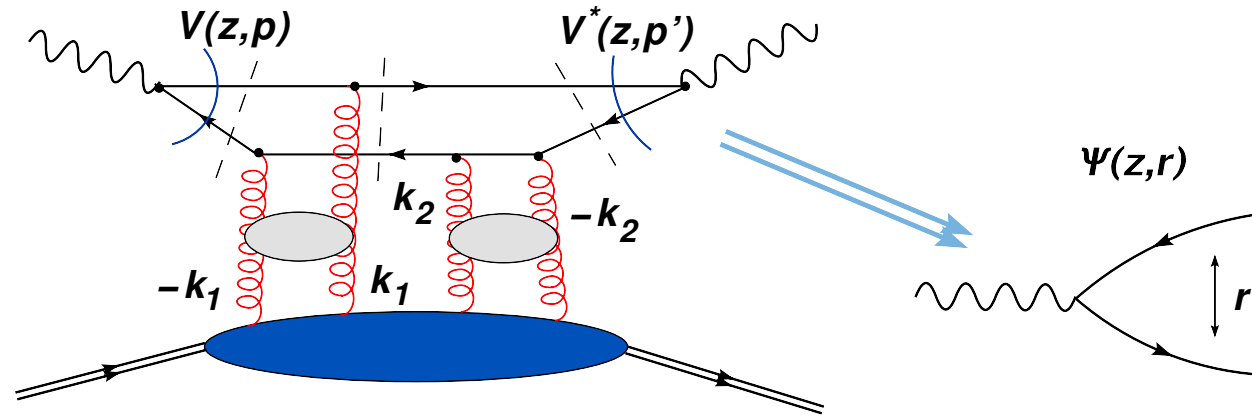
coming from two independent DGLAP evolutions

Corrections — color reconnections between ladders suppressed by $\sim 1/N_c^2$ [Bartels, Ryskin, 1993]



γ* cross section: quark box diagram

Structure:
$$\Delta^{(2n)} \sigma_{\gamma^* p} \sim \int \prod_{i=1}^{2n} \frac{d^2 k_i}{k_i^4} \delta \left(\sum_i \mathbf{k}_i \right) G_{2n}^{\{a_i\}}(x, \{\mathbf{k}_i\}) \Phi_{2n}^{\{a_i\}}(\{\mathbf{k}_i\})$$



Multi-gluon coupling in high energy limit \longrightarrow photon-gluon vertex fusion governs all couplings

$$\Phi_{2n} \sim \alpha_s^n \int d^2 p \int dz \sum_F \text{Color}(F) V^*(z, \mathbf{p}'(F)) V(z, \mathbf{p})$$

After projection on symmetric multiple color singlet, and the Fourier transform result is simple

$$\Phi_{2n} \sim \alpha_s^n \int d^2 r \int dz \Psi^*(z, \mathbf{r}) \prod_{i=1}^n \left[2 - e^{i\mathbf{k}_i \mathbf{r}} - e^{-i\mathbf{k}_i \mathbf{r}} \right] \Psi(z, \mathbf{r})$$

Resumming multi-gluon effects

Taking factorized and symmetric form of unintegrated multi-gluon density

$$G_{2n}^{\{a_i\}}(x, \{k_i^2\}) \sim \sum_{\sigma} \delta^{a_{\sigma(1)} a_{\sigma(2)}} \dots \delta^{a_{\sigma(2n-1)} a_{\sigma(2n)}} f(x, \mathbf{k}_{\sigma(1)}, \mathbf{k}_{\sigma(2)}) \dots f(x, \mathbf{k}_{\sigma(2n-1)}, \mathbf{k}_{\sigma(2n)})$$

Invoking AGK rules one obtains the Glauber-Mueller form used by GBW

$$\Delta^{(2n)}_{\sigma} \sim \frac{(-1)^{n+1}}{n!} R^2 \int d^2r dz |\Psi(z, \mathbf{r})|^2 \prod_{i=1}^n \underbrace{\left\{ \int \frac{d^2k_i}{k_i^4} \frac{\alpha_s f(x, \mathbf{k}_i^2)}{R^2} [2 - e^{i\mathbf{k}_i \mathbf{r}} - e^{-i\mathbf{k}_i \mathbf{r}}] \right\}}_{\text{single dipole scattering xs: } \sigma_1(x, r^2)/R^2}$$

In collinear limit ($k^2 \ll C/r^2$) dipole cross section coincides with DGLAP improved saturation model **[Bartels, Golec-Biernat, Kowalski]**

$$\sigma_1(x, r^2) \simeq \alpha_s(C/r^2) \int^{C/r^2} \frac{dk^2}{k^4} f(x, k^2) (k^2 r^2) \simeq r^2 \alpha_s(C/r^2) xg(x, C/r^2)$$

Resummed cross section:

$$\sigma_d(x, r^2) \simeq R^2 [1 - \exp(-\sigma_1(x, r^2)/R^2)]$$

Recovering saturation model

Combining together QCD information on quark box diagram and multi-gluon density:

$$\sigma_{L,T}(x, Q^2) = \int_0^\infty \frac{dr^2}{r^2} \underbrace{\left[r^2 \int dz \left| \Psi_{L,T}(z, r^2) \right|^2 \right]}_{H_{T,L}(Q^2 r^2)} \sigma_d(x, r^2),$$

with multi gluon evolution in: $\sigma_d(x, r^2) = \sigma_0 [1 - \exp(-\sigma_1(x, r^2)/\sigma_0)]$

Perturbative part of “dipole cross section” combined with modelled soft part (\longrightarrow quark input):

$$\sigma_1 \propto \begin{cases} r^2 \alpha_s(C/r^2) xg(x, C/r^2) & \text{for } C/r^2 < Q_0^2 \\ r^2 \alpha_s(Q_0^2) xg(x, Q_0^2) & \text{for } C/r^2 > Q_0^2 \end{cases}$$

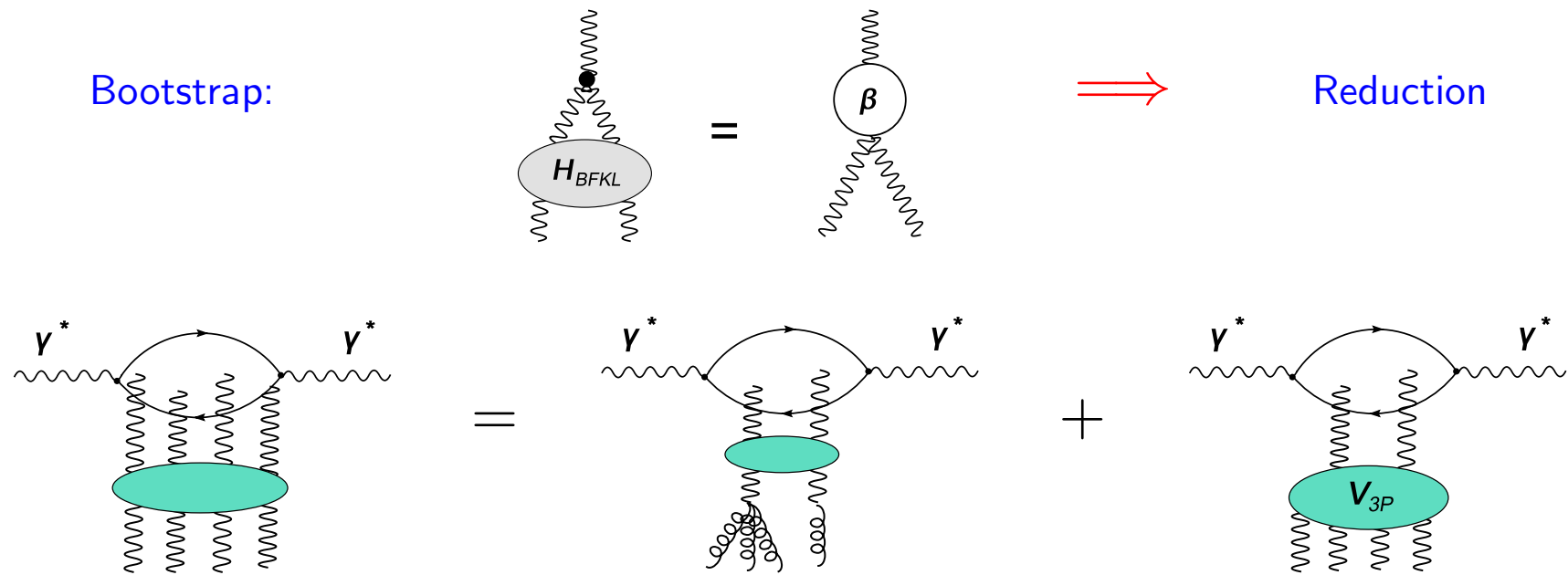
Twist analysis in the Mellin moment space:

$$\tilde{f}(s) = \int_0^\infty dr^2 f(r^2) (r^2)^{s-1},$$

Decomposition will be performed using Parsival formula

$$\sigma_{T,L} p(x, Q^2) = \int_{C_s} \frac{ds}{2\pi i} \tilde{\sigma}(x, s) \tilde{H}_{T,L}(-s, Q^2)$$

Warning: message from small- x : BFKL / BK



At LL($1/x$):

- (1) No direct coupling of four Reggeized gluons to quark loop
- (2) Triple Pomeron vertex — entirely anticollinear! [BFKL'], [Bartels, Kutak]

The discrepancy still to be understood (coupling to the target?)

Multi-gluon density evolution

The gluon density obeys the LO DGLAP equation,

$$\mu^2 \frac{\partial x g(x, \mu^2)}{\partial \mu^2} = \frac{\alpha_s(\mu^2)}{2\pi} \int_x^1 dz P_{gg}(z) \frac{x}{z} g\left(\frac{x}{z}, \mu^2\right)$$

Gluon density $\alpha_s(\mu^2) g(x, \mu^2) = \int \frac{d\omega}{2\pi i} x^{-\omega} \int \frac{d\gamma}{2\pi i} \left(\frac{\mu^2}{\Lambda^2}\right)^\gamma \tilde{g}_0(\omega) \gamma^{1-\tilde{p}(\omega)}$

Double gluon density

$$[\alpha_s(\mu^2) x g(x, \mu^2)]^2 = \int \frac{d\omega}{2\pi i} \int \frac{d\omega'}{2\pi i} x^{-\omega} \hat{G}_{4,0}(\omega', \omega - \omega') \int \frac{d\gamma}{2\pi i} \left(\mu^2 / \Lambda^2\right)^\gamma \gamma^{1-\tilde{p}(\omega')-\tilde{p}(\omega-\omega')}$$

n -gluon density: $[\alpha_s(\mu^2) x g(x, \mu^2)]^n = \int \frac{d\omega}{2\pi i} x^{-\omega} \int \prod_{i=1}^n \frac{d\omega_i}{2\pi i} \hat{G}_{n,0}(\omega_i) \delta\left(\sum_i \omega_i - \omega\right)$

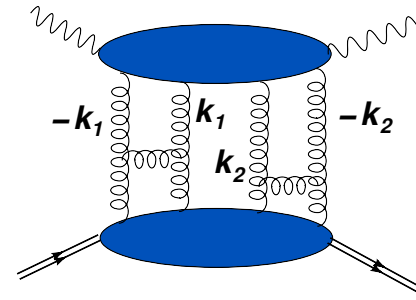
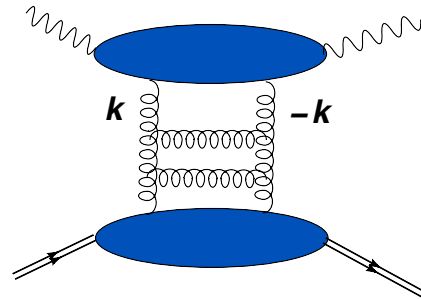
$$\times \int \frac{d\gamma}{2\pi i} \gamma^{n-1-\sum_i \tilde{p}(\omega_i)} \left(\frac{\mu^2}{\Lambda^2}\right)^\gamma$$

Twist structure of γ^* impact factor

2 gluon impact factor $D(k) = \Phi_2(k^2/Q^2) \longrightarrow$ Mellin transform: $\tilde{D}(\gamma)$

Double poles of $\tilde{D}_{T,L}(\gamma)$: for integer γ except of single poles for $\gamma = 1$ (\tilde{D}_L) and $\gamma = 2$ (\tilde{D}_T)

Twist-2 double pole of $\tilde{D}_T(\gamma) \longrightarrow D(k) \sim (k^2/Q^2) \left[a^{(2)} \log(Q^2/k^2) + b^{(2)} \right] \longrightarrow \tilde{P}_{q/g} \otimes g$



Coupling of 4 gluons in two forward ladders:

$$\Phi_4 \sim \int \frac{d\gamma}{2\pi i} \left\{ \tilde{D}(\gamma) (1/Q^2)^\gamma \left[2(\mathbf{k}_1^2)^\gamma + 2(\mathbf{k}_2^2)^\gamma - ((\mathbf{k}_1 + \mathbf{k}_2)^2)^\gamma - ((\mathbf{k}_1 - \mathbf{k}_2)^2)^\gamma \right] \right\}$$

At twist 4: $\Phi_4 \sim k_1^2 k_2^2 \log(Q^2/k_i^2) \longrightarrow$ twist-4 evolution

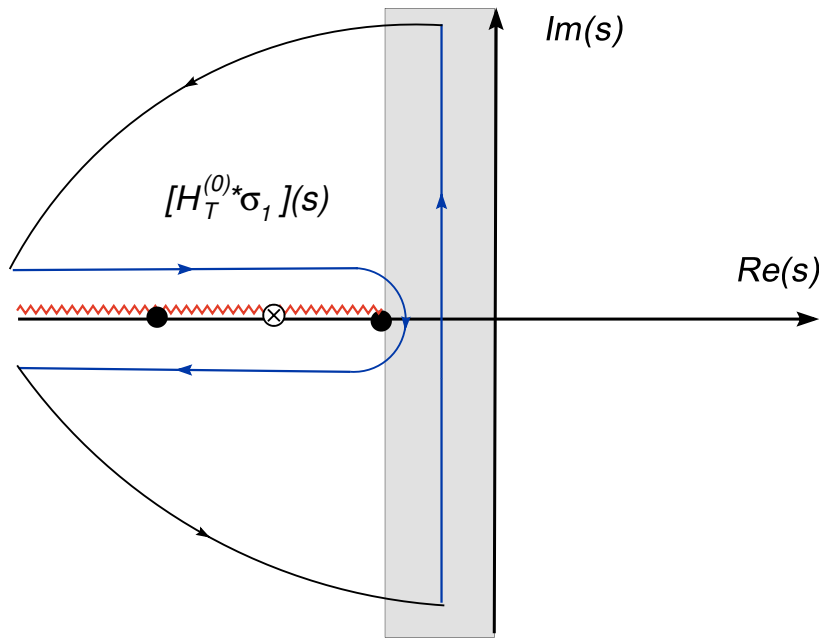
$$\int_{Q_0^2}^{Q^2} \frac{d^2 k_1}{k_1^4} f(x, k_1^2) \int_{Q_0^2}^{Q^2} \frac{d^2 k_2}{k_2^4} f(x, k_2^2) \Phi_4(\mathbf{k}_1, \mathbf{k}_2, Q) \sim \alpha_s^2(Q^2) [xg(x, Q^2)]^2 \log(Q^2/Q_0^2)$$

Mellin structure of DGLAP improved saturation model

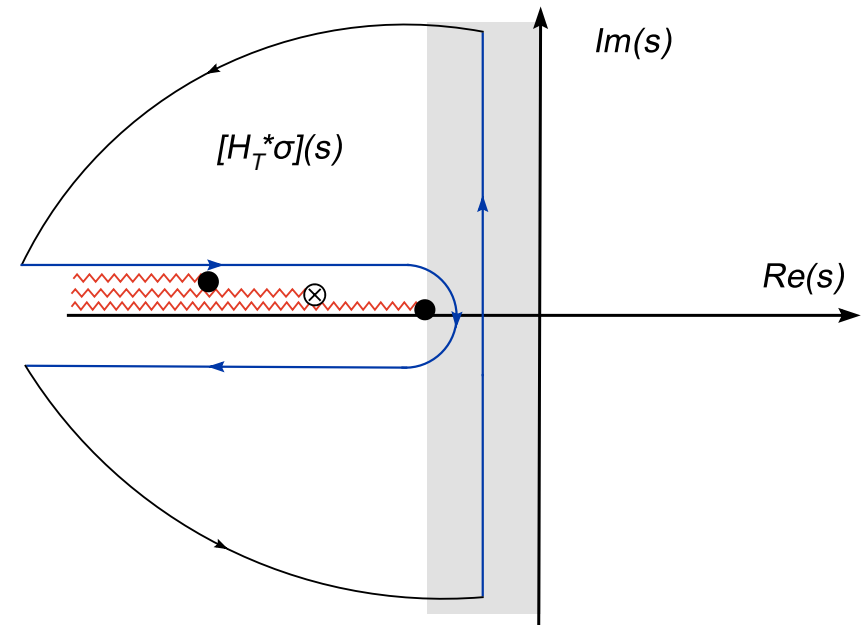
Mellin transform of perturbative part of dipole cross section – term by term

$$\mathcal{M}[\sigma](x, s) = \mathcal{M} \left[\sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \sigma_1^n \right] (x, s) = \sum_{n=1}^{\infty} \mathcal{M}[\sigma_n](x, s)$$

$$\mathcal{M}_{r,2}[\sigma_1^n](x, s) \propto \mathcal{M}_{\mu,2}[(\alpha_s^n (xg)^n)](x, s + n)$$



Single ladder exchanges



Multiple ladder exchange

Generic features on twists 2 and 4 in F_2 , F_T and F_L

Key information: twist 2 and twist 4 poles of the box diagram

$$\tilde{\Phi}_T(\gamma) \sim \frac{+a_T^{(2)}}{(\gamma-1)^2} \Rightarrow \sigma_T^{(2)} \sim \frac{a_T^{(2)}}{Q^2} \sum_f e_f^2 \frac{\alpha_{em}}{\pi} \int_{Q_0^2}^{Q^2} \frac{dQ'^2}{Q'^2} \alpha_s(Q'^2) xg(x, Q'^2)$$

$$\tilde{\Phi}_L(\gamma) \sim \frac{+b_L^{(2)}}{\gamma-1} \Rightarrow \sigma_L^{(2)} \sim \frac{b_L^{(2)}}{Q^2} \sum_f e_f^2 \frac{\alpha_{em}}{\pi} \alpha_s(Q^2) xg(x, Q^2)$$

$$\tilde{\Phi}_T(\gamma) \sim \frac{+b_T^{(4)}}{\gamma-2} \Rightarrow \sigma_T^{(4)} \sim \frac{+b_T^{(4)}}{Q^4} \sum_f e_f^2 \frac{\alpha_{em}}{\pi} [\alpha_s(Q^2) xg(x, Q^2)]^2$$

$$\tilde{\Phi}_L(\gamma) \sim \frac{-a_L^{(4)}}{(\gamma-2)^2} \Rightarrow \sigma_L^{(4)} \sim \frac{-a_L^{(4)}}{Q^4} \sum_f e_f^2 \frac{\alpha_{em}}{\pi} \int_{Q_0^2}^{Q^2} \frac{dQ'^2}{Q'^2} [\alpha_s(Q'^2) xg(x, Q'^2)]^2$$

F_T : twist-2: $\alpha_s x^{-\lambda} \log(Q^2)/Q^2$ — large, twist-4 : $\alpha_s^2 x^{-2\lambda}/Q^4$ — suppressed correction

F_L : twist-2: $\alpha_s x^{-\lambda}/Q^2$ — small, twist-4: $-\alpha_s^2 x^{-2\lambda} \log(Q^2)/Q^4$ — enhanced correction

F_2 : twist-2: $\alpha_s x^{-\lambda} \log(Q^2)/Q^2$

F_2 : twist-4 : $[b_T^{(4)} - a_T^{(4)} \log(Q^2)] \alpha_s^2 x^{-2\lambda}/Q^4$ — correction suppressed by the sign structure

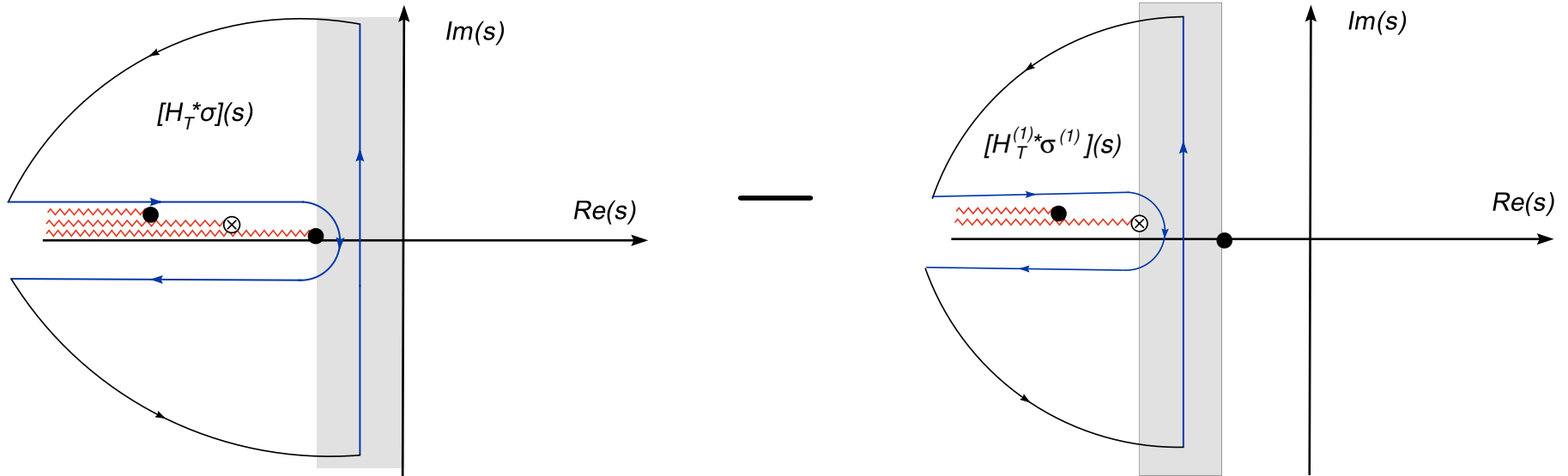
Direct twist extraction in position space: B-GB-K

We define series of subtracted functions

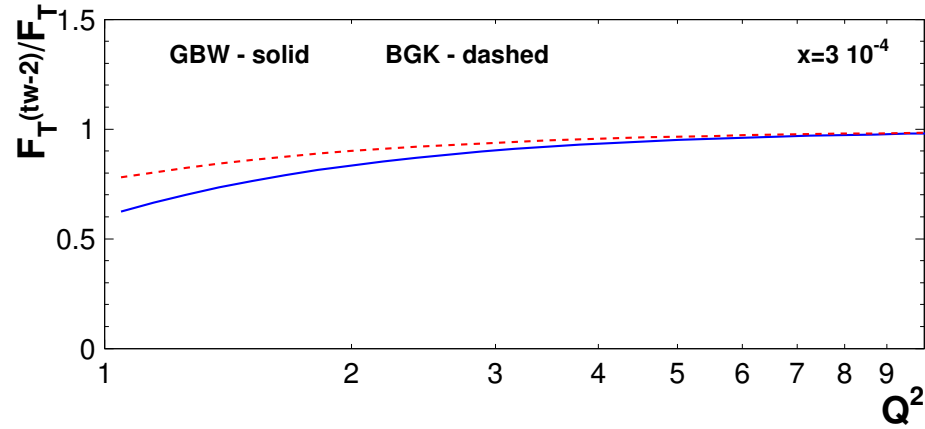
$$\sigma^{(n)}(x, r^2) = \sigma_d(x, r^2) - \sum_{k=1}^n \sigma_k(x, r^2)$$

$$H_{T,L}^{(n)}(Q^2 r^2) = H_{T,L}(Q^2 r^2) - \sum_{k=1}^n \frac{h_k^{T,L}}{(Q^2 r^2)^k},$$

$$\Delta^{(n)}[\sigma H_{T,L}] = \int_{\mathcal{C}_s^{(n-1)}} \frac{ds}{2\pi i} \tilde{\sigma}^{(n-1)}(x, s) \tilde{H}_{T,L}^{(n-1)}(-s, Q^2) - \int_{\mathcal{C}_s^{(n)}} \frac{ds}{2\pi i} \tilde{\sigma}^{(n)}(x, s) \tilde{H}_{T,L}^{(n)}(-s, Q^2),$$

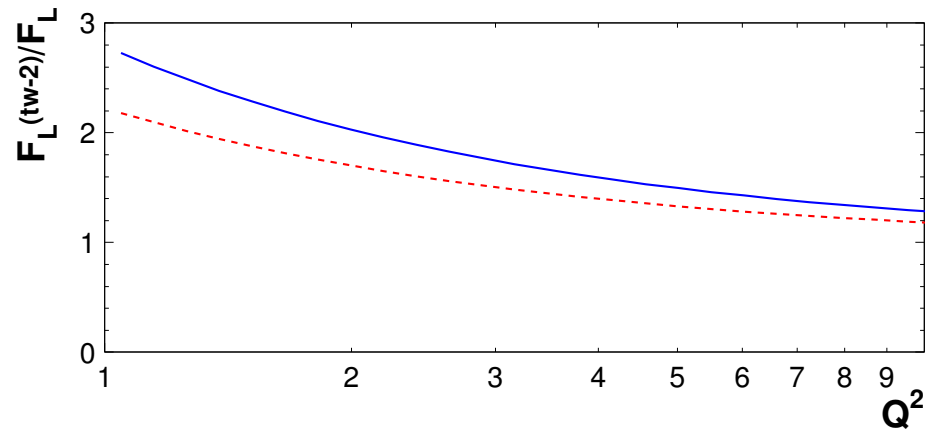


Twist ratios: tw-2/exact

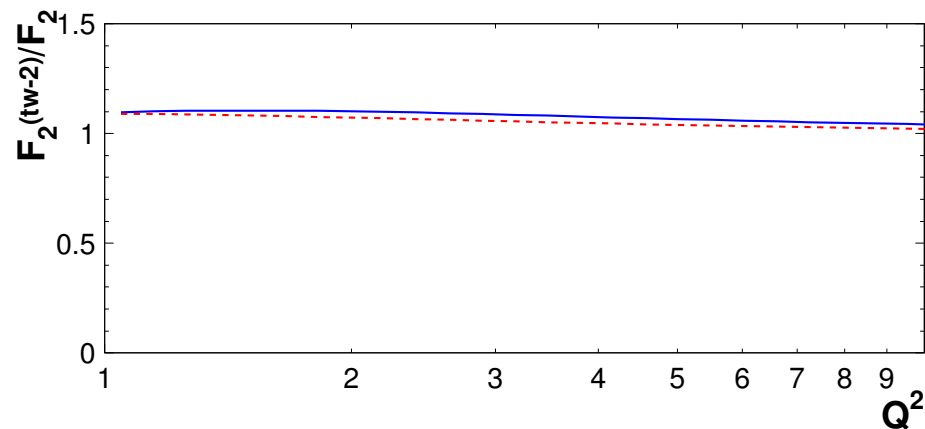


Higher twist contribution at
 $x = 3 \cdot 10^{-4}$ and
 $Q^2 = 10 \text{ GeV}^2$:

$$F_T: \sim 1\%$$

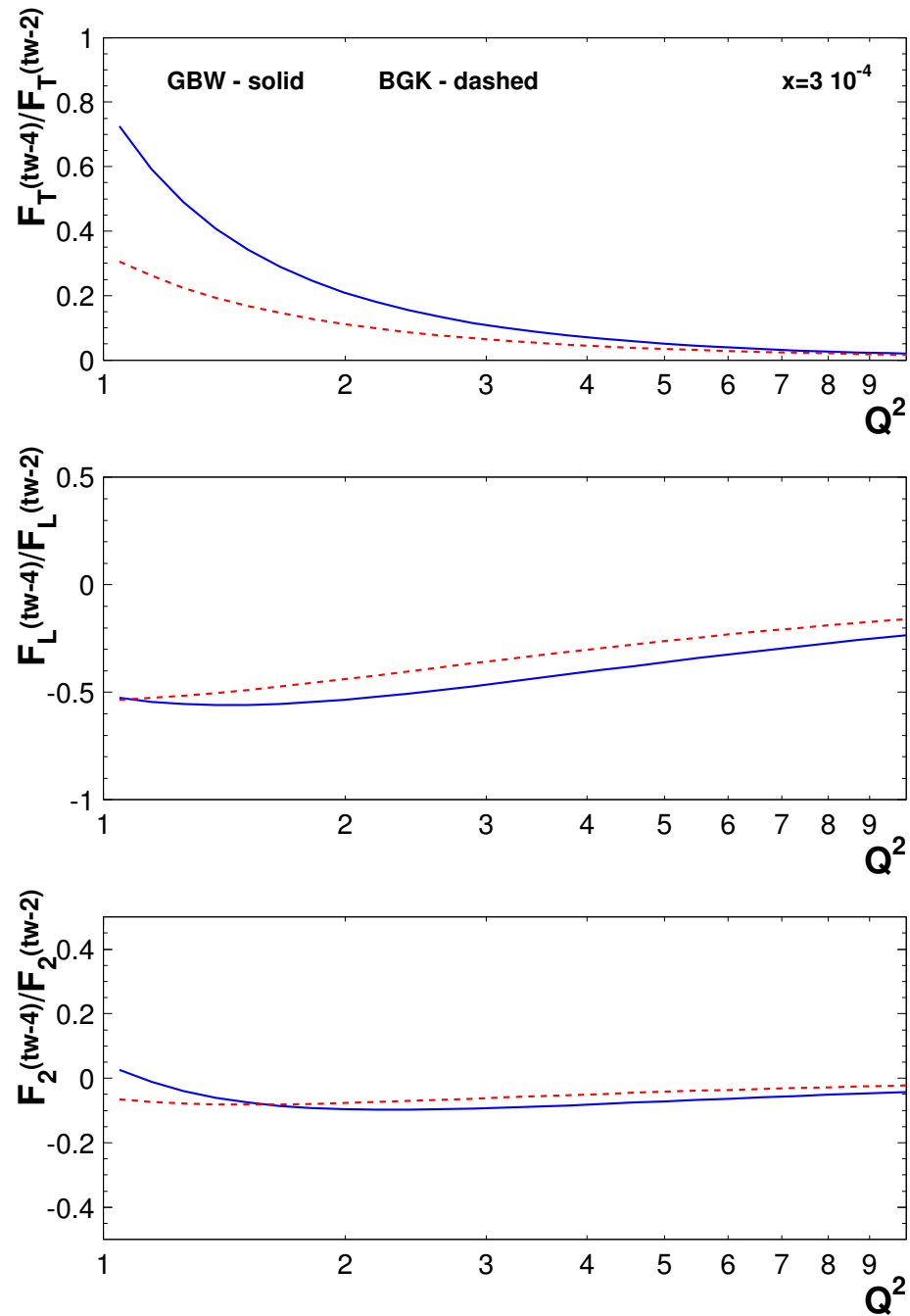


$$F_L: \sim 20\%$$



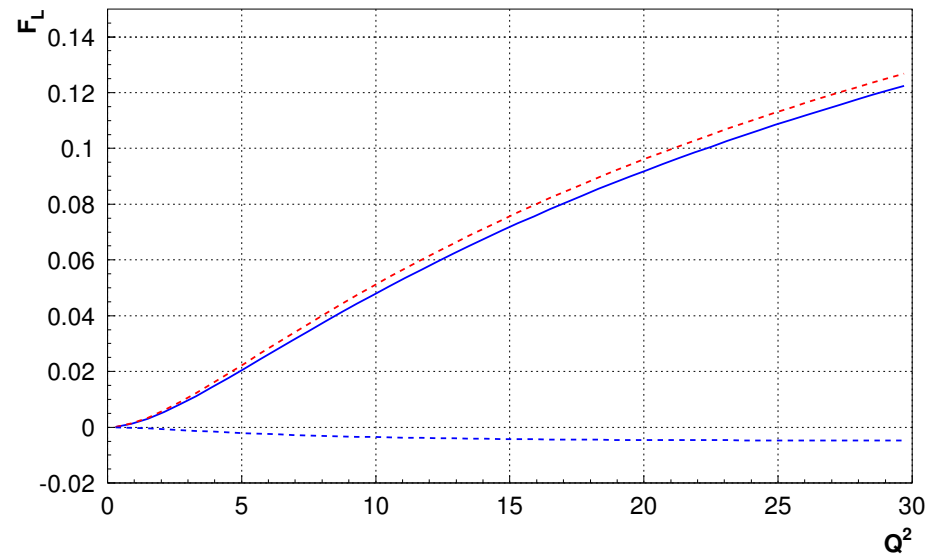
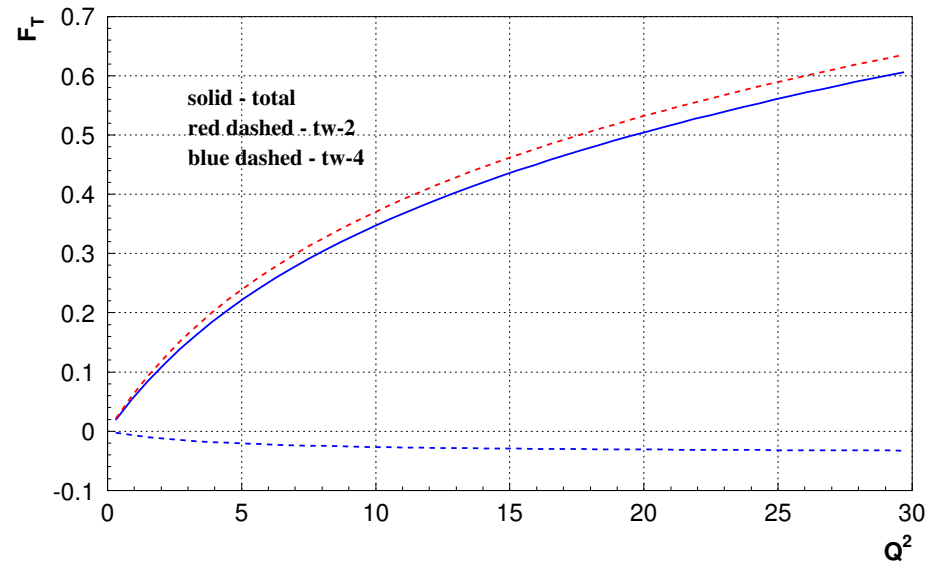
$$F_2: \sim 1\%$$

Twist ratios: tw-4/tw-2



Charm contribution

Charm sf



Summary

- Approximations leading from evolution of higher twist gluon correlators to DGLAP improved saturation model were described
- Twist content of saturation model was analyzed
- Twist 2 and twist 4 contributions to F_L and F_T following from saturation model were explicitly determined
- Twist-4 relative corrections at small x were found to be enhanced for F_L and suppressed for F_T and F_2 . This conclusion is generic and does not rely on model details
- Saturation model gives surprisingly fine cancellations of higher twist corrections in F_2 at all Q^2 : higher twist effects in F_2 found to be $\mathcal{O}(5\%)$ down to $Q^2 = 1 \text{ GeV}^2$!

THANKS

BACKUP

QCD decomposition of B-GB-K

At twist $2n$, n -ladder exchange:

$$\frac{\Delta_{ab}\sigma^{\tau=2n}(x, Q^2)}{\sigma_0} = \frac{1}{R^{2n}} \int_{c_\omega} \frac{d\omega}{2\pi i} \tilde{G}_{n,0}(\omega) \int_{c_s^{(n)}} \frac{ds}{2\pi i} [s+n]^{-\tilde{p}(\omega)} \tilde{H}_{T,L}(-s, Q^2),$$

LL twist- $2n$ evolution (gluon to quark splitting): $\tilde{\sigma}_n(s)a_n/(s+n)$

$$\Delta_a\sigma^{\tau=2n}(x, Q^2) \simeq \sigma_0 \frac{a_n}{(Q^2 R^2)^n} \int_{Q_0^2}^{Q^2} \frac{dQ'^2}{Q'^2} [\alpha_s(Q'^2) xg(x, Q'^2)]^n$$

NLO corrections to twist- $2n$ coefficient function of kinematic origin: $\tilde{\sigma}_n(s)[b_n + c_n(s+n) + \dots]$

$$\Delta_b\sigma^{\tau=2n}(x, Q^2) = \sigma_0 \frac{b_n}{(Q^2 R^2)^n} [\alpha_s(Q^2) xg(x, Q^2)]^n (1 + \mathcal{O}(\alpha_s))$$

In addition: soft contribution: *twist- $2n$ quark input* from higher scatterings: $\tilde{\sigma}^{(n)}(s)a_n/(s+n)$

$$\Delta_c\sigma^{\tau=2n}(x, Q^2) = \int_{c_s} \frac{ds}{2\pi i} \tilde{\sigma}^{(m>n)}(s) \frac{a_n}{s+n} (Q^2)^s$$