Formation of Longitudinal Fields in Eikonal Approximation

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Introduction

- Eikonal approximation has been used in the study of hadronic scattering at high energies.
- So it is natural to ask whether formation of the longitudinal fields is taken into account in the eikonal approximation, or whether it is pure non eikonal effect.
- We will answer this question in two cases:
  - in symmetric frame
  - in asymmetric frame.

Formation of Longitudinal Fields in Symmetric Frame

Consider a symmetric situation, such as scattering of two small or two large objects.

In Lagrangian formulation this has been done for "p-p" and "A-A" scattering.

[I. Balitsky, Phys.Rev.D70:114030,2004; Phys.Rev.D72:074027,2005; Nucl.Phys.Proc.Suppl.152:275-278,2006.]

[L. McLerran, A. Kovner and H. Weigert, Phys.Rev.D52:6231-6237,1995; Phys.Rev.D52:3809-3814,1995.]

[T.Lappi and L. McLerran, Nucl.Phys.A772:200-212,2006.]

We will consider the problem in Hamiltonian formulation.

[T.Altinoluk, A.Kovner and J.Peressutti, Nucl.Phys.A818:232-245,2009]

Using the symmetric gauge

$$A_0 = 0$$

impose a subsidiary gauge fixing condition

$$A_3(\mathbf{x}, t = 0) = 0 \tag{1}$$

then QCD Hamiltonian can be written as

$$\mathcal{H} = \frac{1}{2}(\Pi^i)^2 + \frac{1}{2}(\Pi^3)^2 + \frac{1}{4}(F_{ij})^2 + \frac{1}{2}(F_{i3})^2 \tag{2}$$

We are intersted in the situation where one large object with large momentum moves to the right and another moves to the left. So we split the vector potential A into the sum of three fields

$$A = B + C + \widetilde{A} \tag{3}$$

where

$$B(\mathbf{x}) = \int_{\Lambda}^{\infty} \frac{d^3k}{2\pi} \frac{1}{\sqrt{2\omega_k}} \left[a(k)e^{ikx} + a^{\dagger}(k)e^{-ikx}\right] \tag{4}$$

$$C(\mathbf{x}) = \int_{-\infty}^{-\Lambda} \frac{d^3k}{2\pi} \frac{1}{\sqrt{2\omega_k}} \left[a(k)e^{ikx} + a^{\dagger}(k)e^{-ikx}\right] \tag{5}$$

$$\widetilde{A}(\mathbf{x}) = \int_{-\Lambda}^{\Lambda} \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_{\mathbf{k}}}} \left[a(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}} + a^{\dagger}(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}\right] \tag{6}$$

The momenta in B and C are much larger than Λ, we can approximate $\omega_k \approx k_3$.

$$B(\mathbf{x}) = \int_{\Lambda}^{\infty} \frac{d^3k}{2\pi} \frac{1}{\sqrt{2|k_3|}} \left[a(k)e^{ikx} + a^{\dagger}(k)e^{-ikx}\right] \quad (7)$$

$$C(\mathbf{x}) = \int_{-\infty}^{-\Lambda} \frac{d^3k}{2\pi} \frac{1}{\sqrt{2|k_3|}} \left[a(k)e^{ikx} + a^{\dagger}(k)e^{-ikx}\right] \quad (8)$$

$$\widetilde{A}(\mathbf{x}) = \int_{-\Lambda}^{\Lambda} \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_{\mathbf{k}}}} \left[a(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}} + a^{\dagger}(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}\right] \quad (9)$$

We are interested in the "classical" situation where the number of gluons in each of the incoming hadrons is of order $\frac{1}{g^2}$.

Our parametric counting is defined as

$$\int_{-\Lambda}^{\infty} dk_3 d^2 k_\perp a^\dagger(k) a(k) \sim \int_{-\infty}^{\Lambda} dk_3 d^2 k_\perp a^\dagger(k) a(k) \sim \frac{1}{g^2} \tag{10}$$

Assuming that the integral over longitudinal momenta is saturated on some large momentum scale κ, we have

$$a(k) \sim a^\dagger(k) \sim \frac{1}{\sqrt{\kappa}} \frac{1}{g k_\perp} \tag{11}$$

Using this counting and keeping only the leading order and the first subleading terms in the Hamiltonian, we have

$$H = H_0 + H_1 \tag{12}$$

with

$$H_0 = \int d^3x \left[(\partial_3 B)^2 + (\partial_3 C)^2 \right] \tag{13}$$

and

$$\begin{aligned} H_1 &= \frac{1}{2}(\Pi^i)^2 + \frac{1}{4}(F_{ij})^2 + \frac{1}{2}(\partial_3 A_i)^2 \\ &+ \frac{1}{2}\left[\frac{1}{\partial_3}\left(D_i\Pi_i + J_a^+ + J_a^-\right)\right]^2 \end{aligned} \tag{14}$$

The residual Gauss's law becomes

$$\int dx_3 D_i \Pi_i + J^+ + J^- = 0 \tag{15}$$

with

$$J_a^+(x_i) = g \int dx_3 f^{abc} \partial_3 B_i^b B_i^c; \qquad J_a^-(x_i) = g \int dx_3 f^{abc} \partial_3 C_i^b C_i^c \tag{16}$$

In the interaction picture the Hamiltonian is

$$H_I = \frac{1}{2}\int_x \left\{ \left[\frac{1}{\partial_3}\left(D_i \cdot \Pi_i + \delta(x-t)J^+ + \delta(x+t)J^-\right)\right]^2 + \Pi_i^2 + \frac{1}{2}F_{ij}^2 + (\partial_3 A_i)^2 \right\} \tag{17}$$

The equations of motion that follow from the Hamiltonian are

$$\dot{A}_i^a = \left[D_i \frac{1}{\partial_3^2}\left(D \cdot \Pi + \delta(x-t)J^+ + \delta(x+t)J^-\right)\right]^a + \Pi_i^a \tag{18}$$

$$\begin{aligned} \dot{\Pi}_i^a &= +\left(D_j F_{ji}\right)^a + \partial_3^2 A_i^a \\ &- g f^{abc} \Pi_i^b \frac{1}{\partial_3^2}\left[D \cdot \Pi + \delta(x-t)J^+ + \delta(x+t)J^-\right]^c \end{aligned} \tag{19}$$

A solution for $t < 0$ is

$$A^i_{0a} = -\theta(-x+t)b_a^{+i} - \theta(x+t)b_a^{-i} \quad (20)$$
$$\Pi^i_{0a} = -\delta(-x+t)b_a^{+i} - \delta(x+t)b_a^{-i} \quad (21)$$

with the classical field $b_i^{\pm}$ defined by

$$\partial_i b_i^+ = J^+ \; ; \qquad \partial_i b_i^- = J^-$$
$$F^{ij}(b^+) = F^{ij}(b^-) = 0 \quad (22)$$

Finite contribution to the solutions short time after the collision can be given by $\theta(t)$ type terms in A and $\delta(t)$ type terms in Π.

For arbitrary time t, assume a solution of the form

$$A_i^a(x,t) = A_{0i}^a(x,t) + \delta A_i^a \theta(t) \quad (23)$$
$$\Pi_i^a(x,t) = \Pi_{0i}^a(x,t) + \delta \Pi_i^a \theta(t) \quad (24)$$

One can check equations of motion that electromagnetic fields change smoothly-no jumps in the field at $t = 0$, i.e.

δA has no $\theta(x_3 \pm t)$ type terms and $\delta\Pi$ has no $\delta(x_3 \pm t)$ type terms.

So there are no finite contributions coming from the corrections and therefore

$$A^i_{0a} = -\theta(-x + t)b^{+i}_a - \theta(x + t)b^{-i}_a \quad (25)$$
$$\Pi^i_{0a} = -\delta(-x + t)b^{+i}_a - \delta(x + t)b^{-i}_a \quad (26)$$

is valid for arbitrary time t.

The solutions of the classical equations after collision contain longitudinal electric and magnetic fields.

Short time after the collision the longitudinal fields are given as

$$\begin{aligned} E_3^a &= \frac{1}{\partial_3}\left[D\cdot\Pi + \delta(x_3 - t)J^+ + \delta(x_3 + t)J^-\right] \\ &= -gf^{abc}b_i^{+b}(x)b_i^{-c}(x)\theta(x_3+t)\theta(-x_3+t)\theta(t) \end{aligned} \quad (27)$$

$$B_3^a = \frac{1}{2}\epsilon_{ij}F_{ij}^a = -gf^{abc}b_i^{+b}(x)b_j^{-c}(x)\theta(x_3+t)\theta(-x_3+t)\theta(t) \quad (28)$$

After the collision, the space between receding fast particles is filled with the longitudinal fields.

Formation of Longitudinal Fields in Asymmetric Frame

Consider the frame where eikonal approximation is formulated. In this frame only the projectile is fast moving, target is a distribution of static color fields.

The wavefunction of the right moving projectile is

$$|\psi\rangle_{in} = \Omega[a, a^\dagger, J^+]|\psi\rangle_{J^+}$$

$|\psi\rangle_{J^+}$ is the wave function that only depends on the valance charge density.

Soft gluon degrees of freedom a and $a^\dagger$ are anologous to soft field A.

Charge density J^+ is due to fast modes and analogous to field B.

No dynamical modes C as they are represented by the distribution of static color fields.

Ω is a Bogoliubov type operator but for simplicity consider the classical approximation where it reduces to a coherent operator

$$\Omega = exp\left\{ i \int d\eta d^2 x b_i^a(x) \left[a_i^a(\eta, x) + a_i^{\dagger a}(\eta, x) \right] \right\}$$

where classical field b satisfies

$$\partial_i b_i^a(x) = J^+(x)$$

$$\partial_i b_j^a - \partial_j b_i^a - g f^{abc} b_i^b(x) b_j^c(x) = 0$$

Ω acting on the soft gluon vacuum creates soft fields of the form

$$A_i^a = -b_i^a(x)\theta(-x^-)$$

Transverse electromagnetic field is

$$F_a^{+i} = b_a^i(x)\delta(x^-)$$

Longitudinal electric field is

$$E_3 = \frac{1}{\partial^+}\left[D_i F^{+i} - J^+\delta(x^-)\right]$$

before the collision

- transverse field equation + $\partial_i b_i^a(x) = J^+(x) \Rightarrow E_3 = 0.$
- b_i is a 2 dimensional pure gauge $\Rightarrow B_3 = 0.$

After the Collision

The projectile emerges from the interaction region with the wavefunction

$$|\psi\rangle_{out} = \Omega[Sa, Sa^\dagger, SJ^+]|\psi\rangle_{SJ^+}$$

where the single gluon S-matrix S is expressed in terms of the target field

$$S(x) = exp\left\{i\int dx^- T^a\alpha^a\right\}$$

and

$$\Omega[Sa, Sa^\dagger, SJ^+] = exp\left\{i\int d\eta d^2x\bar{b}_i^a(x)\left[a_i^a(\eta,x) + a_i^{\dagger a}(\eta,x)\right]\right\}$$

with $\bar{b}_i^a = S^\dagger b_i^a[SJ^+]$.

The transformed operator Ω now creates the transverse field

$$F_a^{+i} = \bar{b}_i^a \delta(x^-)$$

Transverse fields accompany the rotated color charge density SJ^+ but

$$\partial \bar{b}_i \neq SJ^+$$

Longitudinal electric field is given by

$$E_3 = \frac{1}{\partial^+}\left[D_i F^{+i} - SJ^+ \delta(x^-)\right] = \theta(t - x_3)\partial_i \left(S^\dagger b_i[SJ^+] - b_i[SJ^+]\right) \neq 0$$

Longitudinal magnetic field is given by

$$F_{ij}^a = \left[\partial_i \bar{b}_j^a - \partial_j \bar{b}_i^a - g f^{abc} \bar{b}_i^b(x) \bar{b}_j^c(x)\right] \theta(t - x_3) \neq 0$$

Summary

Eikonal approximation does take into account formation of the longitudinal fields.

At the given impact parameter all the gluons of the projectile scatter by the same field thus acquire exactly the same phase and this is the reason of existence of longitudinal fields .