Heavy ion collisions and thermalization from AdS/CFT.

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Outline

- Introduction:

  - Heavy ion collisions and thermalization.

  - Basics of AdS/CFT.

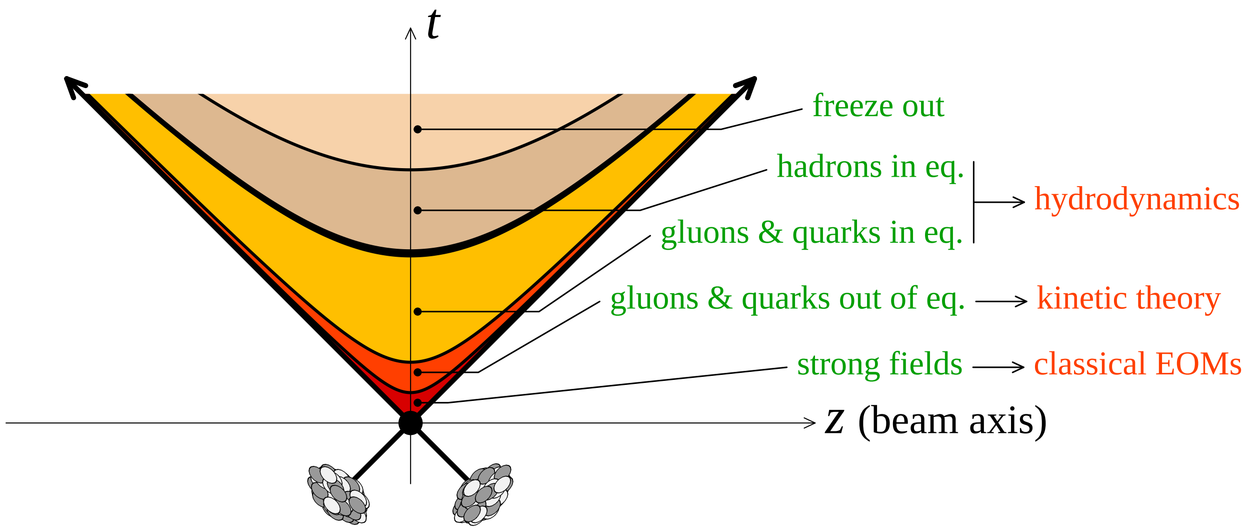
- Exact gravitational waves solutions in AdS_5 background, as dual model to ultrarelativistic nuclei.

  G. B., arXiv:0903.1047.

- Early time dynamics of heavy ion collisions from AdS/CFT.

  G. B., M. Heller, R. Janik, R. Peschanski, arXiv:0906.4423.

Motivation: heavy ion collisions



Early stages of the collision: Color Glass Condensate and Glasma → out-of-equilibrium and weakly coupled.

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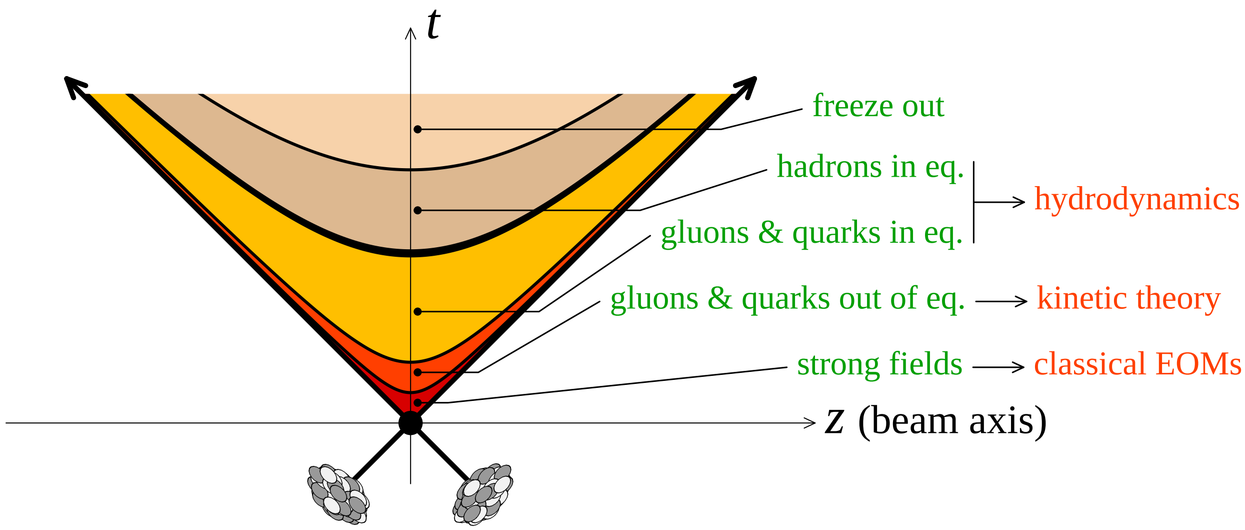

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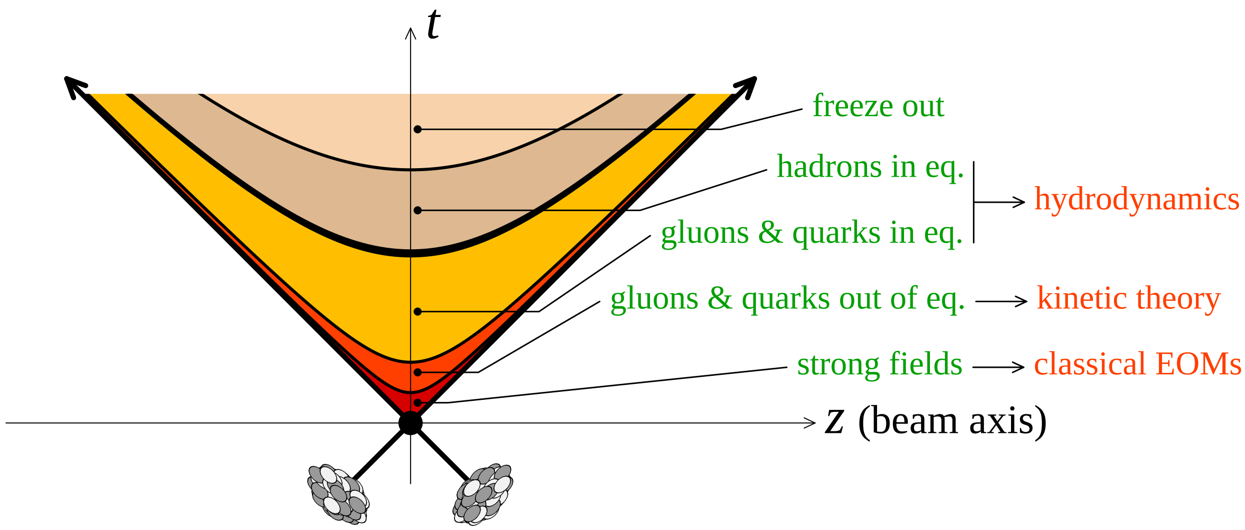

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Intermediate stage: thermalization. Weakly or strongly coupled dynamics?

A few words about AdS/CFT

Holographic duality between:

- Type IIB strings theory in $AdS_5 \times S_5$ background, and
- $\mathcal{N} = 4$ Super Yang-Mills theory in 4 dimensions,

with the relationships between the parameters:

$$G_N \propto \frac{1}{N_c^2}$$

$$\left(\frac{L}{l_s}\right)^4 = \lambda \equiv g_{YM}^2 N_c$$

Maldacena (1998)
Gubser, Klebanov, Polyakov (1998)
Witten (1998)

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Semiclassical strings	$\Leftrightarrow$	Planar limit $N_c \to \infty$
Supergravity	$\Leftrightarrow$	Strong coupling $\lambda \to \infty$

A few words about AdS/CFT

Allowing a perturbation of the AdS_5 metric:

$$ds^2 = \frac{L^2}{z^2}\left[dz^2 + (\eta_{\mu\nu} + h_{\mu\nu}(x^\rho, z))\, dx^\mu dx^\nu\right] \quad \text{with } z > 0$$

Duality: $h_{\mu\nu}$ Field $\Leftrightarrow$ $T_{\mu\nu}$ Operator of $\mathcal{N} = 4$ SYM

$$\langle e^{-i\int d^4x\; T_{\mu\nu} j^{\mu\nu}} \rangle_{\text{SYM}} = e^{iS_{5d\,grav}[h_{\mu\nu}]}$$

$h_{\mu\nu}$: classical solution, with the boundary condition $\lim_{z\to 0} h_{\mu\nu}(x^\rho, z) = j_{\mu\nu}(x^\rho)$.

How to study thermalization from AdS

Through AdS/CFT, $\mathcal{N}=4$ SYM at finite T is described by a black brane metric.

Generalization to local thermal equilibrium:
Ok, with a slowly evolving black brane.

Janik, Peschanski (2006)
Bhattacharyya, Hubeny , Minwalla, Rangamani (2008)

$\Rightarrow$ AdS dual of thermalization: collide two gravitational waves and look for black brane formation.

How to study thermalization from AdS

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  Previous attempts:
  Kajantie, Louko, Tahkokallio (2008)
  Grumiller, Romatschke (2008)
  Albacete, Kovchegov, Taliotis (2008)

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- Step 1: Model incoming nuclei by gravitational waves solutions.
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- Step 2: Find initial conditions at $\tau = 0^+$ from the superposition of the incoming waves.
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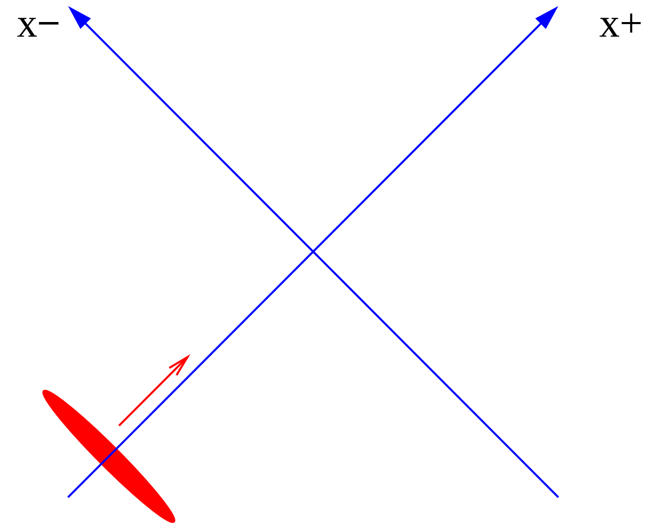
- Step 3: Calculate at $\tau > 0$ the relaxation towards a black brane metric.
  Analytical study:
  G. B., M. Heller, R. Janik, R. Peschanski, arXiv:0906.4423.
  Numerical study:
  Chesler, Yaffe (2009).

Ansatz



$\langle T_{\mu\nu} \rangle$ of an ultrarelativistic nucleus: dominated by $\langle T_{--} \rangle$

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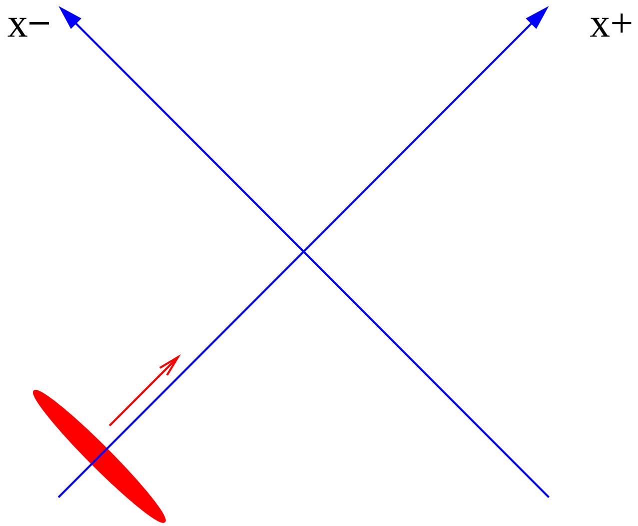

$\langle T_{\mu\nu}\rangle$ of an ultrarelativistic nucleus: dominated by $\langle T_{--}\rangle$

$\Rightarrow$ let us choose the Ansatz $h_{\mu\nu} = \delta_{\mu-}\,\delta_{\nu-}\,b(x^-, \mathbf{x}_\perp, z)$

$$ds^2 = \frac{L^2}{z^2}\left[dz^2 - 2dx^+dx^- + d\mathbf{x}_\perp{}^2 + b(x^-, \mathbf{x}_\perp, z)dx^{-\,2}\right]$$

AdS gravitational wave solutions

$$
\begin{aligned}
ds^2 &= G_{IJ}dx^I dx^J \\
&= \frac{L^2}{z^2}\left[dz^2 - 2dx^+dx^- + d\mathbf{x}_\perp{}^2 + b(x^-,\mathbf{x}_\perp,z)dx^{-2}\right]
\end{aligned}
$$

Here: only one non-trivial Einstein equation, which is linear!

$$
\partial_z^2 b(x^-,\mathbf{x}_\perp,z) - \frac{3}{z}\partial_z b(x^-,\mathbf{x}_\perp,z) + \Delta_\perp b(x^-,\mathbf{x}_\perp,z) = 0
$$

Similar to the result of Siklos (1985).

AdS gravitational wave solutions

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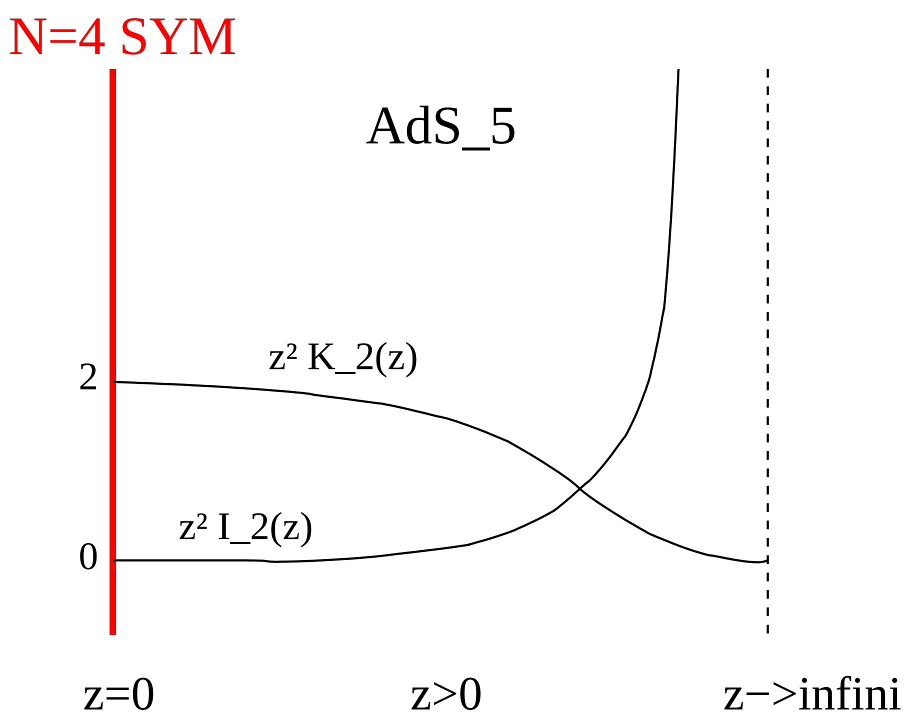
Transverse Fourier transform: $\mathbf{x}_\perp \mapsto \mathbf{k}_\perp$:

$\Rightarrow$ 2 solutions for each mode:

$$\tilde{b}(x^-,\mathbf{k}_\perp,z) = \tilde{c}_1(x^-,\mathbf{k}_\perp)\,\frac{k_\perp^2 z^2}{2}\,K_2(k_\perp z) + \tilde{c}_2(x^-,\mathbf{k}_\perp)\,\frac{z^2}{k_\perp^2}\,I_2(k_\perp z)$$

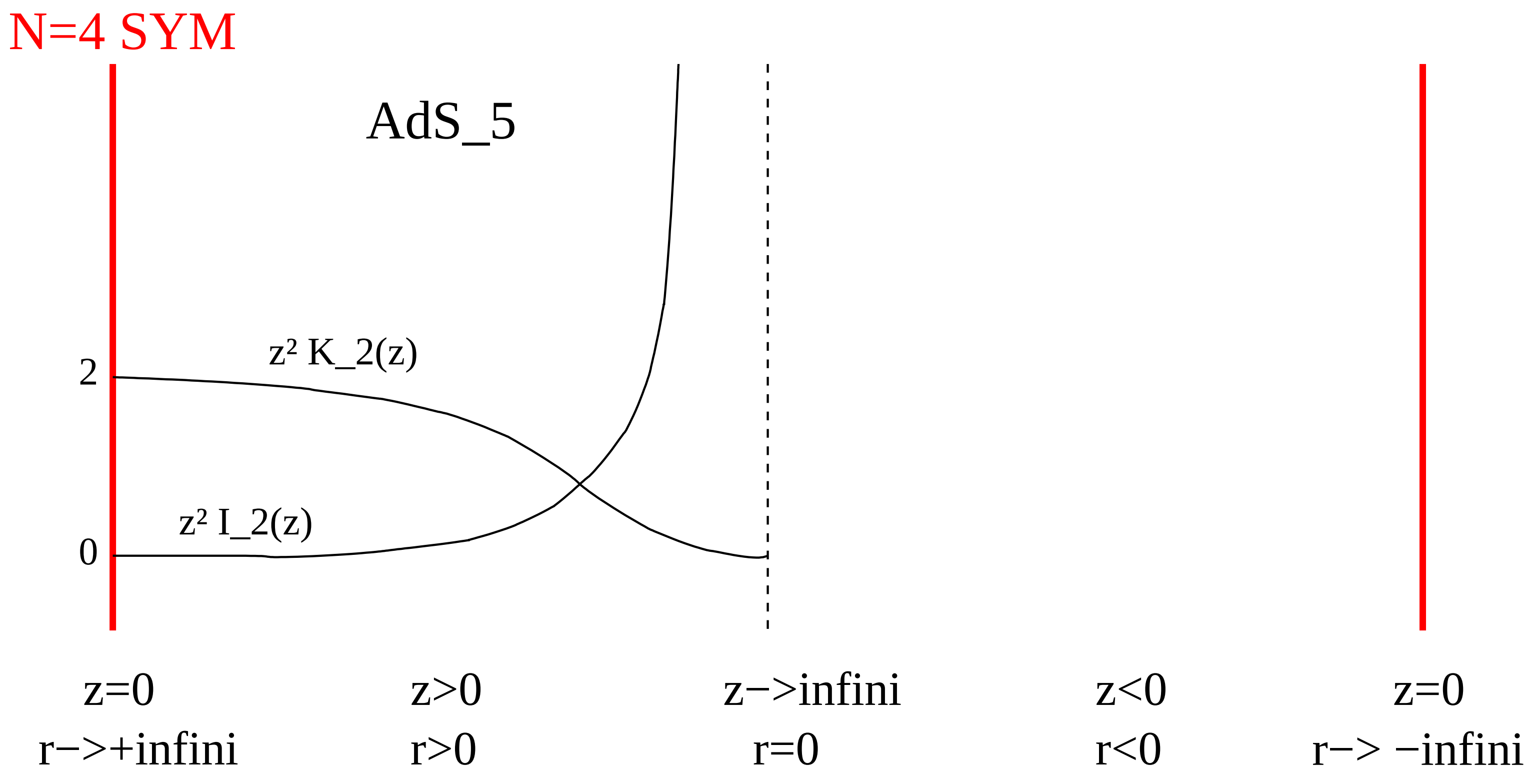
AdS gravitational wave solutions



Poincaré coordinates: only describe half of the AdS_5 space.

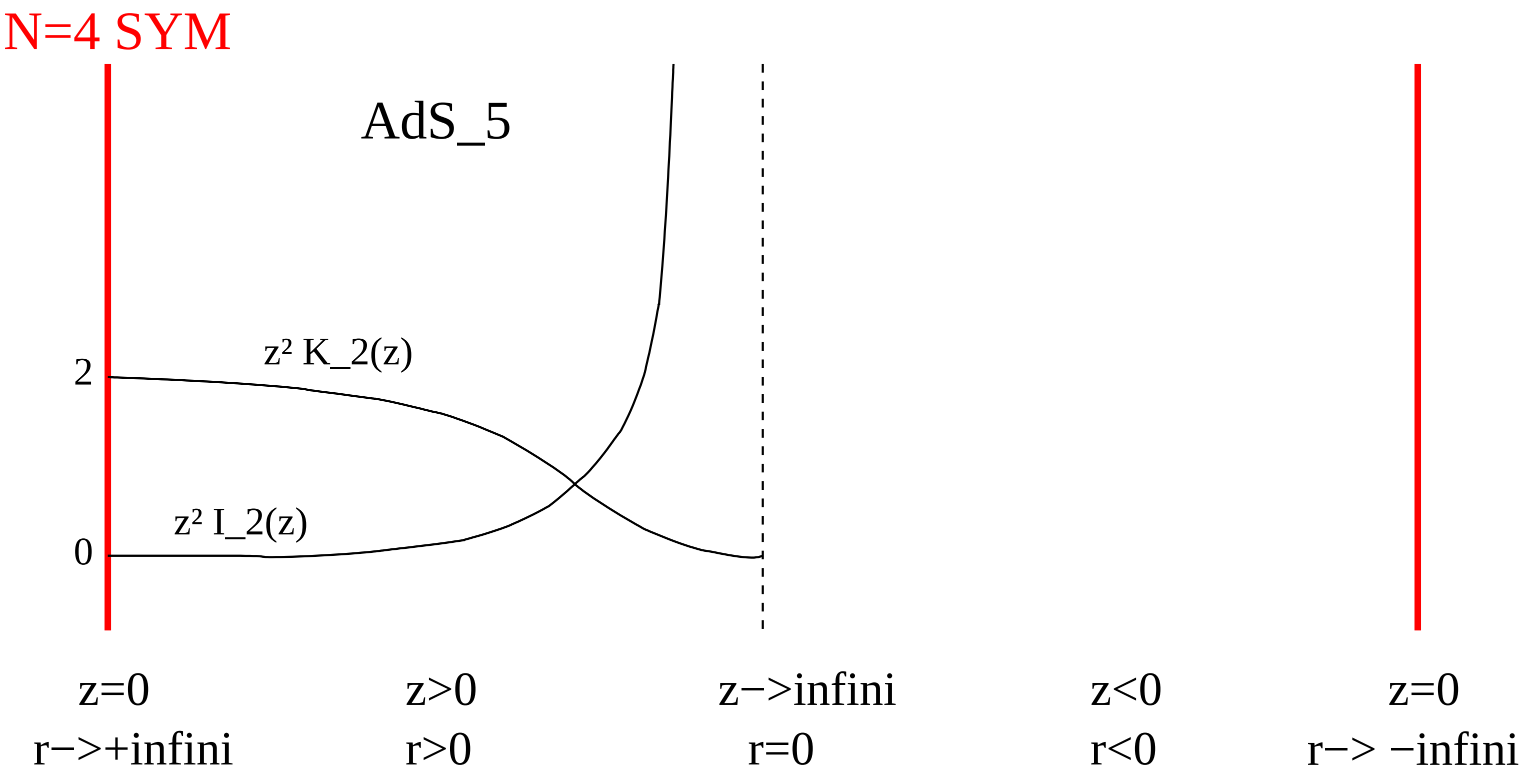
$$r \equiv \frac{L^2}{z}$$

AdS gravitational wave solutions



$\frac{z^2}{k_\perp^2}\ I_2(k_\perp z)$ solution $\Rightarrow$ 5d space-time with a pp curvature singularity at $z \to \infty$.

AdS gravitational wave solutions



Let us keep only the regular solution:

$$\tilde{b}(x^-, \mathbf{k}_\perp, z) = \tilde{c}_1(x^-, \mathbf{k}_\perp) \, \frac{k_\perp^2 z^2}{2} \, K_2(k_\perp z)$$

Analysis of the **metric**

Near the boundary, for $z \to 0$:

$$h_{\mu\nu}(x^\rho, z) = h_{(0)\mu\nu}(x^\rho) + z^2\, h_{(2)\mu\nu}(x^\rho) + z^4 \left[h_{(4)\mu\nu}(x^\rho) + \log\left(z^2 \mu_R^2\right) l_{(4)\mu\nu}(x^\rho)\right] + \mathcal{O}\left(z^6 \log(z^2)\right)$$

$$j_{\mu\nu} = h_{(0)\mu\nu}$$

$$\langle T_{\mu\nu}\rangle_j = \frac{L^3}{4\pi G_N}\left\{h_{(4)\mu\nu} + \text{ non-linear terms in } h_{(2)\mu\nu} \text{ and } h_{(0)\mu\nu}\right\}$$

de Haro, Skenderis, Solodukhin (2001)

Analysis of the metric

Near the boundary, for $z \to 0$:

$$\tilde{b}(x^-, \mathbf{k}_\perp, z) = \tilde{c}_1(x^-, \mathbf{k}_\perp) \Bigg[1 - \frac{k_\perp^2 z^2}{4} + \frac{k_\perp^4 z^4}{32} \left[-\log\left(\frac{k_\perp^2 z^2}{4}\right) + \frac{3}{2} - 2\gamma_E \right] + \mathcal{O}\left(z^6 \log(z^2)\right) \Bigg]$$

$$\tilde{j}_{--}(x^-, \mathbf{k}_\perp) = \tilde{c}_1(x^-, \mathbf{k}_\perp)$$

$$\langle \tilde{T}_{--}(x^-, \mathbf{k}_\perp) \rangle_j = \frac{N_c^2}{64\pi^2}\, k_\perp^4\, \log\left(\frac{\mu_R^2}{k_\perp^2}\right)\, \tilde{j}_{--}(x^-, \mathbf{k}_\perp)$$

Discussion

A distribution of sources $\tilde{j}_{--}(x^-, \mathbf{k}_\perp)$ gives:

- An exact gravitational wave in AdS_5 background:

$$\tilde{b}(x^-, \mathbf{k}_\perp, z) = \frac{k_\perp^2 z^2}{2} K_2(k_\perp z)\, \tilde{j}_{--}(x^-, \mathbf{k}_\perp)$$

- A response of the $\mathcal{N}=4$ SYM fields with an energy-momentum:

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One can tune $\tilde{j}_{--}$, in order to get a not so crazy $\langle \tilde{T}_{--}\rangle_j$ for an ultrarelavistic QCD nucleus.

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If one takes a random $\tilde{j}_{--}$: similarity with the CGC framework (more precisely: with the MV model).

Early time dynamics

G. B., M. Heller, R. Janik, R. Peschanski, arXiv:0906.4423.

Analytic study of the evolution after the collision (for proper time $\tau > 0$), for generic initial conditions at $\tau = 0$.

Assumptions:

- Homogeneity in the transverse plane.
- Boost invariance: space-time rapidity $=$ rapidity $= \eta$.

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$\Rightarrow$ Ansatz for the metric at $\tau > 0$

$$ds^2 = \frac{L^2}{z^2}\left[dz^2 - e^{a(\tau,z)}\, d^2\tau + e^{b(\tau,z)}\, \tau^2 d^2\eta + e^{c(\tau,z)}\, d\mathbf{x}_\perp{}^2\right],$$

with the boundary conditions for $z \to 0$:

$$a(\tau,z) \propto -z^4\, \epsilon(\tau)\,, \quad b(\tau,z) \propto z^4\, p_{||}(\tau)\,, \quad c(\tau,z) \propto z^4\, p_\perp(\tau)$$

Einstein equations

$$ds^2 = \frac{L^2}{z^2}\left[dz^2 - e^{a(\tau,z)}\, d^2\tau + e^{b(\tau,z)}\, \tau^2 d^2\eta + e^{c(\tau,z)}\, d\mathbf{x}_\perp{}^2\right],$$

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One gets 5 non-trivial Einstein equations: nonlinear, with up to 2nd derivative in τ and in z.

They are not all independent: one can reduce the system to three equations:

- 2 evolution equations, with only 1st derivative in τ.
  One of them: holographic counterpart of energy conservation.
- 1 constraint equation: ordinary differential equation in z.
  Holographic counterpart of $T^\mu_\mu = -\epsilon(\tau) + p_\|(\tau) + 2p_\perp(\tau) = 0$.

Constraint equation

From the constraint equation, one has the property:

At each value of τ, some of the metric coefficients have to vanish or to blow up at finite z.

$\Rightarrow$ presence of an horizon (already at $\tau = 0$) for all non-trivial physical solutions.

No threshold on black hole formation (*i.e.* plasma thermalization) in this geometry!

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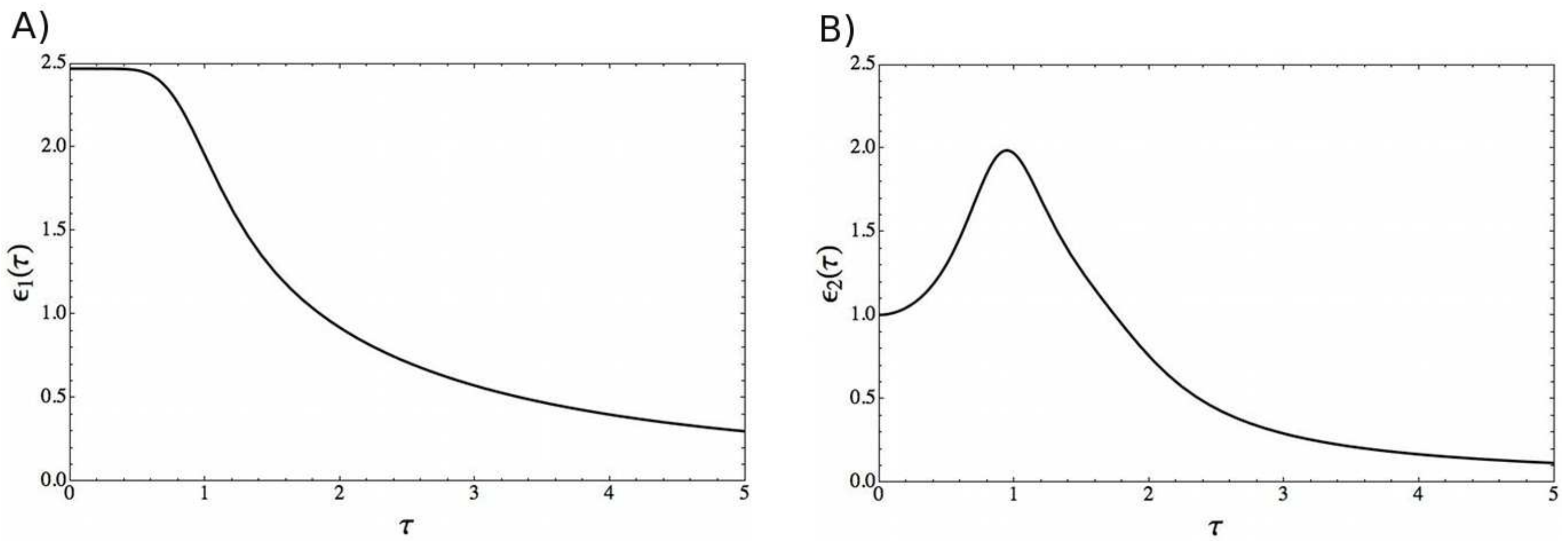
Is it an artifact due to the assumption of infinite transverse size and no impact parameter dependence?

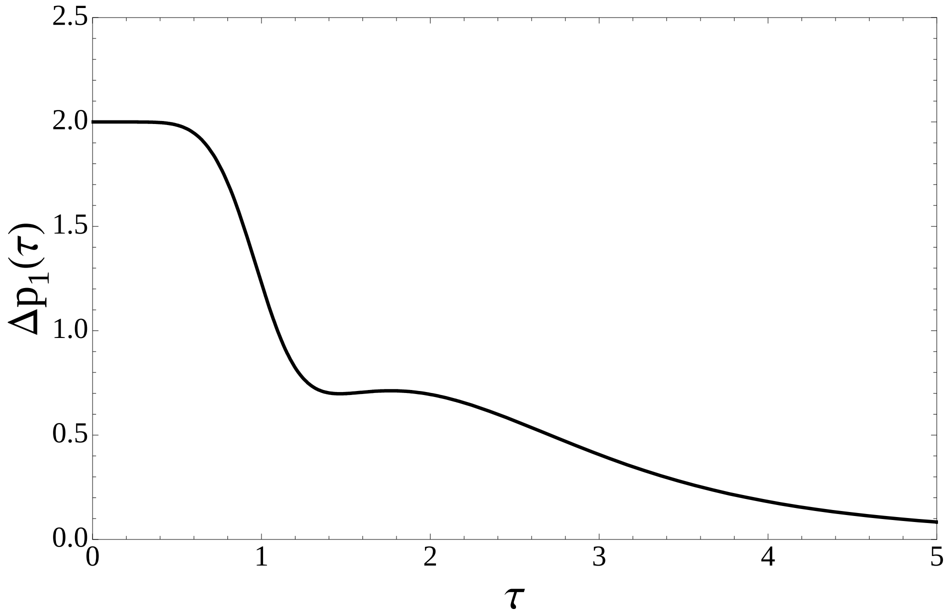
Examples of solution



Left: Solution (Padé) for $\epsilon(\tau)$, with the simplest initial condition solving the constraint equation.

Right: Other solution solving the constraint, but violating the $T_{\mu\nu}$ positivity condition.

Isotropization



Anisotropy of the pressure $\Delta p\,(\tau) = 1 - \frac{p_{\parallel}(\tau)}{p_{\perp}(\tau)}$ for the former solution.

$\Rightarrow$ Isotropization: onset of hydrodynamics.

Outlook

For $\tau = 0$: Initial condition given by the collision of 2 gravitational schockwaves. If possible: include arbitrary $\mathbf{x}_\perp$ dependance.

For $\tau > 0$: Understand better the dynamical horizon imposed by the constraint equation.

Relax homogeneity in the transverse plane.

Relax boost-invariance.

In general: Link with the CGC and Glasma physics?