AN EXACT HYDRODYNAMIC SOLUTION FOR THE ELLIPTIC FLOW

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Goal

- Understanding the dynamical mechanisms of elliptic flow and extraction of exact solutions

- Tools:

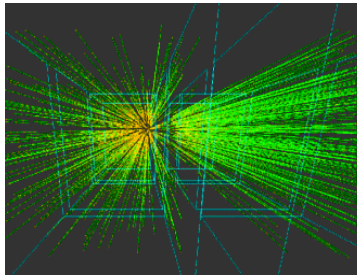
The formulation of the transverse flow in terms of a hydrodynamic potential

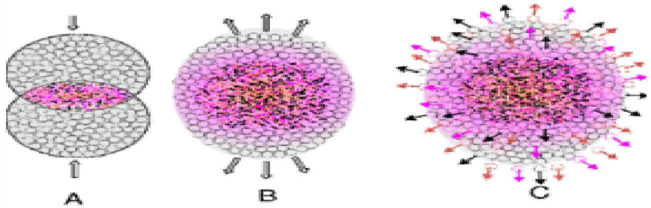
Talk Plan

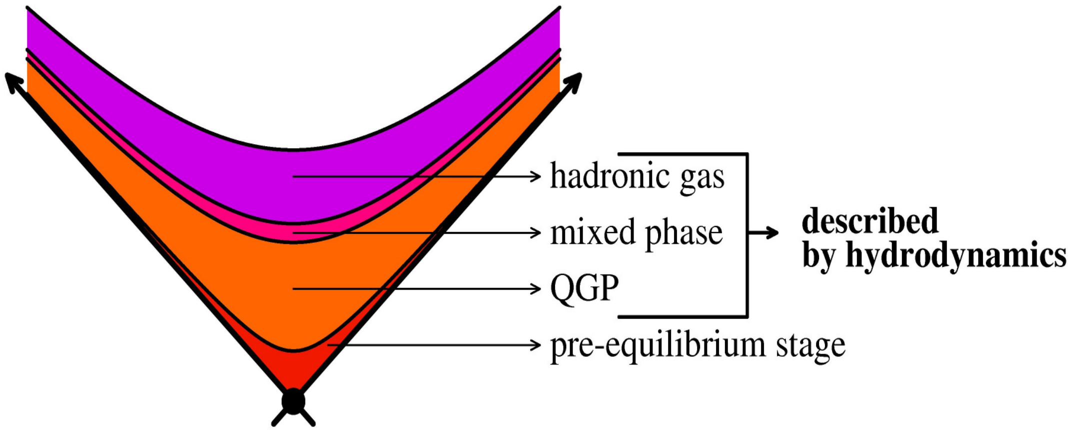
- 1) Introduction (why hydrodynamics?)
- 2) Assumptions and potential formulation of perfect fluid relativistic hydrodynamics
- 3) General and exact solution for spatial anisotropy and elliptic flow
- 4) Applications: Comparison with experiment and simulations
- 5) Assumptions Check
- 6) Conclusions-Prospects

Introduction

- Evidence that hydrodynamics of nearly perfect fluid may be relevant for the description of medium created in heavy-ion collisions







- Almost perfect fluid
  (low viscosity)

Assumptions

- 1) Transversally isentropic:

$$\partial_{x_1}\left(su_1\right)+\partial_{x_2}\left(su_2\right)=0$$

- 2) Quasi-stationary: Transverse flow smooth enough to be driven only by temperature change. Thus, Bernoulli equation:

$$Tu_0=T_0$$

Hydrodynamic Potential $\chi(l,\varphi)$

$$\partial_{x_1}\left(Tu_2\right)-\partial_{x_2}\left(Tu_1\right)=0$$

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- Hodograph Transform:

(kinematical variables) $\rightarrow$ (thermodyknamical variables)

$$(x_1, x_2) \rightarrow (l,\varphi)$$

$$l = \frac{1}{2}\log\left[1-\left(\frac{T}{T_0}\right)^2\right] = \frac{1}{2}\log\left[\frac{u_\perp^2}{1+u_\perp^2}\right] \qquad u_\perp^2 \equiv u_1^2 + u_2^2$$

$$\tan\varphi = \frac{u_2}{u_1}$$

Khalatnikov-Kamenshchik equation

$$\left(1-\frac{e^{2l}}{c_s^2}\right)\frac{\partial^2\chi}{\partial\varphi^2}+\left(1-e^{2l}\right)\frac{\partial^2\chi}{\partial l^2}+\left(1-\frac{1}{c_s^2}\right)e^{2l}\frac{\partial\chi}{\partial l}=0$$

- Solvable!

$$c_s^2=\frac{dp}{d\varepsilon}=\frac{sdT}{Tds}$$

Potential equation

$$\left(1-\frac{e^{2l}}{c_s^2}\right)\frac{\partial^2\chi}{\partial\varphi^2}+\left(1-e^{2l}\right)\frac{\partial^2\chi}{\partial l^2}+\left(1-\frac{1}{c_s^2}\right)e^{2l}\frac{\partial\chi}{\partial l}=0$$

- **Solvable!**

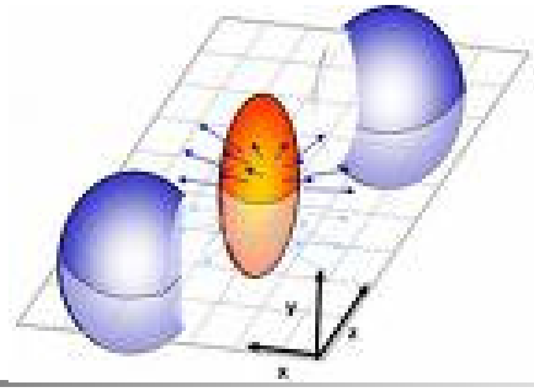
$$c_s^2=\frac{dp}{d\varepsilon}=\frac{sdT}{Tds}$$

$$x_1^2+x_2^2=\frac{e^{-2l}}{T_0^2}\left[\left(\frac{\partial\chi}{\partial l}\right)^2+\left(\frac{\partial\chi}{\partial\varphi}\right)^2\right]$$

$$\arctan\frac{x_2}{x_1}=\varphi+\arctan\left[\frac{\partial\chi}{\partial\varphi}\left(\frac{\partial\chi}{\partial l}\right)^{-1}\right]$$

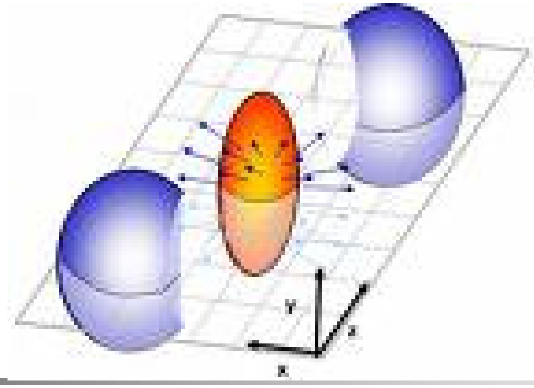
- (1+1)
  [Khalatnikov, 1954]
- Transverse (cosmology)
  [Khalatnikov-Kamenshchik, 2004]

Elliptic Flow



- Cause $\Rightarrow$ Effect

(Initial Spatial Anisotropy) (Entropy Anisotropy)



Elliptic Flow

- **Cause** $\Rightarrow$ **Effect**

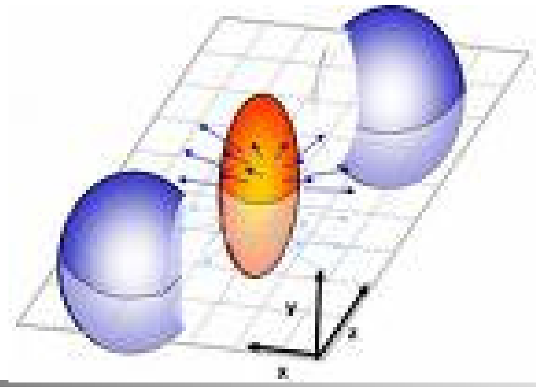
(Initial Spatial Anisotropy) (Entropy Anisotropy)

Eccentricity

$$\varepsilon = \frac{\langle x_2^2 - x_1^2 \rangle}{\langle x_2^2 + x_1^2 \rangle}$$

Elliptic Flow coefficient

$$v_2 = \frac{\int d\varphi \cos(2\varphi) \frac{dS}{d\varphi}}{\int d\varphi \frac{dS}{d\varphi}}$$



Elliptic Flow

- **Cause** $\Rightarrow$ **Effect**

(Initial Spatial Anisotropy) (Entropy Anisotropy)

Eccentricity

$$\varepsilon = \frac{\langle x_2^2 - x_1^2 \rangle}{\langle x_2^2 + x_1^2 \rangle}$$

$$\varepsilon = \frac{\int d\varphi \left\{ \cos 2\varphi \left[\left(\frac{\partial\chi}{\partial\varphi}\right)^2 - \left(\frac{\partial\chi}{\partial l}\right)^2 \right] + 2\sin 2\varphi \frac{\partial\chi}{\partial\varphi}\frac{\partial\chi}{\partial l} \right\}}{\int d\varphi \left[\left(\frac{\partial\chi}{\partial\varphi}\right)^2 + \left(\frac{\partial\chi}{\partial l}\right)^2 \right]}$$

Elliptic Flow coefficient

$$v_2 = \frac{\int d\varphi \cos(2\varphi) \frac{dS}{d\varphi}}{\int d\varphi \frac{dS}{d\varphi}}$$

$$\frac{dS}{d\varphi} = \frac{sT}{T_0\left(1 - e^{2l}/c_s^2\right)} \left[\frac{\partial\chi}{\partial l} - \frac{\partial^2\chi}{\partial l^2} \right]$$

[E.N.S, R.Peschanski '09]

General and exact solution

$$\chi(l,\varphi) = c_0\beta_0(l) + \sum_{p=1}^{\infty} \beta_p(l)\cos(2p\varphi)$$

$$\beta_0'(l) = \left(1 - e^{2l}\right)^{\frac{1-\frac{1}{c_s^2}}{2}}$$

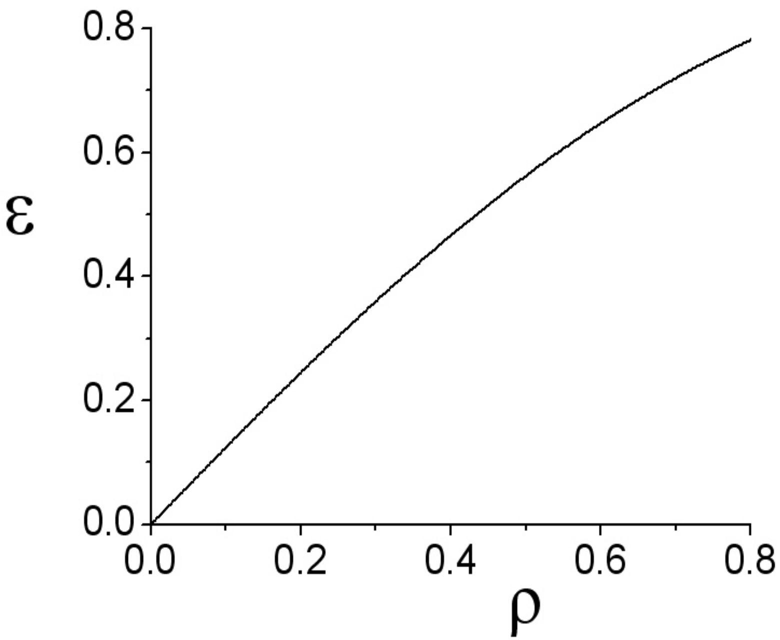
$$\beta_p(l) = c_p^{(1)}(-1)^{p+1}e^{2pl}\;{}_2F_1\left(p+\frac{1}{4}\left(c_s^{-2}-1\right)-\sqrt{\frac{\left(c_s^{-2}-1\right)^2}{16}+p^2c_s^{-2}},\, p+\frac{1}{4}\left(c_s^{-2}-1\right)+\sqrt{\frac{\left(c_s^{-2}-1\right)^2}{16}+p^2c_s^{-2}},1+2p;e^{2l}\right)+$$
$$+c_p^{(2)}\,G^{2,0}_{2,2}\left(e^{2l}\left|\begin{matrix}\frac{5-c_s^{-2}}{4}-\sqrt{\frac{\left(c_s^{-2}-1\right)^2}{16}+p^2c_s^{-2}}, & \frac{5-c_s^{-2}}{4}+\sqrt{\frac{\left(c_s^{-2}-1\right)^2}{16}+p^2c_s^{-2}} \\ -p & p\end{matrix}\right.\right)$$

$$\frac{c_1^{(1)}}{c_0} \equiv \rho$$

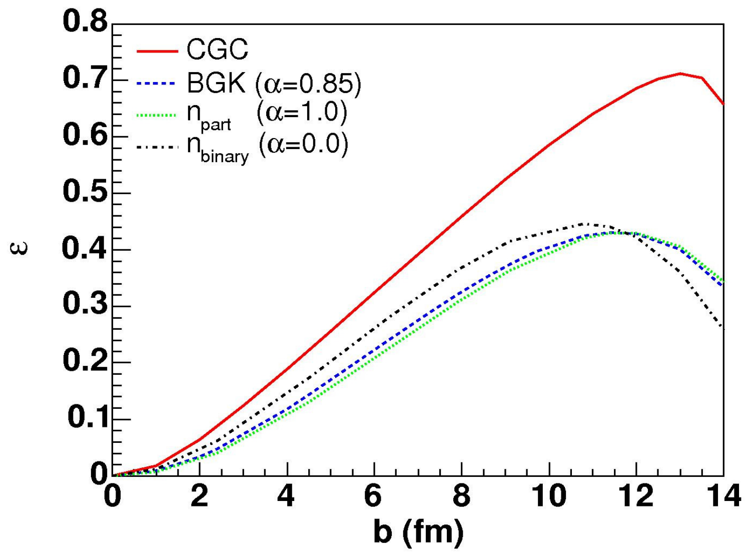
$$\frac{c_1^{(2)}}{c_1^{(1)}} \equiv \lambda$$

Centrality dependence ('Callibration')

- For fixed $T \Rightarrow \dfrac{\rho}{\rho_{\max}} \propto \dfrac{b}{b_{\max}} \propto 1-c \propto 1-\dfrac{N}{N_{\max}}$



[E.N.S, R.Peschanski '09]



[Hirano, *et al* '06]

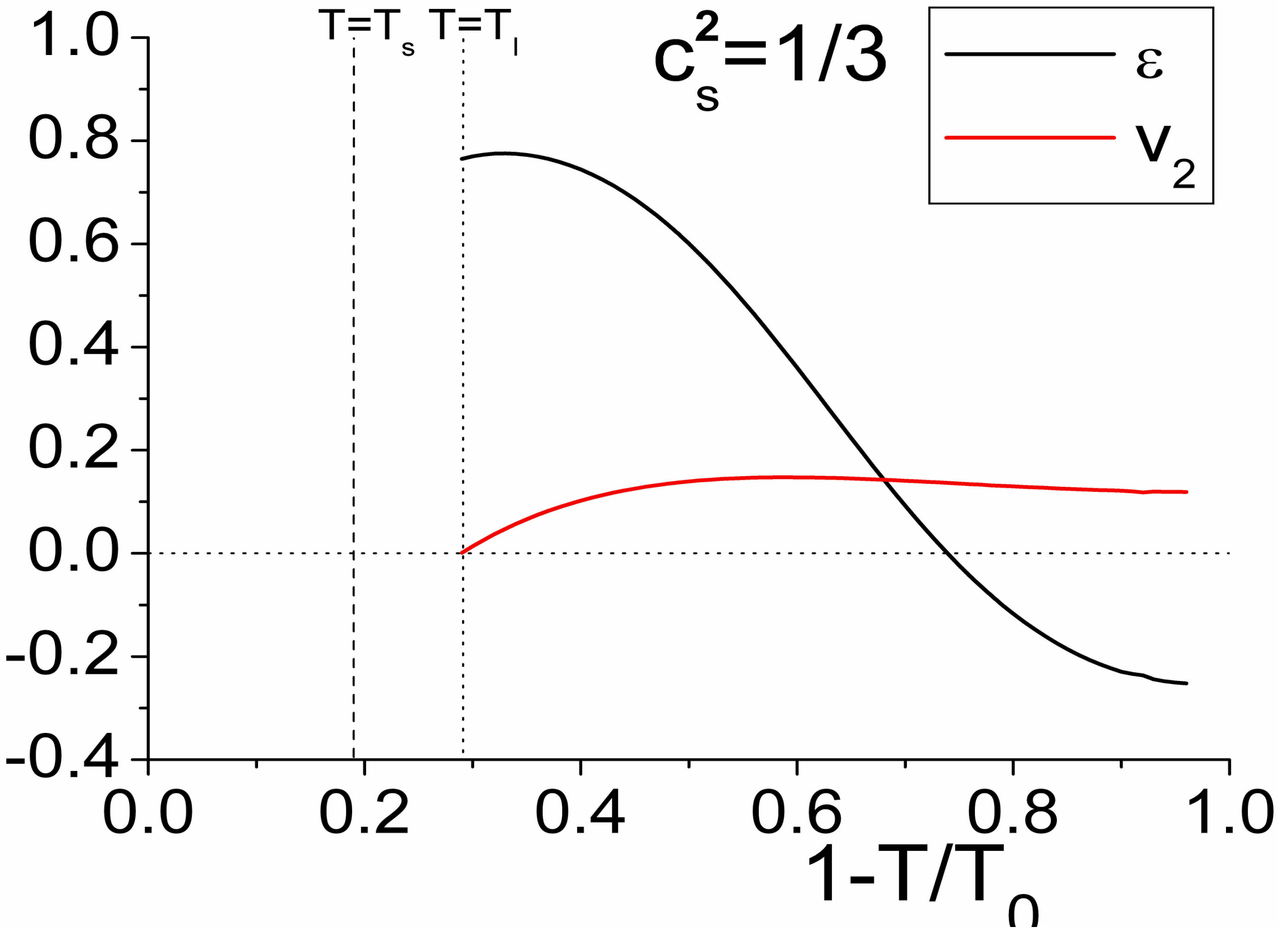
Initial conditions

- Source at a given temperature T_I

- $\boxed{\varepsilon(T_I) = \max}$ (fixes ρ)

- $\boxed{v_2(T_I) = 0}$ (fixes λ)

Temperature dependence

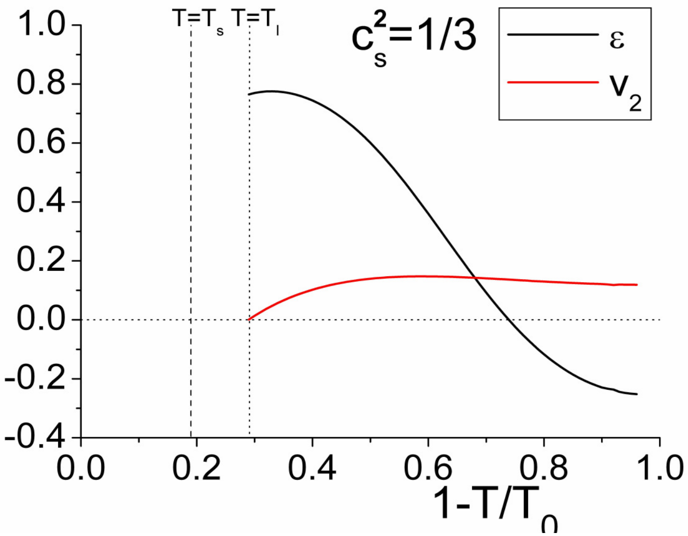


Time dependence

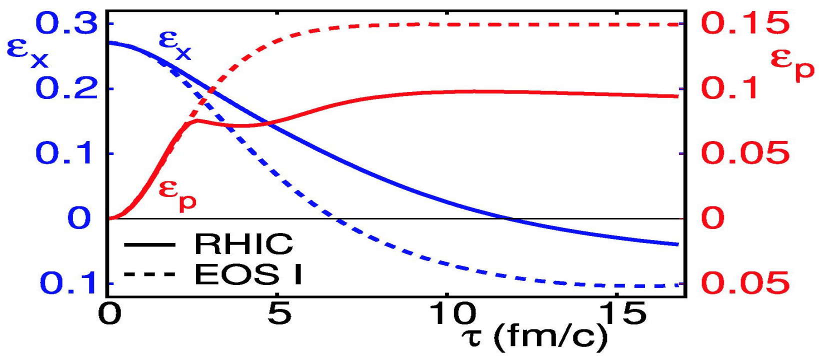
- Temperature evolution $\Longleftrightarrow$ Time evolution

$$\frac{T}{T_0} = \left(\frac{\tau_0}{\tau}\right)^{c_s^2}$$

$$\tau \equiv \sqrt{x_0^2 + x_3^2}$$



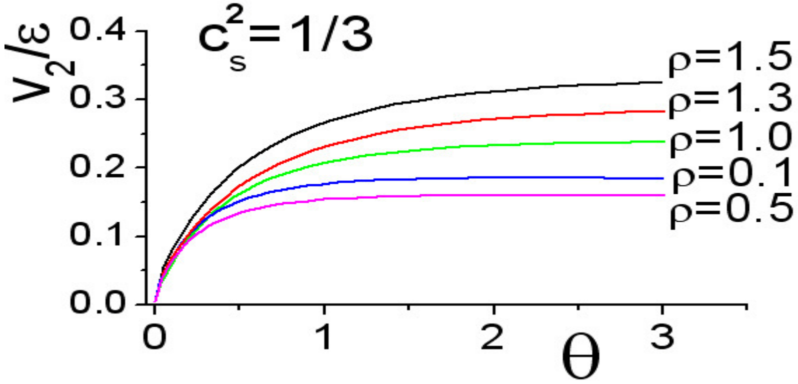
[E.N.S, R.Peschanski '09]

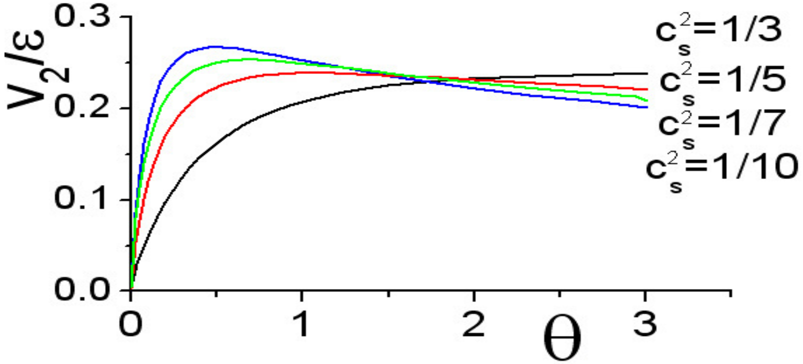


[Kolb, Heinz '03]

Time dependence

$$\theta \equiv \left(\frac{T_0}{T}\right)^{c_s^{-2}} - \left(\frac{T_0}{T_I}\right)^{c_s^{-2}}$$





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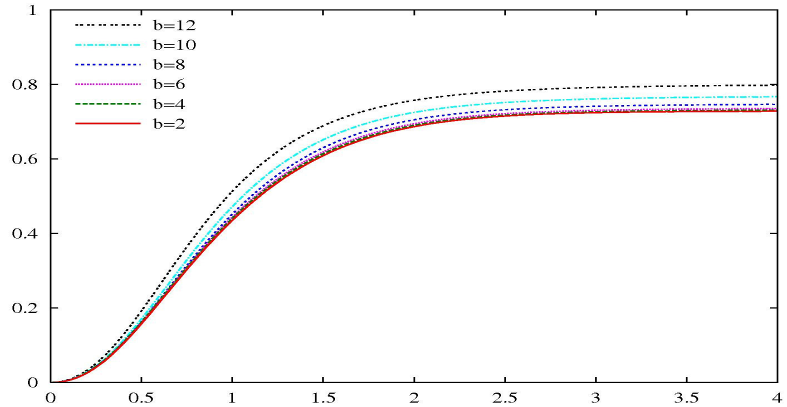
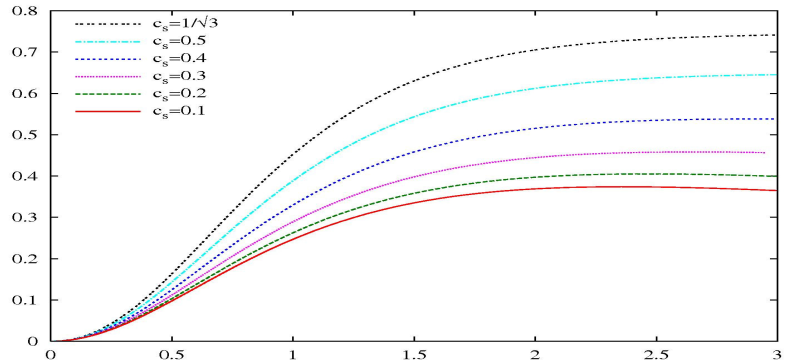




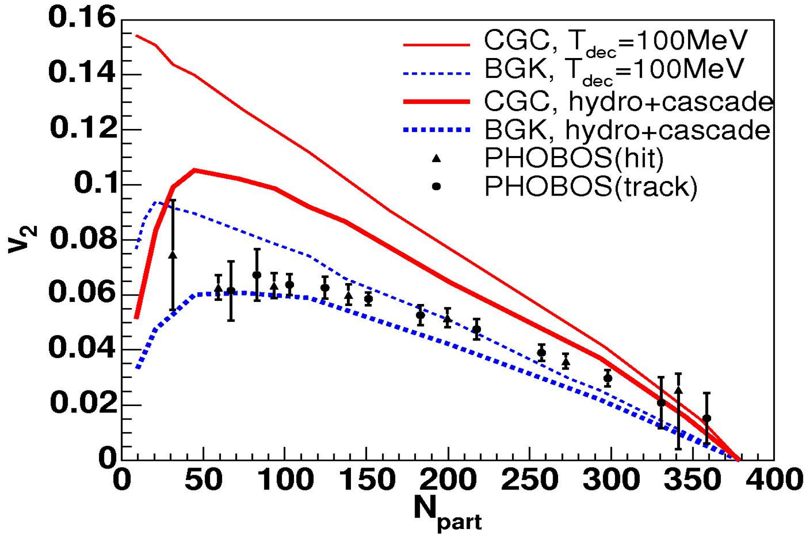
[Bhalerao, *et al* '05]

Centrality dependence

- For fixed T,

$$\frac{\rho}{\rho_{\max}} \propto 1-c \propto 1-\frac{N}{N_{\max}}$$

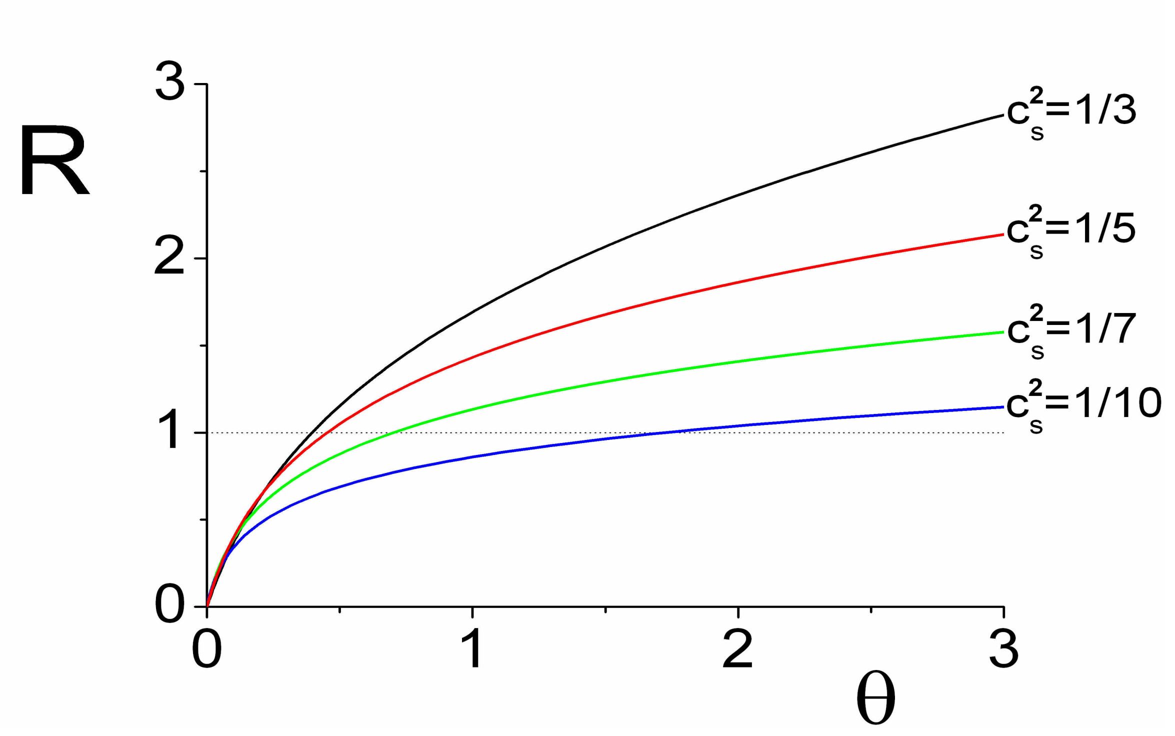
$$\Rightarrow v_{2final} \propto \rho \propto 1-c$$



[Hirano, *et al* '06]

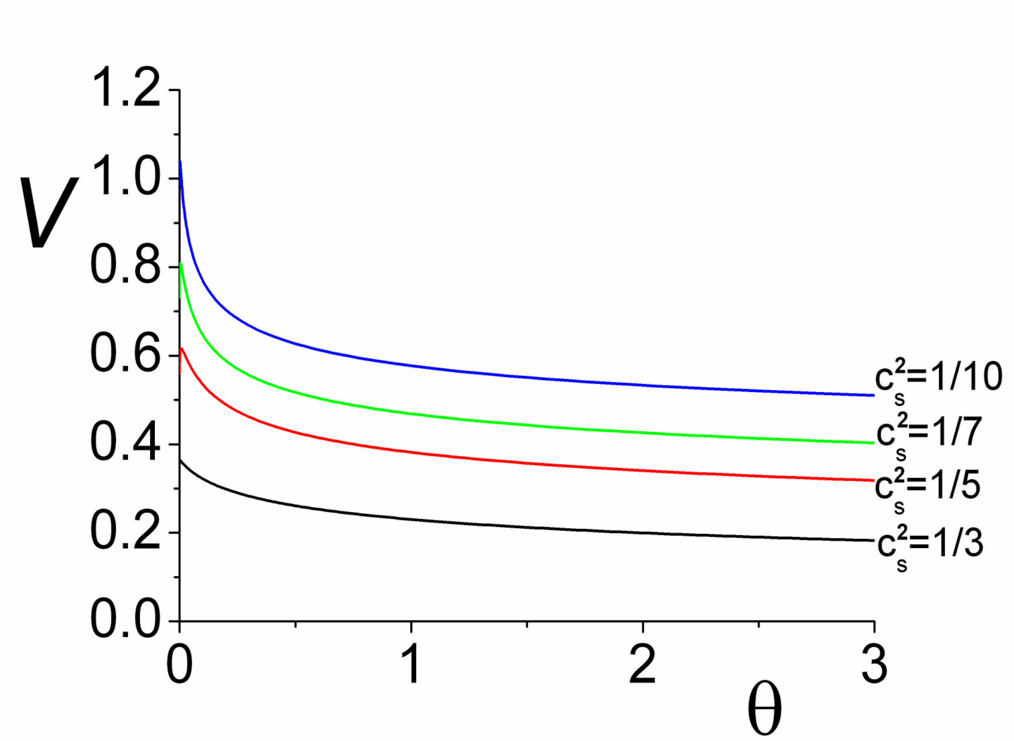
Assumptions check

- 1) Transversally isentropic: $R = \dfrac{\partial_{x_\perp}(su_\perp)}{\partial_\tau(su_0)} >> 1$ Transverse over time typical entropy gradient



Assumptions check

- 2) Quasi-stationarity: $V = \dfrac{\partial_T \ x_\perp(T)}{\partial_T \ \tau(T)} << 1$ Transverse over time, temperature-dependent expansion rate



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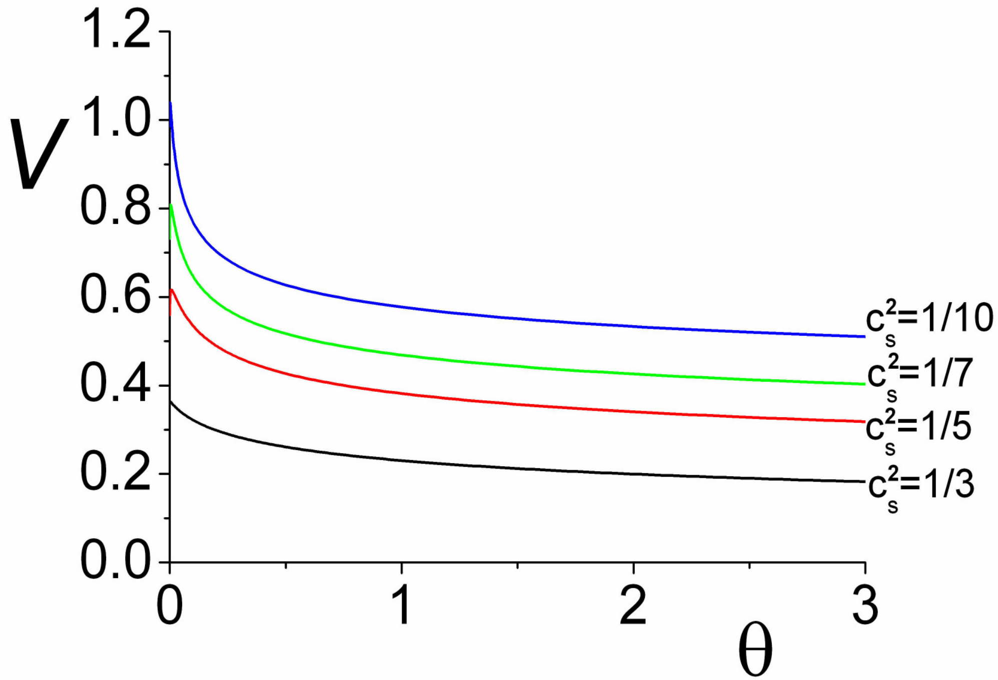

$$R = \frac{\partial_{x_\perp}(su_\perp)}{\partial_\tau(su_0)} = \frac{\frac{\partial_T(su_\perp)}{\partial_T(su_0)}}{\frac{\partial_T \ x_\perp(T)}{\partial_T \ \tau(T)}} = \frac{1}{V}\frac{\partial_T(su_\perp)}{\partial_T(su_0)}$$

The two assumptions are connected

Conclusions

- i) Hydrodynamic potential χ allows to bypass the non-linearities of the initial equations.
- ii) Under transverse isentropicity and quasi-stationarity we obtain the general solution for spatial eccentricity and elliptic flow coefficient.
- iii) Time-evolution is acquired through temperature-evolution.
- iv) Assumptions are verified qualitatively, but not exactly

Outlook

- i) Describe the $p_{\perp}$-dependence of elliptic flow.
- ii) Extend to more physical initial conditions (beyond fixed-temperature)
- iii) Extend to weak viscosity.
- iv) The rather simple mechanisms, may facilitate AdS/CFT approach of elliptic flow.