AdS/CFT and Heavy Ion Collisions

Edmond Iancu
Institut de Physique Théorique, Saclay & CNRS

*Collaborators: Y. Hatta, Al Mueller G. Giecold, E. Avsar,
L. McLerran, D.N. Triantafyllopoulos*

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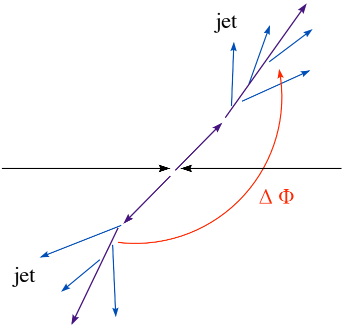
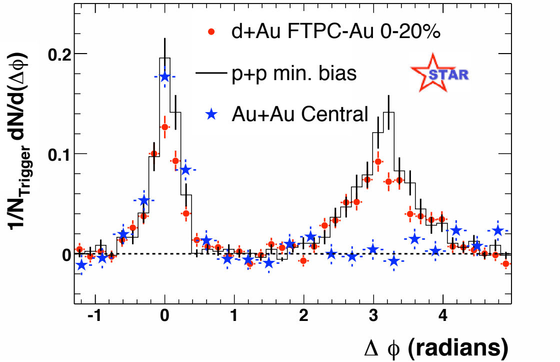
Introduction

- High–energy scattering at strong coupling ...
  Why should we worry about it ??
  - soft Pomeron, total cross–sections, Froissart bound ... are anyway not accessible to the method we know (AdS/CFT)
  - QCD is never in the strong coupling limit $\alpha_s N_c \gg 1$
- Some data at RHIC suggest strong coupling–like behaviour
  - large elliptic flow, early thermalization, large jet quenching
  - ‘strongly coupled Quark–Gluon Plasma’ (sQGP)
- Theoretical interpretation difficult and subject to ambiguities
- The strong–coupling assumption can be tested in lattice QCD
  E.I. and A.H. Mueller, arXiv:0906.3175 → discussion session
- A selfish viewpoint: For a theorist, strong-coupling = challenge

1. Motivations from RHIC
2. Partons at strong coupling
3. Phenomenology
4. Conclusions

Jets in proton–proton collisions



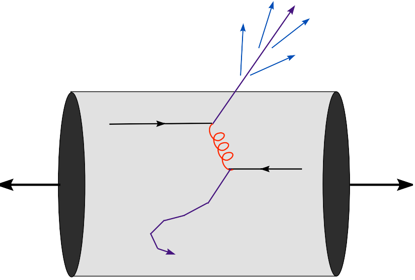


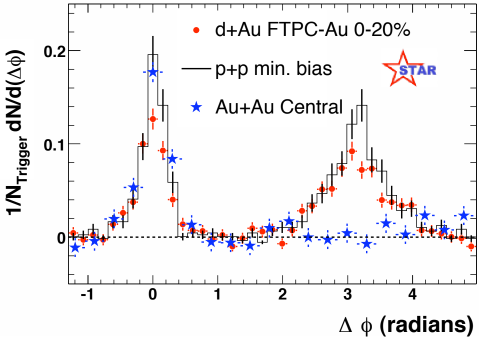
[Nucl.Phys.A783:249-260,2007]

- Azimuthal correlations between the produced jets:

p+p or d+Au : a peak at $\Delta\Phi = 180^\circ$

Nucleus–nucleus collisions at RHIC





- The "away–side" jet has disappeared !

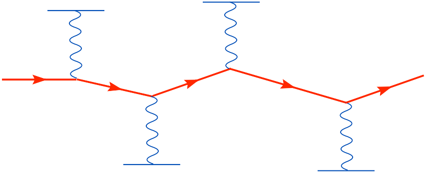
  absorbtion (or energy loss, or "jet quenching") in the medium

- The matter produced in a heavy ion collision is opaque

  high density, strong interactions, ... or both

Jet quenching in pQCD

- Medium rescattering $\Longrightarrow$ transverse momentum broadening



$$\frac{\mathrm{d}\langle k_\perp^2\rangle}{\mathrm{d}t} \equiv \hat{q} \simeq \alpha_s N_c \, xg(x, Q^2)$$

- $xg(x, Q^2)$: gluon distribution per unit volume in the medium

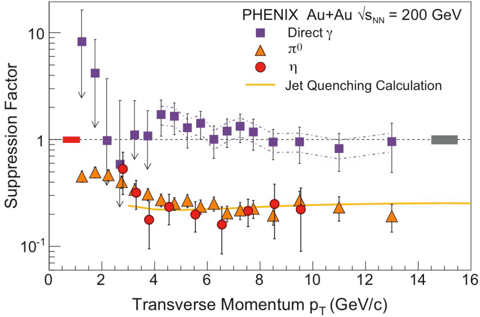
  on the resolution scales $Q^2 \sim \langle k_\perp^2\rangle$ and $1/x \sim \Delta t_{\rm coh} T$

  $xg(x, Q^2) \simeq n_q(T)\, xG_q + n_g(T)\, xG_g$ with $n_{q,g}(T) \propto T^3$

- This requires parton evolution from T up to $Q \gg T$

Nuclear modification factor

- How to measure $\hat{q}$? Compare *AA* collisions at RHIC to *pp* !



$$R_{AA}(p_\perp) \equiv \frac{Yield(A+A)}{Yield(p+p) \times A^2}$$

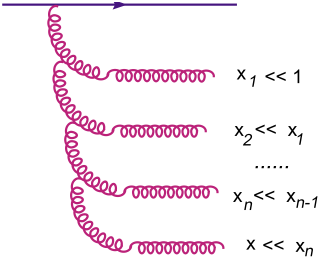
- RHIC data seem to prefer a rather large value for $\hat{q}$:

$\hat{q}_{\mathrm{RHIC}} \simeq 5 \div 15$ vs. $\hat{q}_{\mathrm{pQCD}} \simeq 0.5 \div 1\,\mathrm{GeV}^2/\mathrm{fm}$

$\Longrightarrow$ 5 to 10 times larger than the pQCD estimate !

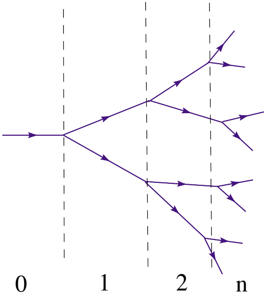
Parton evolution: weak vs. strong coupling

- Weak coupling



- Bremsstrahlung
- Soft & collinear emissions
- Slow process : $\Delta t \sim k_z/k_\perp^2 \gg 1/k_z$
- Low multiplicity : $N \propto \ln E$

- Strong coupling



- Quasi–democratic branching :

$$\omega_n \sim \omega_{n-1}/2$$

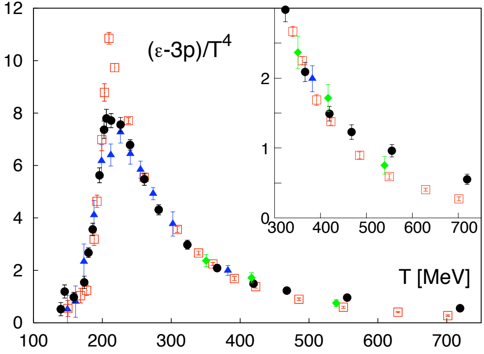
- Hard & fast $\Longrightarrow$ very efficient
- High multiplicity : $N \propto E/\Lambda$

The AdS/CFT correspondance *(Maldacena, 1997)*

- How to study parton evolution in a strongly coupled plasma ?
- A 'duality' (equivalence) between 2 very different theories:
- A gauge theory ($\mathcal{N} = 4$ SYM) in $D = 3 + 1$ at strong coupling
  - SU(N_c), conformal invariance, fixed coupling g, no confinement
- A string theory in $D = 9 + 1$ ($AdS_5 \times S^5$) at weak coupling
  - a representation of the gauge theory useful at strong coupling
- Strong 't Hooft coupling : $\lambda \equiv g^2 N_c \gg 1 \quad \& \quad g^2 \ll 1$
  - the massive string excitations decouple (very heavy)
  - string theory reduces to classical (super)gravity in $AdS_5 \times S^5$

Heating AdS_5 (Witten, 1998)

- $\mathcal{N}=4$ SYM at finite temperature $\Longleftrightarrow$ Black Hole in AdS_5
  - A Black Hole has entropy and thermal (Hawking) radiation
- For $T \gtrsim 2T_c$, the QCD plasma itself is nearly conformal



- 'Trace anomaly' from lattice QCD

$$\beta(g)\,\frac{\mathrm{d}p}{\mathrm{d}g} = \langle T^{\mu}_{\mu}\rangle = \mathcal{E}-3p$$

- $(\mathcal{E}-3p)/\mathcal{E}_0 \lesssim 10\%$

  for any $T \gtrsim 2T_c \simeq 400$ MeV

DIS off the Black Hole *(Hatta, E.I., Mueller, 07)*

- AdS_5 : Our physical world ($D = 4$) × a 'radial' dimension χ
- Virtual photon in 4D ⟷ Maxwell wave A_μ in AdS_5 BH
- DIS cross section ⟷ absorption of the wave by BH

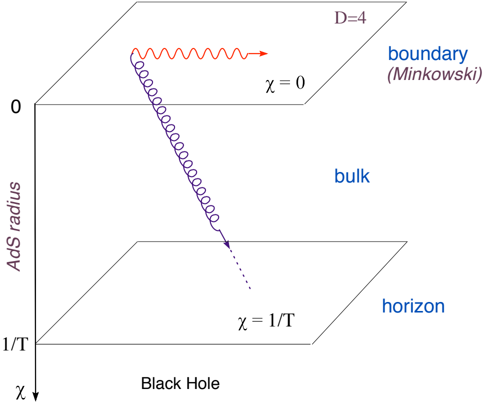
- Physical world: $\chi = 0$

  Black Hole horizon: $\chi = 1/T$

- Maxwell equations in AdS_5 BH

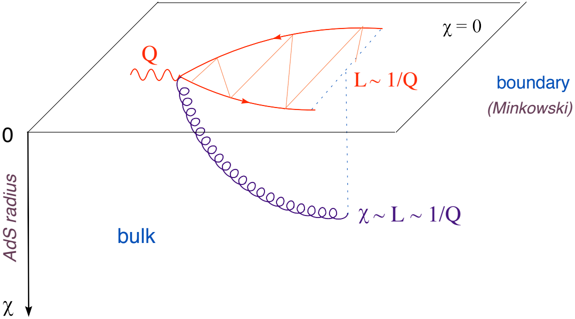
$$\partial_m\left(\sqrt{-g}\,g^{mn}g^{pq}F_{nq}\right) = 0$$

$$F_{mn} = \partial_m A_n - \partial_n A_m$$



The 5th dimension: A reservoir of quantum fluctuations

- Dual to the ‘loop’ momenta in the usual Feynman graphs (the momenta of the quantum fluctuations)



- Radial penetration χ of the wave packet in AdS_5 $\longleftrightarrow$ transverse size L of the partonic fluctuation on the boundary

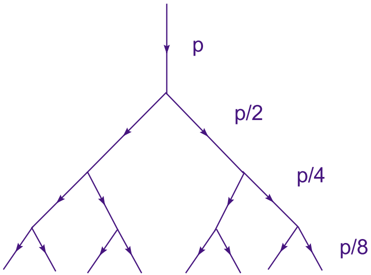
Saturation line

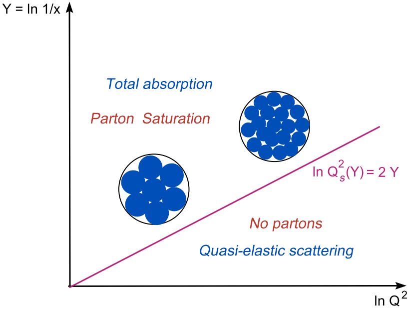
- Gravitational interactions are proportional to the energy density in the wave (ω) and in the plasma (T)
- Large ωT is tantamount to small Bjorken's $x \equiv \frac{Q^2}{2\omega T}$
- Critical ('saturation') value $x_s(Q) \simeq \frac{T}{Q} \ll 1$ for given $Q \gg T$
  - $x > x_s \simeq T/Q$: $F_2(x, Q^2) \approx 0$: no partons
  - $x < x_s \simeq T/Q$: $F_2(x, Q^2) \sim x N_c^2 Q^2$

  $\Longrightarrow$ Parton saturation with occupation numbers $\mathcal{O}(1)$
- The energy of the plasma is carried mostly by the partons along the saturation line, i.e. those with $x = x_s(Q) \ll 1$

Parton evolution at strong coupling

- All partons branch down to the smallest value of x consistent with energy conservation $\Longrightarrow$ no pointlike constituents



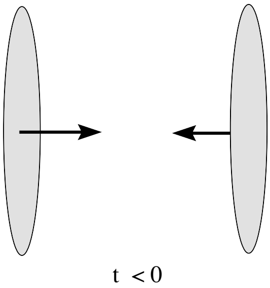


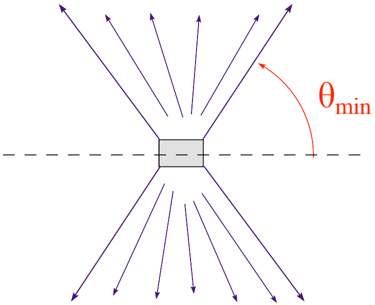
- Q_s grows very fast with the energy: graviton exchanges

$$Q_s^2 \sim \frac{T^2}{x^2} \quad (\text{plasma}) \quad \longrightarrow \quad \frac{LT^3}{x} \quad (\text{`nucleus'})$$

No forward/backward jets !

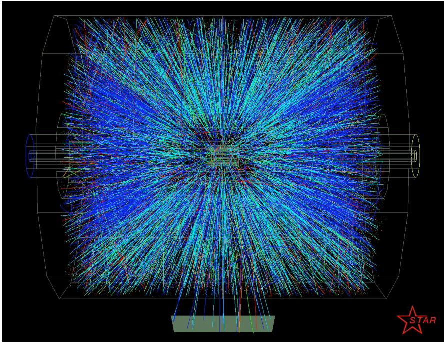
- No large–x partons $\Longrightarrow$ no hard ($Q \gg \Lambda$) particle production at forward/backward rapidities





- Tremendously large multiplicities
  (all the energy would be carried by soft particles with $p \sim \Lambda$)

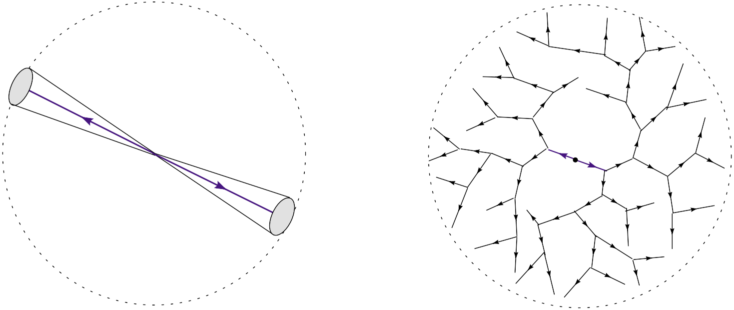
Partons at RHIC



- Partons are actually ‘seen’ (liberated) in the high energy hadron–hadron collisions
  - central rapidity: small–x partons
  - forward/backward rapidities: large–x partons

No jets at strong coupling !

- No jets in e^+e^- annihilation at strong coupling !

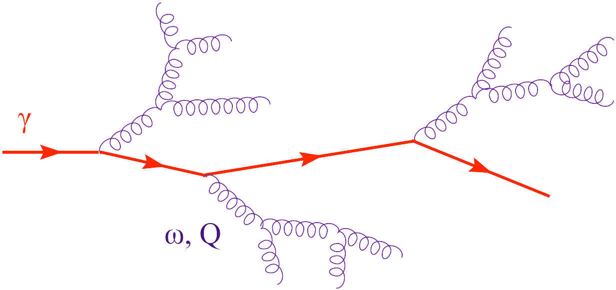


weak coupling strong coupling

- An isotropic distribution of soft hadrons in the detector

  (similar conclusions by Hofman and Maldacena, 2008)

Heavy Quark in a strongly–coupled plasma



- Medium–induced radiation
  - virtual quanta with $Q \lesssim Q_s$ are liberated into the plasma
  - energy loss, momentum broadening
- Different mechanism as compared to pQCD !

  radiation vs. rescattering *(see talk by C. Marquet)*

Conclusions

- High-energy physics appears to be quite different at strong coupling as compared to real–life QCD
- Are AdS/CFT methods useless for HIC ? Not necessarily so !
  - long–range properties (hydro, thermalization, etc) might still be controlled by strong coupling

    cf. the talks by E. Saridakis and G. Beuf
  - most likely, the coupling is moderately strong, so it useful to approach the problems from both perspectives

    cf. the talk by C. Marquet
- One can test the strong–coupling hypothesis in lattice QCD

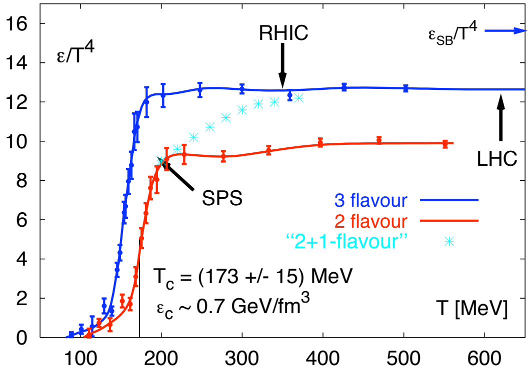
  E.I. and A.H. Mueller, arXiv:0906.3175 (discussion session ?)

Thank you !



Thank you to Christophe, Roberto, Alessandro ... and all the other 20 members of the Organizing Committee !

QCD thermodynamics on the lattice *(Bielefeld Coll.)*

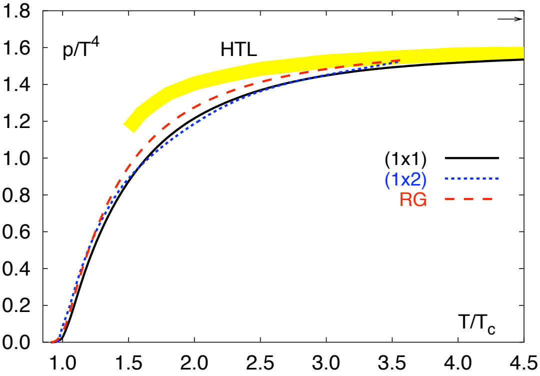


$$\mathcal{E}/\mathcal{E}_0 \approx 0.85 \qquad \text{for} \qquad T = 3T_c$$

- Is this suggestive of weak interactions ? Or of strong ones ?

Resummed perturbation theory

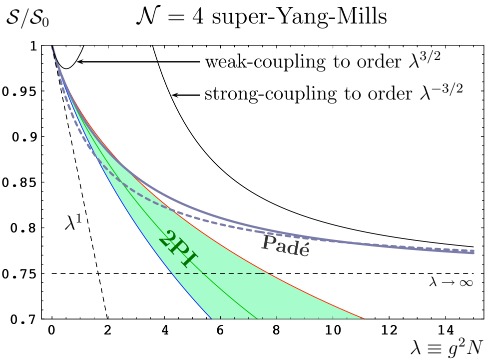
- For $T \gtrsim 2.5\,T_c$, the lattice results are well reproduced by resummed perturbation theory! *(Blaizot, Rebhan, E. I., 2000)*



- Weakly coupled quasiparticles (quarks and gluons)

$\mathcal{N} = 4$ SYM plasma: weak vs. strong coupling

- Weak–coupling to $\mathcal{O}(\lambda^{3/2})$, strong–coupling to $\mathcal{O}(\lambda^{-3/2})$
- Unique Padé approximant *(J.-P. Blaizot, A. Rebhan, E. I., 06)*



- $\mathcal{S}/\mathcal{S}_0 = 0.85$ corresponds to intermediate coupling ($\lambda \simeq 4$)
- $\mathcal{N} = 4$ SYM plasma: A convenient theoretical laboratory

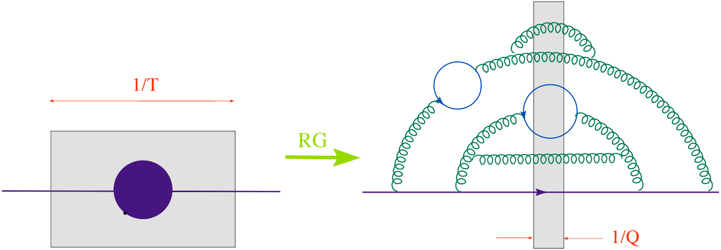
A lattice test of strong coupling *(E.I., A. Mueller 09)*

- Leading–twist, spin n operators (OPE for DIS) :

$$\mathcal{O}_f^{(n)\,\mu_1\cdots\mu_n} \equiv \bar{q}\,\gamma^{\mu_1}(iD^{\mu_2})\cdots(iD^{\mu_n})q$$

$$\mathcal{O}_g^{(n)\,\mu_1\cdots\mu_n} \equiv -\,F^{\mu_1\nu}(iD^{\mu_2})\cdots(iD^{\mu_{n-1}})F^{\mu_n}{}_{\nu}$$

- The operators depend upon the resolution scale



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A lattice test of strong coupling *(E.I., A. Mueller 09)*

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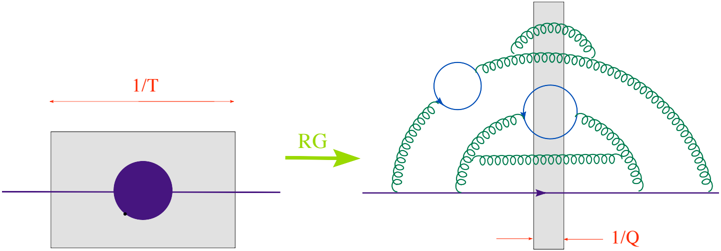


$$\langle\mathcal{O}^{(n)}\rangle_{Q^2} \propto \langle x^{n-1}\rangle_{Q^2}\,, \qquad x = \text{longitudinal momentum fraction}$$

Renormalization group flow

- RG flow $\Longrightarrow$ negative anomalous dimensions

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2}\, \mathcal{O}^{(n)} = \gamma^{(n)} \mathcal{O}^{(n)} \quad \text{with} \quad \gamma^{(n)} \leq 0$$

- Only exception: energy momentum tensor for which $\gamma_T^{(2)} = 0$

$$T^{\mu\nu} = \mathcal{O}_f^{(2)\,\mu\nu} + \mathcal{O}_g^{(2)\,\mu\nu}$$

- QCD at weak coupling: slow evolution

$$\gamma^{(n)}(\mu^2) = -a^{(n)} \frac{\alpha_s(\mu^2)}{4\pi} \quad \Longrightarrow \quad \frac{\mathcal{O}^{(n)}(Q^2)}{\mathcal{O}^{(n)}(\mu_0^2)} = \left[\frac{\ln(\mu_0^2/\Lambda^2)}{\ln(Q^2/\Lambda^2)}\right]^{a^{(n)}/b_0}$$

- Conformal theory, arbitrary coupling: $\dfrac{\mathcal{O}^{(n)}(Q^2)}{\mathcal{O}^{(n)}(\mu_0^2)} = \left[\dfrac{\mu_0^2}{Q^2}\right]^{|\gamma^{(n)}|}$

Anomalous dimensions from lattice QCD

- $\mathcal{N} = 4$ SYM at strong 't Hooft coupling: $\lambda \equiv g^2 N_c \gg 1$

$$\gamma^{(n)} \simeq -\sqrt{\frac{n}{2}}\,\lambda^{1/4} \quad \text{for} \quad 1 \ll n \ll \sqrt{\lambda}$$

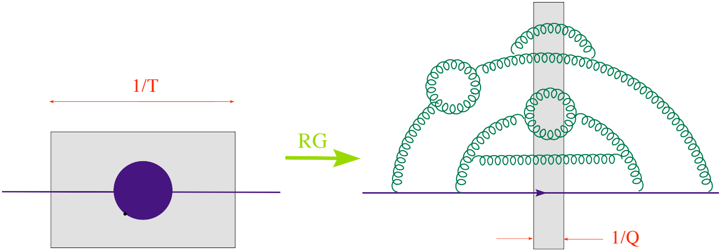
- All the unprotected leading–twist operators are strongly suppressed in the continuum limit $Q \equiv a^{-1} \rightarrow \infty$
- Measure unprotected operators in lattice thermal QCD !
- High–spin operators with $n \geq 4$ are difficult to measure ☹
- One $n = 2$ unprotected operator: orthogonal to $T^{\mu\nu}$ ☺

$$\Theta^{\mu\nu}(\mu^2) = \mathcal{O}_f^{(2)\,\mu\nu}(\mu^2) + C(\mu^2)\mathcal{O}_g^{(2)\,\mu\nu}(\mu^2)$$

- ... but we cannot compute $C(\mu^2)$ except at weak coupling ☹

Quenched QCD

- ... or in quenched QCD (no quark loops), where $C(\mu^2) = 0$ ☺

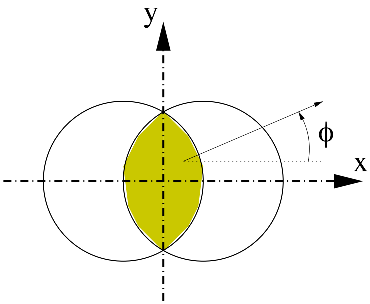


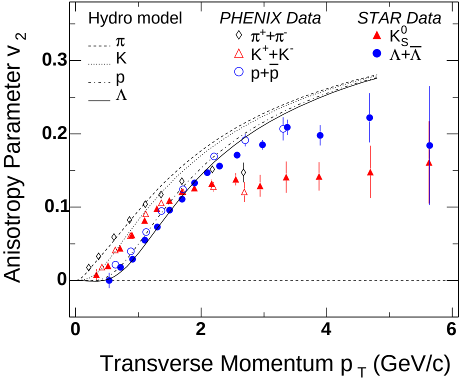
- Measure the quark energy density in quenched lattice QCD

  ...compare the result with the weak coupling expectation (SB)

  - If the difference is less than 30% $\Longrightarrow$ weak coupling
  - A reduction by a large factor $\gtrsim 5 \Longrightarrow$ strong coupling

Elliptic flow



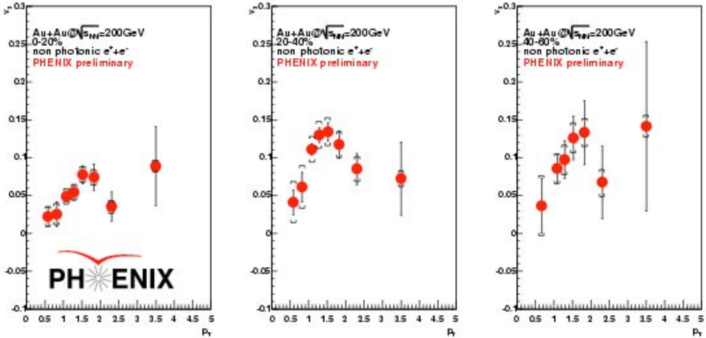


- Non–central *AA* collision: Pressure gradient is larger along x

$$\mathrm{d}N/\mathrm{d}\phi \propto 1 + 2v_2 \cos 2\phi\,, \qquad v_2 = \text{“elliptic flow”}$$

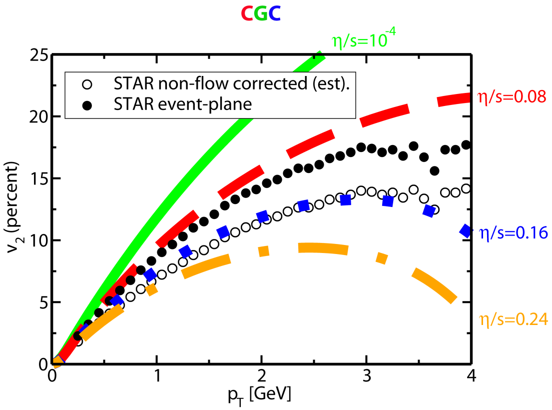
- Large observed flow ! Inconsistent with weak coupling

Elliptic flow



- Even heavy quarks (c, b) seem to flow !

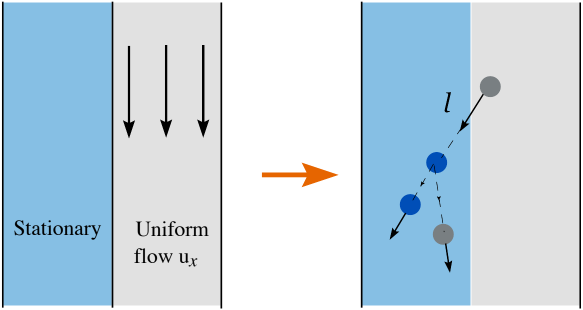
Elliptic flow



- Well described by hydrodynamical calculations with very small viscosity/entropy ratio: "perfect fluid", or "sQGP"

Viscosity

- Shear viscosity η : a measure of a fluid ability to transfer p_x momentum in the y direction



$$\frac{1}{A}\frac{\mathrm{d}p_x}{\mathrm{d}t} = -\eta\,\frac{\mathrm{d}u_x}{\mathrm{d}y}$$

- Proportional to the mean free path $\ell \sim 1/g^4$

$\Longrightarrow$ larger at weak coupling !

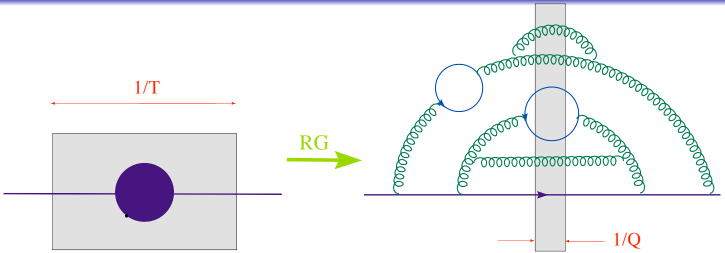
Viscosity over entropy density ratio

- Uncertainty principle: $\eta/s \gtrsim \hbar$
- Weakly interacting systems have $\eta/s \gg \hbar$
- A small η/s ratio is a hint towards strong coupling
- AdS/CFT *(Kovtun, Son, Starinets, 2003)*

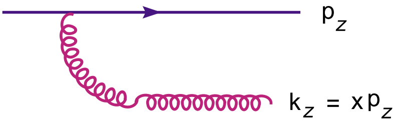
$$\frac{\eta}{s} \to \frac{\hbar}{4\pi} \quad \text{when} \quad \lambda \equiv g^2 N_c \to \infty$$

- This limiting value is believed to be 'universal'
  "any gauge theory which admits a gravity dual"
- The RHIC value is at most a few times $\hbar/4\pi$!
  "strongly–coupled quark–gluon plasma", or sQGP

Parton evolution



- A ‘quasiparticle’ on the scale T may reveal itself as highly composite on the harder scale $Q \gg T$
- Weak coupling: Bremsstrahlung



$$\mathrm{d}\mathcal{P}_{\mathrm{Brem}} \sim \alpha_s N_c \frac{\mathrm{d}^2 k_\perp}{k_\perp^2} \frac{\mathrm{d}x}{x}$$

$$xG(x, Q^2) \simeq \alpha_s N_c \ln(Q^2/T^2)$$

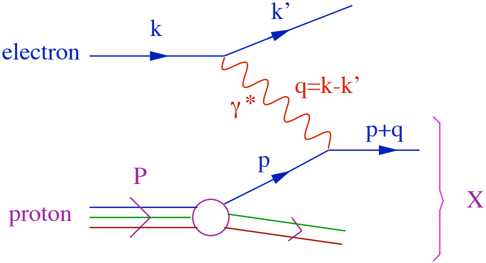
Deep inelastic scattering

- The most direct device to probe parton evolution

- Space–like photon
- 2 independent variables:

$$Q^2 \equiv -q^\mu q_\mu \geq 0$$

$$x \equiv \frac{Q^2}{2P \cdot q}$$

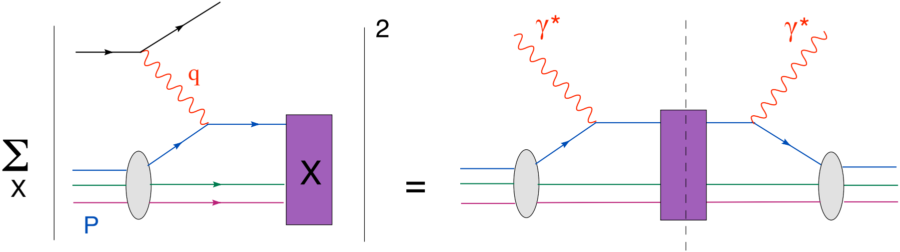


- Physical picture: γ^* absorbed by a quark excitation with
  - transverse size $\Delta x_\perp \sim 1/Q$
  - and longitudinal momentum $p_z = xP$

- Structure function $F_2(x, Q^2)$: quark distribution

Current–current correlator

- Total cross–section (“structure functions”): optical theorem



$$F_{1,2}(x,Q^2) \;\sim\; \mathrm{Im} \int \mathrm{d}^4x\, \mathrm{e}^{-iq\cdot x}\, i\, \langle P\,|\mathrm{T}\,\{J_\mu(x)J_\nu(0)\}\,|\,P\rangle$$

$$J^\mu \;=\; \sum_f e_f\, \bar q_f\, \gamma^\mu\, q_f$$: quark electromagnetic current

- Valid to leading order in α_{em} but all orders in α_s

DIS off the strongly coupled plasma

- Thermal expectation value $(Q^2 \equiv |q^2| \gg T^2)$

$$\Pi_{\mu\nu}(q) \equiv \int \mathrm{d}^4x\, \mathrm{e}^{-iq\cdot x}\, i\theta(x_0)\, \langle\, [J_\mu(x), J_\nu(0)]\, \rangle_T$$

- $\mathcal{N} = 4$ SYM at finite temperature & $\lambda \equiv g^2 N_c \to \infty$: classical gravity in the $AdS_5 \times S^5$ Black Hole geometry

$$\mathrm{d}s^2 = \frac{R^2}{\chi^2}\big(-f(\chi)\mathrm{d}t^2 + \mathrm{d}\boldsymbol{x}^2\big) + \frac{R^2}{\chi^2 f(\chi)}\mathrm{d}\chi^2 + R^2\mathrm{d}\Omega_5^2$$

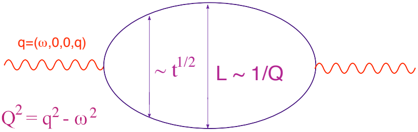
where $f(\chi) = 1 - (\chi/\chi_0)^4$ and $\chi_0 = 1/T =$ BH horizon

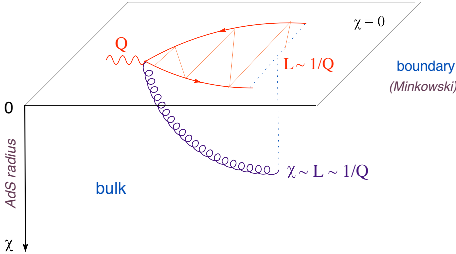
- A Black Hole has entropy and thermal (Hawking) radiation

Space–like photon in the vacuum

- A space–like photon cannot decay in the vacuum :

  virtual fluctuation with size $L \sim 1/Q$ and lifetime $\Delta t \sim \omega/Q^2$



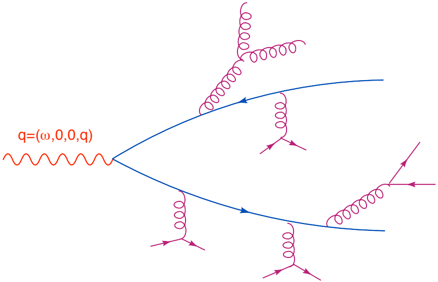


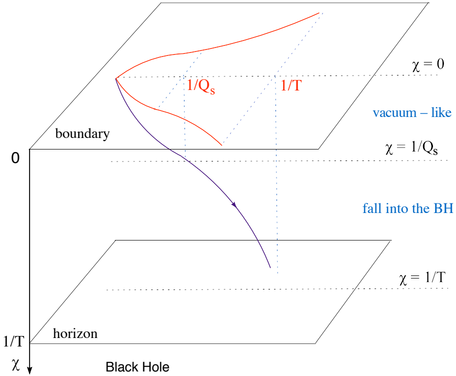
- AdS : The Maxwell wave penetrates into AdS_5 up to a

  radial distance $\chi \sim 1/Q$

Space–like photon in the plasma

- … but it can decay in the presence of the plasma





- This is what happens in the strongly coupled plasma

  … but only for sufficiently high energy ω

Space–like photon in the plasma

- Gravitational interactions are proportional to the energy density in the wave (ω) and in the plasma (T)

- High Q^2/large Bjorken x

  The wave gets stuck near the boundary

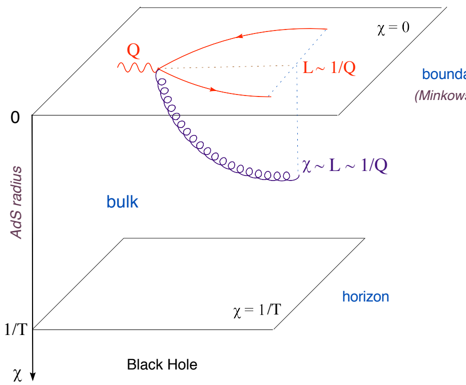
  $$\chi \lesssim 1/Q \ll 1/T$$

  $\Longrightarrow$ No interaction with the BH

- Low Q^2/small x

  $$x \equiv \frac{Q^2}{2\omega T} \lesssim x_s(Q) \simeq \frac{T}{Q}$$

  $\Longrightarrow$ The wave falls into the BH



The energy–momentum sum rule

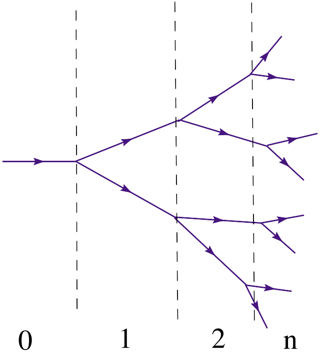
$$\int_0^1 \mathrm{d}x\, F_2(x, Q^2) \;=\; const. \qquad \text{as} \quad Q^2 \to \infty$$

- ... is still dominated by the few partons remaining at $x \sim \mathcal{O}(1)$
- As $x \to 0$, F_2 rises ‘only’ like $F_2(x, Q^2) \sim x^{-\lambda}$ with $\lambda \lesssim 0.3$
- The small–x gluons are numerous, but carry very little energy
- Pointlike valence quarks

... to be contrasted with the situation at strong coupling !

Parton branching at strong coupling

- At strong coupling, branching is fast and quasi–democratic



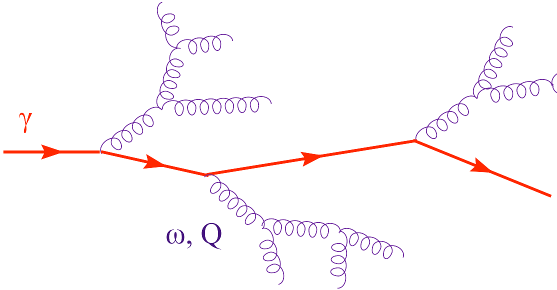
$$\omega_n \sim \frac{\omega_{n-1}}{2} \sim \frac{\omega}{2^n}$$

$$Q_n \sim \sim \frac{Q_{n-1}}{2}$$

$$\Delta t_n \sim \frac{\omega_n}{Q_n^2}$$

- When $\omega_n \sim Q_n \sim T$, the quanta disappear into the plasma
- Dominant mechanism for energy loss and momentum broadening at strong coupling

Heavy Quark in a strongly–coupled plasma

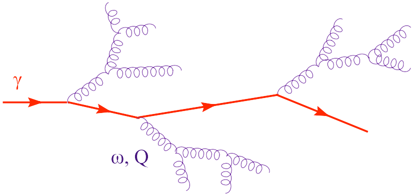


- Medium–induced radiation
    - virtual quanta with $Q \lesssim Q_s$ are liberated into the plasma
    - energy loss, momentum broadening
    - Langevin equation from AdS/CFT

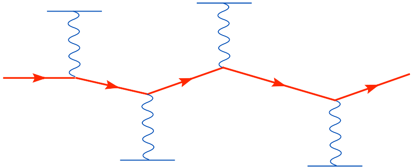
Casalderrey-Solana, Teaney, 2006; Gubser, 2006; Dominguez et al, 2008

Momentum broadening

- Strong coupling : fluctuations in the emission process



- pQCD : thermal rescattering



See talk by Cyrille Marquet