

Dipole models beyond HERA data

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- HERA data offer compelling evidence for the presence of saturation effects at low (x, Q^2):
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$$Q_s^2(x = 10^{-4}) \approx 1 \text{ GeV}^2$$

- **DGLAP** works fine for $Q^2 > 10 \text{ GeV}^2$.

Valence-like or negative gluons, tension in the fits when pushed down to $Q^2 \sim 1 \text{ GeV}^2$

- Saturation-based approaches describe well **high Q^2 data (for $x < 10^{-2}$)** for F2 and FL

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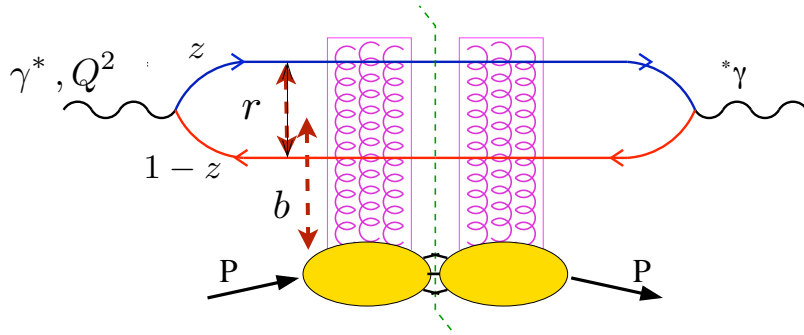
- Can we see DGLAP (or any other linear approach) fail in a region where it is supposed to work?

How low in x does one need to go? What are the best suited observables?

This question is not purely academic: LHeC Working group.

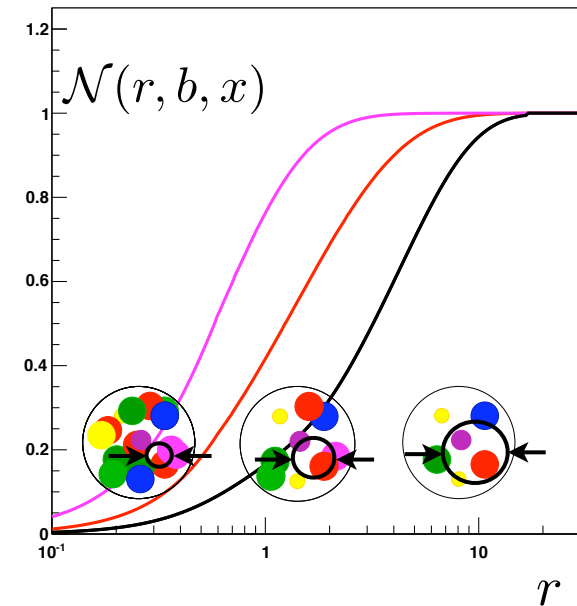
I'll assess this question just for inclusive observables: FL and F2

Saturation in the dipole model of DIS



$$\sigma_{T,L}^{\gamma^* P}(x, Q^2) = \int_0^1 dz \int d^2 \mathbf{r} \left| \Psi_{T,L}^{\gamma^* \rightarrow q\bar{q}}(z, Q, r) \right|^2 \sigma^{dip}(x, r)$$

$$\sigma^{dip}(x, r) = 2 \int d^2 b \mathcal{N}(x, b, r)$$



(My) Classification of dipole models in the market

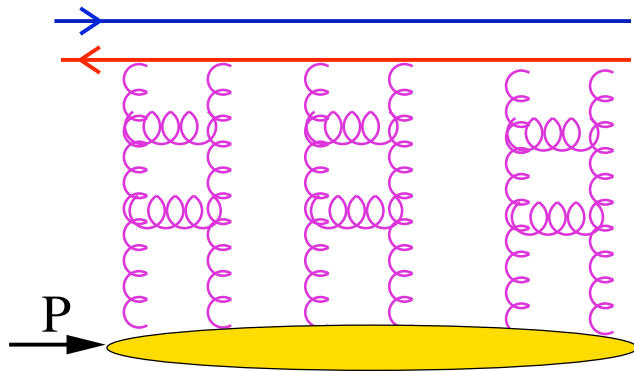
⇒ According to the physical mechanism driving saturation, i.e (x, Q^2, r) -dynamics:

- Eikonalization of leading twist result + DGLAP
- BK or BFKL+saturation
- Phenomenological models: Regge Theory; non-perturbative input.

⇒ According to phenomenological details:

- Impact parameter dependence
- quark masses, inclusion of charm or beauty contributions

⇒ **DGLAP-based models:** Saturation results from eikonalization of two-gluon exchange:



$$\frac{d\sigma^{dip}}{d^2b} = 1 - \exp \left[-\frac{\pi^2}{2 N_c} r^2 xg(x, Q^2) T_p(b) \right]$$

initial gluon d.f. fitted to data

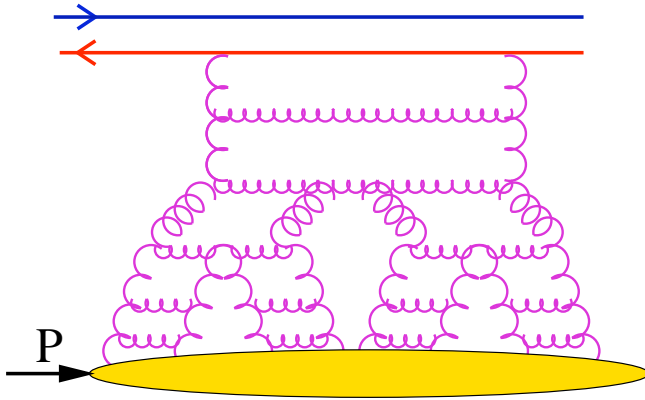
$$xg(x, Q_0^2 = 1 \text{ GeV}^2) = A_g x^{-\lambda_g} (1-x)^{5.6}$$

Leading $\ln Q^2$ terms in each cascade resummed through DGLAP

- **BGBK (Bartels-Golec-Biernat-Kowalski):** Trivial impact parameter dependence
- **IPSat (Kowalski-Teaney):** Gaussian profile $T_p(b) \sim \exp(-b^2/2B)/(2\pi B)$
- **Good description of HERA data (F2, F2charm, diffractive J/Ψ)**
- **Admits a trivial extension to nuclear targets. Good description of shadowing**
- **No dynamical input for extrapolation to small-x**
- **Fits to data allow for growing or “valence like” initial gluon d.f:**

$$-0.41 < \lambda_{glue} < 0.3$$

Models based on the Balitsky-Kovchegov equation or Color-Glass-Condensate:



$$\frac{\partial \mathcal{N}(r, x)}{\partial \ln(x_0/x)} = \int d^2 r_1 K(r, r_1, r_2) [\mathcal{N}(r_1, x) + \mathcal{N}(r_2, x) - \mathcal{N}(r, x) - \mathcal{N}(r_1, x)\mathcal{N}(r_2, x)]$$

A) Calculations based on numerical solutions of BK eqn **with running coupling**
JLA-Armesto-Milhano-Salgado (AAMS), Kuokkanan-Rummukainen-Weigert (KRW).

- Trivial impact parameter dependence. Overall normalization fitted to data
- Input: Initial conditions for the evolution, $\mathcal{N}(x_0, r)$ (GBW, G-M, scaling)
- **KRW**: Energy conservation (i.e., large-x) effects considered.

B) Models based on analytical solutions of BFKL+ absorptive barrier
Iancu-Itakura-Munier-Soyez (CGC), Kowalski-Motyka-Watt (b-CGC)

- Evolution speed λ fitted to data
- b-CGC: Impact parameter dependence.

χ /d.o.f \sim 1.6. Lowest evolution speed of all models: $\lambda \sim 0.16$

⇒ “Phenomenological” models

A) Golec-Biernat-Wusthoff
$$\left\{ \begin{array}{l} \mathcal{N}^{GBW}(x, r) = \theta(R_p - b) \left(1 - \exp \left[-\frac{r^2 Q_s^2(x)}{4} \right] \right) \\ Q_s^2(x) = Q_0^2 \left(\frac{x_0}{x} \right)^\lambda \end{array} \right.$$

B) Models based on Regge Theory.
Forshaw-Shaw (FS04).

FS04:
$$\sigma^{dip}(r, x) = \left\{ \begin{array}{l} A^{soft} x^{-\lambda_{soft}}, \quad \text{for } r > r_1 \quad (\lambda_{soft} \sim 0.66) \\ A^{hard} r^2 x^{-\lambda_{hard}}, \quad \text{for } r < r_0 \quad (\lambda_{hard} 0.34) \end{array} \right.$$

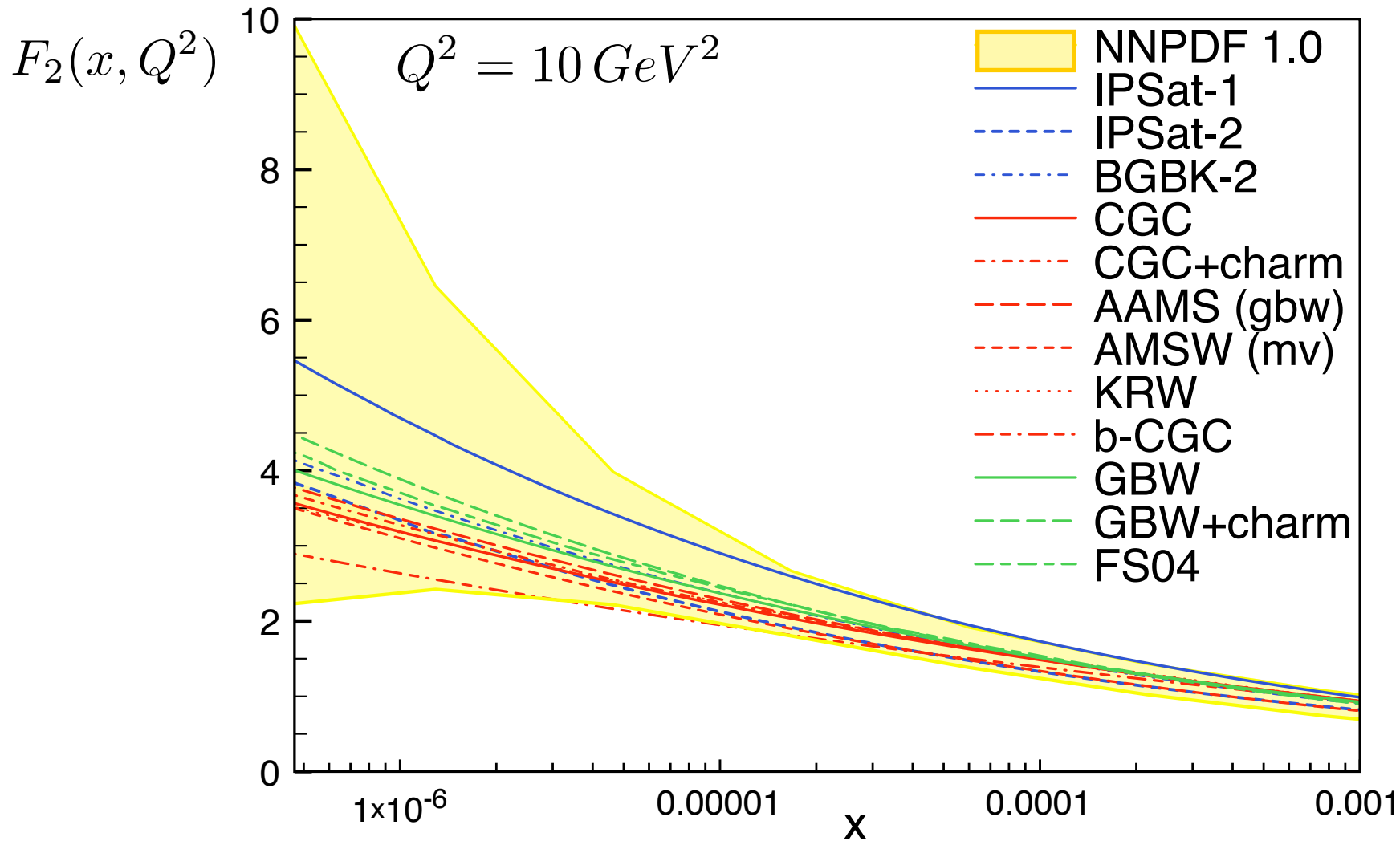
C) Strong coupling dipole from AdS/CFT. Kovchegov-Lu-Rezaeian

- Valid in the photo production region: $Q^2 < 2 \text{ GeV}^2$
- Main feature: “Saturation of saturation”: $Q_s^2(x) \rightarrow \text{constant}, \quad \text{for } x \rightarrow 0$

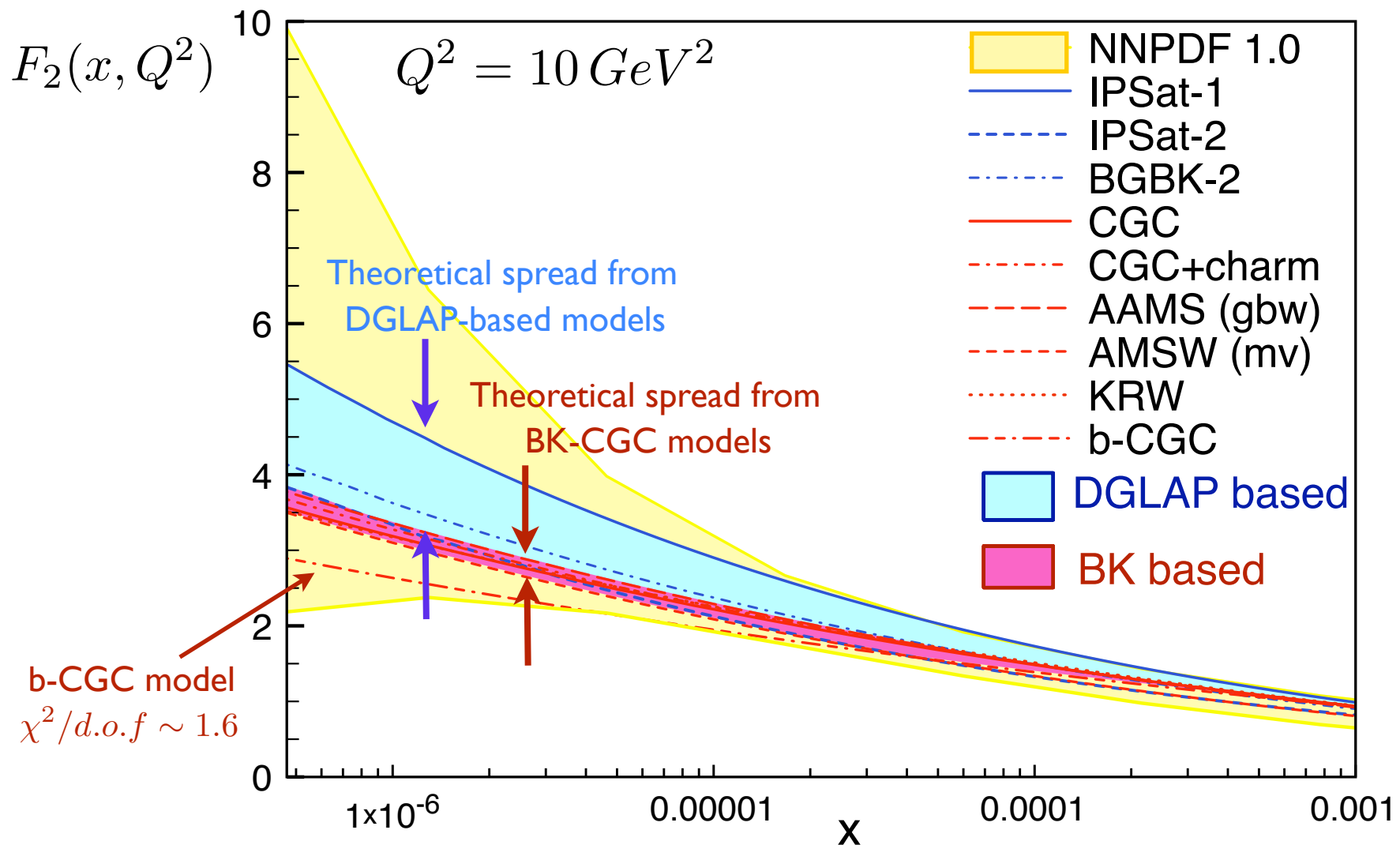
D) Models tuned to fit also RHIC data.

E) Others (my apologies).

Extrapolation for F2 beyond HERA kinematic regime (LHeC?)

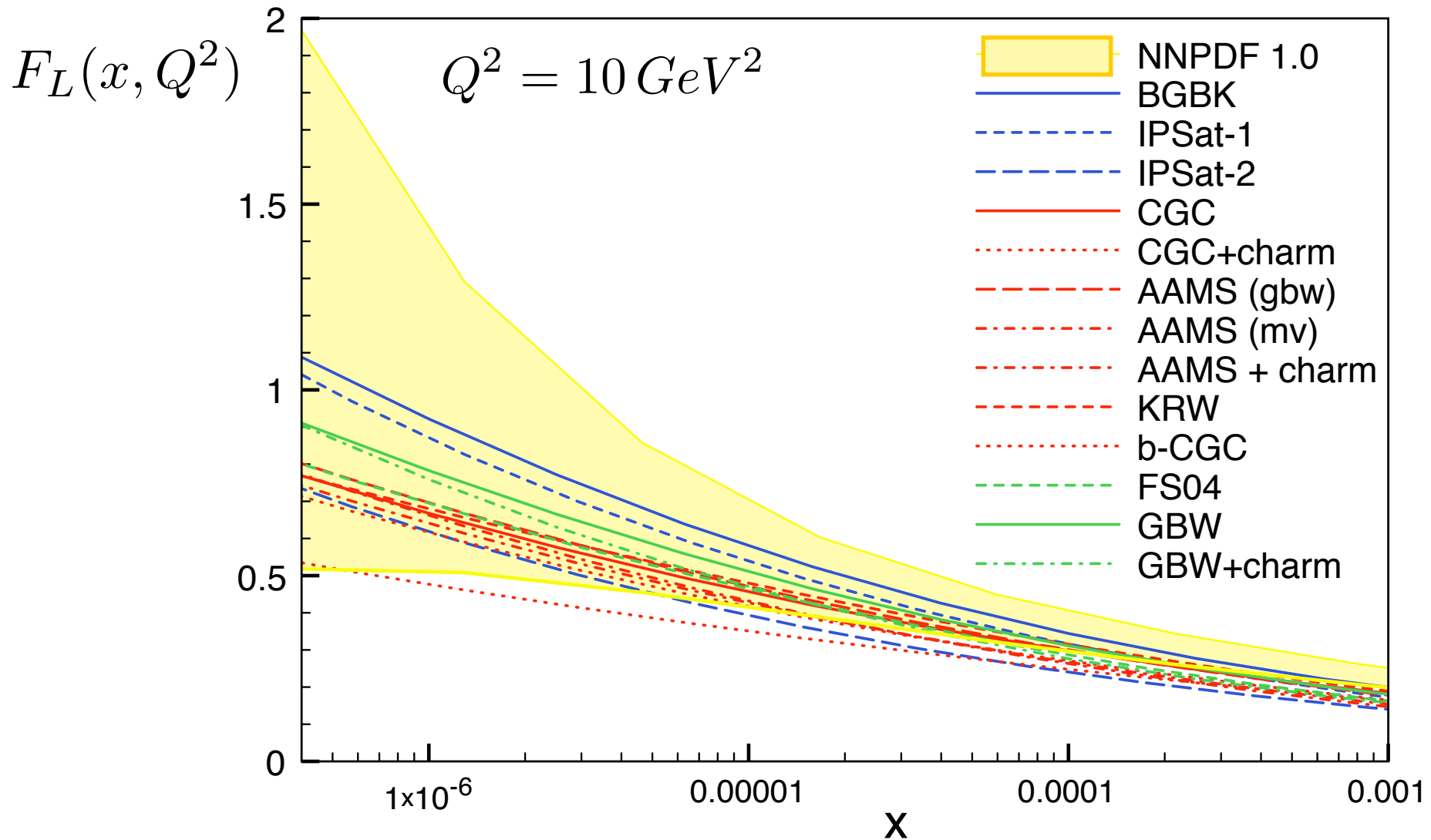


Extrapolation of the models for F2 : Only BK vs DGLAP-based models

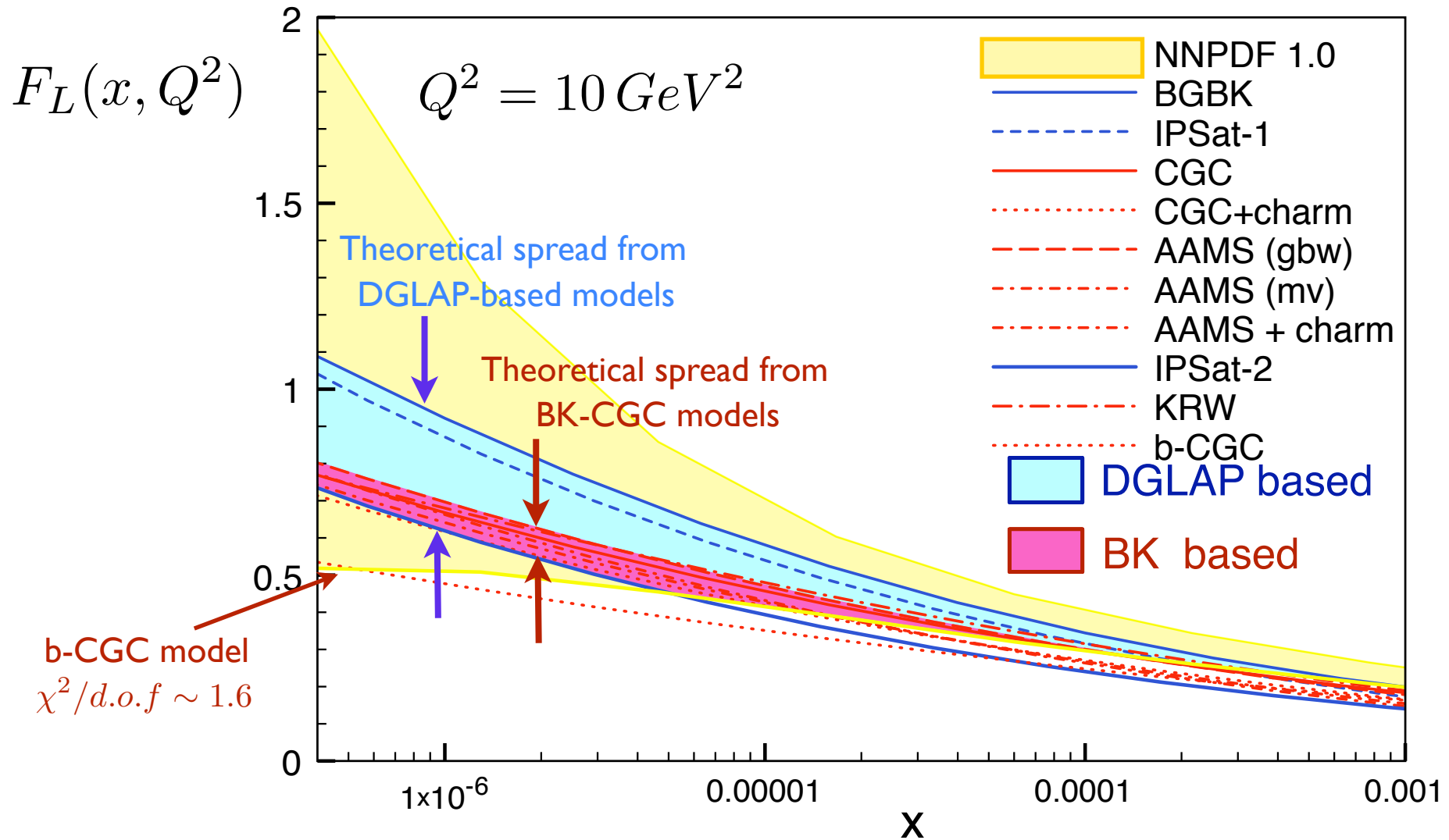


- A democratic treatment of all models results in very large uncertainties ($\sim 100\%$ at $x=10^{-7}$)
- BK-CGC models closer to each other (b-CGC diverges a bit, $\chi^2/d.o.f \sim 1.6$)
- The largest error band is spanned by DGLAP-based models

Extrapolation for FL beyond HERA kinematic regime (LHeC?)



Extrapolation of the models for FL : Only BK vs DGLAP-based models



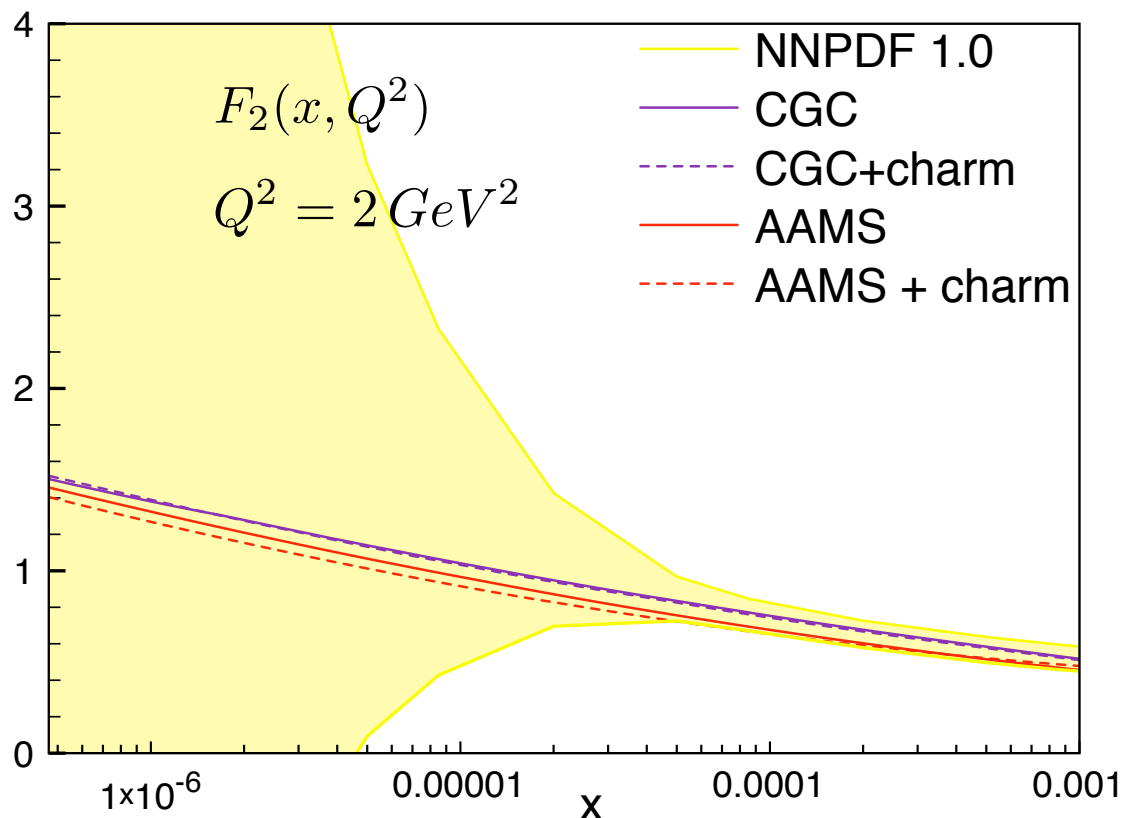
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Quark masses

$$\sigma_{T,L}^{\gamma^* P}(x, Q^2) = \sum_{flavors} \int_0^1 dz \int d^2\mathbf{r} \left| \Psi_{T,L}^{\gamma^* \rightarrow q\bar{q}}(z, Q, r, m_f) \right|^2 \sigma^{dip}(\tilde{x}, r)$$

$$\tilde{x} = x \left(1 + \frac{4m_f^2}{Q^2} \right)$$

- **Light flavors (u,d,s):** $0 < m_{u,d,s} < 140 \text{ MeV}$. Fits in the photoproduction region demand a large (pion) mass.
- **Charm (& beauty):** Needed in order to reproduced measured F_2^{charm}
- Extrapolations are stable after switching on the charm. Value of the saturation scale change.



$$Q_s^2(x) = \left(\frac{x_0}{x} \right)^\lambda \text{ GeV}^2$$

$x_0 / 10^{-4}$	charm	no charm
GBW	0.41	3
CGC	0.1	0.27

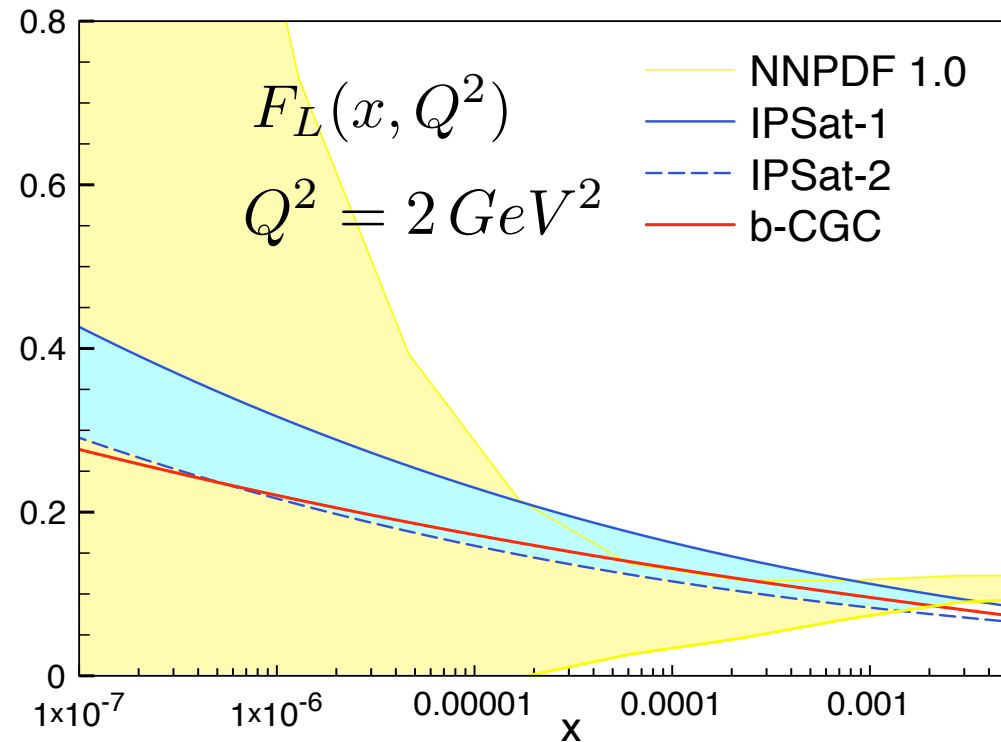
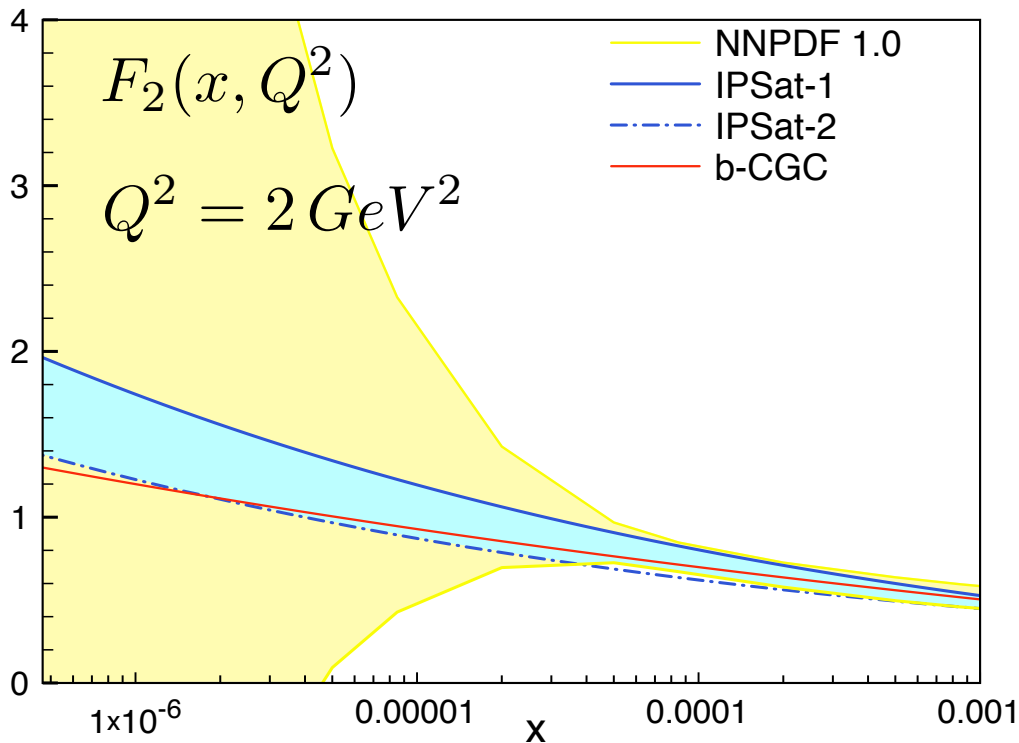
Impact parameter dependence

- Only two of the models shown include impact parameter dependence (crucial physical ingredient for exclusive measurements, diffraction ...)

- IPSat: Good description of exclusive observables (VM, Diffraction) at HERA

Poor extrapolation to small-x

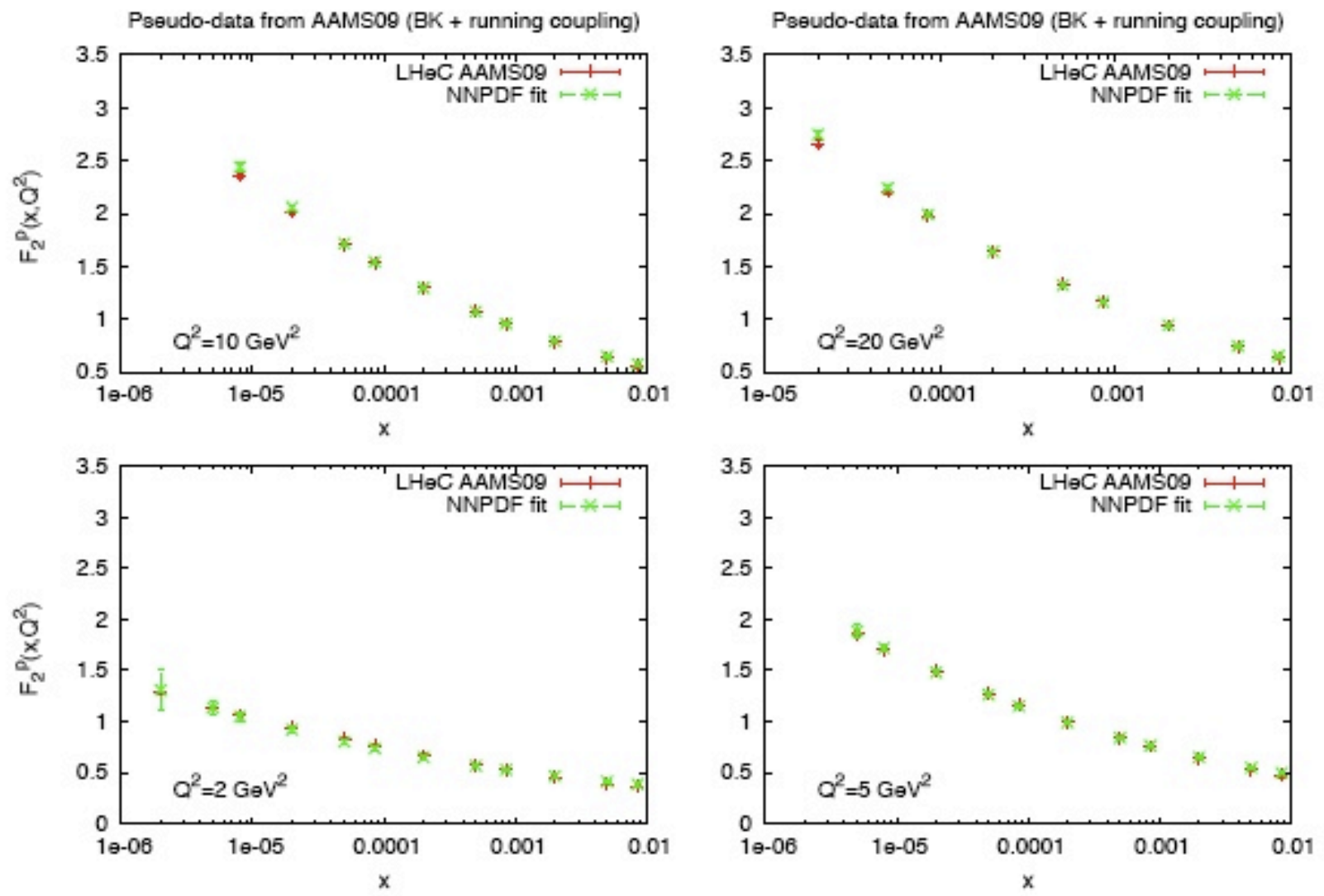
- b-CGC: Not so good description of HERA data. $\chi^2/\text{d.o.f} \sim 1.6$. Lowest evolution speed of all models: $\lambda \sim 0.16$



⇒ Could DGLAP account for data compatible with the previous predictions?

- 1- Generate pseudo-data for F2 & FL using AAMS input (BK with running coupling)
- 2- Include it in the DGLAP data set, and run the fits

DGLAP NNPDF (fits and figure by Juan Rojo) can fit the pseudo-data for F2

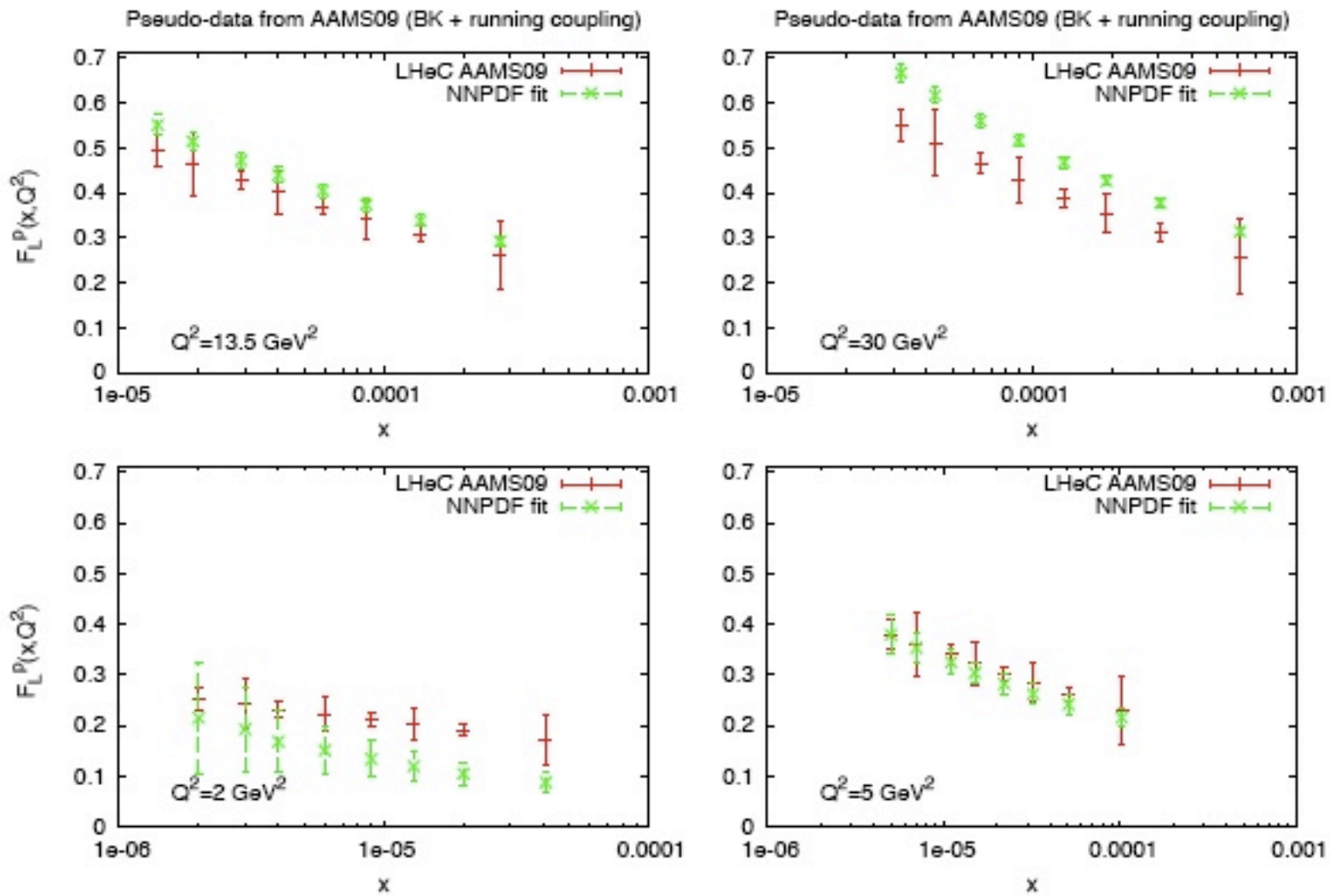


DGLAP can adjust its initial condition to account for a “fully saturated” F2

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DGLAP NNPDF (fits and figure by Juan Rojo) fails to fit the pseudo-data for FL



The divergence between linear DGLAP analyses and non-linear small-x dynamics is visible in FL already for $x \sim 10^{-4}$. Around the corner!!

SUMMARY

- Little spread in extrapolations towards small- x for FL and F2 from BK-CGC dipole models
- F2 is not well suited to tell DGLAP from non-linear QCD dynamics
- A measurement of FL at $x \sim 10^{-4} \div 10^{-5}$ could suffice to pin down the onset of non-linear effects (provided a large lever arm in Q^2)

SUMMARY

- Little spread in extrapolations towards small- x for FL and F2 from BK-CGC dipole models
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OUTLOOK

- Not all linear approaches are DGLAP. The checks should be extended to other approaches: small- x resummed DGLAP, k_t -factorization...)
- This studies should be extended to other observables: diffraction, vector mesons...
- In order to do so, a more satisfactory inclusion of charm and impact parameter dependence in BK-based models is needed