# non-linear QCD meets data [and this is good news ...]

## *a global analysis of lepton-proton scattering with running coupling BK evolution*

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[AAMS]

*PRD 80:034031, 2009*
*DIS2009 Proceedings*

Low x Meeting 2009, CERN
– 9 Sep 2009 –

# ◌ non-linear QCD [ingredients]

- dipole formulation of high energy QCD
  - $k_T$ factorized expression for physical observables
    - ↪ like all ‘dipole models’
    - ↪ applicability of our approach ‘highly correlated’ with validity of dipole formulation/factorization

- first principle QCD calculation of x-evolution of dipole scattering amplitude
  - running coupling BK
    - ↪ unlike most ‘dipole models’

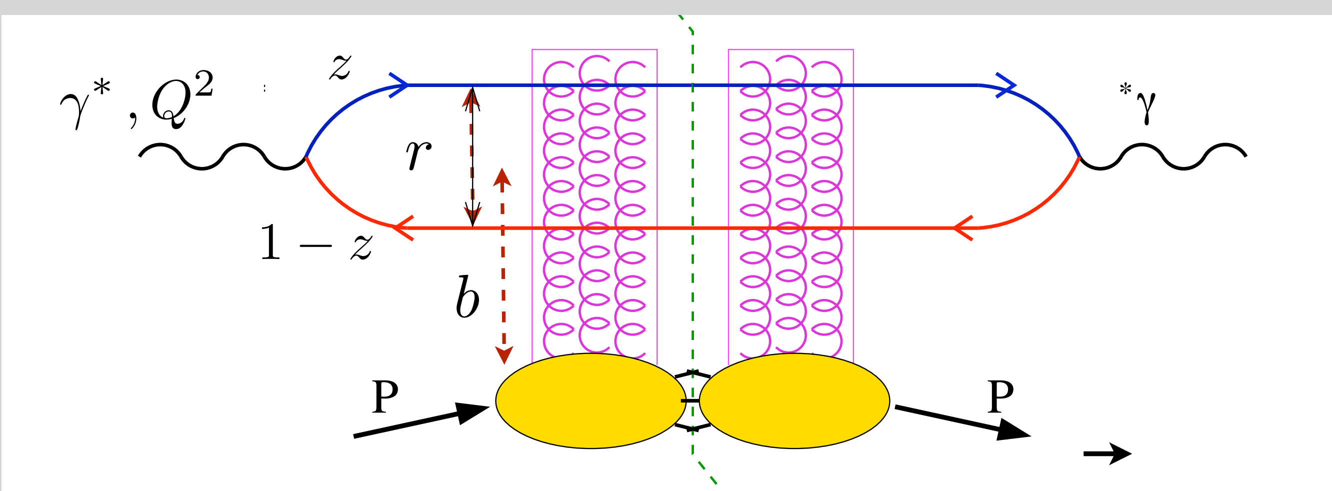
# dipole QCD [in DIS]

- at high energy [x << 1] the coherence length of the virtual photon fluctuation

$$l_c \sim (2m_N x)^{-1} \simeq 0.1/x \text{ fm} \gg R_N$$

- total virtual photon-proton cross section can be factorized as

$$\sigma_{T,L}(x,Q^2) = \int_0^1 dz \int d\mathbf{b}\, d\mathbf{r}\, |\Psi_{T,L}(z,Q^2,\mathbf{r})|^2 \mathcal{N}(\mathbf{b},\mathbf{r},x)$$




QED calculation

[imaginary part of]
dipole-target scattering amplitude
:: all QCD information
:: all x dependence
:: non-perturbative, but x-evolution computable from first principles

# impact parameter dependence

- b-dependence governed by long-distance non-perturbative physics
- AAMS 1.0 resorts to translational invariance approximation
  - proton homogeneous in transverse plane

$$\sigma_{T,L}(x,Q^2) = \int_0^1 dz \int d\mathbf{b}\, d\mathbf{r}\, |\Psi_{T,L}(z,Q^2,\mathbf{r})|^2 \mathcal{N}(\mathbf{b},\mathbf{r},x)$$

$$\sigma_{T,L}(x,Q^2) = \sigma_0 \int_0^1 dz \int d\mathbf{r}\, |\Psi_{T,L}(z,Q^2,\mathbf{r})|^2 \mathcal{N}(r,Y)$$

'b-integration'
fit parameter
:: if factorized structure [of x, r and b dependence] unchanged by evolution, then related to t-dependence in diffractive events

  - exclusive observables and nuclei requires more sophisticated treatment of b-dependence

# ⥁ from A to B

- want the best possible, numerically tractable, incarnation of non-linear evolution
  - with Dense-Dense effects and NLO
    - ↪ RFT-QCD [Kovner's talk] contains all Dense-Dense effects
      - no known strategy for numerical implementation
      - NLO [running coupling] should trump Dense-Dense [toy model]

*[Dumitru, Iancu, Portugal, Soyez and Triantafyllopoulos, JHEP 0708:062, 2007]*

- 'safely' neglect Dense-Dense if NLO formulation available
  - B-JIMWLK
    - ↪ no Dense-Dense, no NLO, numerically challenging ...
    - ↪ but BK [large N, mean field] solutions deviate only 0.1% from full B-JIMWLK

*[Kovchegov, Kuokannen, Rummukainen, Weigert, NPA 823, 47 (2009)]*

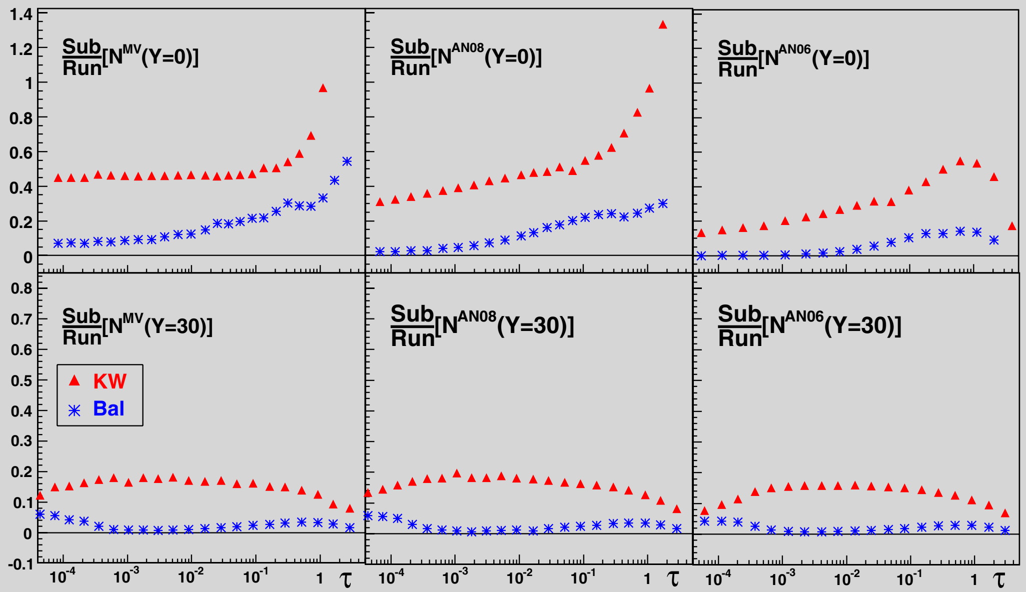
- 'safely' replace full B-JIMWLK by BK
  - LO-BK not consistent with data [unless coupling very small]
  - NLO-BK computed *[Balitsky, Chirilli, Kovchegov, Weigert]*
  - running coupling part numerically tractable

# ⟳ on why B is B’

- NLO-BK = all orders in $\alpha_s N_f$ + other *[Chirilli’s talk]*
  - all orders in $\alpha_s N_f$ = rc + subtraction

$$\frac{\partial \mathcal{N}(r,Y)}{\partial Y} = \mathcal{R}[\mathcal{N}] - \mathcal{S}[\mathcal{N}]$$

  - subtraction piece numerically demanding
    - ↪ running coupling contribution dominant over conformal piece in $\alpha_s N_f$ piece




*[Albacete, Kovchegov]*

- scheme dependent definition of ‘subtraction’ piece
  - Balitsky’s (Bal) scheme minimizes ‘subtraction’ contribution

- [other] yet to be numerically computed :: challenging ::

# BK evolution with running coupling [Bal scheme]

$$\frac{\partial \mathcal{N}(r,Y)}{\partial Y} = \int d\mathbf{r_1}\, K^{\mathrm{Bal}}(\mathbf{r},\mathbf{r_1},\mathbf{r_2}) \quad [\mathcal{N}(r_1,Y)+\mathcal{N}(r_2,Y)-\mathcal{N}(r,Y)-\mathcal{N}(r_1,Y)\mathcal{N}(r_2,Y)]$$

same structure as LO-BK

modified kernel

$$K^{\mathrm{Bal}}(\mathbf{r},\mathbf{r_1},\mathbf{r_2}) = \frac{N_c\,\alpha_s(r^2)}{2\pi^2}\left[\frac{r^2}{r_1^2\,r_2^2}+\frac{1}{r_1^2}\left(\frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)}-1\right)+\frac{1}{r_2^2}\left(\frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)}-1\right)\right]$$

$$\Lambda_{QCD} = 0.241\ \mathrm{GeV}$$

$$\alpha_s(M_Z) = 0.1176$$

- $r < r_{fr}, \quad \alpha_s(r^2) = \dfrac{12\pi}{(11N_c - 2N_f)\,\ln\left(\frac{4\,C^2}{r^2\Lambda^2_{QCD}}\right)}$
- $r > r_{fr}, \quad \alpha_s(r^2_{fr}) \equiv \alpha_{fr} = 0.7$

*regulated IR behaviour*

$C^2$ — uncertainty in FT from momentum to coordinate space

*:: fit parameter*

# ⥁ initial condition[s]

- 2 families of initial conditions
  - generalized with anomalous dimension GBW and MV forms

$$\mathcal{N}^{GBW}(r, Y=0) = 1 - \exp\left[-\left(\frac{r^2\, Q_{s\,0}^2}{4}\right)^{\gamma}\right]$$

$$\mathcal{N}^{MV}(r, Y=0) = 1 - \exp\left[-\left(\frac{r^2 Q_{s\,0}^2}{4}\right)^{\gamma} \ln\left(\frac{1}{r\,\Lambda_{QCD}} + e\right)\right]$$

  - differ in UV behaviour
  - fit parameters
    - ↪ initial saturation scale $Q_{s\,0}^2$
    - ↪ anomalous dimension $\gamma$
  - anomalous dimension in GBW form set to one after initial tests

# data

- all available $F_2(x,Q^2)$ data
  - with :: $x \leq 10^{-2}$
  - no cut on $Q^2$ :: $0.045 \text{ GeV}^2 \leq Q^2 \leq 800 \text{ GeV}^2$
- 847 data points
  - statistical and systematic uncertainties added in quadrature
  - normalization uncertainties not considered
- redefinition of Bjorken x as to go smoothly to photoproduction

$$\tilde{x} = x \left(1 + \frac{4m_f^2}{Q^2}\right)$$

with $m_f = 0.14 \text{ GeV}$ , only light quarks

# summary [for fit]

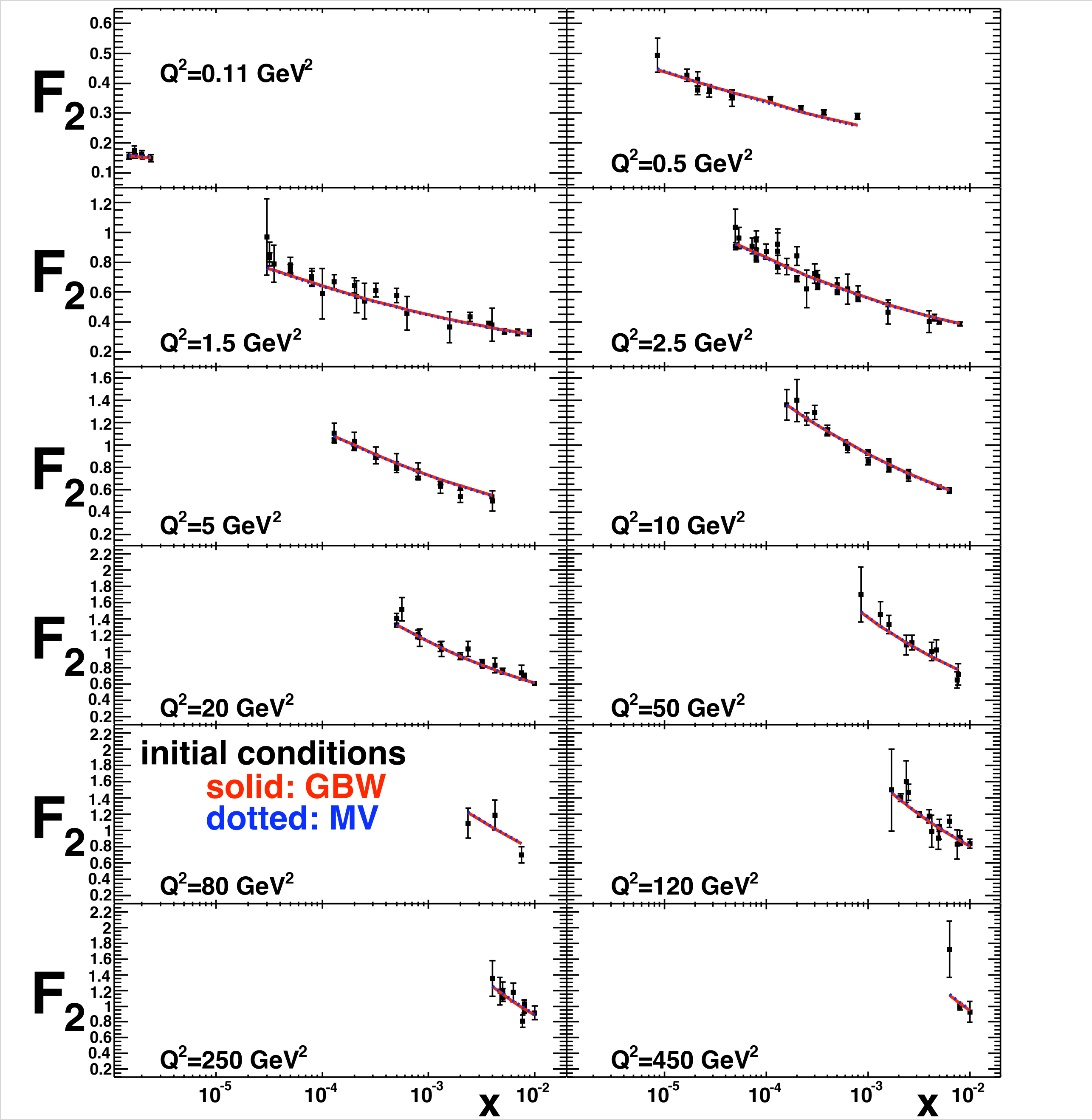
- $F_2$ calculated from

$$F_2(x,Q^2) = \frac{Q^2}{4\,\pi^2\alpha_{em}}\,(\sigma_T + \sigma_L)$$

$$\sigma_{T,L}(x,Q^2) = \sigma_0 \int_0^1 dz \int d\mathbf{r}\,|\Psi_{T,L}(z,Q^2,\mathbf{r})|^2\,\mathcal{N}(r,Y)$$

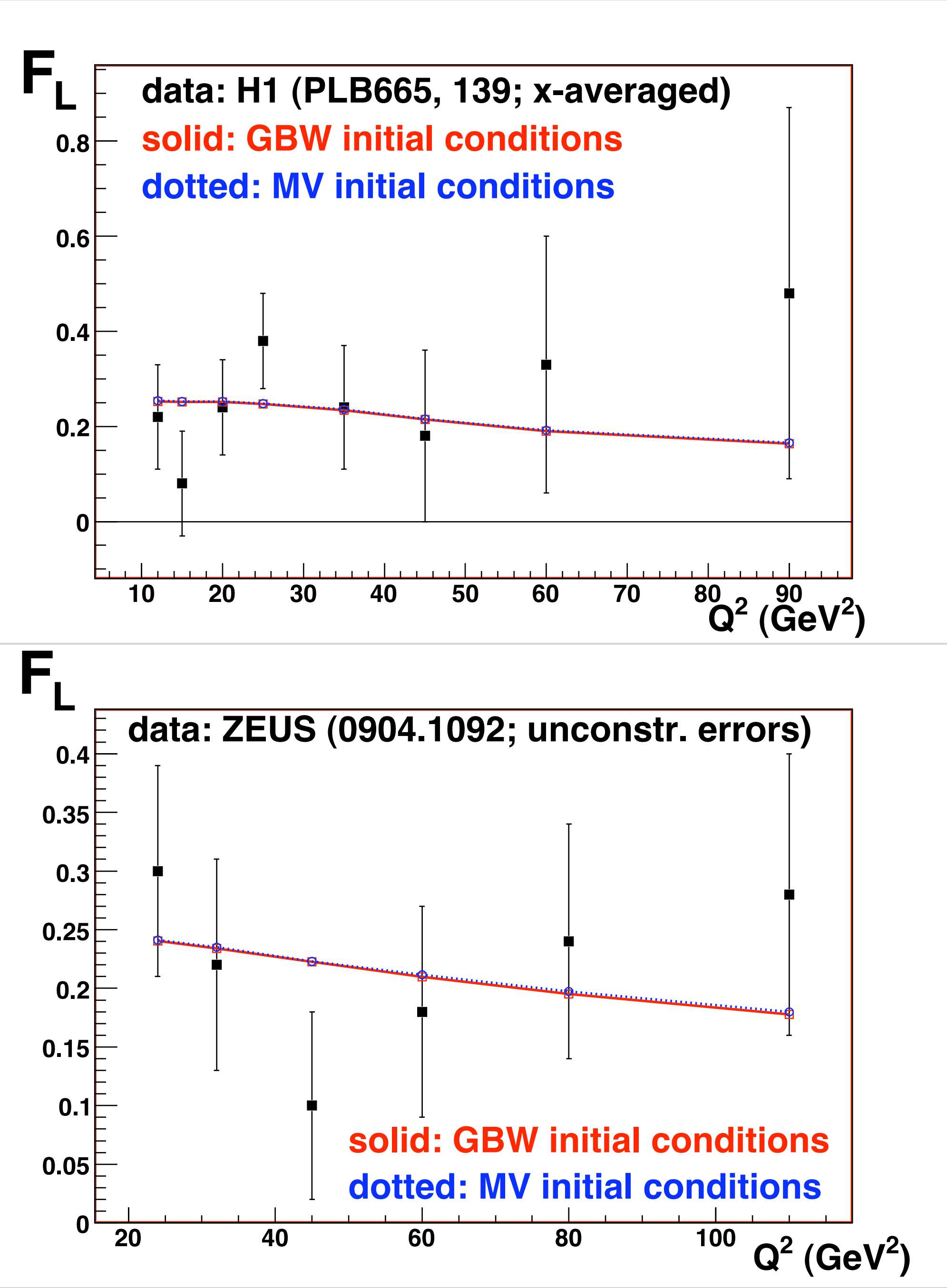
- 4 fit parameters [3 for GBW ic]
  - total normalization of cross section [b-integration]
  - IR uncertainty in running coupling [from FT]
  - initial saturation scale [in ic]
  - anomalous dimension [in ic] :: MV only

# fit results




| Initial condition | $\sigma_0$ (mb) | $Q^2_{s0}$ (GeV$^2$) | $C^2$ | $\gamma$ | $\chi^2$/d.o.f. |
|---|---|---|---|---|---|
| GBW | 31.59 | 0.24 | 5.3 | 1 (fixed) | 916.3/844=1.086 |
| MV | 32.77 | 0.15 | 6.5 | 1.13 | 906.0/843=1.075 |

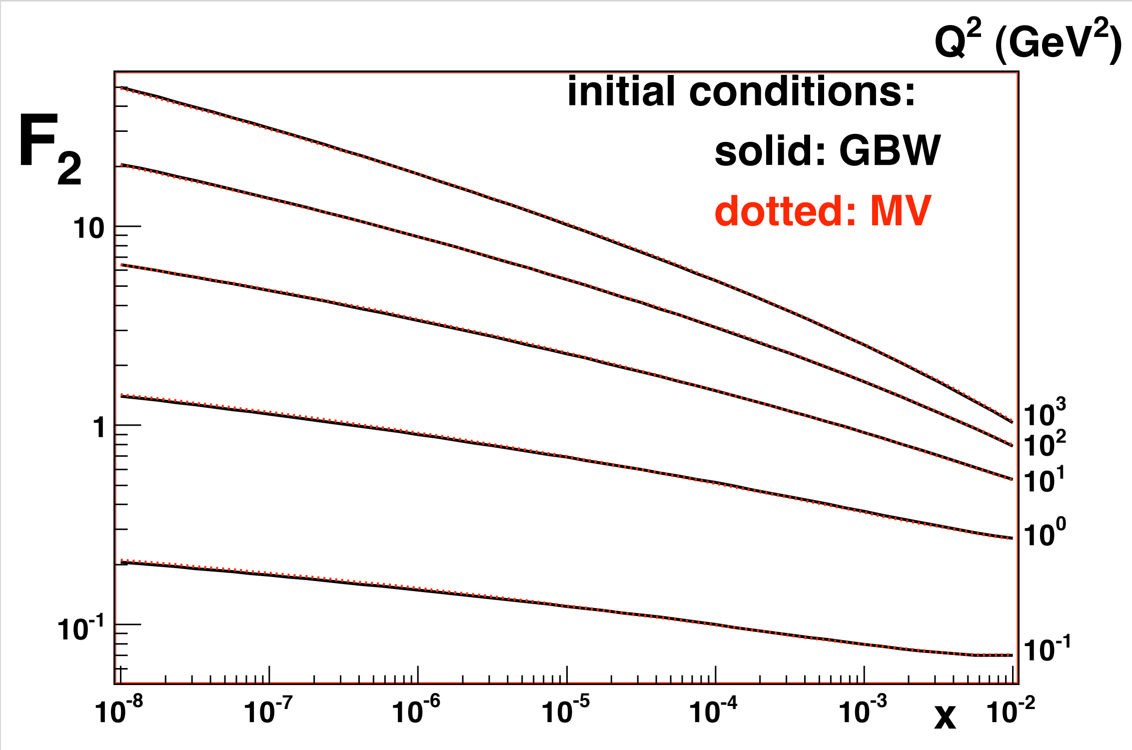
# $F_L$

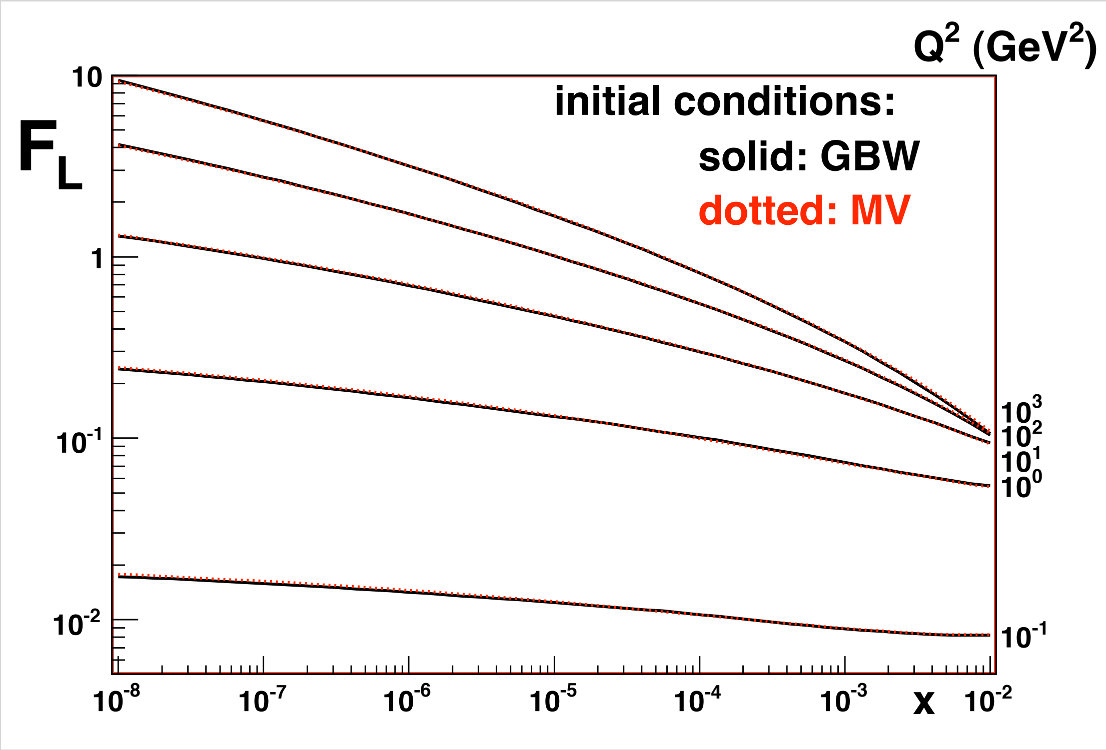



$$F_L(x, Q^2) = \frac{Q^2}{4\,\pi^2 \alpha_{em}}\,\sigma_L$$

- $F_L$ data not included in the fit
  - consistently described [error bars too large for meaningful statement]

# ⟳ predictions

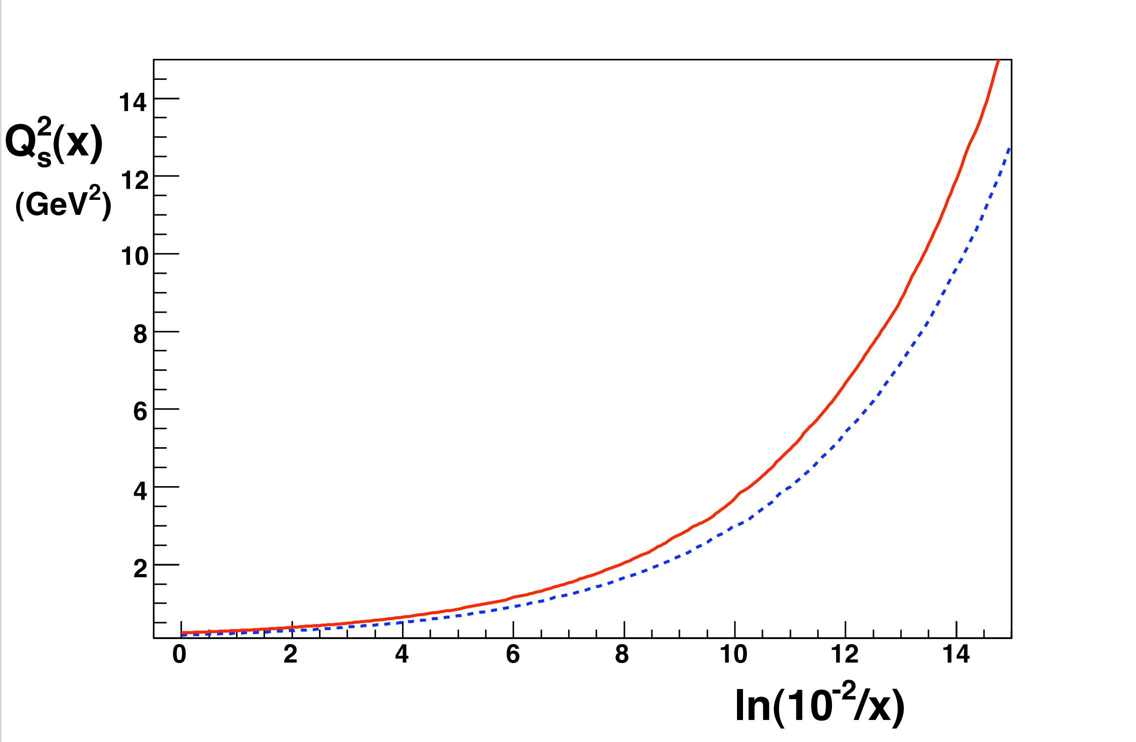



- $F_2$ and $F_L$ extrapolated to LHeC and UHECR kinematical conditions
  - near independence on [tested] initial conditions
  - first principle approach allows for credible extrapolation
    - ↪ ‘all’ relevant physics included

- AAMS $F_2$ and $F_L$ cannot be fitted by NLO-DGLAP [Albacete’s talk]
  - reported deviations from NLO-DGLAP in HERA data ! [Caola, Forte, Rojo]
    - ↪ ressumation, heavy quark, non-linearities? not NLLO

# saturation momentum, geometric scaling




$$\mathcal{N}(r = 1/Q_s(x), x) = 1 - \exp[-1/4]$$

- large [perturbative] saturation scale for forward region in pp at the LHC

$$x = (2\,M/\sqrt{s})e^{-y} \qquad Q_s^2 \simeq 3 \div 4\ \mathrm{GeV}^2 \qquad y = 6$$

- geometric scaling in DGLAP ?? [no scale]

# AAMS 1.0

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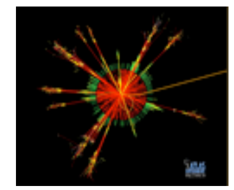
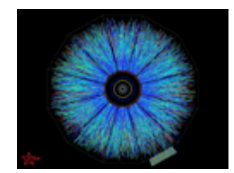
## Dipole-proton cross section





The imaginary part of the dipole-proton scattering amplitude is available as a FORTRAN routine for public used. This quantity has been fitted to lepton-proton data using the Balitsky-Kovchegov evolution equations with running coupling. More details can be found at

J. L. Albacete, N. Armesto, J. G. Milhano and C. A. Salgado, arXiv:0902.1112

Please refer to this publication when using the routine.

In order to compute the dipole cross section, simply multiply the output from the routine by the corresponding constant values

sigma0=31.59 mb for GBW initial conditions

sigma0=32.77 mb for MV initial conditions

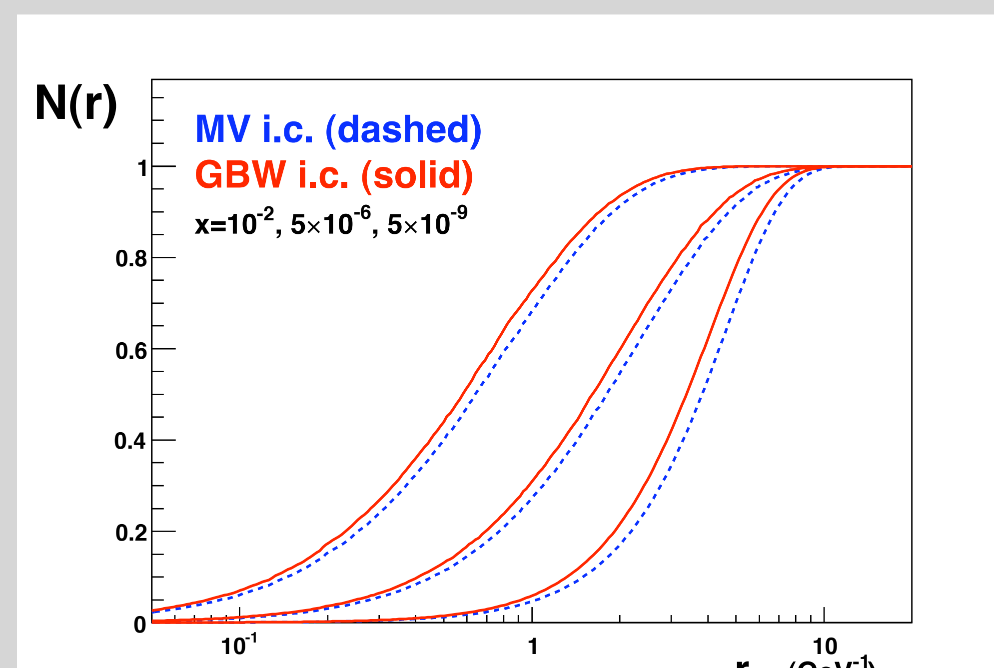
To download the code, please follow this link

NEWS

The code has been updated to work properly with some old compilers. If you find any problem, please, let us know

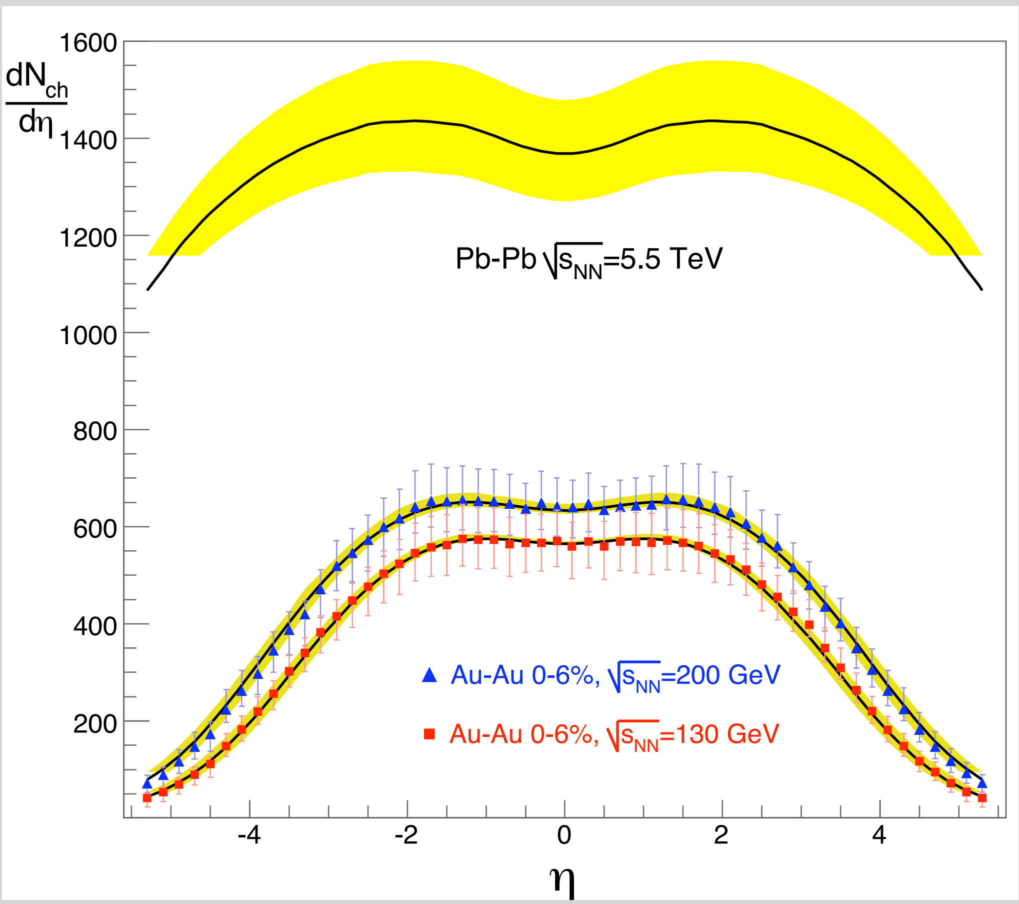
http://www-fp.usc.es/phenom/rcbk/

$$10^{-12} \leq x \leq 10^{-2}$$



# also used in ...

- mid-rapidity multiplicities in AA *[Albacete PRL 99, 262301]*
  - preceded present work [is its basis]




$$\frac{dN_{ch}}{dy\,d^2b} = C\frac{4\pi N_c}{N_c^2-1}\int\frac{d^2p_t}{p_t^2}\int^{p_t} d^2k_t\,\alpha_s(Q)\quad\varphi\left(x_1,\frac{|\underline{k_t}+\underline{p_t}|}{2}\right)\varphi\left(x_2,\frac{|\underline{k_t}-\underline{p_t}|}{2}\right)$$

$$\varphi(Y,k) = \int\frac{d^2r}{2\pi\,r^2}\exp\{i\,\underline{r}\cdot\underline{k}\}\,\mathcal{N}(Y,r)$$

- diffractive and forward hadron production in pp *[Betemps, Gonçalves and Santana Amaral]*
- hadron and direct photon production in ep and pA *[Rezaeian and Schafer]*
- comparisons with NLO-DGLAP NNPDF [Albacete's talk]
- independent related work [Weigert's talk]

# ⥁ to do

- charm
  - refit
  - $F_2^c$
- nuclei
  - decent treatment of impact parameter dependence
  - tool for extrapolation for PbPb at the LHC
- understand $Q^2$ dependence
- is PDF extraction possible ?