QCD evolution and correlations in impact parameter

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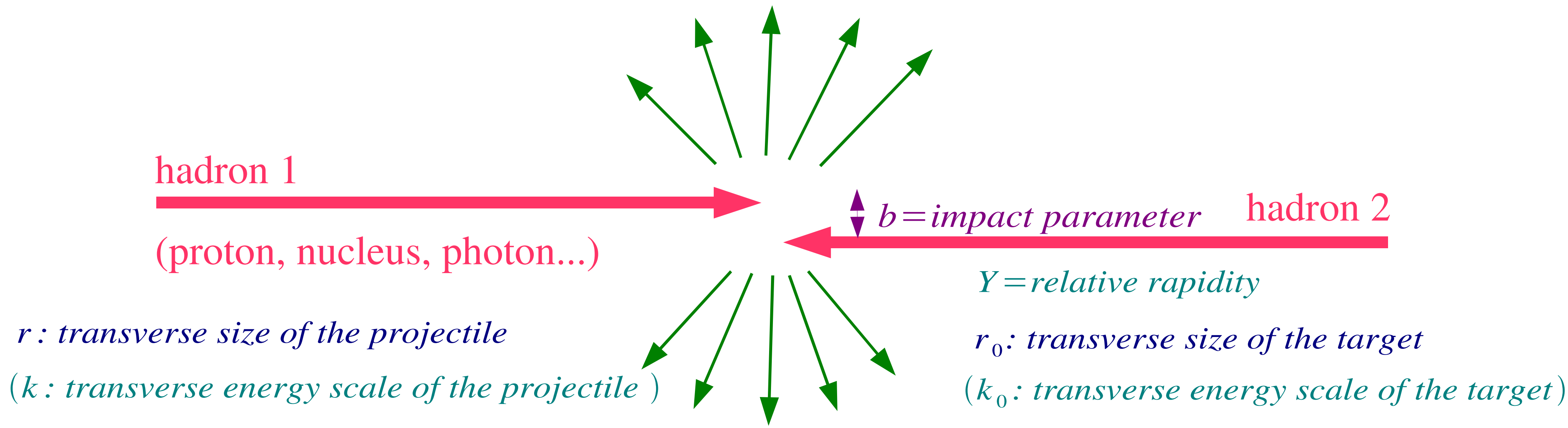
Work done in collaboration with G. Salam and G. Soyez

ÉCOLE POLYTECHNIQUE

Low-x meeting 2009, Ischia

cnrs dépasser les frontières

High energy QCD



$$A(Y, \overset{k}{r}) = \int d^2 b\, A(Y, b, \overset{k}{r}) = \textit{elastic amplitude}$$

$$A(Y, b, \overset{k}{r}) = \textit{fixed impact parameter amplitude} \quad \leq 1$$

(High) energy dependence of QCD amplitudes?

Outline

- ✯ High energy QCD evolution identified with a one-dimensional stochastic process

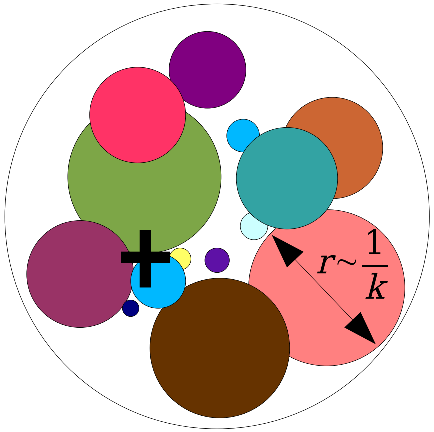
  This identification is still a conjecture!

  One important assumption: impact parameters evolve independently.

- ✯ Numerical check of the independence of different impact parameters in a toy model

How a high rapidity hadron looks

Iancu, Mueller, Munier (2004)



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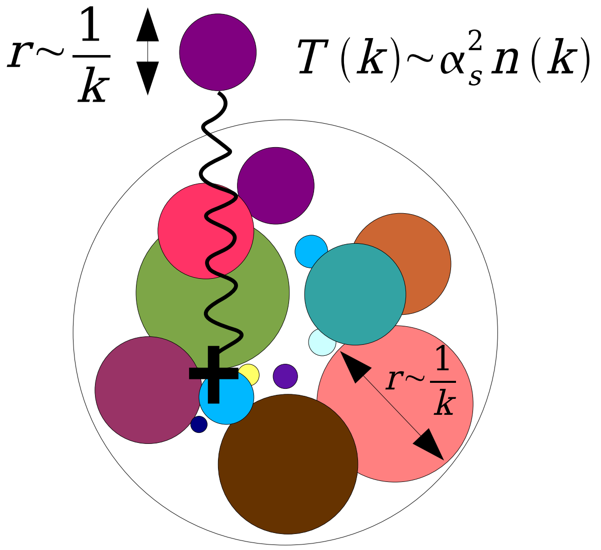

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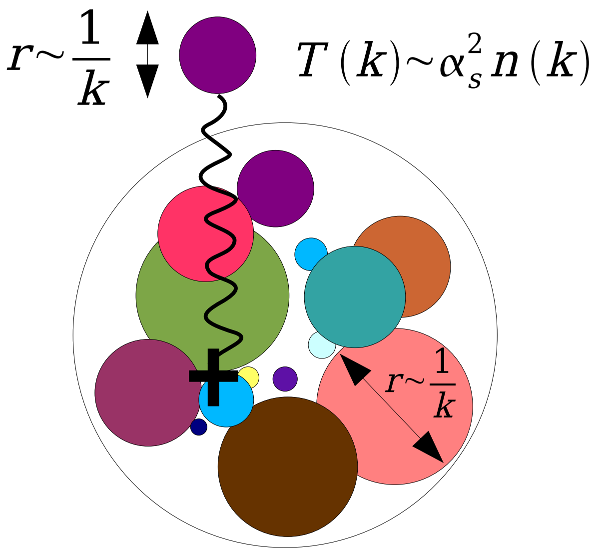

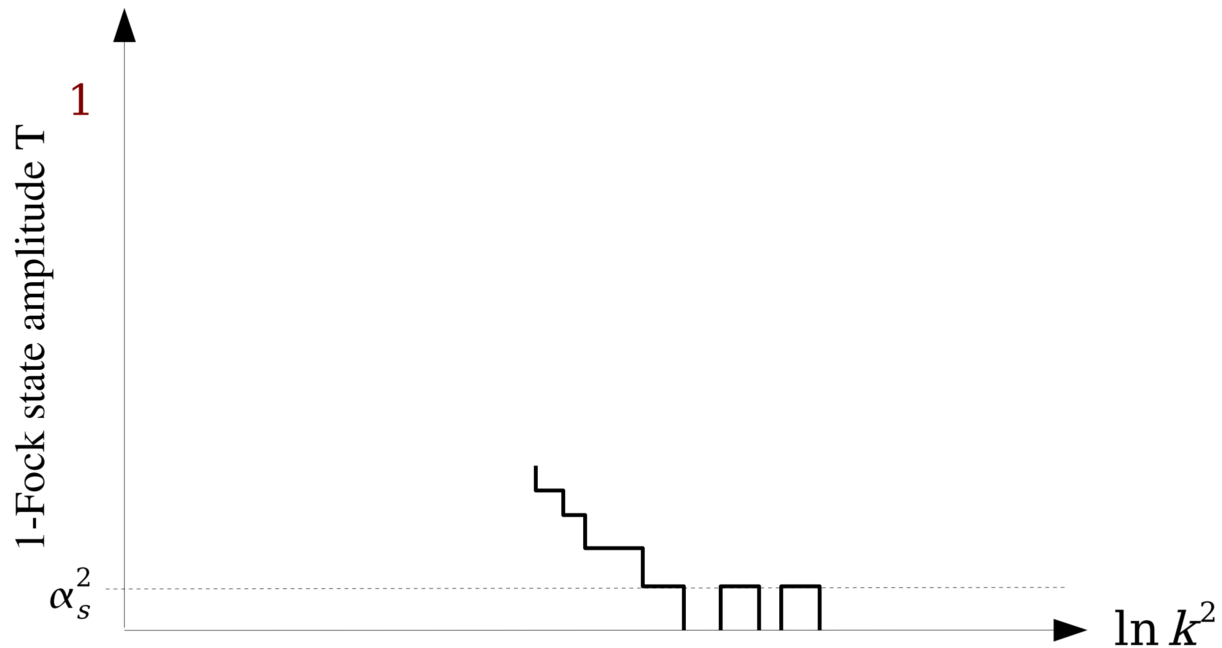

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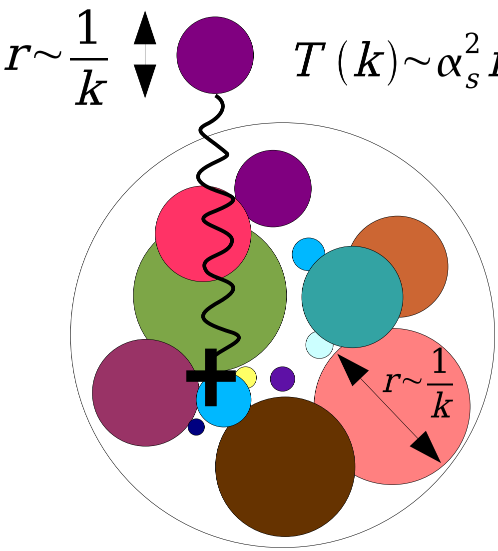
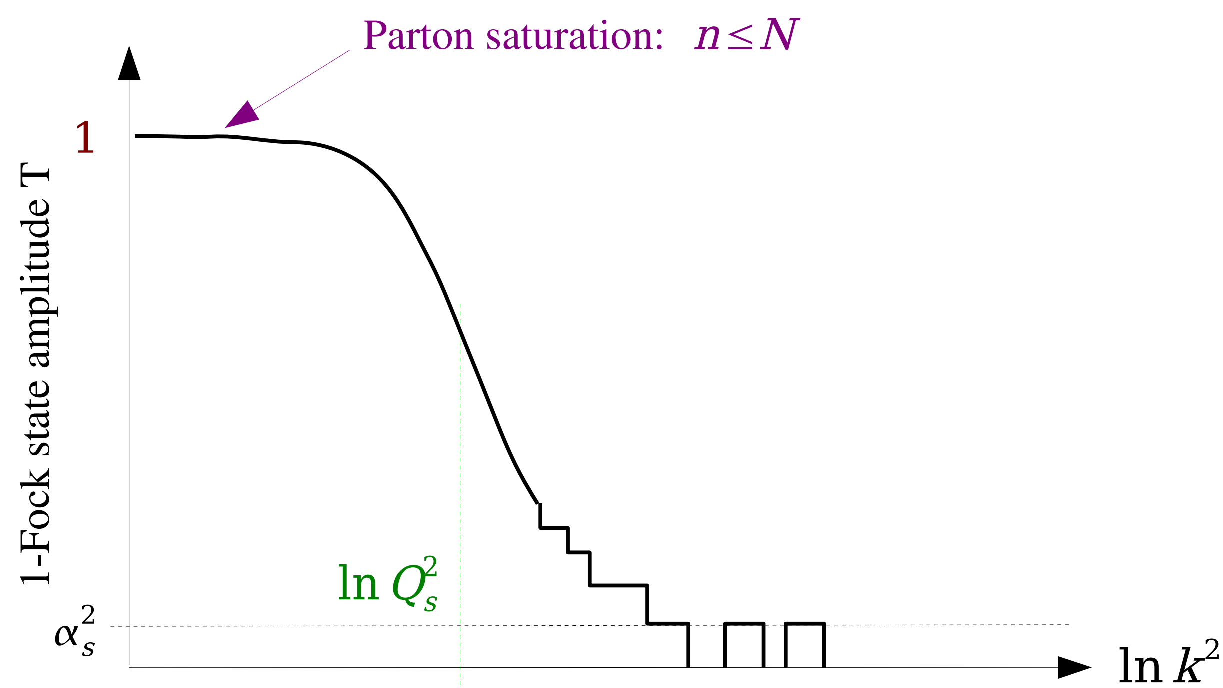


$T(k) \sim \alpha_s^2 n(k)$

unitarity: $T(r) \le 1 \;\; \Rightarrow \;\; N = \frac{1}{\alpha_s^2}$



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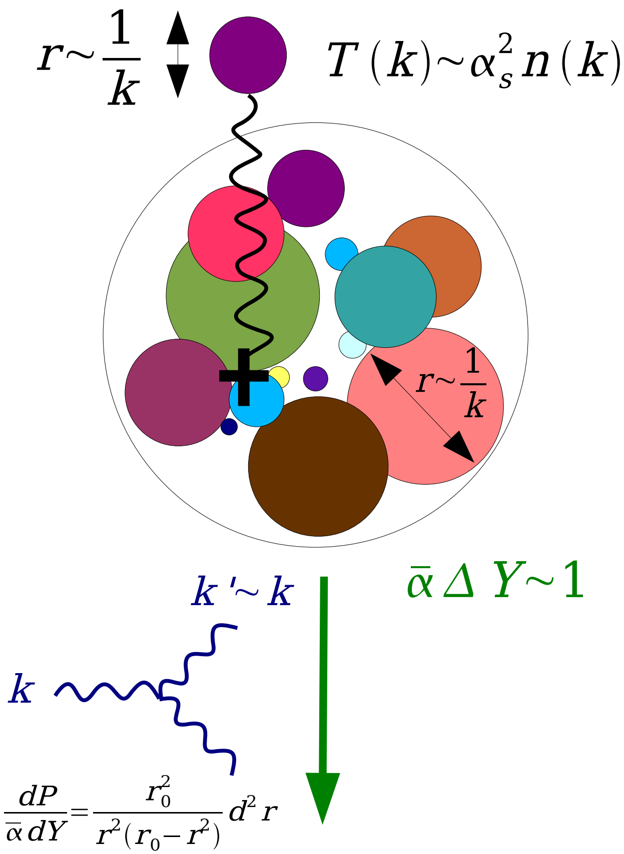

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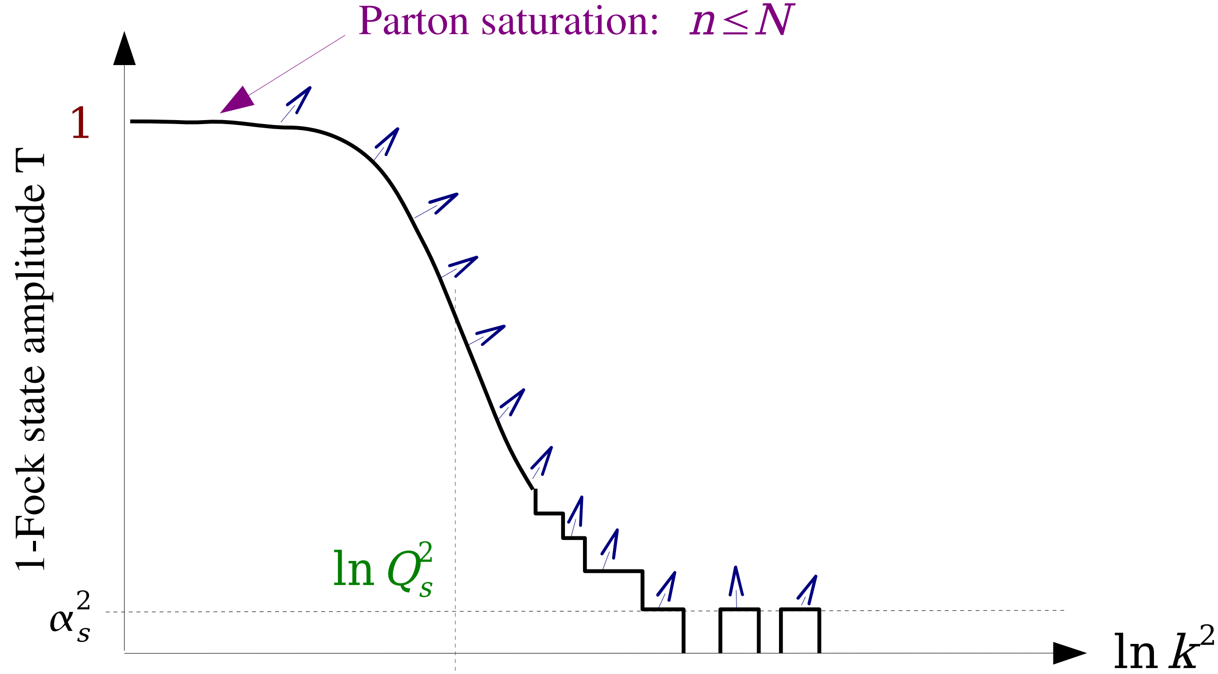

$$\partial_{\bar{\alpha} Y} T = \chi(-\partial_{\ln k^2}) T \qquad + \alpha_s \sqrt{T} \; \nu$$

$BFKL \sim \partial^2_{\ln k^2} T + T$

Noise term
due to discreteness

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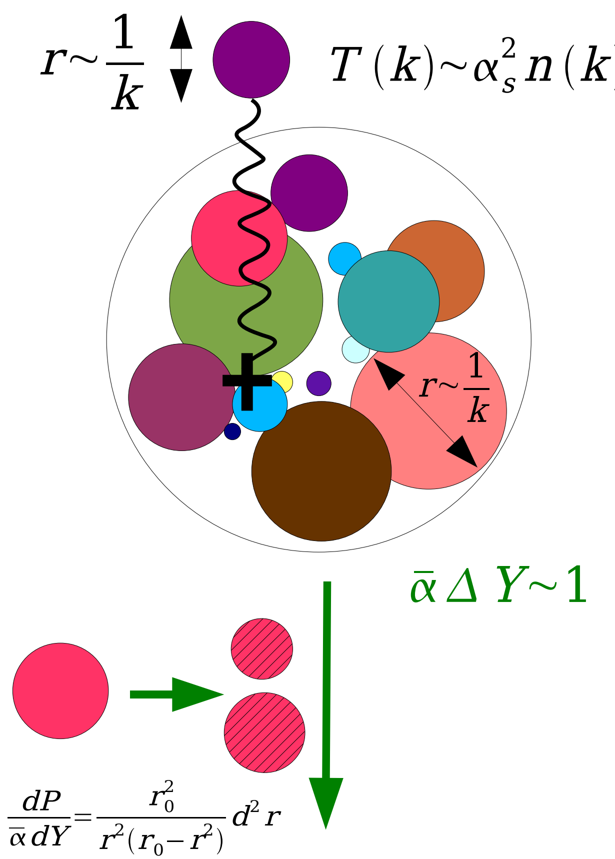

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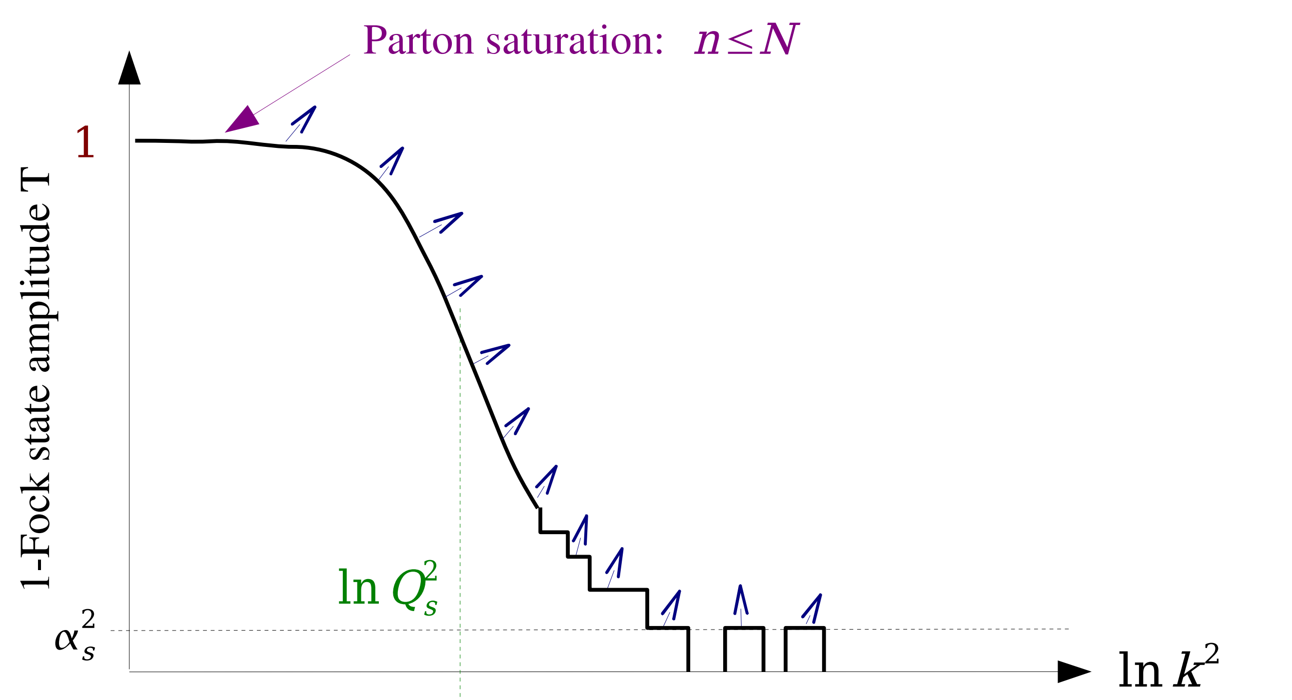

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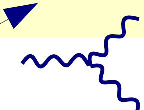

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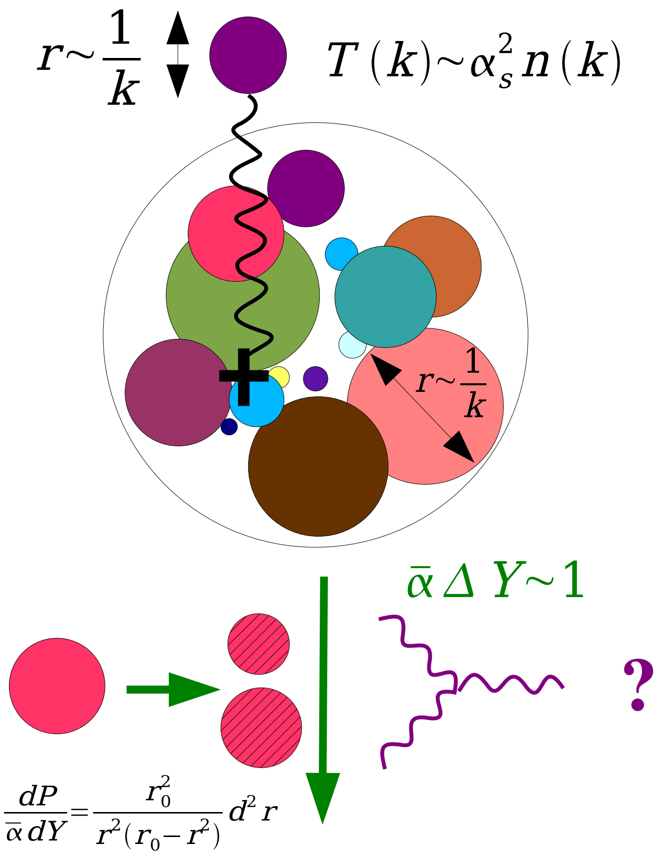

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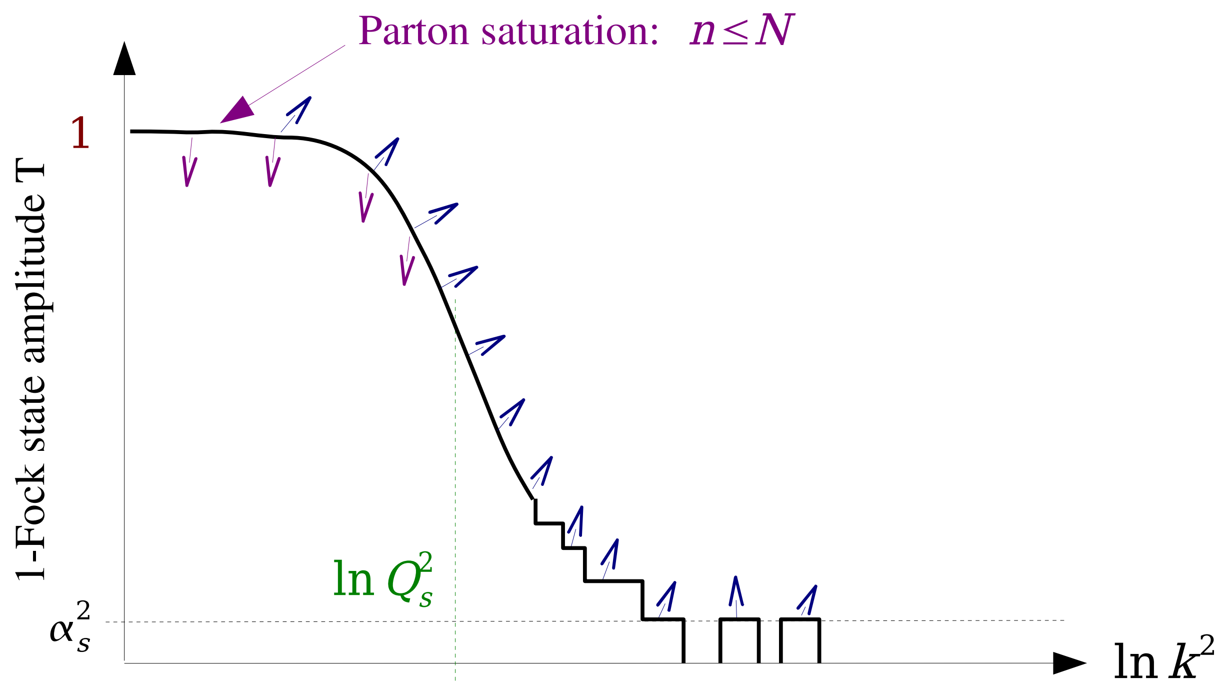

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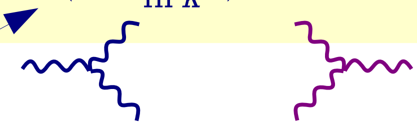

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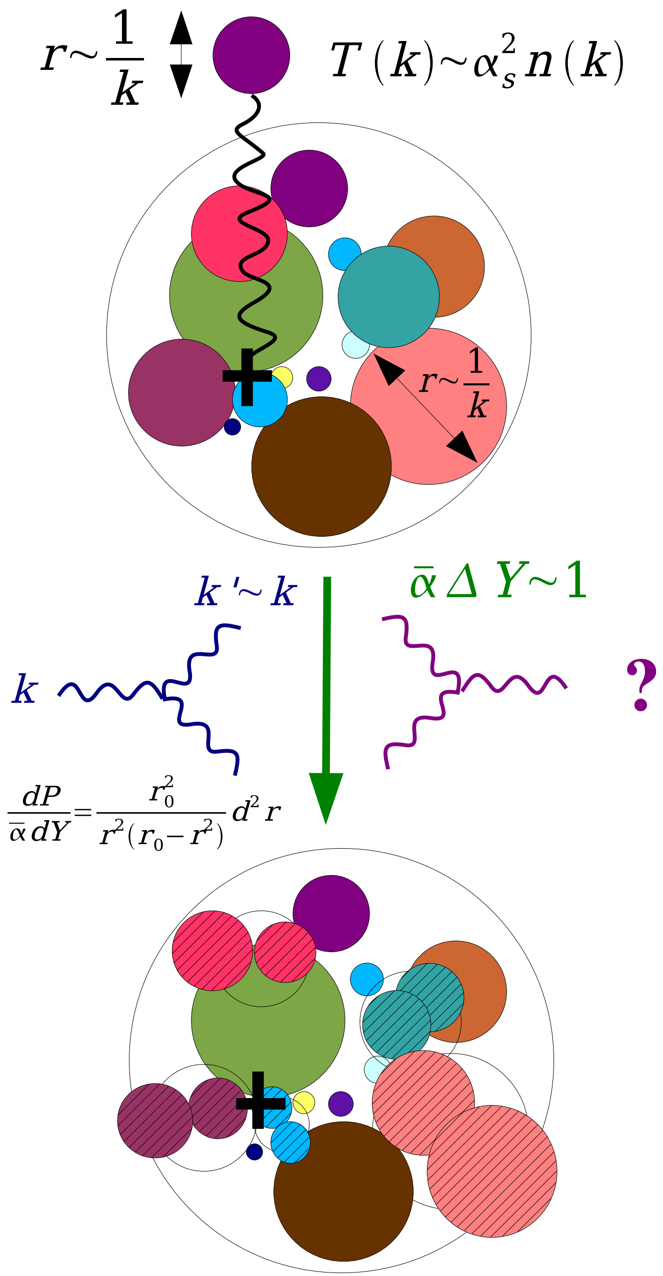

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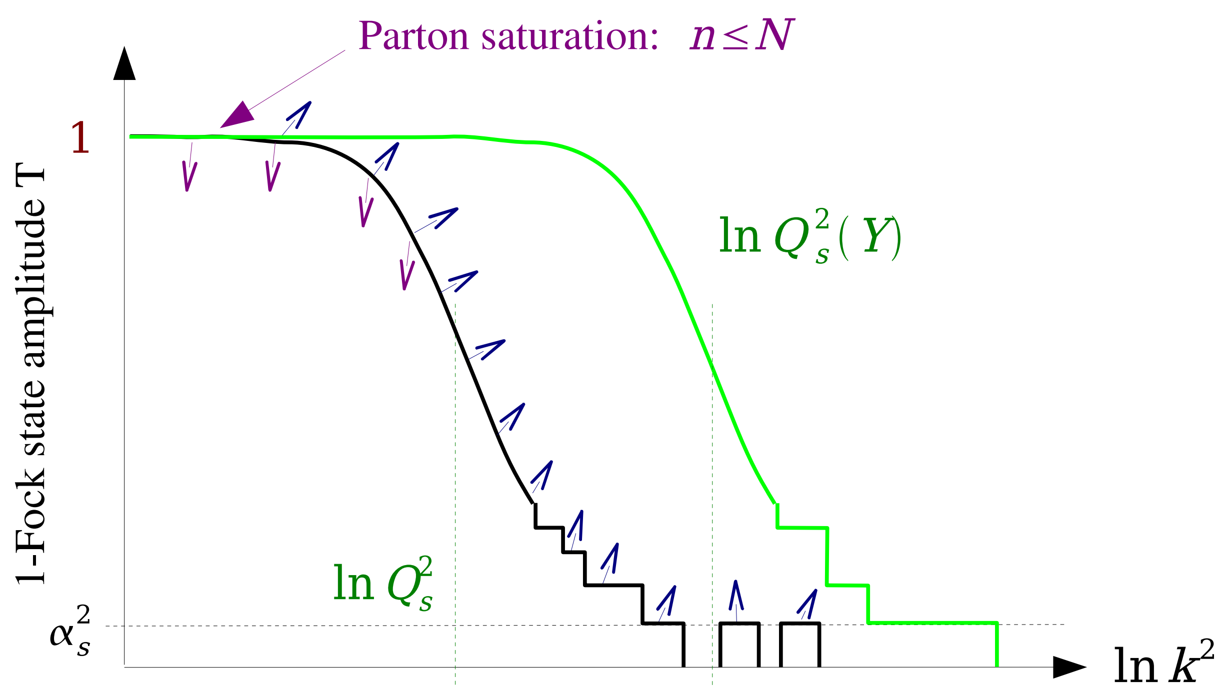

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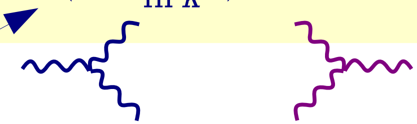

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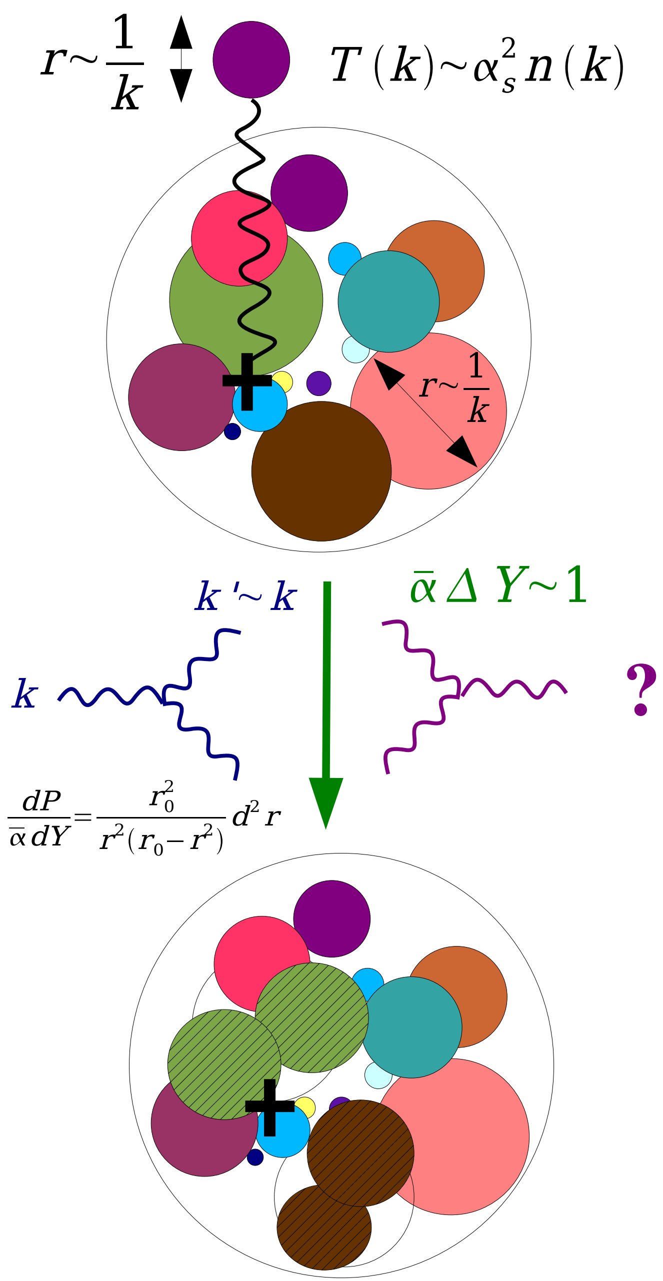

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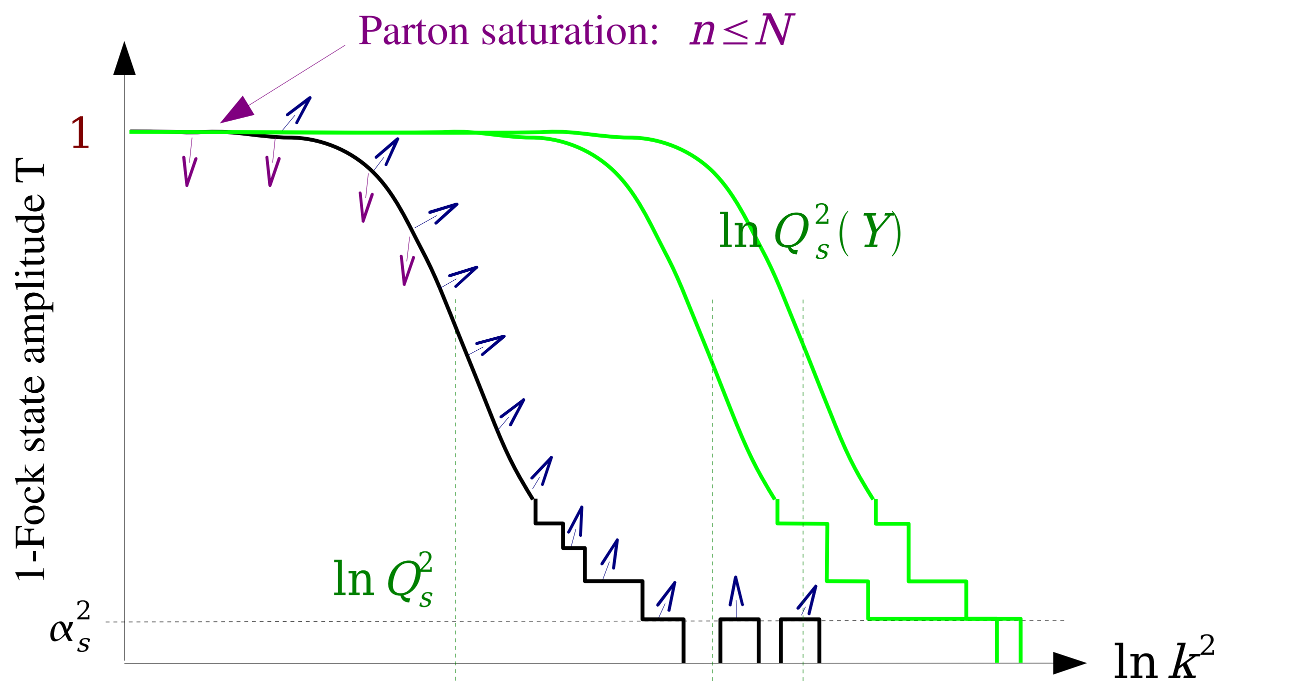

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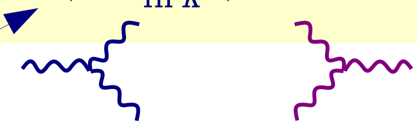

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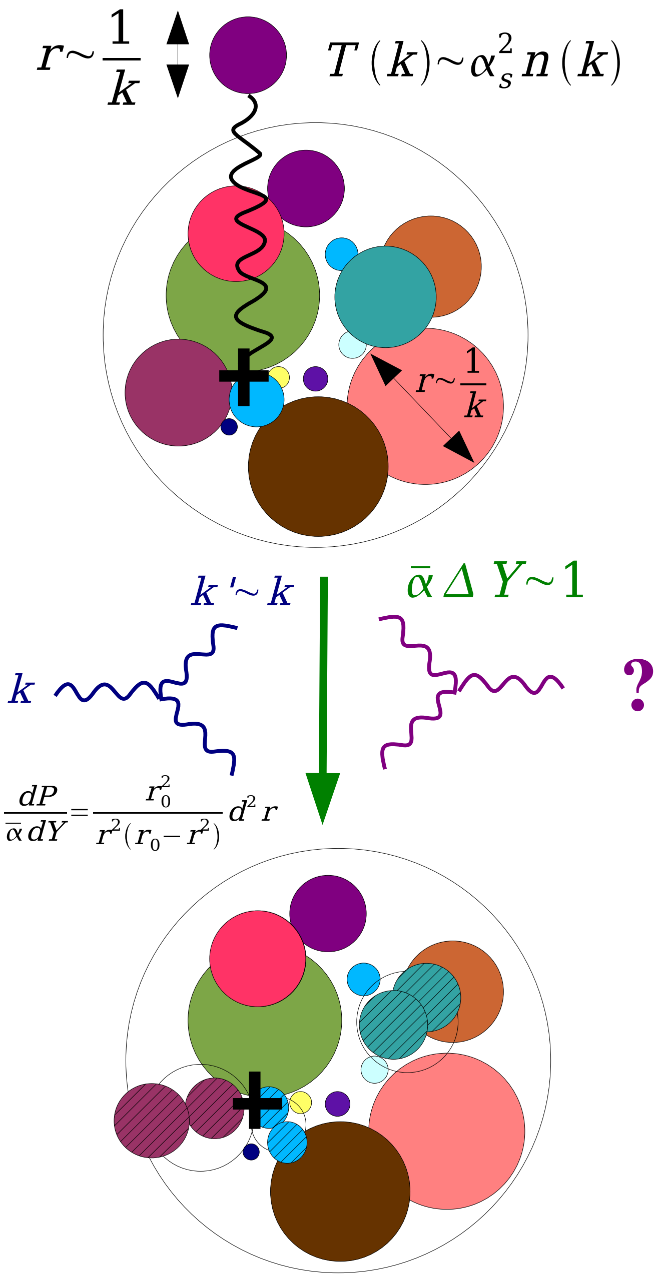

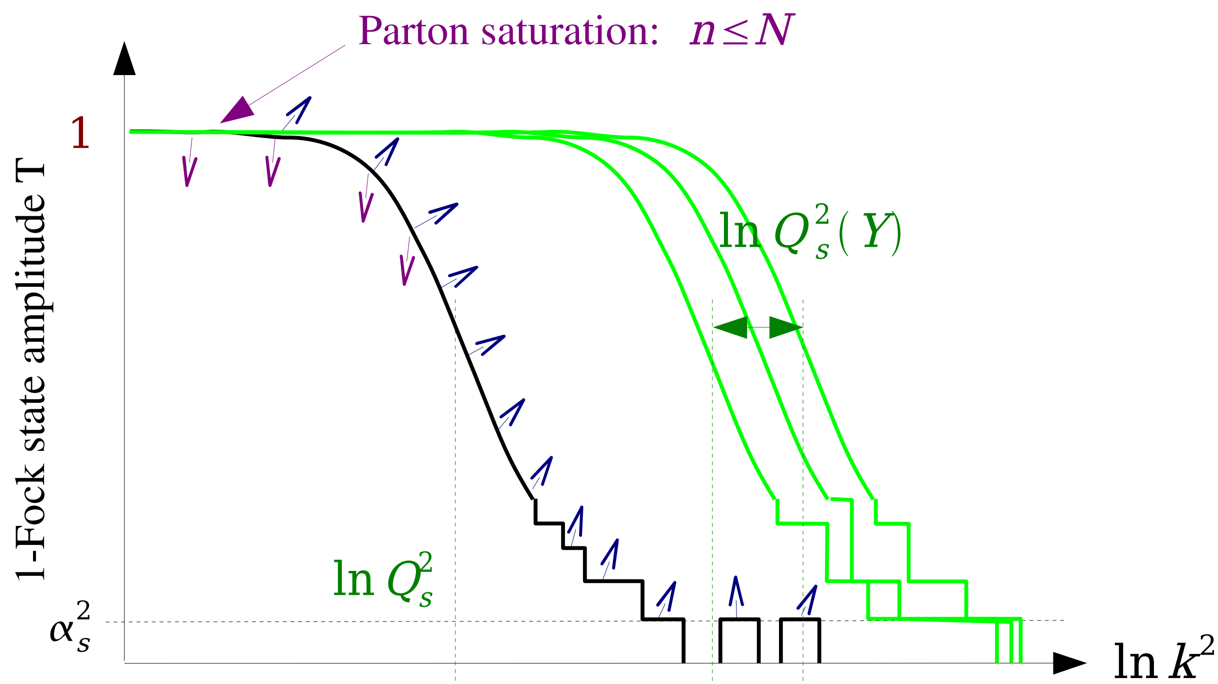

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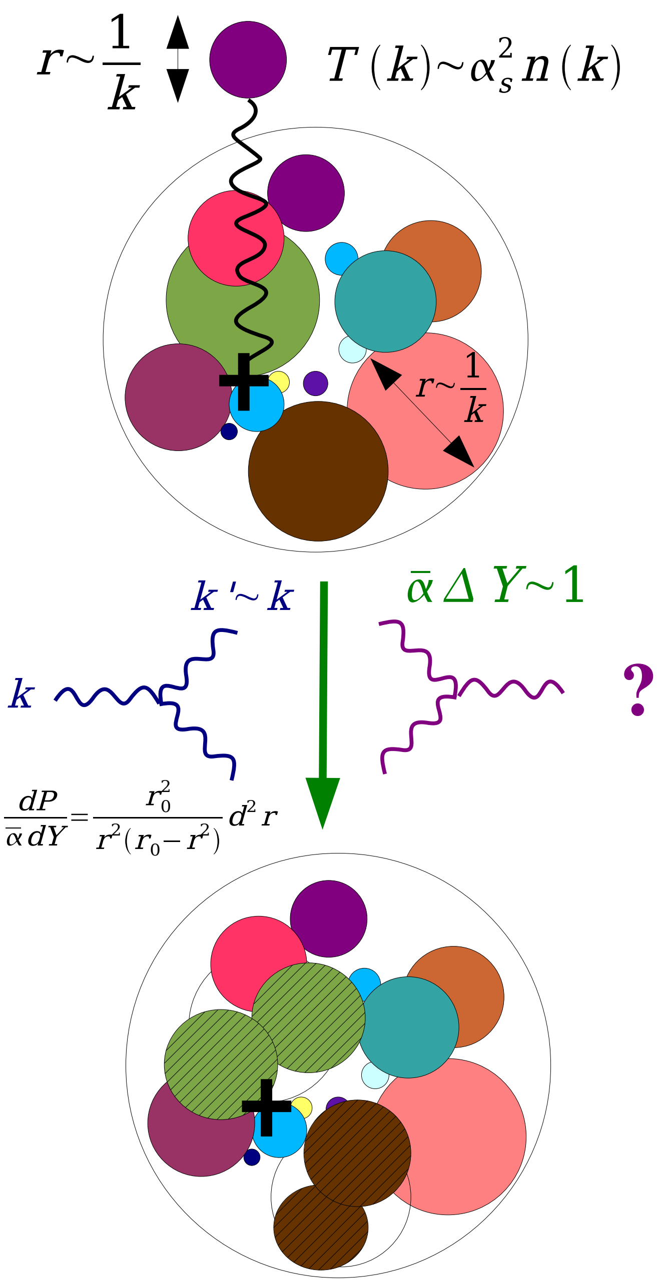

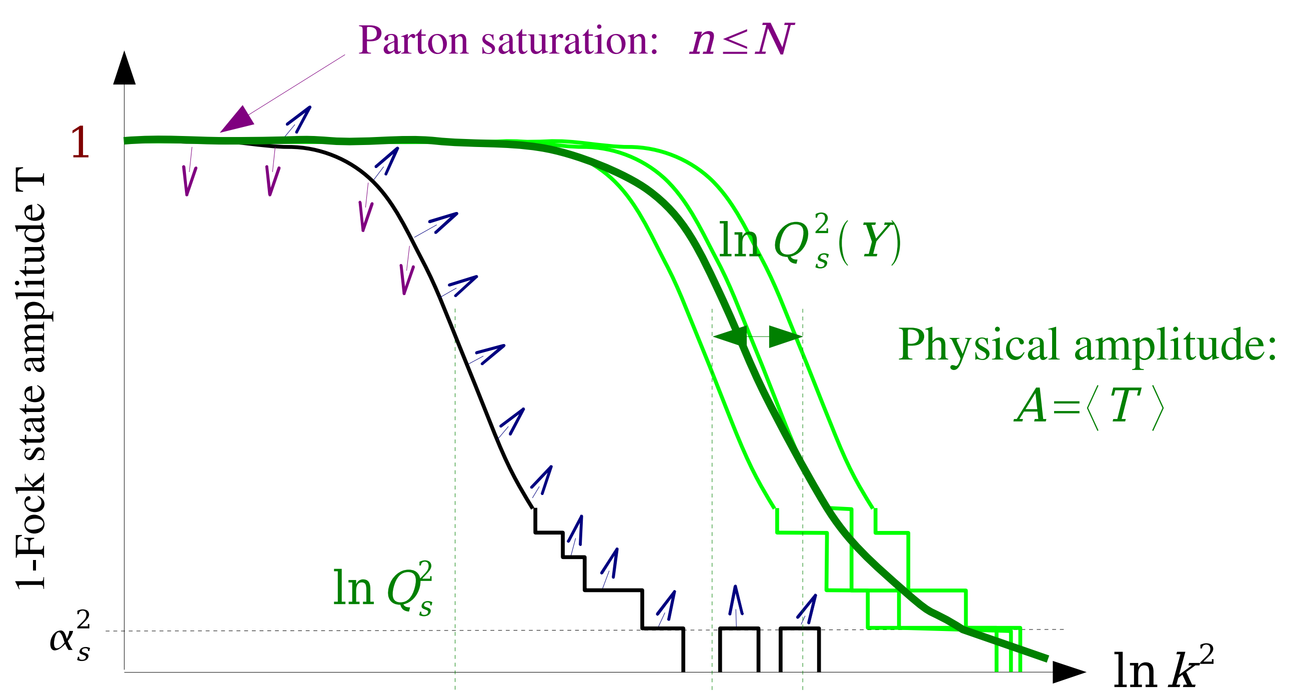

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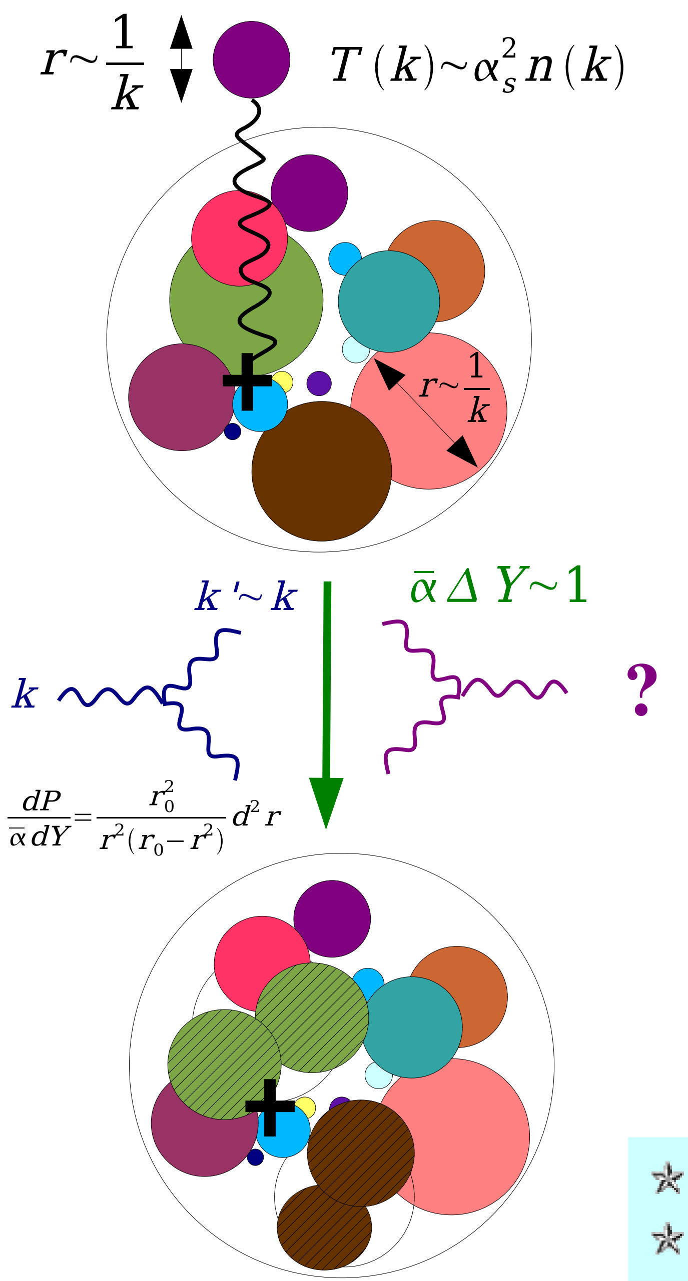

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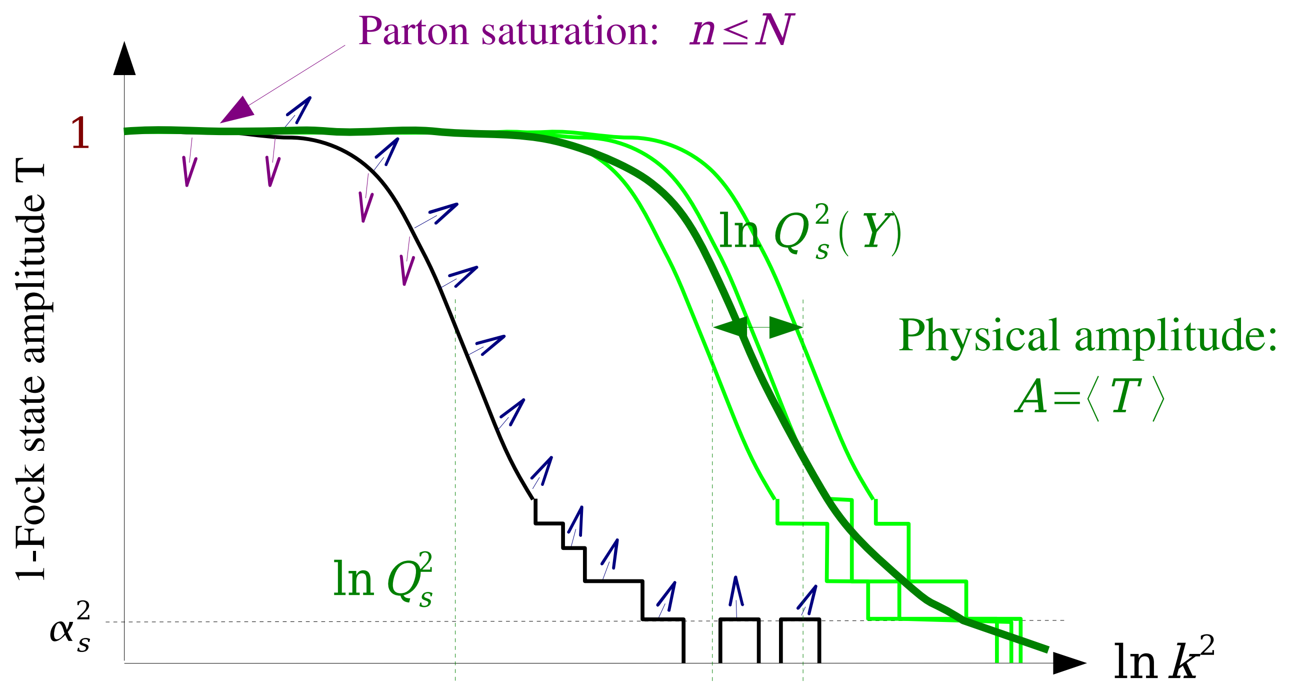

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Noise term due to discreteness

- simple picture of high energy scattering, based on the parton model
- connects the QCD problem to more general physics and mathematics
- new results for QCD amplitudes!

QCD and reaction-diffusion

$$\partial_{\bar{\alpha} Y} T = \chi(-\partial_{\ln k^2})\, T - T^2 + \alpha_s \sqrt{T}\;\nu$$

Similar to the sFKPP equation $\quad \partial_t T = \partial_x^2\, T + T - T^2 + \sqrt{\frac{2}{N} T}\;\nu$

Fisher; Kolmogorov, Petrovsky, Piscunov (1937)

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Predictions for QCD amplitudes

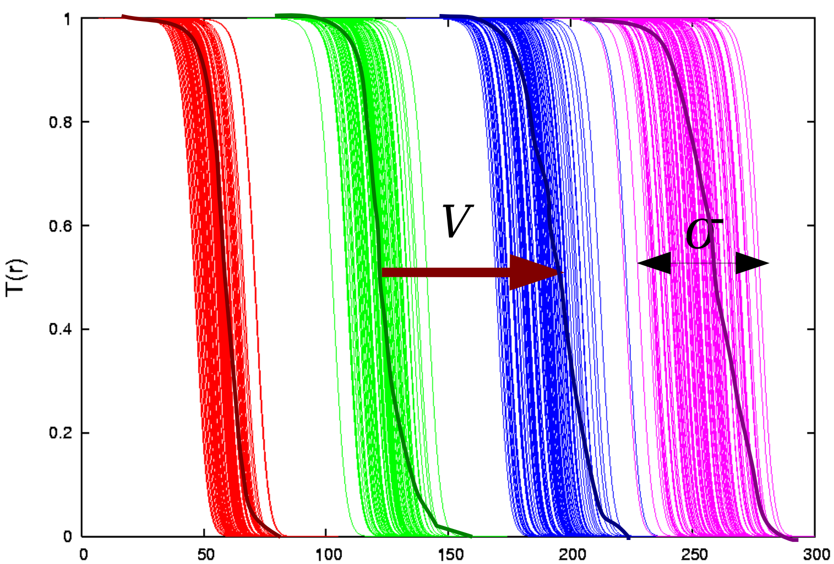
Shape of the *partonic* amplitude: $T \sim (r Q_s(Y))^{2\gamma_0}$

Saturation scale:

$$V = \frac{d}{d(\bar{\alpha} Y)} \langle \ln Q_s^2 \rangle = \frac{\chi(\gamma_0)}{\gamma_0} - \frac{\pi^2 \gamma_0 \chi''(\gamma_0)}{2 \ln^2(1/\alpha_s^2)} + \pi^2 \gamma_0^2 \chi''(\gamma_0) \frac{3 \ln\ln(1/\alpha_s^2)}{\gamma_0 \ln^3(1/\alpha_s^2)}$$

$$\langle \ln^n Q_s^2 \rangle_{cumulant} = \pi^2 \gamma_0^2 \chi''(\gamma_0) \frac{n!\,\zeta(n)}{\gamma_0^n} \left[\frac{\bar{\alpha} Y}{\ln^3(1/\alpha_s^2)} \right]$$

$$\Rightarrow A \sim A \left(\frac{r^2 Q_s^2(Y)}{\sqrt{\frac{\bar{\alpha} Y}{\ln^3(1/\alpha_s^2)}}} \right)$$

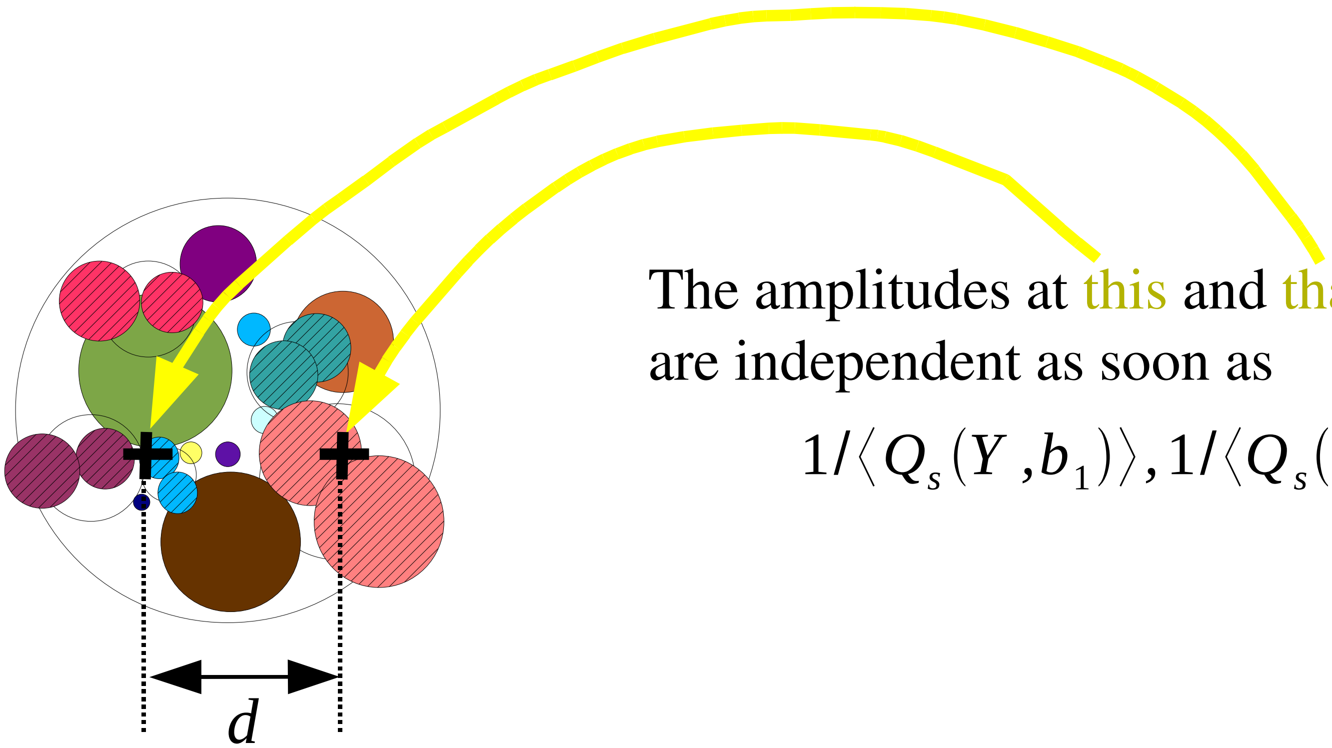


Traveling waves

Brunet, Derrida, Mueller, Munier (2004)

These formulas are independent of the precise form of the stochasticity and of the nonlinearity.

Independence of the different impact parameters?



The amplitudes at this and that impact parameters are independent as soon as

$$1/\langle Q_s(Y,b_1)\rangle, 1/\langle Q_s(Y,b_2)\rangle < d$$

As soon as the distance between the probed impact parameters is larger than the relevant distance scale of the evolution (=the inverse saturation scale), the amplitudes measured at the two impact parameters should be independent.

Supported by (too) simple analytical estimates
(fluctuations neglected...)

Outline

☆ High energy QCD evolution identified with a one-dimensional stochastic process

This identification is still a conjecture!

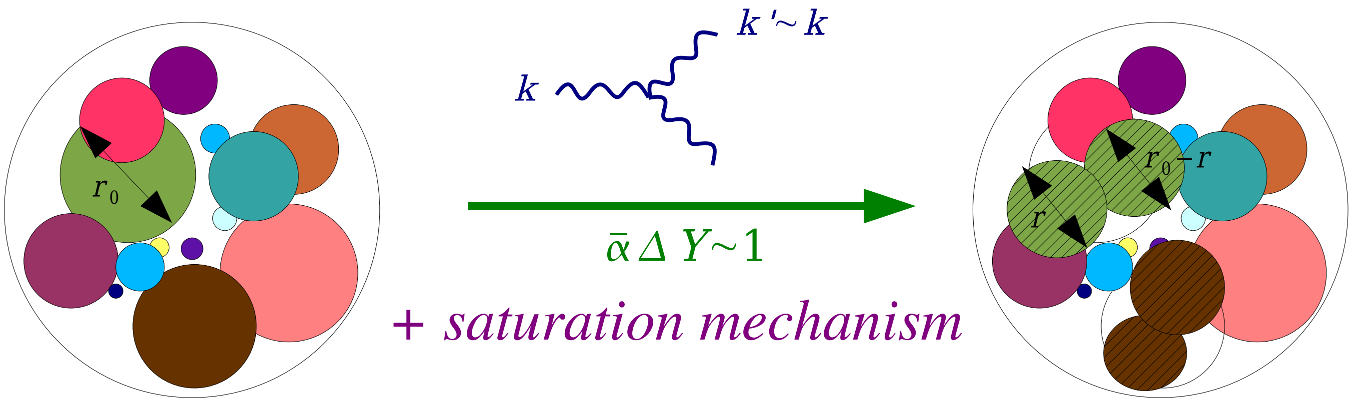
One important assumption: impact parameters evolve independently.

☆ Numerical check of the independence of different impact parameters in a toy model

Toy model with impact-parameter dependence

QCD:

$$\frac{dP}{\bar{\alpha}\, dY} = \frac{r_0^2}{r^2 (r_0 - r^2)} d^2 r$$



Too complicated!

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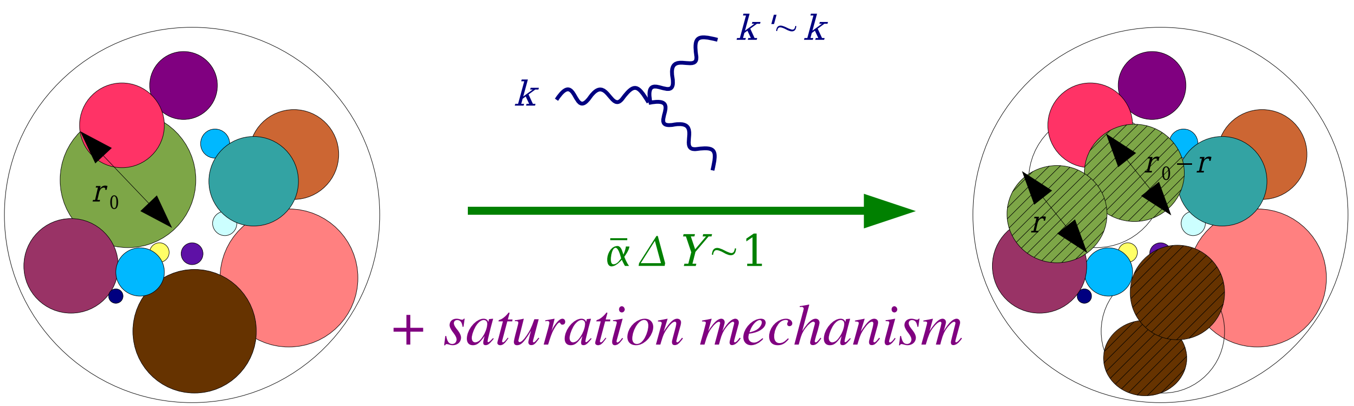


Too complicated!

Toy model:

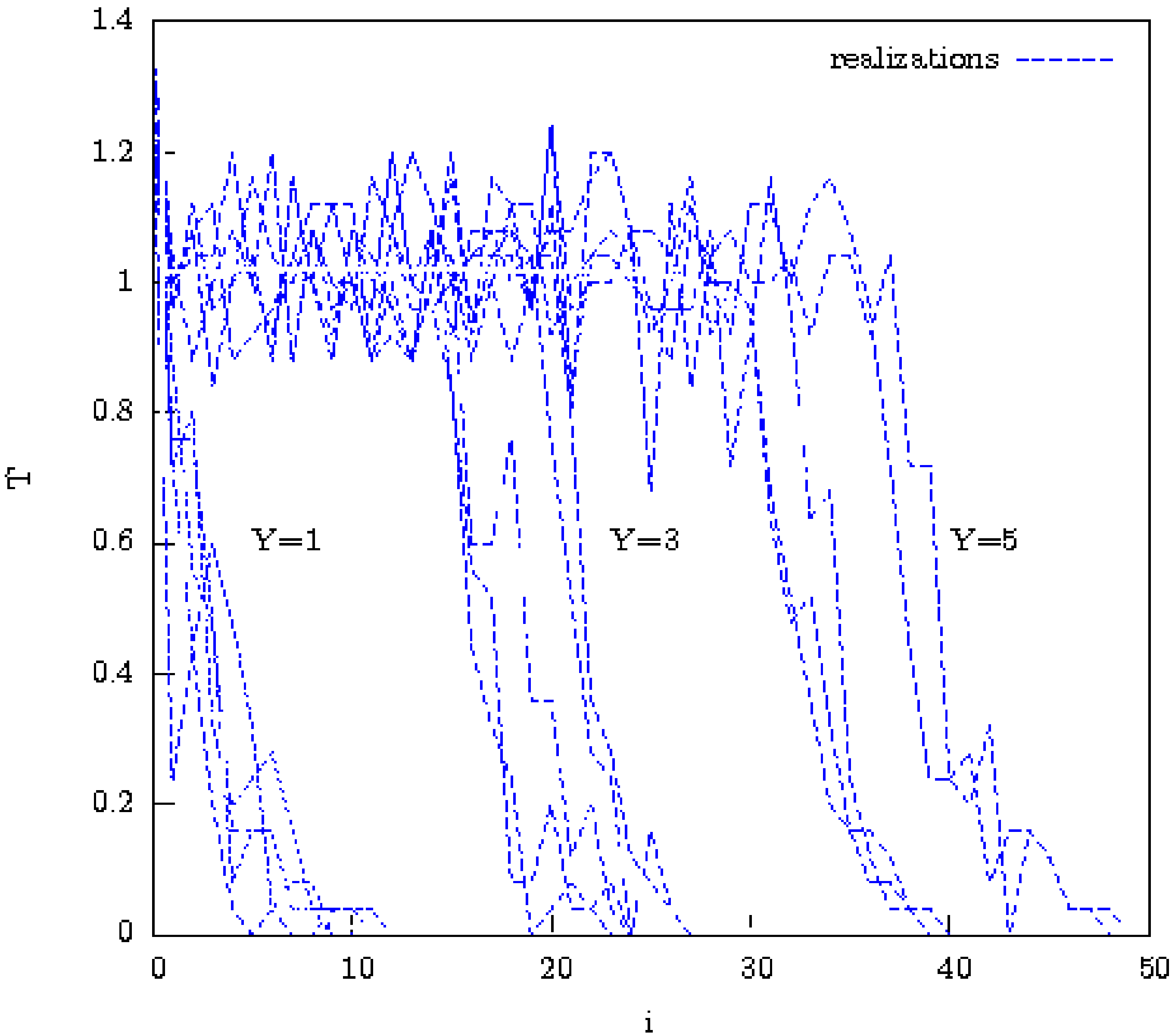
$$\frac{dP}{dY} = \frac{|r_0|}{|r|\,|r_0 - r|}\, dr$$

+ discretized sizes + saturation mechanism

We have 2 variables (r,b), and we keep the singularity structure of QCD

Traveling waves

N=25



One given impact parameter

Traveling waves

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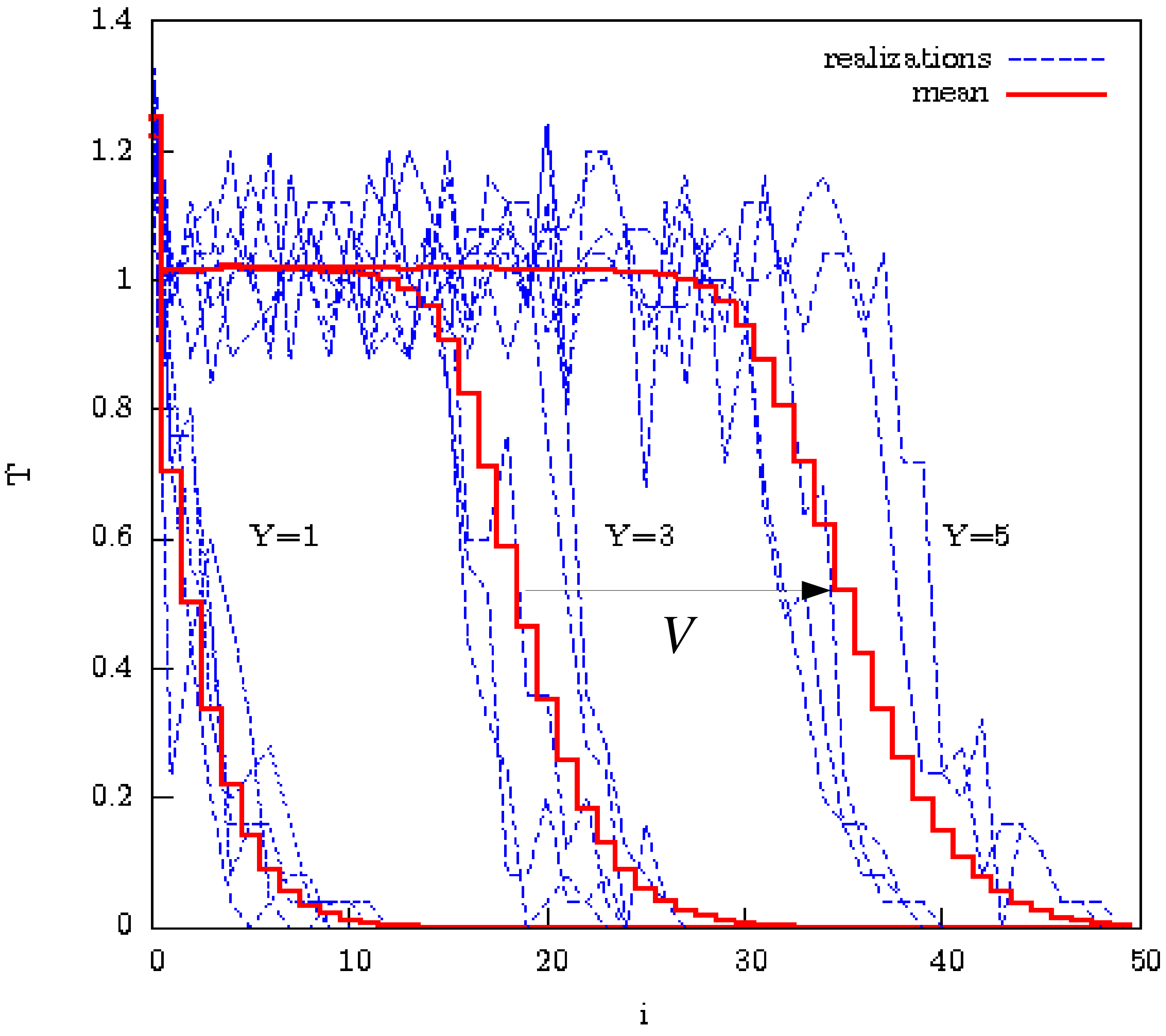

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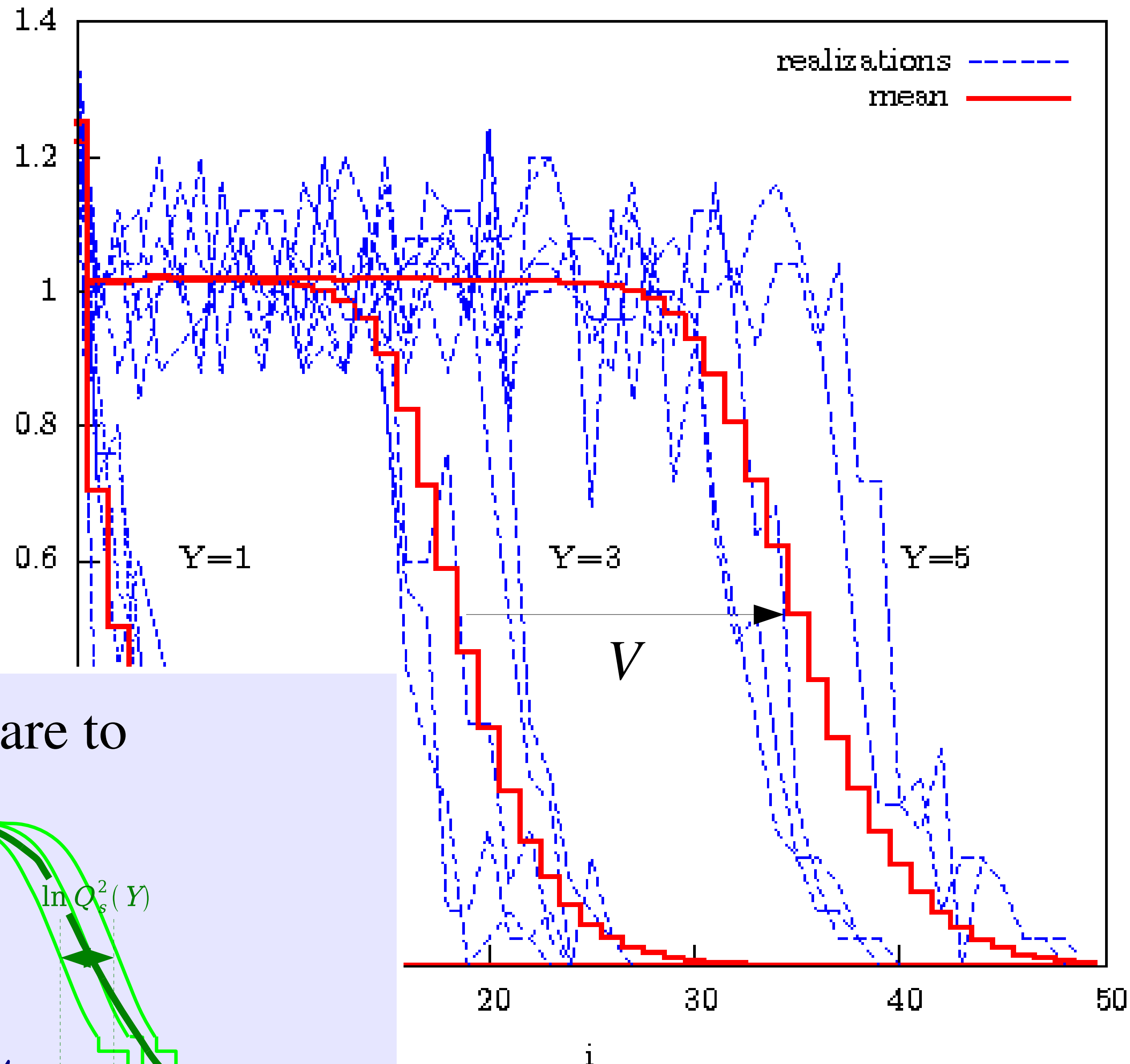
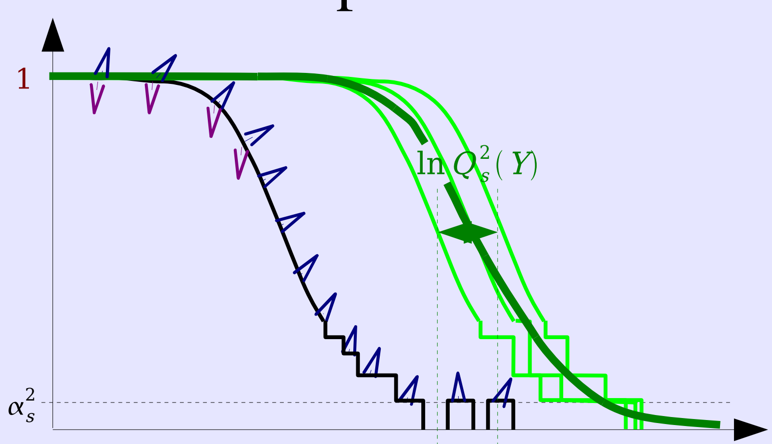


N=25

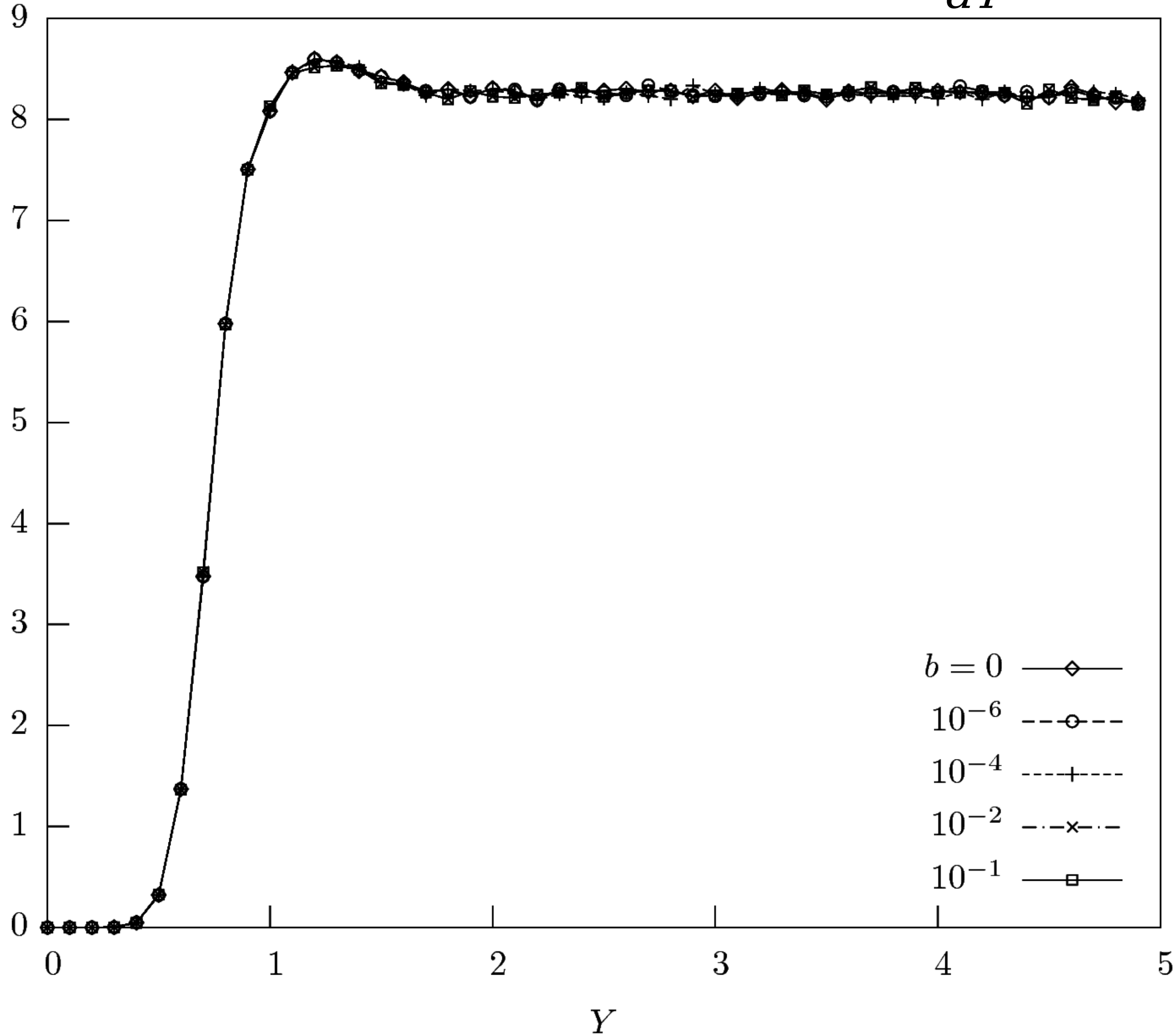
Compare to



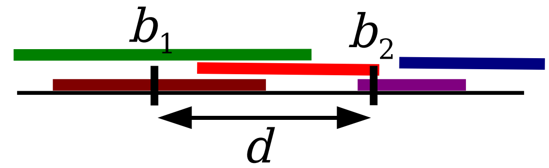
en impact parameter

Traveling wave velocity

$$V = \langle \rho_s(Y + dY, b) - \rho_s(Y, b) \rangle / dY = \frac{d \log Q_s}{dY}$$

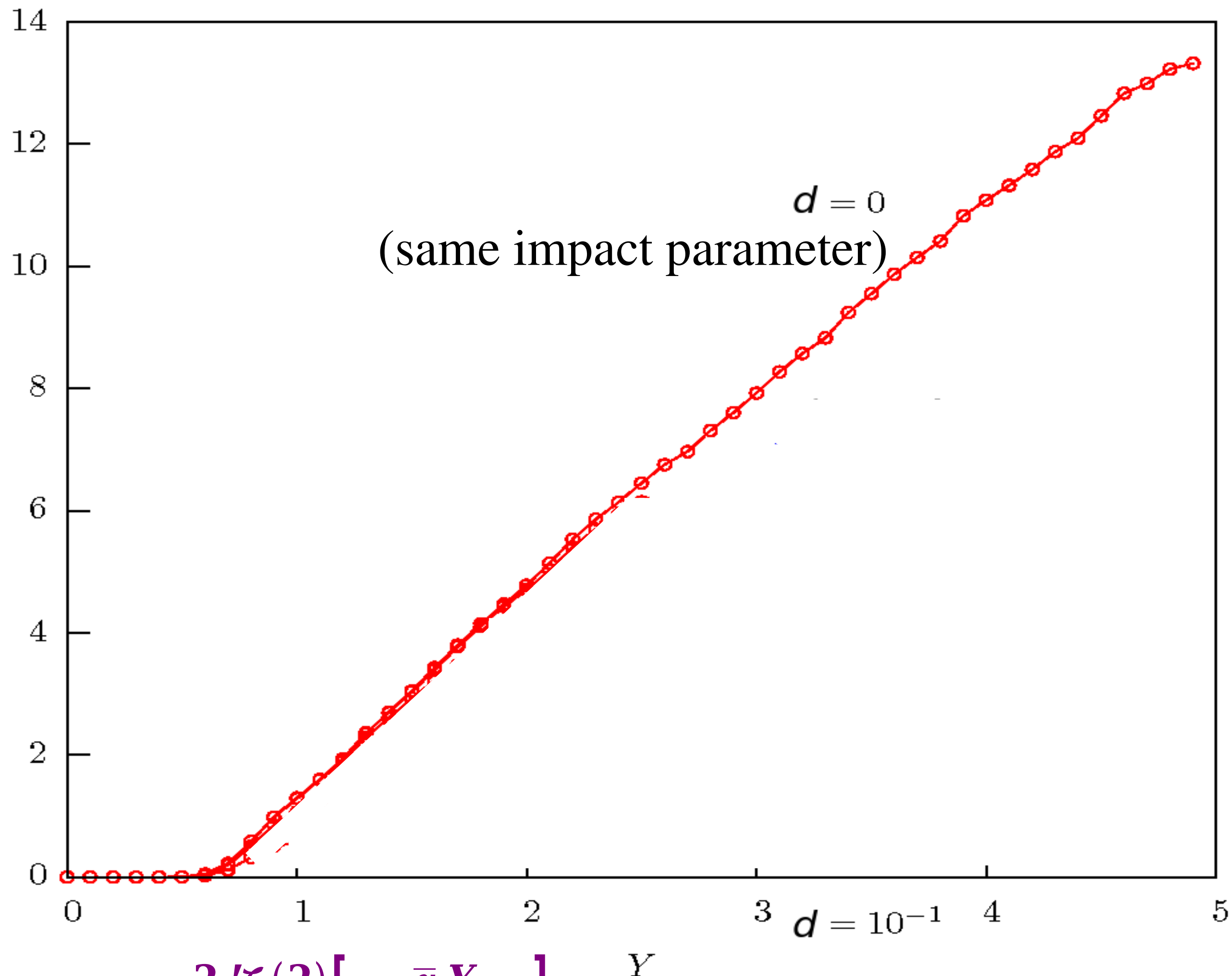


Dispersion of the saturation scales



$\rho_s = \log Q_s$

$$\langle \rho_S(Y,0)\rho_S(Y,b)\rangle - \langle \rho_S(Y,0)\rangle\langle \rho_S(Y,b)\rangle$$



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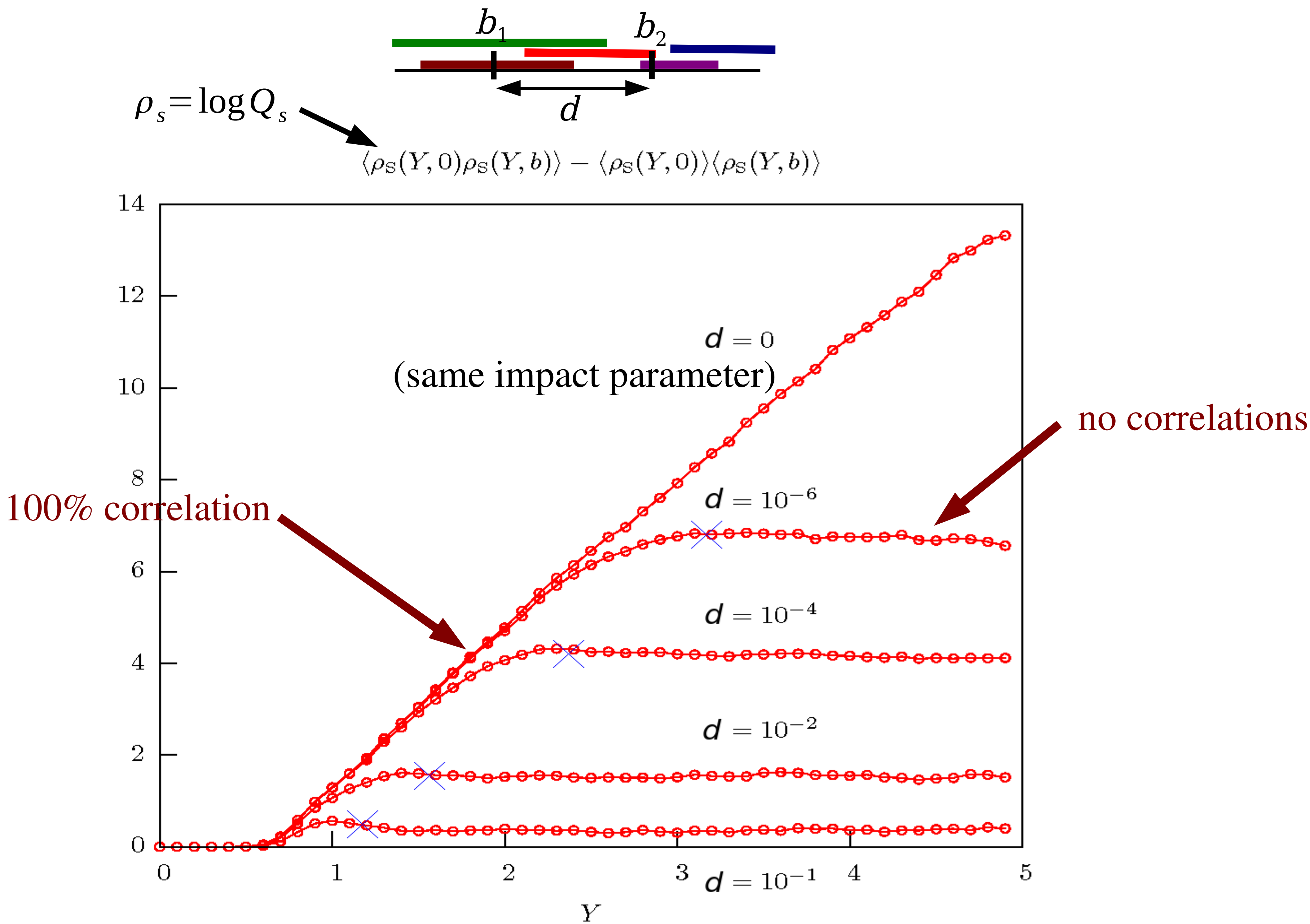
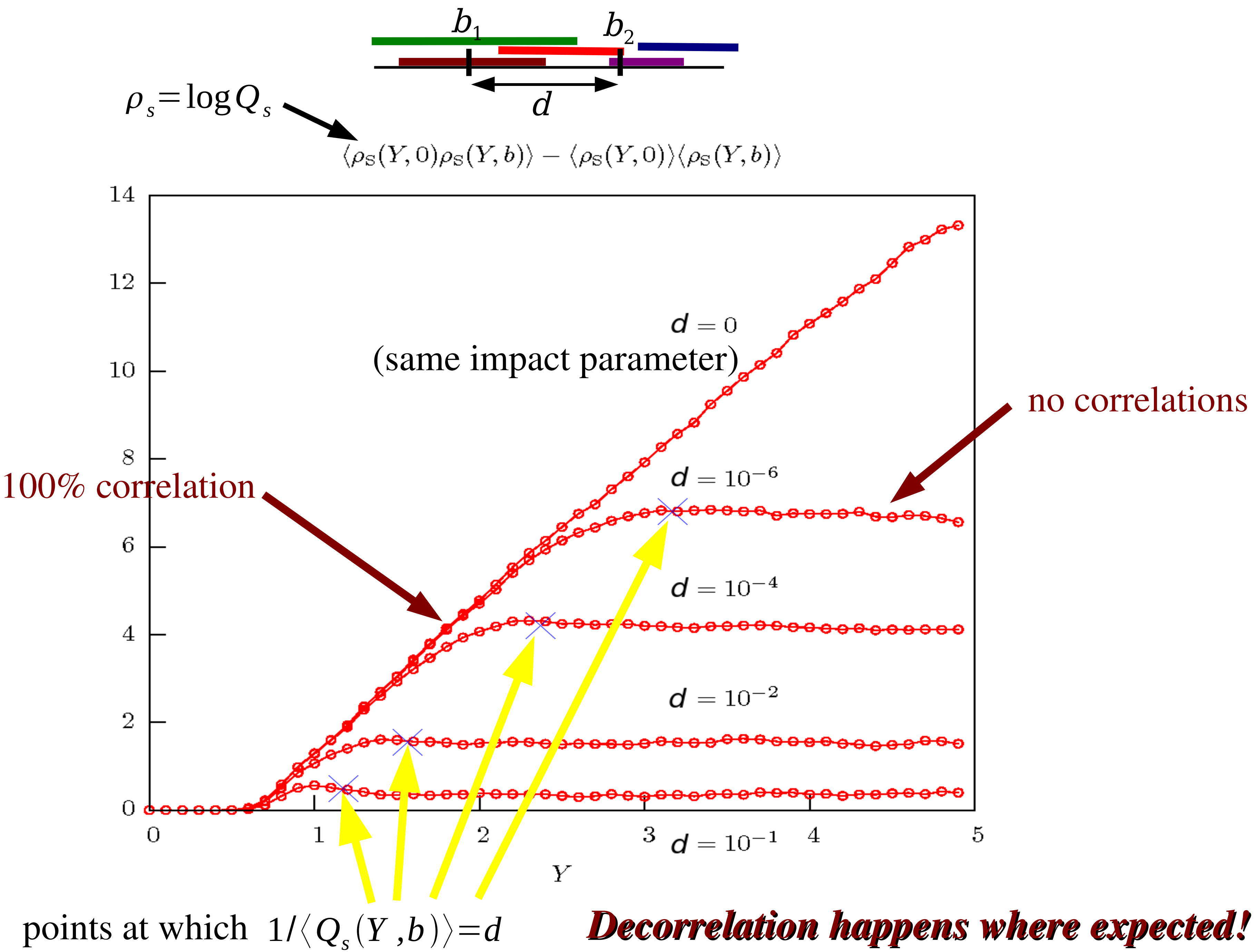
Correlation of the saturation scales



Correlation of the saturation scales



Decorrelation happens where expected!

A more refined look

Comparison with a fixed impact-parameter version of the model

The toy model is defined by its interval splitting rate

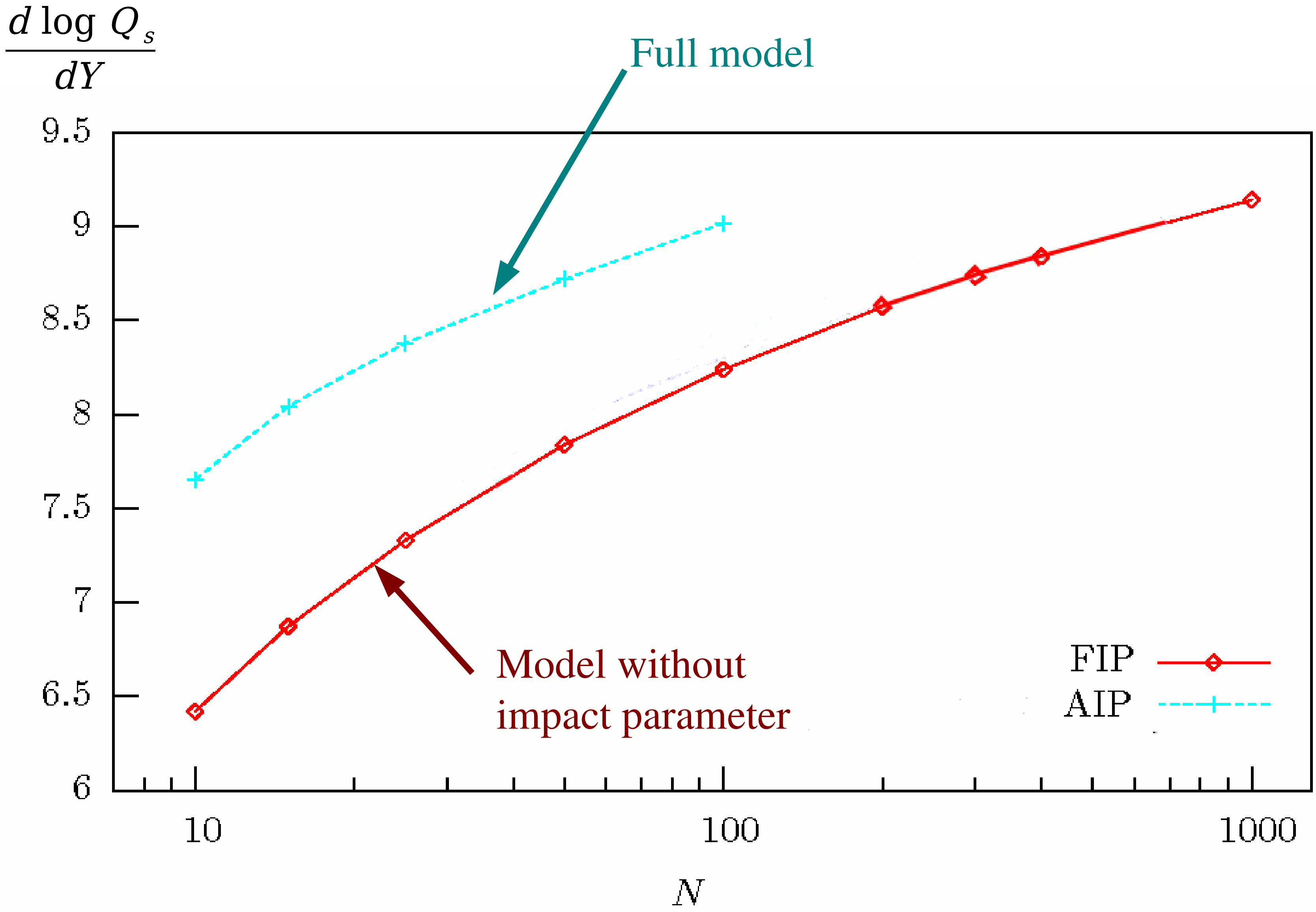
$$dP = \frac{|r_0|}{|r||r_0 - r|} d\,r \quad dY \qquad \text{(+ saturation condition)}$$

which generates a distribution of sizes and impact parameters of intervals.

One may *discard the impact parameter dependence* (this implies a rescaling of the splitting rate) and get a true ***one-dimensional model*** for which only the size matters (like what people usually do for e.g. the Balitsky-Kovchegov equation).

A more refined look

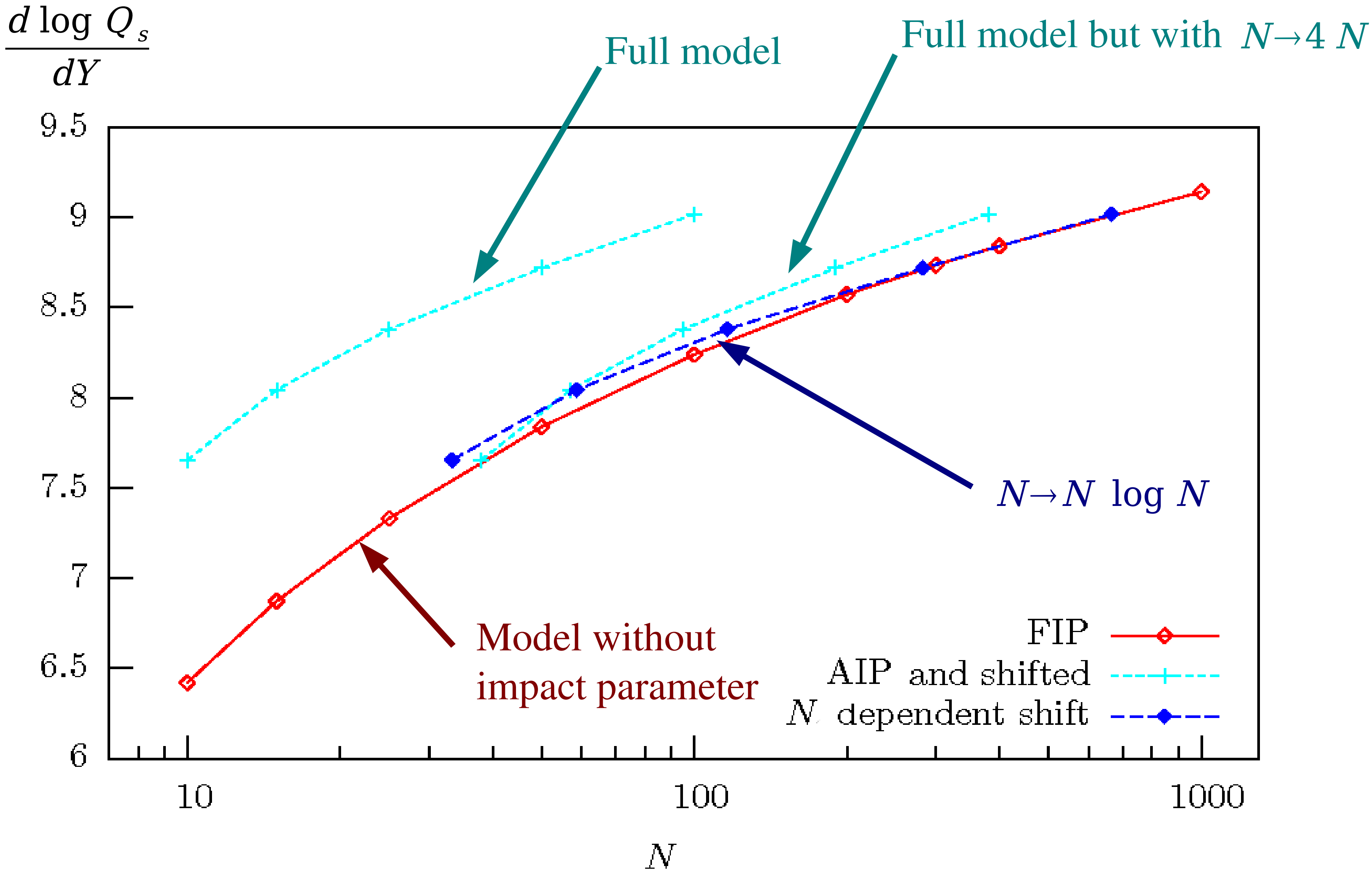
Comparison with a fixed impact-parameter version of the model



Significant disagreement!

A more refined look

Comparison with a fixed impact-parameter version of the model



The disagreement seems to amount to a mere rescaling of N!
(=rescaling of the QCD coupling)

Summary

We have identified, from the physics, the universality class of high energy QCD as the one of *one-dimensional* reaction-diffusion processes, whose dynamics are governed by an equation of the form

$$\partial_{\bar{\alpha} Y} T = \chi(-\partial_{\ln k^2})\, T - T^2 + \alpha_s \sqrt{T}\ \nu$$

We went back to the assumption that the QCD evolutions at different impact parameters decouple.

In a toy model, we have found that this is true.

We are now trying to build a complete picture of the hadron in impact-parameter space:

What is the distribution of the gluons/dipoles in the 2D plane?
Are there very dense regions (« hot spots ») coexisting with empty regions?
Or is the distribution more or less uniform?

Mueller, SM, in progress