Analytic structure of Regge poles and high-energy interactions.

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Introduction.

Most of applications of Regge theory to high-energy interactions are based on linear approximation for Regge-trajectories:

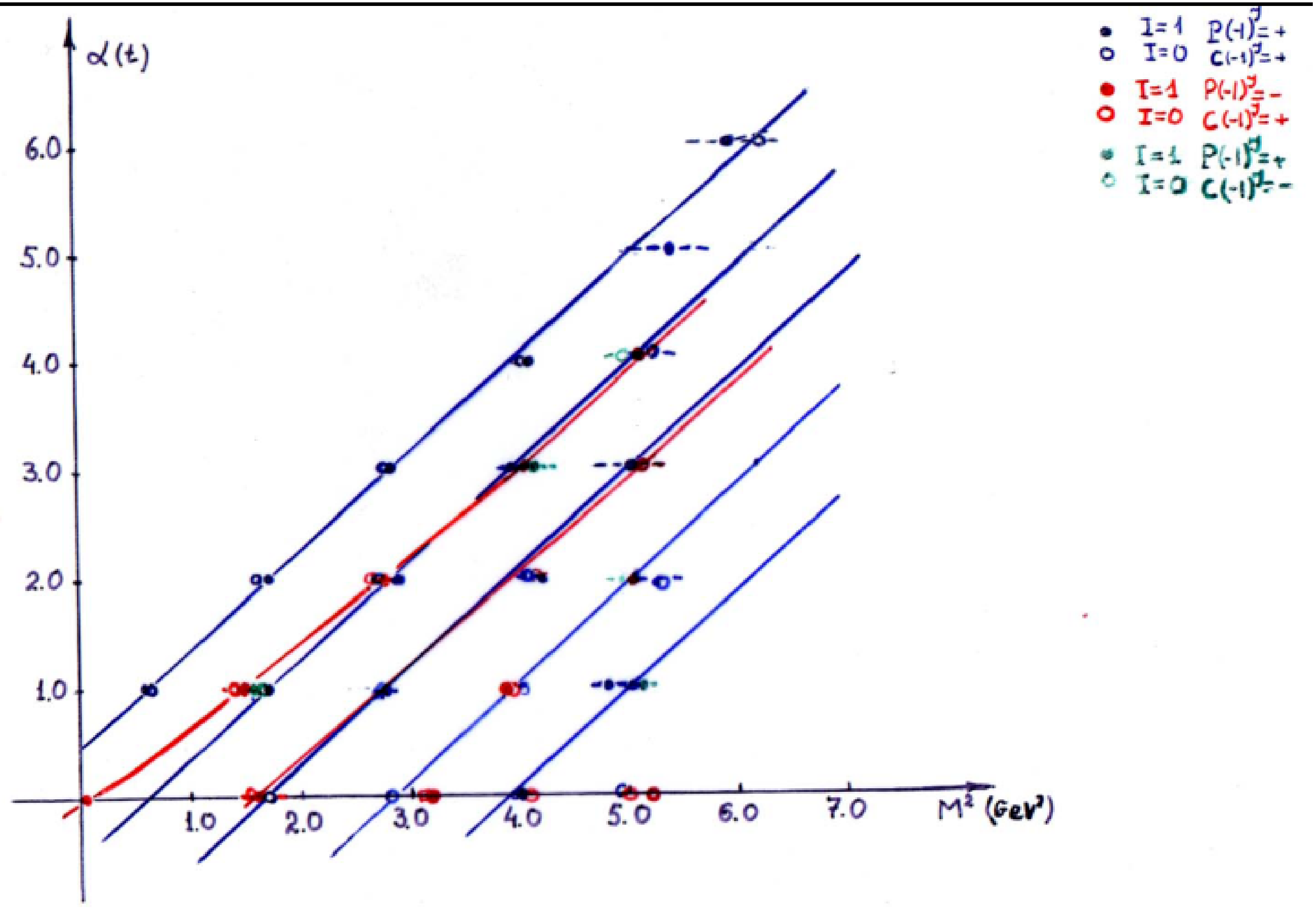
$$\alpha_a(t) = \alpha_a(0) + \alpha_a{}'(0)t$$

This is natural in Regge theory as characteristic $t \sim 1/\alpha' \ln(\frac{s}{s_0})$ are small and $b^2 \sim \alpha' \ln(\frac{s}{s_0})$.

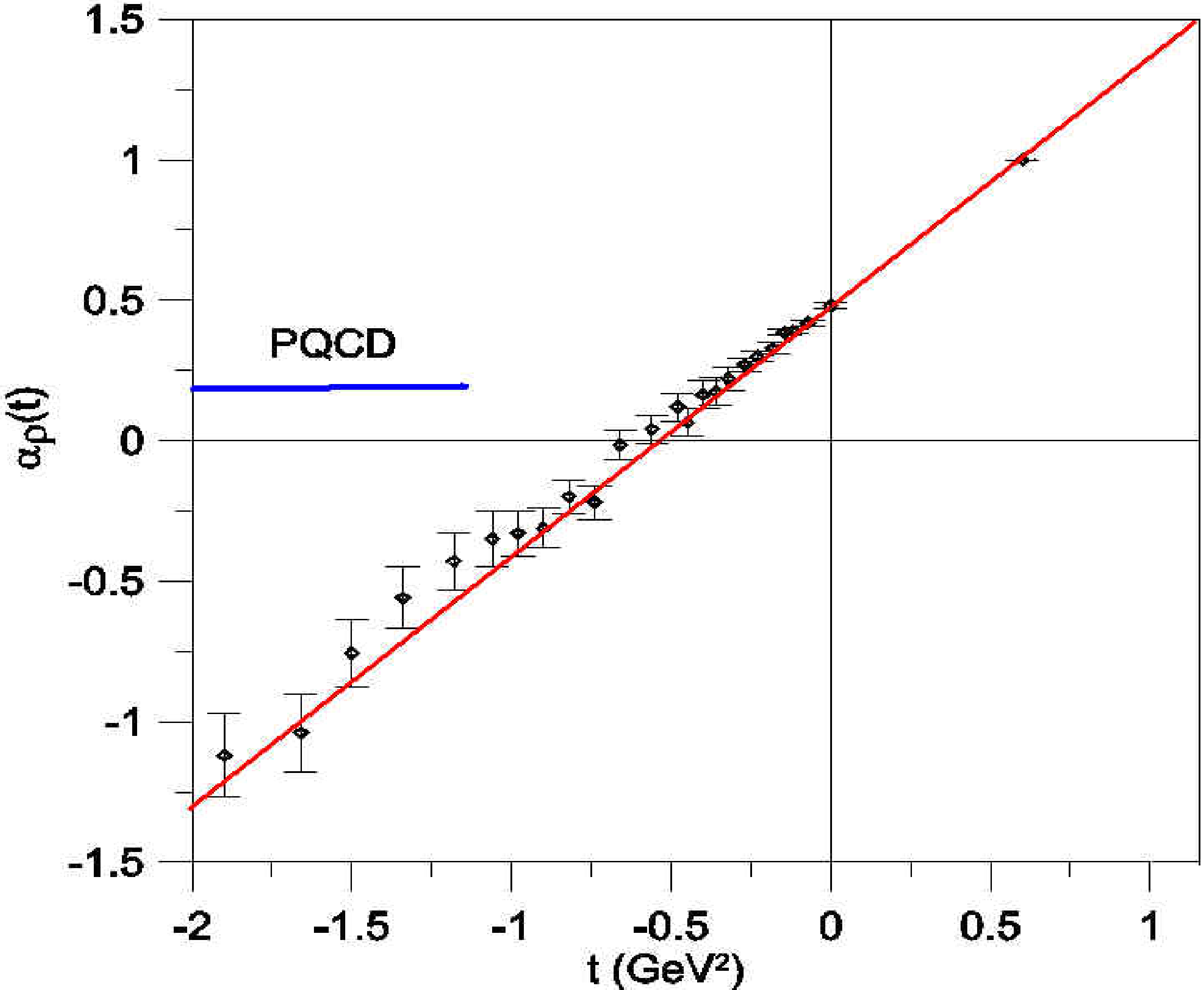
In this b-region linear approximation is valid.

Secondary Regge-trajectories ($\rho, A_2, \omega, ..$) are approximately linear with universal slope.

Spectrum of mesons



Linearity of the effective ρ-trajectory up to t ≈ -2 GeV² from $\pi^- p \rightarrow \pi^0 n$.



Introduction.

For pomeron, however, we do not have such information and $\alpha'_P(0)$ is known to be small (~ 0.1 GeV^{-2}). In supercritical pomeron theory ($\Delta \equiv \alpha_P(0) - 1 > 0$) very large impact parameters $b^2 \sim \ln^2(\frac{s}{s_0})$ are important (Froissart type behavior) and in this region an analytic structure of the P-pole (existence of a branch point at $t = 4\mu^2$) plays a vital role.

Introduction.

Several natural questions:

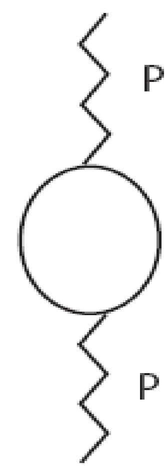
- How large are $\mathrm{Im}\,\alpha_a(t)$ and how they distort linearity of Regge trajectories?
- How slopes of trajectories are related to characteristic scales?
- How b-dependence of Regge amplitudes at large impact parameters is modified?
- What can we learn about relative importance of perturbative versus nonperturbative effects from behavior of Regge-trajectories?

Analytic properties of Regge poles.

It follows from t-channel unitarity that Regge poles have cuts related to production of real intermediate states.

For Regge poles with positive G-parity the lowest branch point is at $t = 4\mu^2$ and is due to $\pi\pi$ - exchange.

Contribution of the Regge pole to the amplitude



$$T_a(s,t) = b_a(t)\eta(\alpha_a(t))\left(\frac{s}{s_0}\right)^{\alpha_a(t)}$$

Imaginary parts of Regge-trajectories.

$$\operatorname{Im}\alpha_a(t) = \frac{b_a(t)(t-4\mu^2)^{(\alpha_a(t)+1/2)}}{16\pi C(\alpha_a(t))(4s_0)^{\alpha_a(t)}\sqrt{t}}$$

$t \to 4\mu^2$ V.N.Gribov,I.Ya.Pomeranchuk(1962)

where

$$C(j) = \frac{\pi\Gamma(2j+2)}{2^{(2j+1)}\Gamma(j+1)^2} \quad , s_0 = 1 GeV^2$$

Imaginary parts can be determined from widths of resonances (for small $\operatorname{Im}\alpha_a(t)$)

$$\operatorname{Im}\alpha_a(M_n^2) = \Gamma_n M_n / \alpha_n'(M_n^2)$$

Dispersion relations for Regge trajectories.

If $\mathrm{Im}\,\alpha(t)$ is known it is possible to restore $\mathrm{Re}\,\alpha(t)$ using dispersion relations. It is important to know behavior of $\mathrm{Im}\,\alpha(t)$ at large t. Data indicate that $\Gamma(t) \approx Const$ and $\mathrm{Im}\,\alpha(t) \to \sqrt{t}$ for $t \geq 1 GeV^2$.

Thus it is enough to make one substraction (it is convenient to make it at t=0)

Dispersion relations for Regge trajectories

$$\mathrm{Re}\,\alpha_a(t) = \alpha_a(0) + \alpha'_{0a}(0)t + \frac{t}{\pi}\int_{4\mu^2}^{\infty} \frac{\mathrm{Im}\,\alpha_a(t')dt'}{(t'-t)t'}$$

where $\alpha'_{0a}(0)$ is the "bare" slope (without hadronic loops). Dispersion integral gives an extra contribution to $\alpha'_a(0)$:

$$\delta\alpha'(0) = \frac{1}{\pi}\int_{4\mu^2}^{\infty} \frac{\mathrm{Im}\,\alpha(t')dt'}{t'^2}$$

It can be especially important for pomeron ($\alpha'_{oP}(0)$ is small).

ρ- trajectory.

Model for $\mathrm{Im}\,\alpha_\rho(t)$.

For t close to $4\mu^2$

$$\mathrm{Im}\,\alpha_\rho(t) \simeq \frac{C}{\sqrt{t}}(t-4\mu^2)^{\alpha_\rho(4\mu^2)+1/2}$$

Note that $\alpha_\rho(4\mu^2)+\frac{1}{2}\approx 1$ and $C=\frac{b_\rho(4\mu^2)}{32\pi}\approx 0.2GeV^{-1}$

This behavior extrapolates well to region of large t (reproduces widths of resonances on ρ- trajectory).

$$\Gamma_\rho = 153MeV, (\Gamma^{\exp}_\rho = 149.4 \pm 1 MeV)$$

$$\Gamma_{\rho 3} = 164MeV, (\Gamma^{\exp}_{\rho 3} = 161 \pm 10 MeV)$$

ρ- trajectory.

In this approximation rho-trajectory can be written in a close form A,A.Anselm,V.N.Gribov (1972)

$$\alpha_\rho(t) = \alpha_\rho(0) + \alpha'_{0\rho} t + \tilde{\alpha}(t)$$

$$\tilde{\alpha}_\rho(t) = \left\{ 2(t - 4\mu^2)\left(\frac{1}{q} arctg \frac{q}{2\mu}\right) + 4\mu \right\} \frac{C}{\pi} \quad , q = \sqrt{-t} \qquad t < 0$$

$$\tilde{\alpha}_\rho(t) = \left\{ (t - 4\mu^2)\left(\frac{1}{q} \ln \frac{1 - \frac{q}{2\mu}}{1 + \frac{q}{2\mu}}\right) + 4\mu \right\} \frac{C}{\pi} \quad , q = \sqrt{t} \qquad t > 0$$

ρ- trajectory.

For $|t| \gg 4\mu^2$ $\tilde{\alpha}_\rho(t) \approx \sqrt{-t}$

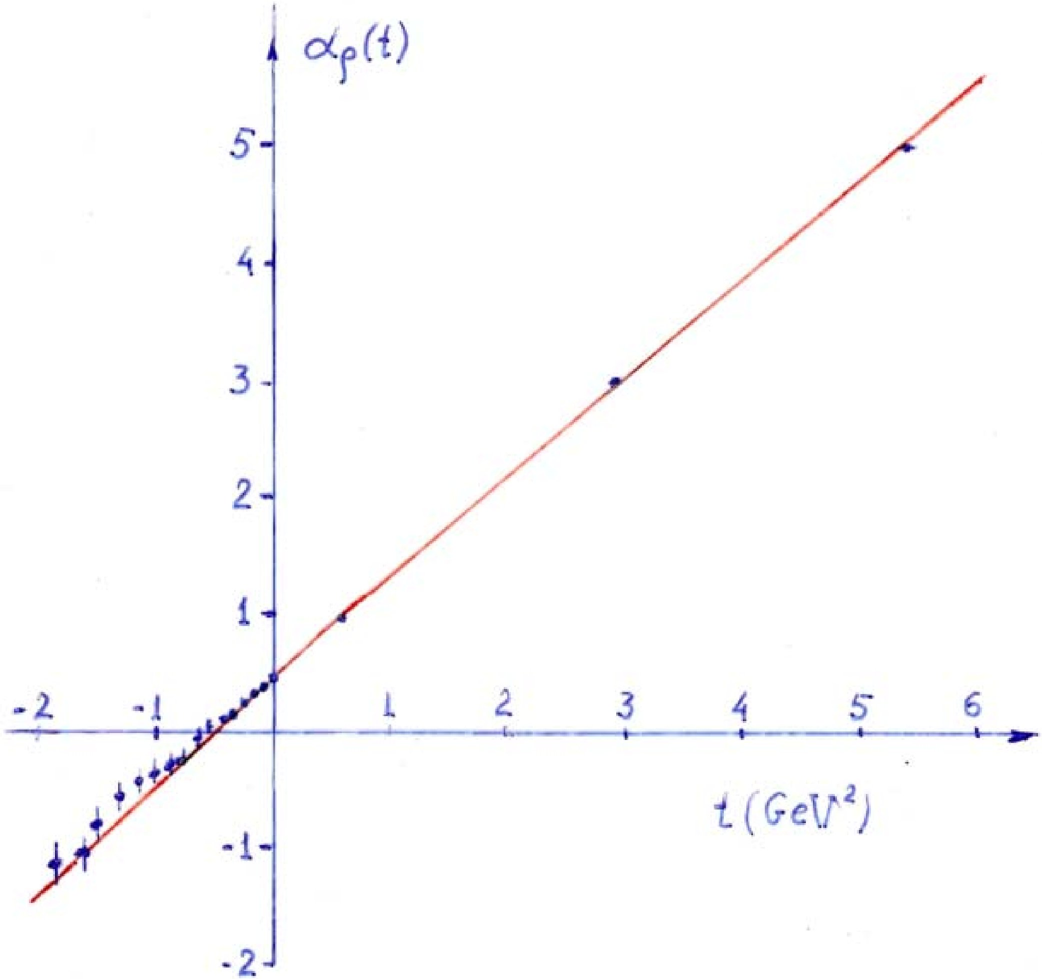
Very small contribution of $\tilde{\alpha}_\rho(t)$ to $\operatorname{Re}\alpha_\rho(t)$ in the resonance region. Explains approximate linearity of secondary trajectories.

Some correction to the slope in the small t region: $\tilde{\alpha}'(0) = 0.25 GeV^{-2}$

General consistency of the $\operatorname{Re}\alpha_\rho(t)$ with experimental data both for t>0 and t<0.

Relation to the paper by R.Fiore et.al (2000)

ρ- trajectory.



Pomeron trajectory.

For pomeron trajectory an information in the resonance region is practically absent.

In the small t region:

$$\mathrm{Im}\,\alpha_P(t) \approx (g^P_{\pi\pi}(t))^2 \frac{(t-4\mu^2)^{\alpha_P(t)+1/2}}{32\pi\sqrt{t}}$$

I shall assume a validity of the dispersion relation with one substraction.

Pomeron trajectory.

In this case $\alpha_P(t) = \alpha_P(0) + \alpha'_{0P} t + \tilde{\alpha}_P(t)$

$$\tilde{\alpha}_P(t) = \frac{t}{\pi} \int_{4\mu^2}^{\infty} \frac{\operatorname{Im} \alpha_P(t') dt'}{(t' - t) t'}$$

Simple expression for $\alpha_P(4\mu^2) = 1$

$\tilde{\alpha}_P(t) = C_P h_P(q^2),$ A.A.Anselm,V.N.Gribov

$$h_P(q^2) = q^2 \left\{ \frac{8\mu^2}{q^2} - \left(1 + \frac{4\mu^2}{q^2}\right)^{3/2} \ln \frac{1 - \sqrt{1 + q^2/4\mu^2}}{1 + \sqrt{1 + q^2/4\mu^2}} + \ln \frac{M_{\max}^2}{\mu^2} \right\}$$

$$C_P = \frac{\sigma_{\pi\pi}^P \mu^2}{32\pi^3} \qquad q^2 = -t > 0 \qquad M_{\max}^2 \sim 1 GeV^2$$

Pomeron trajectory.

Note a small numerical coefficient $1/32\pi^3$.
As a result $\tilde{\alpha}'_P(0) \approx 0.05 GeV^{-2}$ is small (but important) even for the scale $1/\mu$.

The pomeron trajectory is strongly curved.

Another source of curvature for pomeron trajectory is mixing of singlet $q\bar{q}$ and gg -trajectories (due to crossing of trajectories).

A.B.K.,Yu.A.Simonov (2000)

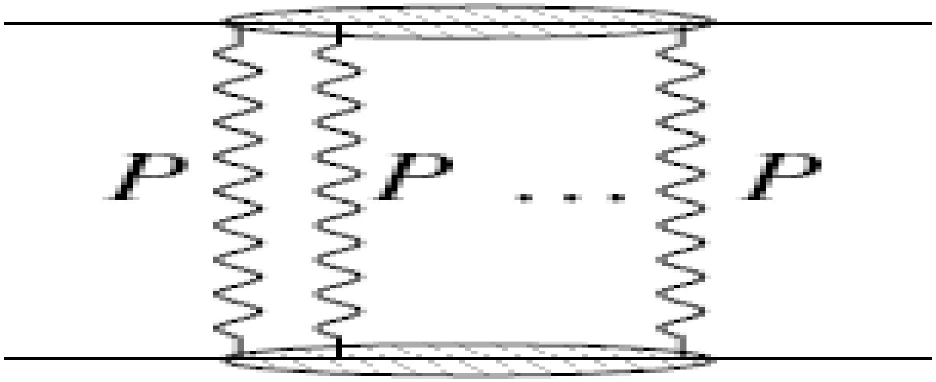
Transition from nonperturbative slope $\alpha'_{gg} = \frac{4}{9}\alpha'_{q\bar{q}}$

at large t>0 to very small BFKL slope at large –t in AdS/CFT approach. R,Brower et al (2006)

Amplitudes of diffractive processes.

Single P-exchange with Δ > 0 violates unitarity: $f_l^{el} \sim s^{\Delta}, \quad |f_l^{el}| \leq 1.$

Multi-pomeron exchanges are necessary to restore unitarity.



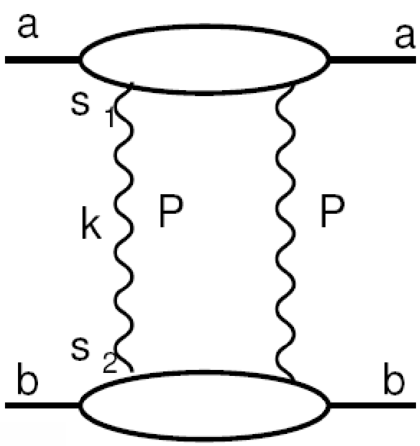
Unitarity effects in Gribov`s approach.

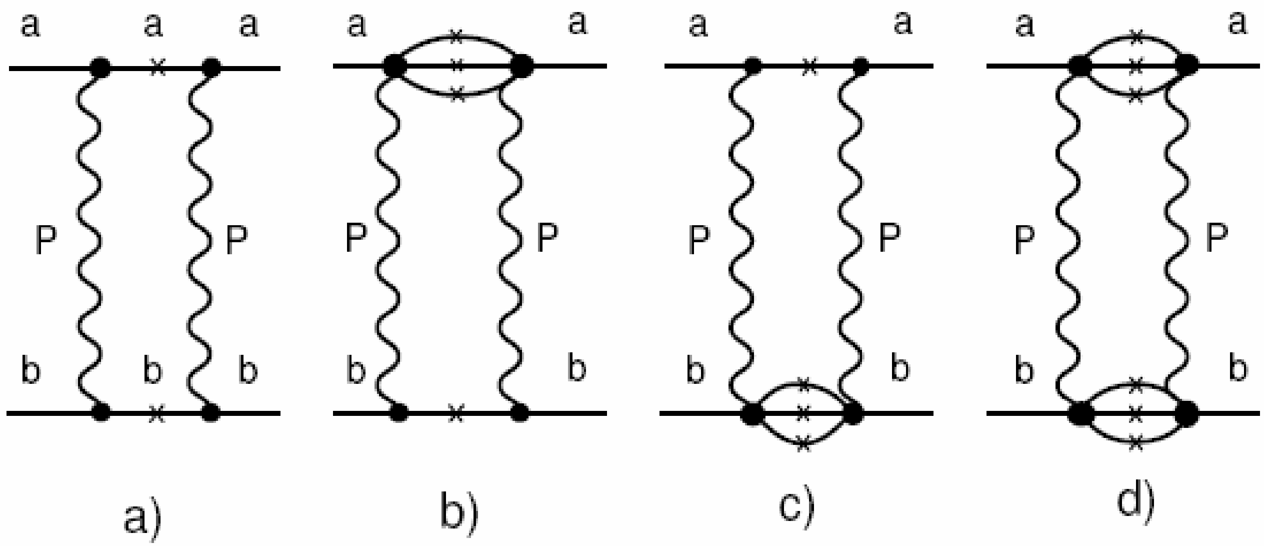
Consider PP-exchange

Amplitudes can be expressed

In terms of contributions of

diffractive intermediate states.





Radius of interaction in eikonal model.

Summation of nP-exchanges with account of poles only leads to the eikonal amplitudes $\mathrm{Im}\, T_{\mathrm{el}}(s,b) \;=\; 1 - \mathrm{e}^{-\Omega/2}$

with $i\Omega \equiv 4\delta_P(s,b)$ -Fourier transform of $T_P(s,t)$.

For linear P-trajectory and gaussian form of the residue

$$\mathrm{Im}\,\delta_P(s,0) \sim (\frac{s}{s_0})^{\Delta} \exp(-\frac{b^2}{\lambda_P(s)}) = \exp(-\frac{b^2}{\lambda_P(s)} + \Delta \ln \frac{s}{s_0})$$

$$\lambda_P(s) = R^2 + \alpha'_P \ln(\frac{s}{s_0}) \;\;,\;\; R^2(s) = \lambda_P(s)\Delta \ln(\frac{s}{s_0}) \simeq \alpha'_P \Delta \ln^2(\frac{s}{s_0})$$

Radius of interaction and singularity in P-trajectory.

Froissart type increase of the radius is a general feature of supercritical P theory.

What happens in the limit $\alpha'_P(0) \to 0$? It is necessary to take into account the branch point in P-trajectory at $t = 4\mu^2$.

$$\alpha_P(t) = \alpha_{0P}(0) + \alpha'_{0P} t + \varepsilon (4\mu^2 - t)^\gamma$$

M.S.Dubovikov, K.A.Ter-Martorosyan (1977)

It is possible to neglect by the last term only in the region $b < 4\alpha'_P \mu \xi$ $\qquad \xi \equiv \ln\left(\frac{s}{s_0}\right)$

Radius of interaction and singularity in P-trajectory.

For $b > 4\alpha'_P \mu \xi$

$$\operatorname{Im}\delta_P(s,b) = \frac{B\xi \exp\left[2\mu(a\xi - b)\right]}{\sqrt{b}(b - 4\alpha'_P \mu \xi)^{\gamma+1}}$$

where $a = \dfrac{\Delta}{2\mu} + 2\alpha'_P \mu$ $\quad B = \dfrac{g^2 \varepsilon_1}{4\pi^{3/2}}(4\mu)^{\gamma+1/2}\Gamma(\gamma+1)$

$$\varepsilon_1 = \varepsilon \sin(\pi\gamma)$$

generic dependence on b at large b.

$$R(\xi) = a\xi - \frac{(\gamma + 1/2)}{2\mu}\ln(\eta\xi) \quad , \quad \eta = \left(\frac{\Delta}{2\mu} - 2\alpha'_P \mu\right)$$

Inelastic diffraction.

The singularity in the P-trajectory is especially important in amplitudes of inelastic diffraction, as impact parameters $b < R(\xi)$ are strongly suppressed. In eikonal model the suppression is given by the factor $\exp\left[-\Omega(b,\xi)\right]$.

Modification of $\delta_P(s,b)$ at large b allows to resolve problems with unitarity in inelastic diffraction. A.B.K. (1979)

Conclusions.

1. Unitarity cut plays a minor role for secondary $(\rho, A_2, \omega, ..)$ Regge trajectories and leads to a small deviation from linearity.
2. For pomeron the cut is important.
3. Pomeron trajectory has a strong curvature in the small t-region.
4. Singularity of the pomeron trajectory plays a vital role for supercritical pomeron and determines radius of interaction at superhigh energies.