REGGEON FIELD THEORY - R WE THERE YET?

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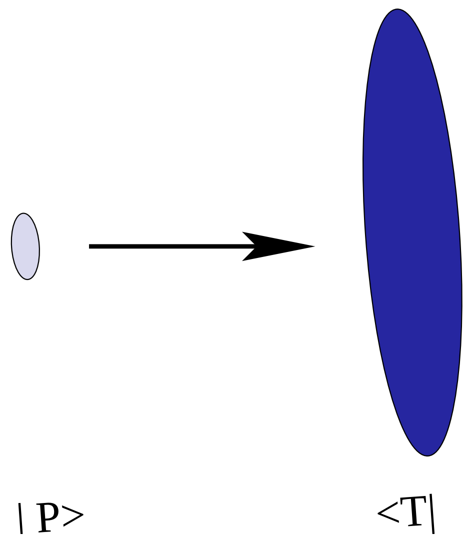
with M. Lublinsky, T. Altinoluk and J. Peressutti; JHEP 2009

THE SCATTERING AMPLITUDE

SCATTER A "'PROJECTILE"' HADRON $|P\rangle$ ON A "'TARGET"' HADRON $|T\rangle$ AT HIGH ENERGY



$|P\rangle$ - A DISTRIBUTION OF COLOR CHARGES DENSITY $j^a(X)$.

$|T\rangle$ - AN ENSEMBLE OF (STRONG) COLOR FIELDS $\alpha^a(X)$.

ENERGY IS HIGH - SCATTERING IS EIKONAL

THE S - MATRIX

THE EIKONAL S - MATRIX:

EVERY PROJECTILE GLUON KEEPS THE TRANSVERSE POSITION BUT ACQUIRES A PHASE

$$|X,a\rangle \to S^{ab}(X)|X,b\rangle$$

WITH

$$S^{ab}(X) = \mathcal{P}\,\exp\left\{i\int dX^- \,T^a\,\alpha^a(X,X^-)\right\}^{ab}\,.$$

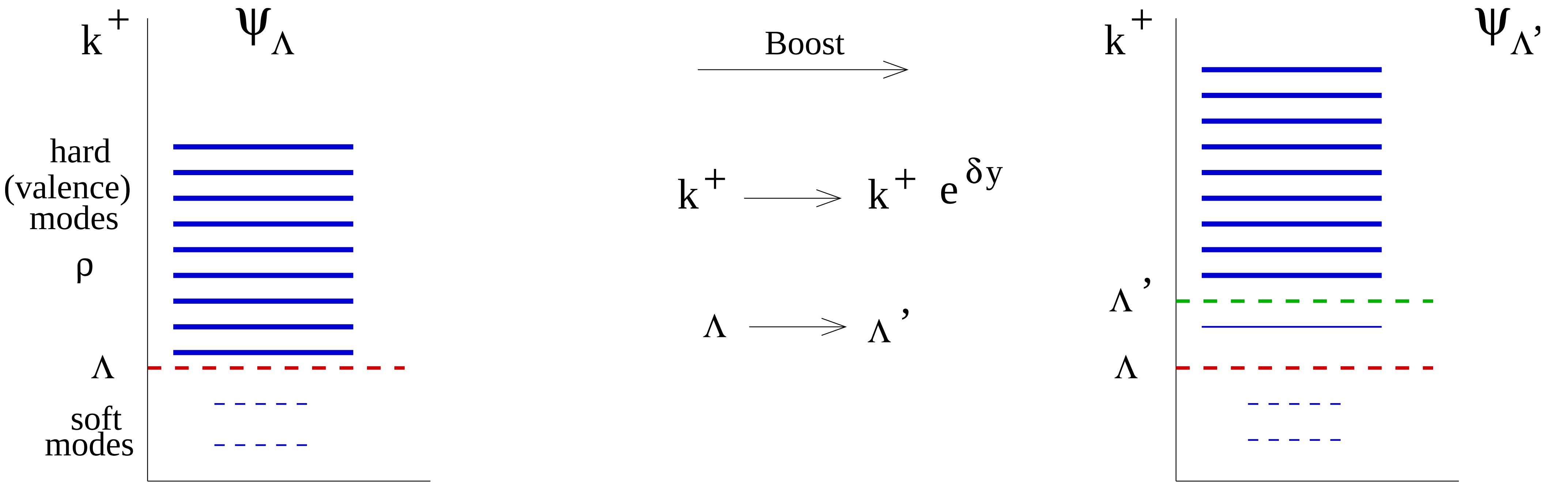
THE FORWARD SCATTERING AMPLITUDE:

$$\mathcal{S} = \langle \mathrm{IN} | \mathrm{OUT} \rangle =< \langle P | \hat{S} | P \rangle >_T$$

$$=< \int dj \, W^P[j] \; \exp\left\{ i \int d^2X \, j^a(X) \; \alpha^a(X) \right\} >_T$$

$W^P[j]$ - PROBABILITY DISTRIBUTION OF THE PROJECTILE COLOR CHARGE DENSITY

AS THE HADRON IS BOOSTED, ITS WAVE FUNCTION AND THE PROBABILITY DISTRIBUTION CHANGE



UNDER BOOST THE LONGITUDINAL MOMENTA SCALE .

NEW GLUONS RISE FROM THE "BOTTOMLESS PIT" WHICH IS THE ZERO MODE.

COLOR FIELD BECOMES STRONGER BECAUSE OF THESE EXTRA WEIZSACKER-WILIAMS GLUONS

NEED TO KNOW THE "'SOFT VACUUM"' WAVEFUNCTION

THEN:

$$\mathcal{S} = \langle \mathrm{IN}|\mathrm{OUT}\rangle = \langle P_{\mathrm{valence}}|\langle P_{\mathrm{soft}}|\hat{S}|P_{\mathrm{soft}}\rangle|P_{\mathrm{valence}}\rangle$$

ALSO DEFINES THE RFT HAMILTONIAN

(substitute $\alpha^a(X) \rightarrow \delta/\delta j^a(X)$):

$$\langle P_{\mathrm{soft}}|\hat{S}|P_{\mathrm{soft}}\rangle = 1 - H_{RFT}[j, \delta/\delta j]\ \Delta\eta + ...$$

H_{RFT} EVOLVES THE PROBABILITY DISTRIBUTION WITH RAPIDITY η:

$$\frac{d}{d\eta} W^P[j] = -H_{RFT}[j, \delta/\delta j]\ W^P[j]$$

THE "'SOFT VACUUM"' - DIAGONALIZE H_{QCD}

$H_{QCD}[a, a^\dagger, j]$ - QCD HAMILTONIAN ON THE SOFT HILBERT SPACE WITH VALENCE BACKGROUND COLOR CHARGE

HERE:

$a,\ a^\dagger$ - SOFT GLUE CREATION AND ANNIHILATION OPERATORS

j - VALENCE COLOUR CHARGE DENSITY

FIND $\Omega[a, a^\dagger, j]$ SUCH THAT

$$\Omega^\dagger H_{QCD} \Omega \ = \ H_{\rm diagonal}$$

FOUND Ω!

M. Lublinsky, U. Wiedemann, A.K.

$$\Omega = CB$$

$$C \; = \exp\left\{ i \int d^2 X b_i^a(X) \int d\eta [a_i^a(X,\eta) + a_i^{a\dagger}(X,\eta)] \right\}$$

$$\partial_i b_i^a(X) = j^a(X), \qquad \partial_i b_j^a(X) - \partial_j b_i^a(X) - g f^{abc} b_i^b(X) b_j^c(X) = 0$$

C COHERENT OPERATOR - CREATES WEIZSACKER-WILLIAMS FIELD

B - BOGOLYUBOV OPERATOR: $B \; = \exp\{\Lambda[j]a^2 + a^{\dagger 2} + ...\}$

B - DEFINES GLUON QUASIPARTICLES ABOVE THE WEISZACKER-WILLIAMS BACKGROUND

FOR "'NUCLEUS"' $b \sim O(1/g)$; $\Lambda \sim O(1)$

Ω DEFINES THE WHOLE SPECTRUM

$$|VAC\rangle = \Omega|0\rangle; \qquad |N\rangle = \frac{1}{\sqrt{N!}}\Omega(a^\dagger)^N|0\rangle$$

Ω IS PERTURBATIVELY ACCURATE: E.G.

$$\langle VAC|a^\dagger a|VAC\rangle = b^2\ (O(1/\alpha_s))\ + \Lambda^2\ (O(1)) + O(\alpha_s) + \ldots$$

GIVEN THIS Ω WE FIND

$$H_{RFT} = \frac{1}{2\pi} [b - \bar{b}\,] R^\dagger\ (1 - l - L)\,(1 - 2l)\,R\,(1 - 2l)\ (1 - l - L)\ [b - \bar{b}]$$

HERE

$R(X) = \exp\{gT^a \frac{\delta}{\delta j^a(X)}\}$; THE SINGLE GLUON SCATTERING MATRIX

$b \equiv b[j]$; THE WW FIELD OF THE INCOMING STATE

$\bar{b} = R^\dagger b[Rj]$; THE FIELD IN THE OUTGOING STATE

PROJECTION OPERATORS:

$$l \equiv \frac{\partial_i \partial_j}{\partial^2}; \qquad L \equiv \frac{D[b]_i D[b]_j}{D[b]^2}$$

The expression in the paper is more complicated, but we modified it for a reason.

WHEN IS THIS APPROXIMATION FOR H_{RFT} VALID?

JIMWLK/KLWMIJ - YES

POMERON LOOPS? ("'DIPOLE-DIPOLE"' SCATTERING)

NUCLEUS-NUCLEUS SCATTERING?

ANSWER

WHENEVER THE $|IN\rangle$ AND $|OUT\rangle$ STATES ARE PERTURBATIVELY CLOSE!

WHEN THE OVERLAP COMES FROM THE AREA OF MOST PROBABILITY WE ARE OK

IF IT'S THE TAILS - WE ARE LOST (WE DON'T KNOW THE TAILS)

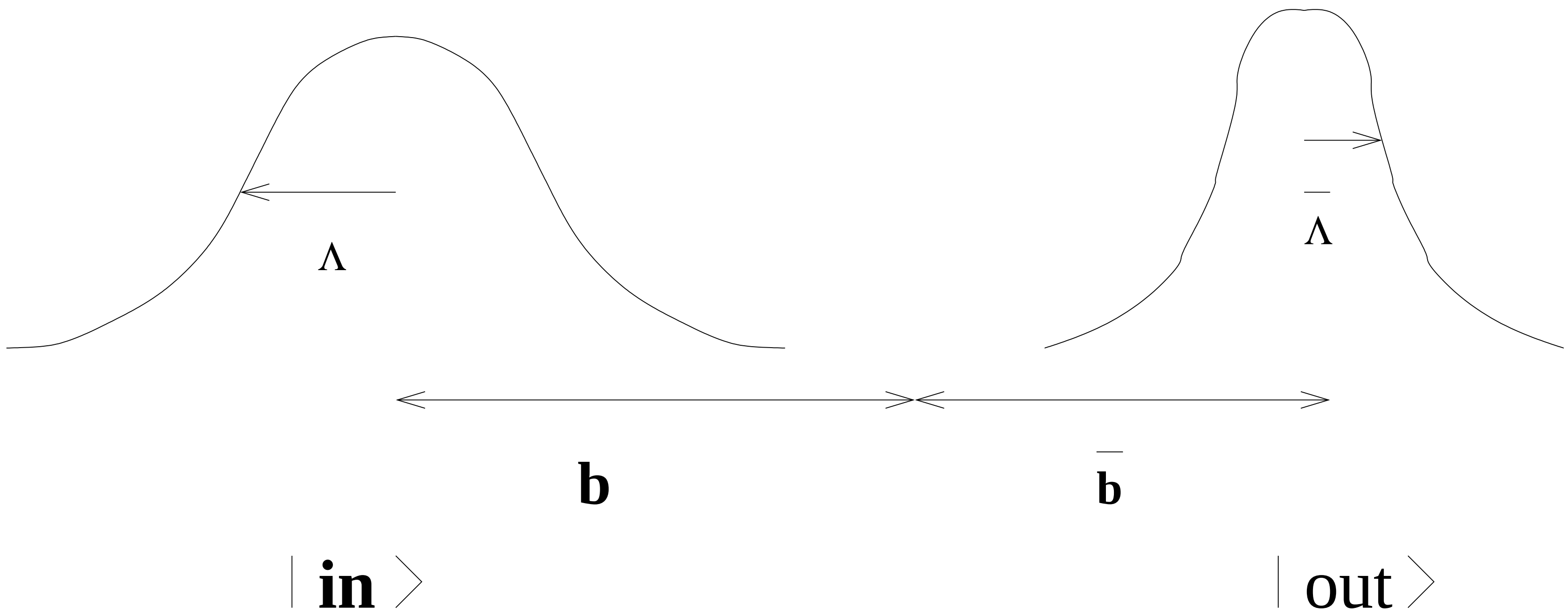


Figure 1: THE TAIL OF THE TWO GAUSSIANS.

E.G.

KLWMIJ: $b \sim O(g); \quad \bar{b} \sim O(g); \quad \Lambda \sim O(1)$

JIMWLK: $\Lambda \sim O(1); \quad b \sim O(1/g); \quad \bar{b} \sim O(1/g);$ BUT $b - \bar{b} \sim O(g)$

DIPOLE-DIPOLE - TARGET IS ALWAYS PERTURBATIVE, SO WE ARE ALWAYS OK

NUCLEUS-NUCLEUS - NOT SO GOOD...

$R \sim 1; \quad b \sim O(1/g) \quad \rightarrow \quad b - \bar{b} \sim O(1/g); \quad \Lambda \sim O(1)$

OVERLAP IS DOMINATED BY THE TAILS OF THE TWO WAVEFUNCTIONS - WE NEED TO KNOW THE TAILS

OUR PERTURBATIVE EXPRESSION FOR Ω IS NOT ENOUGH

WKB APPROXIMATION WOULD BE APPROPRIATE ($A \sim O(1/g)$) - BUT COMPLICATED

SINGLE GLUON INCLUSIVE SPECTRUM

FORMALLY NEED THE SAME Ω

$$N = \langle 0|\Omega^\dagger\ S^\dagger\ \Omega\ a^\dagger\ a\ \Omega^\dagger\ S\ \Omega|0\rangle$$

HERE $\Omega\ a^\dagger\ a\ \Omega^\dagger$ IS THE QUASIPARTICLE NUMBER OPERATOR

NUCLEUS-NUCLEUS COLLISIONS - FINAL STATES $N \sim O(1/\alpha_s)$

WE DO NOT KNOW THESE STATES WITH PERTURBATIVE ACCURACY

$$\langle N|a^\dagger a|N\rangle = \langle 0|a^N \Omega^\dagger a^\dagger a \Omega (a^\dagger)^N|0\rangle$$

SUPPOSE

$$\Omega^\dagger a \Omega = b + \Lambda a + g\Xi a^2 + ...$$

THEN

$$\langle N|a^\dagger a|N\rangle \sim b^2 + \Lambda^2 N + g^2 \Xi^2 N^2 + ...$$

IF $N \sim 1/\alpha_s$ ALL TERMS ARE OF THE SAME ORDER, BUT WE KNOW ONLY THE FIRST TWO!

WKB WOULD BE GOOD HERE TOO...

CONCLUDE

THE IMPORTANT PARAMETER IS THE PHASE SPACE DENSITY OF THE FINAL STATE GLUONS!

OUR H_{RFT} AND SINGLE GLUON SPECTRUM IS VALID WHEN THE **FINAL STATES** ARE **NOT SATURATED**

TO GO BEYOND WE MUST WORK THROUGH THE WKB APPROXIMATION IN QCD...