transverse-momentum dependent parton densities: evolution equations

I.O. Cherednikov & **N.G. Stefanis**

JINR-Dubna (RU) & INFN-Cosenza (IT) &
Moscow-U (RU) & Ruhr-U Bochum (DE)

prepared for the Low-X Meeting, 8 - 13 Sept 2009, Ischia, Napoli, Italy renormalization-group properties and extra divergences of the transverse-momentum dependent (TMD) parton densities, or distribution functions (PDFs) are discussed;

quantitative analysis is given in the light-cone gauge;

generalized definition of the TMD PDF, based on the renormalization procedure for the hadronic matrix elements with the Wilson exponentials, are proposed;

evolution equations (Q^2 and rapidity) are considered.

- **integrated parton densities:** definition; gauge invariance; RG properties

- **unintegrated (TMD) densities:** definition; gauge invariance, extra divergences, problems of renormalization

- **generalized definition:** cancelation of extra divergences; UV and rapidity evolution

- **conclusions and outlook**

integrated parton densities: definition; gauge invariance; RG properties

quark distribution:

$$\mathcal{Q}_{i/h}(x) = \frac{1}{2}\int \frac{d\xi^-}{2\pi}\, \mathrm{e}^{-ik^+\xi^-} \langle h(P)|\bar{\psi}_i(\xi^-, \mathbf{0}_\perp)\gamma^+\psi_i(0^-, \mathbf{0}_\perp)|h(P)\rangle \sim$$

$$\sim \langle h(P)|a_i^\dagger a_i(x)|h(P)\rangle$$

gauge invariance is saved by the insertion of the **gauge link** (in the fundamental representation)

$$[y, x|\Gamma] = \mathcal{P}\exp\left[-ig\int_{x[\Gamma]}^{y} dz_\mu A_a^\mu(z) t_a\right]_{\mathrm{F}}$$

$$\hat{\mathcal{Q}}_{i/h}(x) = \frac{1}{2}\int \frac{d\xi^-}{2\pi}\, \mathrm{e}^{-ik^+\xi^-} \langle h(P)|\bar{\psi}_i(\xi^-, \mathbf{0}_\perp)[\xi^-, 0^-]\gamma^+\psi_i(0^-, \mathbf{0}_\perp)|h(P)\rangle$$

gluon distribution:

$$\mathcal{G}^{\mu\nu}(x) = \frac{n^-_\rho n^-_\sigma}{(P^+)^2} \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} \, \mathrm{e}^{-i\xi^- k^+} .$$

$$\cdot \mathrm{Tr}_c \langle h(P)| \, F^{\mu\rho}(\xi^-, \mathbf{0}_\perp)[\xi^-, 0]_\Gamma \, F^{\nu\sigma}(0^-, \mathbf{0}_\perp)[0, \xi^-]'_\Gamma |h(P)\rangle$$

gauge links are taken in the adjoint representation:

$$[y, x|\Gamma] = \mathcal{P} \exp \left[-ig \int_{x[\Gamma]}^{y} dz_\mu A^\mu_a(z) t_a \right]_{\mathrm{A}}$$

renormalization group properties of the parton ($\mathcal{P}$) distribution are given by the DGLAP equation:

$$\mu \frac{d}{d\mu} \hat{\mathcal{P}}_{i/h}(x,\mu) = \sum_j \int_x^1 \frac{dz}{z} \, P_{ij}\left(\frac{x}{z}\right) \hat{\mathcal{P}}_{j/h}(x,\mu)$$

$P_{ij}\left(\frac{x}{z}\right)$ is the DGLAP integral kernel, which controls the dependence from the UV scale μ—therefore, the (logarithmic) scale Q^2-dependence stems from DGLAP:

$$\hat{\mathcal{P}}_{i/h}(x,\mu) \rightarrow \hat{\mathcal{P}}_{i/h}(x,Q^2)$$

unintegrated (TMD) parton densities

"naive" definition (example of the quark distribution):

$$f_i(x, \boldsymbol{k}_\perp) = \frac{1}{2} \int \frac{d\xi^- d^2\xi_\perp}{2\pi(2\pi)^2} \; \mathrm{e}^{-ik^+\xi^- + ik_\perp \cdot \xi_\perp} .$$

$$\cdot \langle p | \bar{\psi}_i(\xi^-, \xi_\perp) [\xi^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\xi}_\perp;]^\dagger \gamma^+ [\infty^-, \mathbf{0}_\perp; 0^-, \mathbf{0}_\perp] \psi_i(0^-, 0_\perp) | p \rangle \; |_{\xi^+=0}$$

formally (!):

$$\int d^2k_\perp f_i(x, \boldsymbol{k}_\perp) = \mathcal{Q}_i(x)$$

$f_i(x, \boldsymbol{k}_\perp)$ accumulates a lot of the phenomenologically important quantities. E.g., $T-$odd functions, responsible for the single-spin asymmetries in SIDIS, Drell-Yan, etc.

factorization of semi-inclusive processes

$$F(x_B, z_h, p_h, Q^2) = \sum_i e_i^2 \cdot$$

$$\cdot H(Q^2, \mu^2) \otimes \mathcal{F}_D(x_B, \boldsymbol{k}_\perp, \mu^2, \eta) \otimes \mathcal{F}_F(z_h, \boldsymbol{q}_\perp, \mu^2, \hat{\eta}) \otimes S(\mu^2)$$

however: this definition suffers from several shortcomings.

- **gauge invariance** is not complete: in the light-cone gauge, dependence on the pole prescription in the gluon propagator still takes place

- **extra** (rapidity) **divergences** associated with the features of the light-cone gauge, or the light-like Wilson lines (*in the integrated case, these divergences cancel*)

- **reduction to the integrated case**: formal integration doesn't produce correct result because of additional uncanceled UV divergences

towards the solution:

- **gauge invariance** is completely restored by means of the additional transverse Wilson line at light-cone infinity (`Belitsky, Ji, Yuan`). This gauge link contributes only in the light-cone gauge and cancels the pole-prescription dependence

- **extra divergences** can be avoided by using the non-light-like gauge connectors in covariant gauges, or an axial gauge off the light cone (`Collins, Soper`). This entails the introduction of an additional rapidity parameter $\zeta = (p \cdot n)^2/n^2$ (with $n^2 \neq 0$) to encode the deviation from the light cone; the calculations become more complicated; problems with factorization could arise.

- **generalized renormalization** procedure for the light-like Wilson lines (or a subtractive method) (`Collins, Hautmann`): extra divergences cancel by the additional "soft" factor, defined by the vacuum average of particular Wilson lines (demonstrated explicitly in the covariant gauge, in the 1-loop order)

tree approximation:

$$f^{(0)}(x, \boldsymbol{k}_\perp) =$$

$$= \frac{1}{2} \int \frac{d\xi^- d^2\xi_\perp}{2\pi(2\pi)^2} \mathrm{e}^{-ik^+\xi^- + ik_\perp \cdot \xi_\perp} \langle p | \bar{\psi}(\xi^-, \xi_\perp) \gamma^+ \psi(0^-, 0_\perp) | p \rangle =$$

$$= \delta(1-x)\delta^{(2)}(\boldsymbol{k}_\perp)$$

one-gluon exchanges, contributing to the UV-divergences, are described by the diagrams:

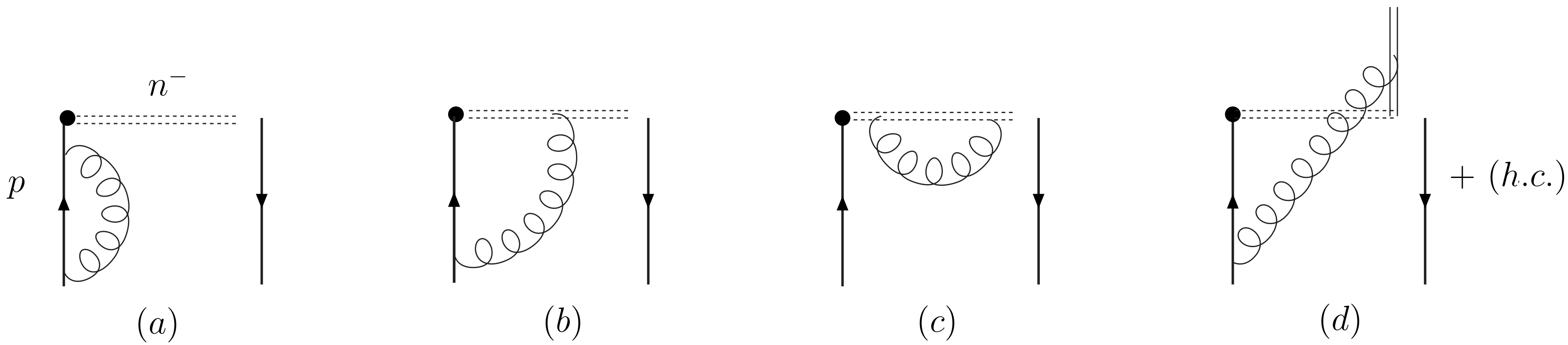

Figure 1: One-gluon exchanges for the TMD PDF: diagrams producing UV divergences. Only (a) and (d) contribute in the light-cone gauge

one-loop **anomalous dimension** is defined via the renormalization constant

$$\gamma = \frac{1}{2}\,\frac{1}{Z^{(1)}}\,\mu\frac{\partial \alpha_s(\mu)}{\partial \mu}\,\frac{\partial Z^{(1)}(\mu, \alpha_s(\mu); \epsilon)}{\partial \alpha_s}$$

and reads

$$\gamma_{\rm LC} = \gamma_{\rm smooth} - \delta\gamma \quad , \quad \gamma_{\rm smooth} = \frac{3}{4}\frac{\alpha_s}{\pi}C_{\rm F} + O(\alpha_s^2)$$

defect of anomalous dimension

$$\delta\gamma = -\frac{\alpha_s}{\pi}C_{\rm F}\ \ln\frac{\eta}{p^+}$$

contains undesirable p^+-dependent term which should be removed by a consistent procedure.

after standard R-operation, TMD PDF is still not renormalized!
problem of “mixed” divergences

$\delta\gamma$ is nothing else, but the cusp anomalous dimension:

$$p^+ = (p \cdot n^-) \sim \cosh \chi$$

defines an angle χ between the direction of the quark momentum p_μ and the light-like vector n^-.

$$\ln p^+ \to \chi, \; \chi \to \infty$$

renormalization of the Wilson operators with obstructions (cusps, self-intersections) requires additional renormalization factor depending on the cusp angle (Korchemsky, Radyushkin)

$$Z_\chi = \left[\langle 0 | \mathcal{P} \exp \left[ig \int_\chi d\zeta^\mu \; \hat{A}^a_\mu(\zeta) \right] |0\rangle \right]^{-1}$$

generalized renormalization:

$$\mathcal{O}_{\rm ren}(\chi, ...) = Z_\chi Z_{\rm R} \, \mathcal{O}(\chi, ...)$$

integration contour:

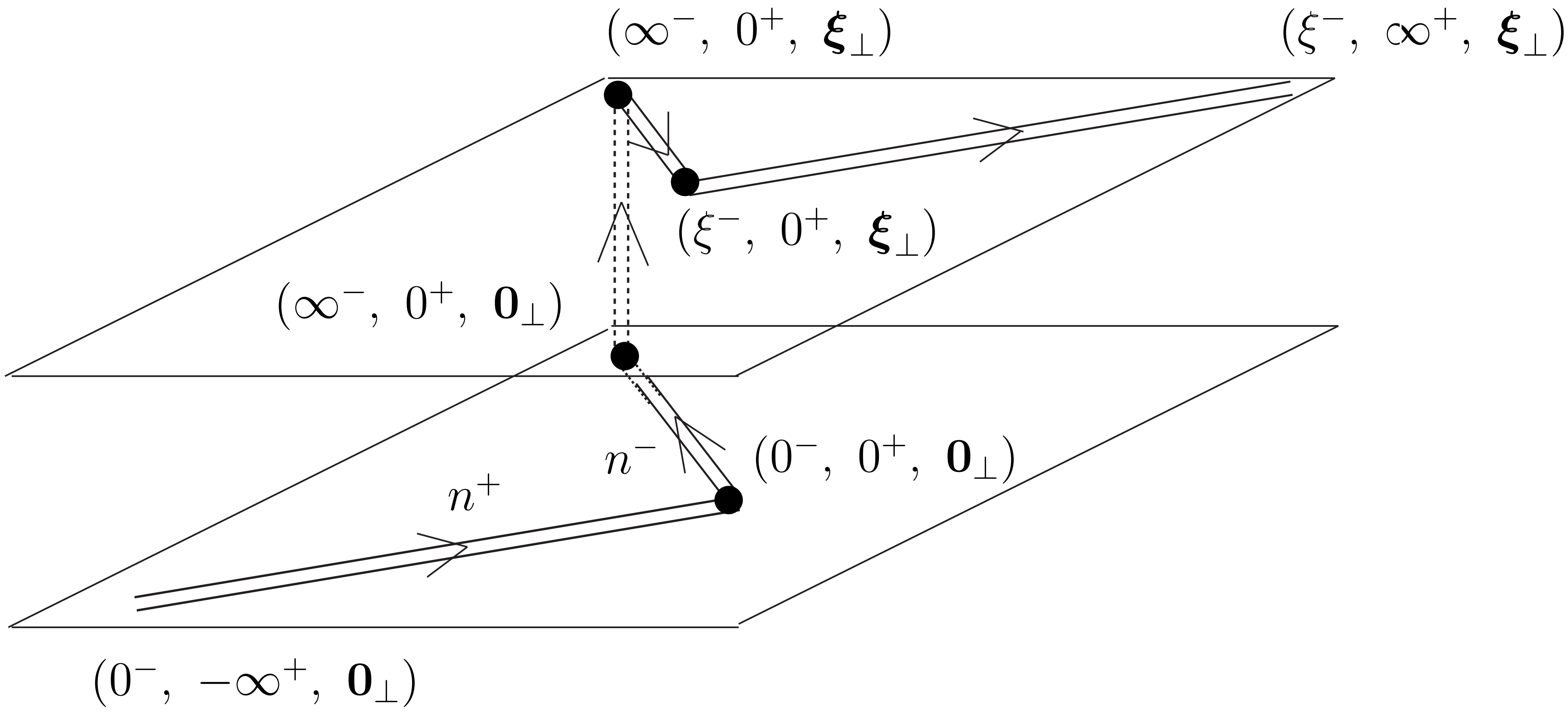

Figure 2: Space-time picture: integration trajectory for the additional cusp-dependent renormalization factor

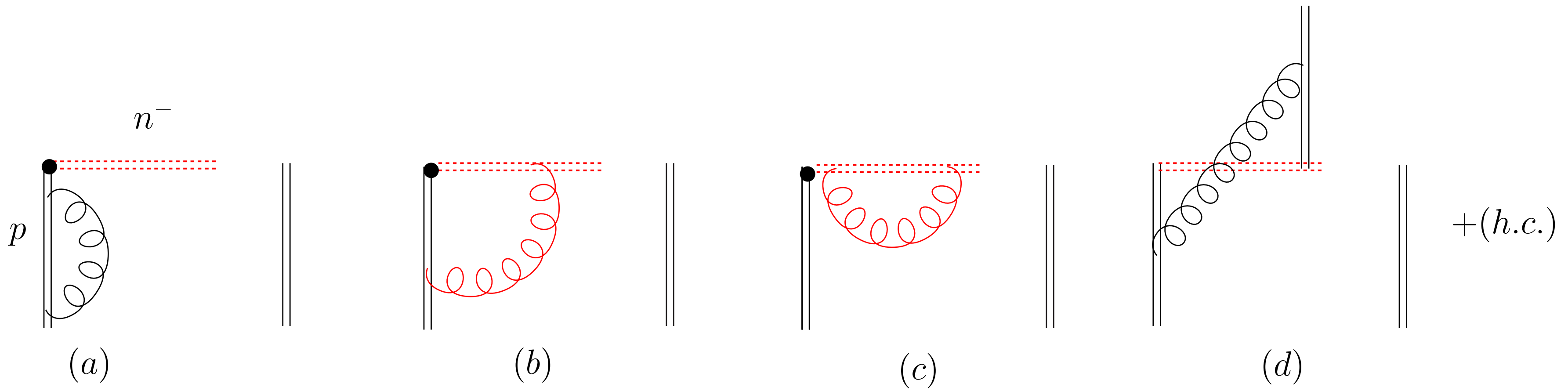

Figure 3: One-gluon exchanges for the generalized multiplicative renormalization factor

generalized **renormalization constant:**

$$\hat{Z}_{\rm mod} = 1 + \frac{\alpha_s}{4\pi} C_{\rm F} \frac{2}{\epsilon} \left(-3 - 4 \ln \frac{\eta}{p^+} + 4 \ln \frac{\eta}{p^+} \right) = 1 - \frac{3\alpha_s}{4\pi} C_{\rm F} \frac{2}{\epsilon}$$

$$\frac{1}{2} \mu \frac{d}{d\mu} \ln \hat{Z}_{\rm mod}(\mu, \alpha_s, p^+) = \frac{3}{4} \frac{\alpha_s}{\pi} C_{\rm F} + O(\alpha_s^2)$$

generalized definition of TMD PDF:

$$\mathcal{F}\left(x, \boldsymbol{k}_\perp; \mu, \eta\right) =$$
$$\frac{1}{2}\int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{2\pi(2\pi)^2} \mathrm{e}^{-ik^+\xi^- + ik_\perp\cdot\xi_\perp} \langle P|\bar{\psi}(\xi^-, \boldsymbol{k}_\perp)[\xi^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\xi}_\perp]^\dagger$$
$$\times[\infty^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\infty}_\perp]^\dagger \gamma^+ [\infty^-, \boldsymbol{\infty}_\perp; \infty^-, \mathbf{0}_\perp][\infty^-, \mathbf{0}_\perp; 0^-, \mathbf{0}_\perp]$$
$$\times\psi(0^-, \mathbf{0}_\perp)|P\rangle \left[\Phi(p^+, n^-|0^-, \mathbf{0}_\perp)\Phi^\dagger(p^+, n^-|\xi^-, \boldsymbol{\xi}_\perp)\right]^{-1}$$

soft factor:

$$\Phi(p^+, n^-|0) = \left\langle 0 \left| \mathcal{P}\exp\left[ig\int_{\mathcal{C}_{\rm cusp}} d\zeta^\mu\, t^a A^a_\mu(\zeta)\right]\right| 0 \right\rangle$$

$$\Phi^\dagger(p^+, n^-|\xi) = \left\langle 0 \left| \mathcal{P}\exp\left[-ig\int_{\mathcal{C}'_{\rm cusp}} d\zeta^\mu\, t^a A^a_\mu(\xi + \zeta)\right]\right| 0 \right\rangle$$

dependence on the dimensional regularization scale μ of the re-defined TMD PDF (**UV-evolution**):

$$\frac{1}{2}\mu\frac{d}{d\mu}\,\mathcal{F}(x,\boldsymbol{k}_\perp;\mu) = \int d^2\boldsymbol{q}_\perp \int_x^1 \frac{dz}{z}\,P_\perp\left(\frac{x}{z},\boldsymbol{q}_\perp,\alpha_s\right)\,\mathcal{F}(z,\boldsymbol{q}_\perp,\mu)$$

$$P_\perp\,(y,\boldsymbol{q}_\perp,\alpha_s) = \gamma_{\rm mod}\,\delta(1-y)\,\delta^{(2)}(\boldsymbol{k}_\perp-\boldsymbol{q}_\perp) + O(\alpha_s^2)\ ,$$

$$\gamma_{\rm mod} = \gamma_{2q} = -\frac{1}{2}\mu\frac{d}{d\mu}\ln\Sigma_{\rm mod}(\alpha_s,\epsilon) = \frac{3}{4}\frac{\alpha_s}{\pi}\,C_{\rm F} + O(\alpha_s^2)\ .$$

consistency equation:

$$\mu\frac{d}{d\mu}\left[\eta\frac{d}{d\eta}\mathcal{F}(x,\boldsymbol{k}_\perp;\mu,\eta)\right] = 0$$

set of evolution equations for TMD PDFs

- **UV-evolution** (in the integrated case—DGLAP equation)

$$\mu\frac{d}{d\mu}\,\mathcal{F}(x,\boldsymbol{k}_\perp,\mu,\eta)=\mathcal{K}_{\rm UV}\otimes\mathcal{F}(x,\boldsymbol{k}_\perp,\mu,\eta)$$

- **rapidity evolution** (Collins-Soper equation) (no correspondence in the integrated case!)

$$\eta\frac{d}{d\eta}\,\mathcal{F}(x,\boldsymbol{k}_\perp,\mu,\eta)=\mathcal{K}_{\rm CS}\otimes\mathcal{F}(x,\boldsymbol{k}_\perp,\mu,\eta)$$

- **BFKL evolution** (relation to the rapidity evolution is not known!)

$$x\frac{d}{dx}\,\mathcal{F}(x,\boldsymbol{k}_\perp,\mu,\eta)=\mathcal{K}_{\rm BFKL}\otimes\mathcal{F}(x,\boldsymbol{k}_\perp,\mu,\eta)$$

reduction to the integrated PDF

$$\int d^{\omega-2}\boldsymbol{k}_\perp \mathcal{F}(x, \boldsymbol{k}_\perp; \mu, \eta) \to \hat{\mathcal{Q}}(x, \mu)$$

restores the DGLAP evolution

conclusions

- generalized **definition of TMD PDF** is proposed (the problem of the UV renormalization of unintegrated PDFs is solved);
- direct connection to the **integrated PDF** is established;
- **UV and rapidity evolution** of the modified TMD PDF is discussed;
- relations of the **BFKL and Collins-Soper** evolutions: to be studied!

I.Ch., N. Stefanis:

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