QCD at small-x
in nucleus-nucleus collisions

Low-x meeting, Ischia, September 2009

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Outline

1. **Introduction**
2. **Multi-gluon correlations**
3. **Dilute limit**

Collaborators:

$T.\ Lappi$ $(Jyvaskyla)$ $R.\ Venugopalan$ (BNL)

$A.\ Dumitru$ $(Baruch\ College)$ $L.\ McLerran$ (BNL)

1 **Introduction**

Hydrodynamics

Correlations at large ΔY

Color Glass Condensate

2 **Multi-gluon correlations**

Power counting

Single gluon spectrum at LO

Leading Log factorization

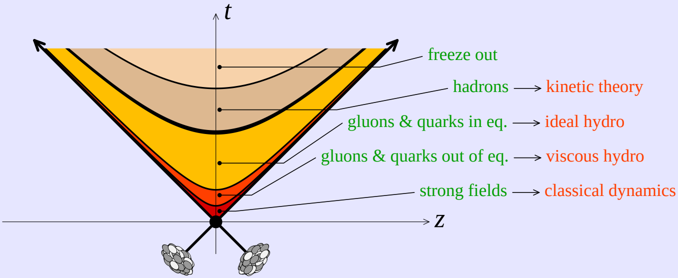
Multi-gluon correlations

3 **Dilute limit**

Short range correlations at LO

Pomeron splittings

Stages of a nucleus-nucleus collision



- The Color Glass Condensate provides a framework to describe nucleus-nucleus collisions up to a time $\tau \sim {Q_s}^{-1}$

Reminder on hydrodynamics

Equations of hydrodynamics :

$$\partial_\mu T^{\mu\nu} = 0 \text{ (energy-momentum conservation)}$$
$$\partial_\mu J^\mu_{\rm B} = 0 \text{ (baryon number conservation)}$$

- These equations contain only first order time derivatives
- Required initial conditions :

$T^{\mu\nu}(\tau=\tau_0,\eta,\vec{\boldsymbol{x}}_\perp)$, $J^\mu_{\rm B}(\tau=\tau_0,\eta,\vec{\boldsymbol{x}}_\perp)$

Additional inputs :

Equation of state: $p = f(\epsilon)$

Transport coefficients: $\eta, \zeta, \cdots$

Initial correlations and hydrodynamics

- The equations of hydrodynamics are non-linear. Therefore, solving hydro evolution for event averaged initial conditions is not the same as solving hydro event-by-event, and averaging observables at the end :

$$\mathrm{HYDRO}\left[\left\langle T^{\mu\nu}_{\mathrm{init}}\right\rangle\right] \neq \left\langle \mathrm{HYDRO}\left[T^{\mu\nu}_{\mathrm{init}}\right]\right\rangle$$

- To study hydrodynamics event by event, one needs an event generator for $T^{\mu\nu}(\tau_0, \eta, \vec{\boldsymbol{x}}_\perp)$

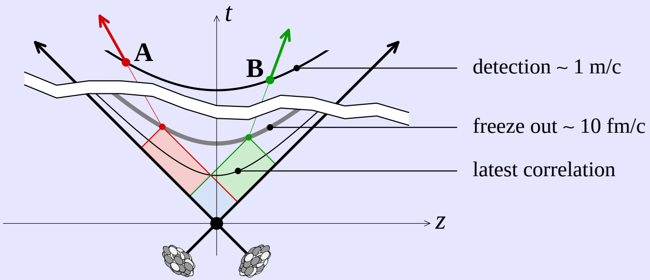
- To achieve this, it is not sufficient to know the average $\left\langle T^{\mu\nu}(\tau_0, \eta, \vec{\boldsymbol{x}}_\perp)\right\rangle$. We also need correlations :

$$\left\langle T^{\mu_1\nu_1}(\tau_0, \eta_1, \vec{\boldsymbol{x}}_{1\perp}) T^{\mu_2\nu_2}(\tau_0, \eta_2, \vec{\boldsymbol{x}}_{2\perp})\right\rangle$$

$$\left\langle T^{\mu_1\nu_1}(\tau_0, \eta_1, \vec{\boldsymbol{x}}_{1\perp}) T^{\mu_2\nu_2}(\tau_0, \eta_2, \vec{\boldsymbol{x}}_{2\perp}) T^{\mu_3\nu_3}(\tau_0, \eta_3, \vec{\boldsymbol{x}}_{3\perp})\right\rangle$$

$$\cdots$$

Long range rapidity correlations probe early dynamics



Long range rapidity correlations are created early

From causality, the latest time at which a correlation between two particles can be created is :

$$t_{\text{correlation}} \le t_{\text{freeze out}} \; e^{-\frac{1}{2}|y_A - y_B|}$$

Example: $t_{\text{freeze out}} = 10$ fm/c, $|y_A - y_B| = 6$: $t_{\text{correlation}} \le 0.5$ fm/c

Effective degrees of freedom

McLerran, Venugopalan (1994)

- The fast partons (large $x > x_0$) are frozen by time dilation
  ▷ described as static color sources on the light-cone :

$$J^\mu = \delta^{\mu +}\rho(x^-, \vec{\boldsymbol{x}}_\perp) \qquad (x^- \equiv (t-z)/\sqrt{2})$$

Note: $\rho(x^-, \vec{\boldsymbol{x}}_\perp) \propto \delta(x^-)$

- Slow partons (small $x < x_0$) are not static over the time-scales of the collision process
  ▷ must be treated as the usual gauge fields
  ▷ coupled to the current J^μ by a term : $J^\mu A_\mu$

- The color sources ρ are random, with a distribution $W_Y[\rho]$ ($Y \equiv \ln(1/x_0)$ is the rapidity separating "slow" and "fast")

Parton evolution

Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner (1997-2001), Blaizot, Iancu, Weigert (2002)

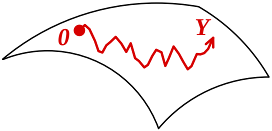
Renormalization group equation (JIMWLK) :

$$\frac{\partial W_Y}{\partial Y} = \mathcal{H} \; W_Y$$

$$\mathcal{H} = \frac{1}{2} \int_{\vec{x}_\perp, \vec{y}_\perp} \frac{\delta}{\delta \mathcal{A}^+(\epsilon, \vec{y}_\perp)} \eta(\vec{x}_\perp, \vec{y}_\perp) \frac{\delta}{\delta \mathcal{A}^+(\epsilon, \vec{x}_\perp)}$$

$$(-\partial_\perp^2 \mathcal{A}^+(\epsilon, \vec{x}_\perp) = \rho(\epsilon, \vec{x}_\perp) \quad , \quad \epsilon \sim 1/x_0)$$

- $\eta(\vec{x}_\perp, \vec{y}_\perp)$: non-linear functional of ρ
- Resums all the powers of $\alpha_s \ln(1/x)$
- Diffusion in the space of mappings $\{\mathbb{R}^2 \mapsto SU(3)\}$:



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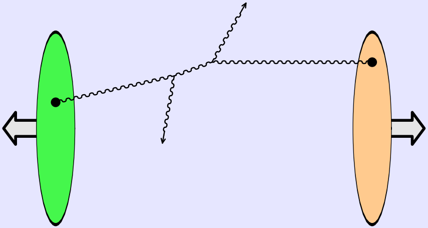
2. **Multi-gluon correlations**
   - Power counting
   - Single gluon spectrum at LO
   - Leading Log factorization
   - Multi-gluon correlations

3. Dilute limit
   - Short range correlations at LO
   - Pomeron splittings

Power counting

$$J^\mu \equiv \delta^{\mu+}\,\rho_1(x^-,\vec{\boldsymbol{x}}_\perp) + \delta^{\mu-}\,\rho_2(x^+,\vec{\boldsymbol{x}}_\perp)$$

$$\mathcal{S} = \underbrace{-\frac{1}{2}\int d^4x\ \mathrm{tr}\,F_{\mu\nu}F^{\mu\nu}}_{\mathcal{S}_{YM}} + \int d^4x\ J^\mu A_\mu$$



- Dilute regime : one parton in each projectile interact

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Power counting

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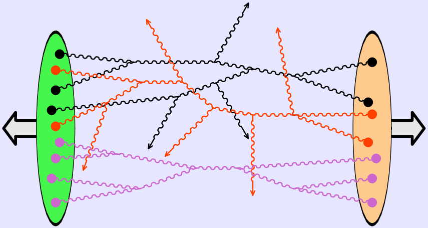
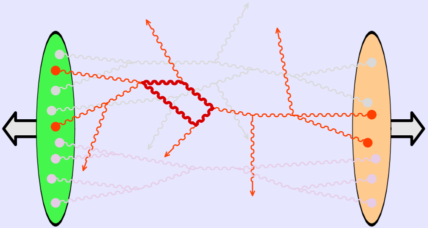


- Dilute regime : one parton in each projectile interact
- Dense regime : multiparton processes become crucial

Power counting

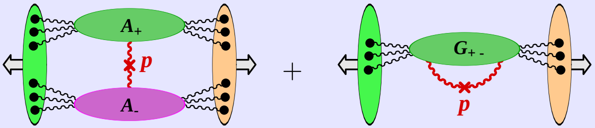


- In the saturated regime, the sources are of order $1/g$ (because $\langle\rho\rho\rangle \sim$ occupation number $\sim 1/\alpha_s$)
- Order of a connected diagram :

$$\frac{1}{g^2}\; g^{\#\ \text{produced gluons}}\; g^{2(\#\ \text{loops})}$$

Diagrammatic expansion of dN_1/d^3p

- The single inclusive spectrum has a simple diagrammatic representation :



- There are only connected graphs (AGK cancellation)

- Perturbative expansion in the saturated regime :

$$\frac{dN_1}{d^3\vec{p}} = \frac{1}{g^2}\Big[\underbrace{c_0}_{\text{LO}} + \underbrace{c_1\, g^2}_{\text{NLO}} + \underbrace{c_2\, g^4}_{\text{NNLO}} + \cdots\Big]$$

Expression in terms of classical fields at LO

Gluon spectrum at LO :

$$\left.\frac{dN_1}{d^3\vec{\boldsymbol{p}}}\right|_{\rm LO} \propto \int_{x,y} e^{ip\cdot(x-y)}\,\square_x\square_y \sum_\lambda \epsilon_\mu^{(\lambda)}\epsilon_\nu^{(\lambda)}\;\mathcal{A}^\mu(x)\mathcal{A}^\nu(y)$$

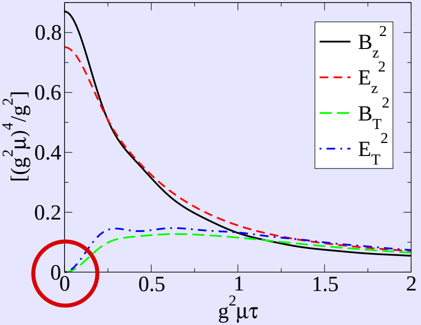
- $\mathcal{A}$ obeys the classical EOM : $\dfrac{\delta\mathcal{S}_{YM}}{\delta\mathcal{A}} + J = 0$
- The boundary conditions are very simple:

$$\lim_{x^0\to-\infty} \mathcal{A}(x) = 0$$

Initial classical fields

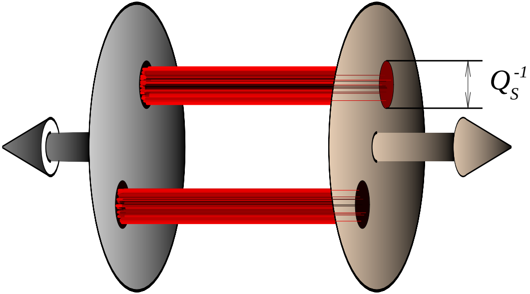
Lappi, McLerran (2006)

- Immediately after the collision, the chromo-$\vec{\boldsymbol{E}}$ and $\vec{\boldsymbol{B}}$ fields are purely longitudinal :



Initial classical fields

- The initial chromo-$\vec{E}$ and $\vec{B}$ fields form longitudinal "flux tubes" extending between the projectiles:

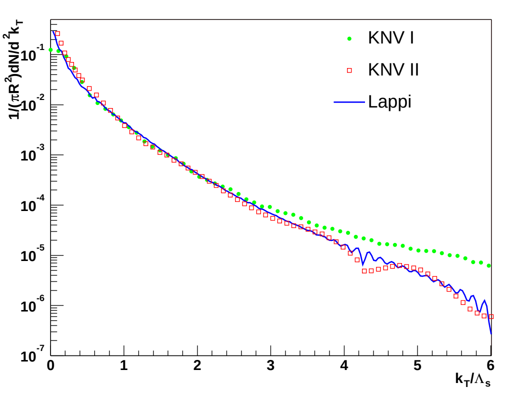


- The color correlation length in the transverse plane is Q_S^{-1}
  ▷ flux tubes of diameter Q_S^{-1}, filling up the transverse area

Single gluon spectrum at LO

Krasnitz, Nara, Venugopalan (1999 – 2001), Lappi (2003)

- No analytic solution for the Yang-Mills equations, but straightforward numerically



What is factorization ?

- The naive perturbative expansion of $dN_1/d^3\vec{p}$,

$$\frac{dN}{d^3\vec{p}} = \frac{1}{g^2}\left[c_0 + c_1\, g^2 + c_2\, g^4 + \cdots\right],$$

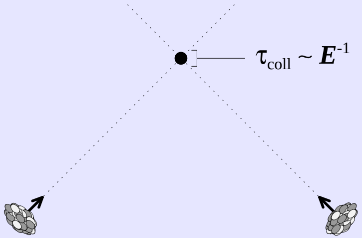
assumes that the coefficients c_n are of order one

- This assumption is upset by large logarithms of $1/x_{1,2}$:

$$\begin{aligned} c_1 &= & d_{10} &+ d_{11}\ln\left(\frac{1}{x_{1,2}}\right) \\ c_2 &= d_{20} + d_{21}\ln\left(\frac{1}{x_{1,2}}\right) &&+ \underbrace{d_{22}\ln^2\left(\frac{1}{x_{1,2}}\right)}_{\text{Leading Log terms}} \end{aligned}$$

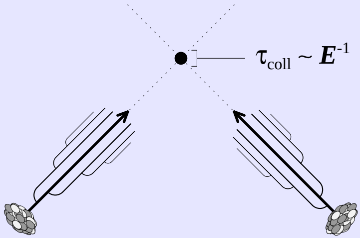
- Factorizability: the logarithms must be universal and resummable into functionals that depend only on the projectiles being collided

Why factorization works: causality



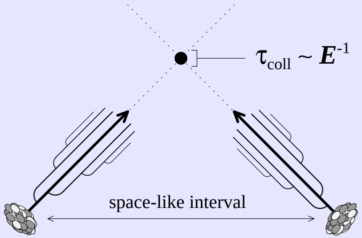
- The duration of the collision is very short: $\tau_{\rm coll} \sim E^{-1}$

Why factorization works: causality



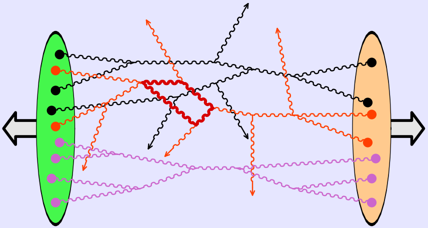
- The duration of the collision is very short: $\tau_{coll} \sim E^{-1}$
- The logarithms we want to resum arise from the radiation of soft gluons, which takes a long time
  ▷ it must happen (long) before the collision

Why factorization works: causality



- The duration of the collision is very short: $\tau_{\rm coll} \sim E^{-1}$
- The logarithms we want to resum arise from the radiation of soft gluons, which takes a long time
  ▷ it must happen (long) before the collision
- The projectiles are not in causal contact before the impact
  ▷ the logarithms are intrinsic properties of the projectiles, independent of the measured observable

Why the proof is complicated: strong fields



- Procedure: **(i)** calculate the 1-loop corrections, **(ii)** disentangle the logarithms from the finite contributions, **(iii)** show that the logs can be assigned to the projectiles

- Problem: strong fields, analytic calculation not feasible

  ▷ Take advantage of the retarded nature of the boundary conditions in order to separate the initial state evolution (calculable analytically) from the collision itself (hopeless)

Factorization in two steps: FG, Lappi, Venugopalan (2008)

I : The NLO gluon spectrum can be written as a perturbation of the initial value of the classical fields on the light-cone :

$$\left.\frac{dN_1}{d^3\vec{p}}\right|_{\rm NLO} = \Big[\frac{1}{2}\int_{\vec{u},\vec{v}\in{\rm LC}} \mathcal{G}(\vec{u},\vec{v})\,\mathbb{T}_{\boldsymbol{u}}\mathbb{T}_{\boldsymbol{v}} + \int_{\vec{u}\in{\rm LC}} \beta(\vec{u})\,\mathbb{T}_{\boldsymbol{u}}\Big]\;\left.\frac{dN_1}{d^3\vec{p}}\right|_{\rm LO}$$

$(\mathbb{T}_{\boldsymbol{u}} \sim \delta/\delta\mathcal{A}_{\rm initial}(\boldsymbol{u}))$, $\mathcal{G},\beta$ are calculable analytically)

II : The operator $[\cdots]$ is related to the JIMWLK Hamiltonian:

$$\frac{1}{2}\int_{\vec{u},\vec{v}\in{\rm LC}} \mathcal{G}(\vec{u},\vec{v})\,\mathbb{T}_{\boldsymbol{u}}\mathbb{T}_{\boldsymbol{v}} + \int_{\vec{u}\in{\rm LC}} \beta(\vec{u})\,\mathbb{T}_{\boldsymbol{u}} = \log\left(\frac{\Lambda^+}{p^+}\right)\times\mathcal{H}_1 + \log\left(\frac{\Lambda^-}{p^-}\right)\times\mathcal{H}_2 + \text{finite terms}$$

▷ Factorization follows easily

Leading Log factorization

- By averaging over all the configurations of the sources in the two projectiles, we get a factorized formula for the resummation of the leading log terms to all orders :

$$\left\langle \frac{dN_1}{d^3\vec{\boldsymbol{p}}} \right\rangle_{\rm LLog} = \int \left[D\rho_1\, D\rho_2\right]\, W_{Y_1}[\rho_1]\, W_{Y_2}[\rho_2]\; \left.\frac{dN_1}{d^3\vec{\boldsymbol{p}}}\right|_{\rm LO}$$

with : $\frac{\partial}{\partial Y} W_Y = \mathcal{H}\, W\ , \quad Y_1 = \log(\sqrt{s}/p^+)\ , \quad Y_2 = \log(\sqrt{s}/p^-)$

- The distributions $W[\rho_{1,2}]$ must be evolved up to the rapidity of the produced gluon

Multigluon spectrum at LO

FG, Lappi, Venugopalan (2008)

- In the saturated regime, the inclusive n-gluon spectrum at Leading Order is the product of n 1-gluon spectra:

$$\left.\frac{dN_n}{d^3\vec{\boldsymbol{p}}_1 \cdots d^3\vec{\boldsymbol{p}}_n}\right|_{\rm LO} = \left.\frac{dN_1}{d^3\vec{\boldsymbol{p}}_1}\right|_{\rm LO} \times \cdots \times \left.\frac{dN_1}{d^3\vec{\boldsymbol{p}}_n}\right|_{\rm LO}$$

- At LO, in a given configuration of the sources $\rho_{1,2}$, the n gluons are not correlated
- Note: this is true for the bulk ($p_\perp \lesssim Q_s$), but not for the tail of the distribution

Multigluon spectrum at NLO

- At NLO, one has again:

$$\left.\frac{dN_n}{d^3\vec{\boldsymbol{p}}_1\cdots d^3\vec{\boldsymbol{p}}_n}\right|_{\rm NLO} =$$

$$= \Big[\frac{1}{2}\int_{\vec{\boldsymbol{u}},\vec{\boldsymbol{v}}\in{\rm LC}} \mathcal{G}(\vec{\boldsymbol{u}},\vec{\boldsymbol{v}})\,\mathbb{T}_{\boldsymbol{u}}\mathbb{T}_{\boldsymbol{v}} + \int_{\vec{\boldsymbol{u}}\in{\rm LC}} \beta(\vec{\boldsymbol{u}})\,\mathbb{T}_{\boldsymbol{u}}\Big]\;\left.\frac{dN_n}{d^3\vec{\boldsymbol{p}}_1\cdots d^3\vec{\boldsymbol{p}}_n}\right|_{\rm LO}$$

- Correlations appear at NLO thanks to the operator $\mathcal{G}(\vec{\boldsymbol{u}},\vec{\boldsymbol{v}})\mathbb{T}_{\boldsymbol{u}}\mathbb{T}_{v}$, which can link two different gluons

- Thanks to their universal structure, we can factorize these correlations into the distributions $W[\rho_{1,2}]$

Leading Log factorization

Factorization formula for the n-gluon spectrum

$$\left\langle \frac{dN_n}{d^3\vec{\boldsymbol{p}}_1 \cdots d^3\vec{\boldsymbol{p}}_n} \right\rangle_{\text{LLog}} = \int \left[D\rho_1\, D\rho_2\right]\, W[\rho_1]\, W[\rho_2] \times \left.\frac{dN_1}{d^3\vec{\boldsymbol{p}}_1}\right|_{\text{LO}} \times \cdots \times \left.\frac{dN_1}{d^3\vec{\boldsymbol{p}}_n}\right|_{\text{LO}}$$

- This formula tells us that (in the Leading Log approximation) all the correlations arise from the $W[\rho]$'s
  ▷ they pre-exist in the wave-function of the projectiles
- Note: some short range correlations will also arise from splittings in the final state (not taken into account here, because does not come with a $\ln(s)$)

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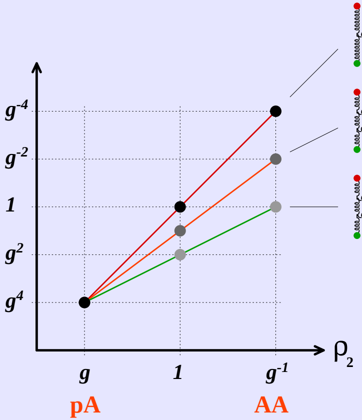
Contributions to the 2-gluon spectrum when $\rho_2 \to g$



- Only the disconnected graph contributes when $\rho_2 \sim g^{-1}$
- Some connected graphs become important when $\rho_2 \sim g$
  ▷ short range correlations between the two gluons (in a fixed configuration of $\rho_{1,2}$)

Pomeron splittings

- In additions to modifications of the pattern of local correlations (visible at LO), the power counting allows new contributions in the leading logarithmic corrections

- The dilute limit of the JIMWLK Hamiltonian is

$$\mathcal{H} \underset{\rho\to g}{\longrightarrow} g^2\rho^2\left(\frac{\delta}{\delta\rho}\right)^2 \sim g^2$$

Note: this operator preserves the number of ρ's

If $\rho \sim g$, ρ-number-changing operators have the same order

$$g^n\rho^2\left(\frac{\delta}{\delta\rho}\right)^n \sim g^2$$

Note: if $\rho \gg g$, the operators with $n > 2$ are suppressed

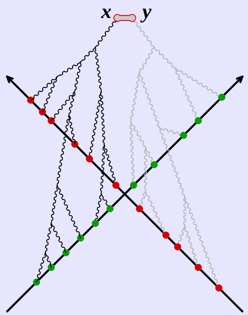
- These new operators correspond to Pomeron splittings (when evolving away from the fragmentation region)

Summary

- In the saturated regime, the correlations that affect the bulk of particle production all come via the evolution of the initial state prior to the collision:
  - long range rapidity correlations ($\Delta Y \sim \alpha_s^{-1}$)
  - provide a natural explanation for the ridge (R. Venugopalan), and for the fact that the multiplicity distribution is a negative binomial (L. McLerran)
- Correlations are more complicated in the dilute regime:
  - short range correlations become important
  - initial state evolution now sensitive to pomeron splittings
- The "dilute limit" is applicable to several situations:
  - collisions involving a small, non saturated, projectile
  - production of high-p_t particles in AA collisions

Expression in terms of classical fields at LO

- Classical fields are sums of tree diagrams :



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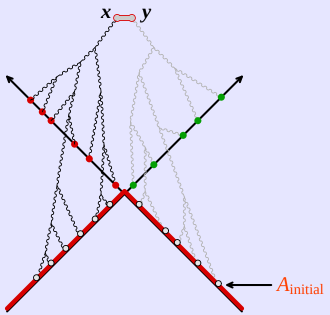
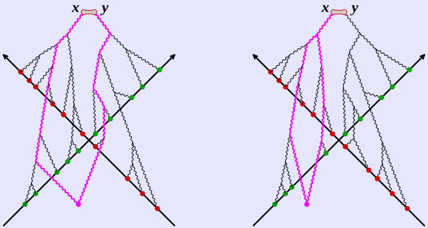


- Note : thanks to the retarded boundary conditions, the gluon spectrum is a functional of the value of the classical field on some initial Cauchy surface :

$$\left.\frac{dN_1}{d^3\vec{\boldsymbol{p}}}\right|_{\rm LO} = \mathrm{F}[\mathcal{A}_{\rm initial}]$$

Single gluon spectrum at NLO

1-loop contributions



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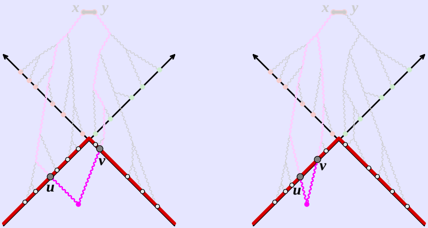

- Can be written as a perturbation of the LC initial fields :

$$\left.\frac{dN}{d^3\vec{\boldsymbol{p}}}\right|_{\rm NLO} = \left[\frac{1}{2}\int\limits_{\vec{\boldsymbol{u}},\vec{\boldsymbol{v}}\in{\rm LC}} \mathcal{G}(\vec{\boldsymbol{u}},\vec{\boldsymbol{v}})\,\mathbb{T}_{\boldsymbol{u}}\mathbb{T}_{\boldsymbol{v}}\right]\left.\frac{dN}{d^3\vec{\boldsymbol{p}}}\right|_{\rm LO}$$

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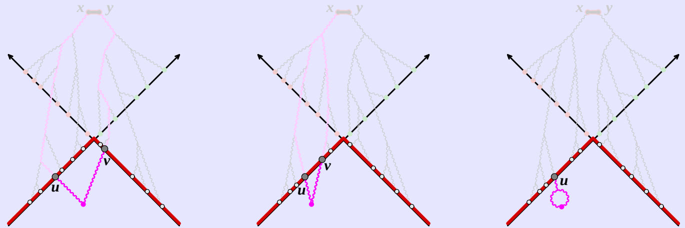

- The loop correction can also be below the light-cone :

$$\left.\frac{dN}{d^3\vec{p}}\right|_{\rm NLO}=\Big[\frac{1}{2}\int\limits_{\vec{u},\vec{v}\in{\rm LC}}\mathcal{G}(\vec{u},\vec{v})\,\mathbb{T}_{u}\mathbb{T}_{v}+\int\limits_{\vec{u}\in{\rm LC}}\beta(\vec{u})\,\mathbb{T}_{u}\Big]\left.\frac{dN}{d^3\vec{p}}\right|_{\rm LO}$$

Contributions to the 2-gluon spectrum when $\rho_2 \to g$

In the limit $\rho_2 \to g$

$$\left.\frac{dN_2}{d^3\vec{\boldsymbol{p}}d^3\vec{\boldsymbol{q}}}\right|_{\rm LO} \propto \frac{1}{|\vec{\boldsymbol{p}}|\,|\vec{\boldsymbol{q}}|}\left|\mathcal{A}^{(+)}(\vec{\boldsymbol{p}})\mathcal{A}^{(+)}(\vec{\boldsymbol{q}})+\Sigma^{(+)}(\vec{\boldsymbol{p}},\vec{\boldsymbol{q}})\right|^2$$

$$\mathcal{A}^{(+)}(\vec{\boldsymbol{p}}) = \int d^4x\; e^{ip\cdot x}\;\square_x\;\mathcal{A}(x)$$

$$0 = \frac{\delta\mathcal{S}_{\rm YM}}{\delta\mathcal{A}} + J\,,\quad \lim_{x^0\to-\infty}\mathcal{A}(x) = 0$$

$$\Sigma^{(+)}(\vec{\boldsymbol{p}},\vec{\boldsymbol{q}}) = \frac{1}{2}\int_{\vec{\boldsymbol{k}}}\Big(a^{(+)}_{+\boldsymbol{k}}(\vec{\boldsymbol{p}})a^{(+)}_{-\boldsymbol{k}}(\vec{\boldsymbol{q}}) + \vec{\boldsymbol{p}}\leftrightarrow\vec{\boldsymbol{q}}\Big)$$

$$a^{(+)}_{\pm\boldsymbol{k}}(\vec{\boldsymbol{p}}) = \int d^4x\; e^{ip\cdot x}\;\square_x\;a_{\pm\boldsymbol{k}}(x)$$

$$0 = \left[\square_x + \frac{\delta^2\mathcal{S}_{\rm YM}}{\delta\mathcal{A}^2}\right]a_{\pm\boldsymbol{k}}\,,\quad \lim_{x^0\to-\infty}a_{\pm\boldsymbol{k}}(x) = e^{\pm ik\cdot x}$$