Small x evolution in the CGC

and how to tailor the final state to expose nonlinear effects

Heribert Weigert
UNIVERSITY of OULU
OULUN YLIOPISTO

Low x, Ischia 2009

Outline

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Outline

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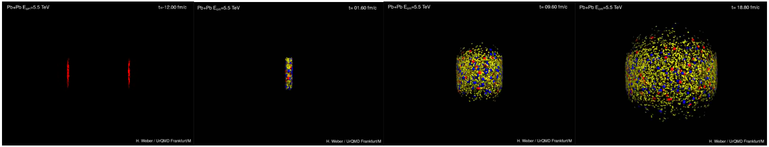
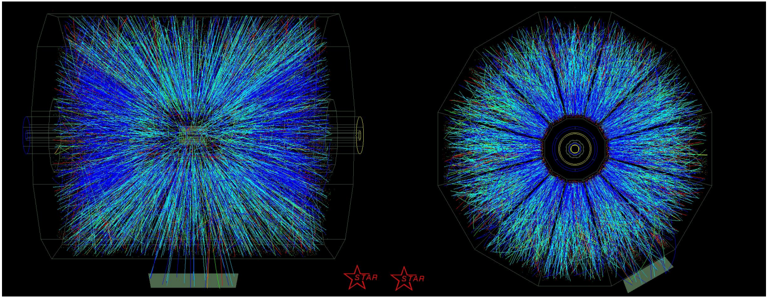
RHIC: searching for the Quark Gluon Plasma





side view front view

Energy dependence: from photons to gluons

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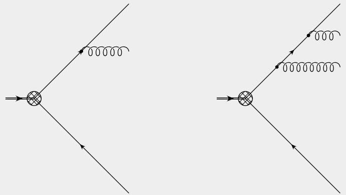
photon-like contributions



- enhanced by phase space integrals $\frac{dE}{E}\frac{d\theta}{\theta} \rightarrow \alpha_s \ln E \ln\theta$
- all orders calculation needed $\sum_{n=0}^{\infty}(\alpha_s \ln E)^n \dots$

Energy dependence: from photons to gluons

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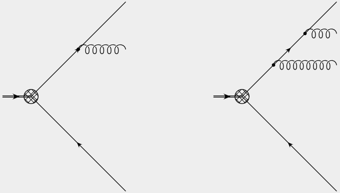
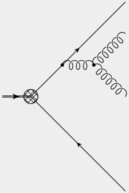


QCD: charged gluons



- enhanced by phase space integrals $\frac{dE}{E}\frac{d\theta}{\theta}$ ⟶ $\alpha_s \ln E \ln\theta$
- all orders calculation needed $\sum_{n=0}^{\infty}(\alpha_s \ln E)^n \ldots$
- gluons charged ⟶ radiation nonlinear in QCD

Variables: transverse resolution vs energy

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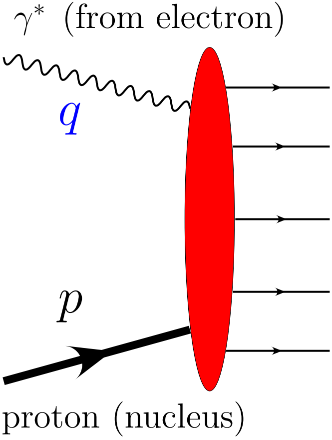

- $Q^2 := -q^2 \gg 0$ spacelike! transverse resolution $\Delta \boldsymbol{r} \sim \frac{1}{Q}$

- $x = x_{\text{Bj}} := \frac{Q^2}{2p.q} = \frac{Q^2}{2m\,E_{\text{rest}}}$

Variables: transverse resolution vs energy

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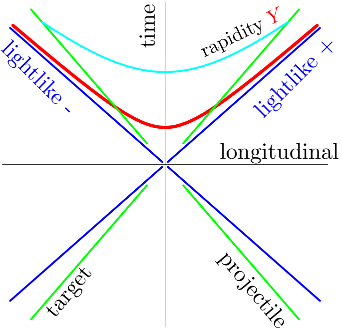

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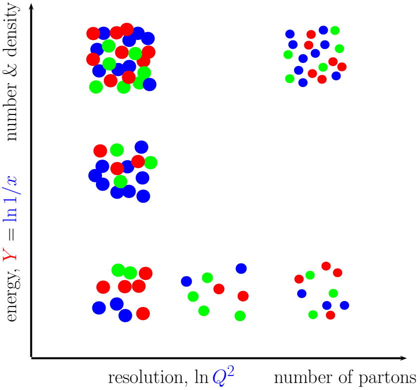
- $x = x_{\mathrm{Bj}} := \frac{Q^2}{2p.q} = \frac{Q^2}{2m\, E_{\mathrm{rest}}}$
- $Y = \ln \frac{1}{x} \propto \ln E_{\mathrm{rest}}$ all used synonymously

Large energies mean large densities



Large energies mean large densities

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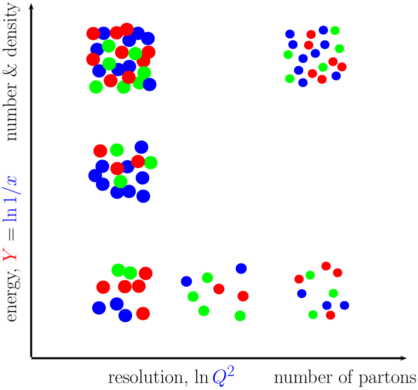
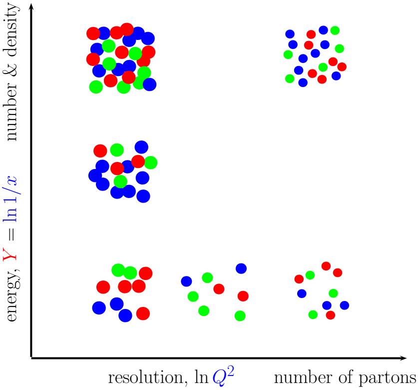
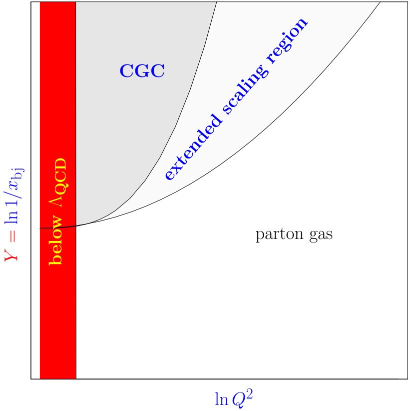


- density → finite correlation length R_s

Large energies mean large densities





- density → finite correlation length R_s

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Gluon production at increasing energy

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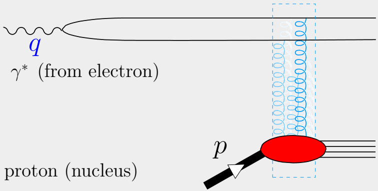
Nikolaev, Zakharov, Frankfurt, Strikman, Levin, Mueller,. . . (dipole picture)

the photon splits



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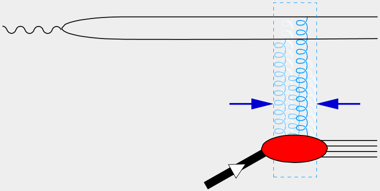

- target gluons Lorentz contracted to $\delta(x^-)$

Gluon production at increasing energy

Nikolaev, Zakharov, Frankfurt, Strikman, Levin, Mueller,... (dipole picture)

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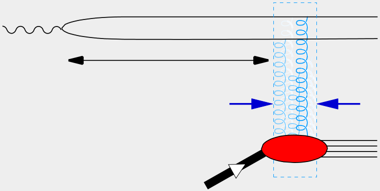

- target gluons Lorentz contracted to $\delta(x^-)$
- creation time dilated $\sim \frac{1}{x}$

Gluon production at increasing energy

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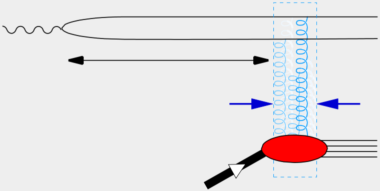

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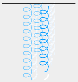
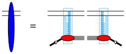
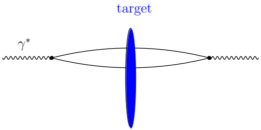


$$\sum_{\text{gluons}} \quad = \mathrm{P} \exp -ig \int dz_\mu A^\mu(z) \qquad = U_{\boldsymbol{x}}$$

Total cross section



$\sigma_{\mathrm{DIS}}(Y, Q^2) = \mathrm{Im}$



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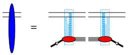
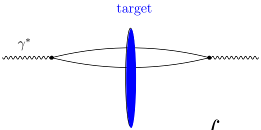
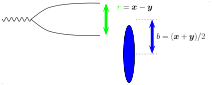


energy, $\ln 1/x$

$$\sigma_{\mathrm{DIS}}(Y, Q^2) = \mathrm{Im}$$





$$= \int d^2 r \; |\psi^2|(r^2 Q^2) \int d^2 b \; \langle \frac{\mathrm{tr}(1 - U_{\boldsymbol{x}} U_{\boldsymbol{y}}^\dagger)}{N_c} \rangle (Y)$$

σ_{dipole}

Total cross section

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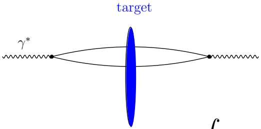
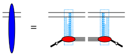
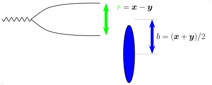
energy, $\ln 1/x$

$\sigma_{\text{DIS}}(Y, Q^2) = \text{Im}$







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- σ_{dipole} contains $U_{\boldsymbol{x}}$

Total cross section

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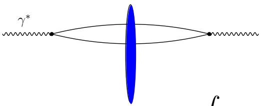

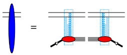

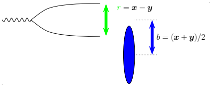

$$= \int d^2 r\, |\psi^2|(r^2 Q^2) \int d^2 b\, \langle \frac{\mathrm{tr}(1 - U_{\boldsymbol{x}} U^\dagger_{\boldsymbol{y}})}{N_c} \rangle (Y)$$

σ_{dipole}

- σ_{dipole} contains $U_{\boldsymbol{x}}$
- $\langle \ldots \rangle (Y)$ difficult
  target wavefunction: non-perturbative

Total cross section

energy, $\ln 1/x$

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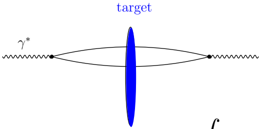

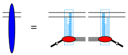

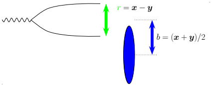

$$= \int d^2r \; |\psi^2|(r^2 Q^2) \int d^2b \; \langle \frac{\text{tr}(1 - U_{\boldsymbol{x}} U^\dagger_{\boldsymbol{y}})}{N_c} \rangle (Y)$$

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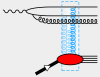
$$\frac{d}{dY} \langle \ldots \rangle (Y)$$



Total cross section

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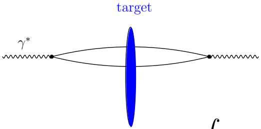

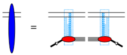

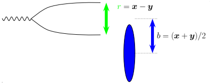

$$= \int d^2r \; |\psi^2|(r^2 Q^2) \int d^2b \; \langle \frac{\text{tr}(1 - U_x U_y^\dagger)}{N_c} \rangle(Y)$$

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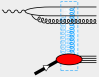
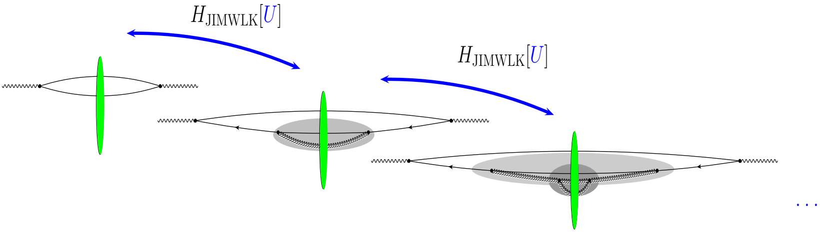


- Bookkeeping device: $\langle \ldots \rangle(Y) = \int \hat{D}[U] \; \ldots \hat{Z}_Y[U]$

The JIMWLK evolution equation

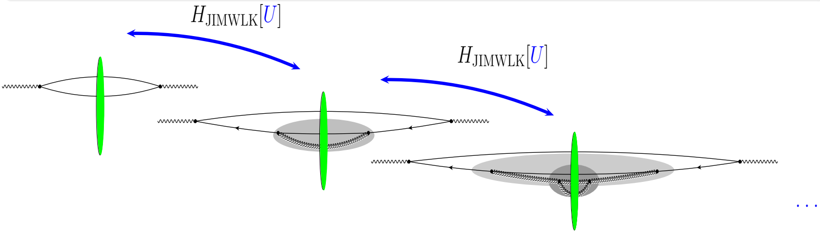


in parallel: Balitsky: inf. hierarchies, Balitsky-Kovchegov (BK): truncations (large N_c)

The JIMWLK evolution equation

Heribert Weigert **Nucl. Phys. A703**, 2002, 823

$$\frac{d}{dY} Z_Y[U] = -H_{\text{JIMWLK}}[U] \; Z_Y[U]$$



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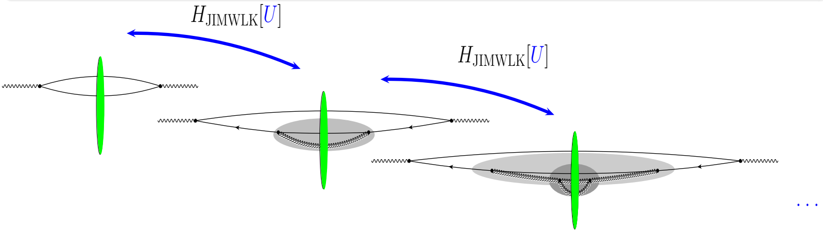

energy dependence of $\langle \ldots \rangle(Y) = \int \hat{D}[U] \; \ldots \hat{Z}_Y[U]$

in parallel: Balitsky: inf. hierarchies, Balitsky-Kovchegov (BK): truncations (large N_c)

Saturation scale and cross section

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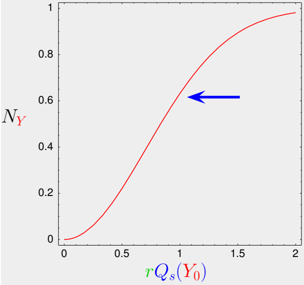
$$\langle \ldots \rangle(Y) \quad \longrightarrow \quad \int d^2b \, \langle \frac{\mathrm{tr}(1 - U_{\boldsymbol{r}} U^\dagger_{\boldsymbol{0}})}{N_c} \rangle(Y) =: N_Y(\boldsymbol{r})$$

Saturation scale and cross section

$$\langle \ldots \rangle(Y) \rightarrow \int d^2b \, \langle \frac{\mathrm{tr}(1 - U_{\boldsymbol{r}} U_{\boldsymbol{0}}^\dagger)}{N_c} \rangle(Y) =: N_Y(\boldsymbol{r})$$

Correlation length shrinks



- $$R_s(Y) \sim \frac{1}{Q_s(Y)}$$

$R_s(Y) \equiv$ correlation length
$Q_s(Y) \equiv$ saturation scale

Saturation scale and cross section

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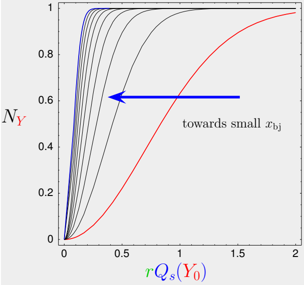

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Saturation scale and cross section

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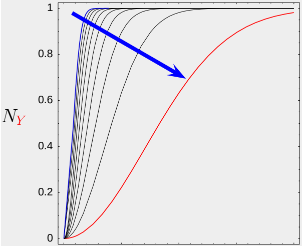

change units: $rQ_s(Y_0) \rightarrow rQ_s(Y)$

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- scaling solutions:
  initial conditions erased
  "attractor"

Saturation scale and cross section

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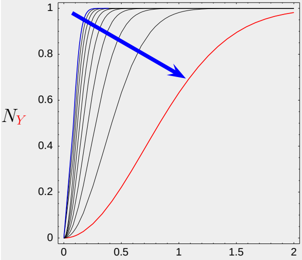

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- scaling solutions:
  initial conditions erased
  "attractor"
- Phenomenology:
  HERA data!

scaling

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Final state cancellations for JIMWLK

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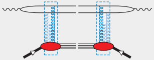
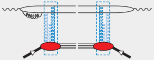
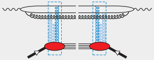
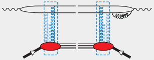
diagrams light cone time ordered

in dipole picture: Mueller, Chen; Kovchegov, Levin

H_{JIMWLK}  $=$  $+$  $+$ 

$+$ alt. attchm.

Final state cancellations for JIMWLK

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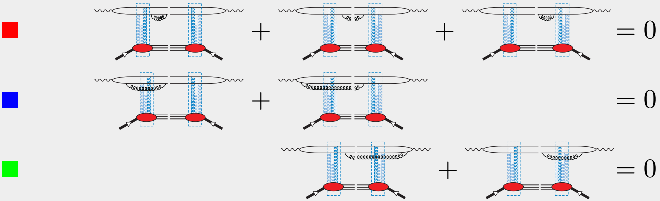
diagrams light cone time ordered

in dipole picture: Mueller, Chen; Kovchegov, Levin

H_{JIMWLK} 

$+$ alt. attchm.

relies on separate final state cancellations



- repeats for alt. attachments of gluon lines

Final state cancellations for JIMWLK

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diagrams light cone time ordered

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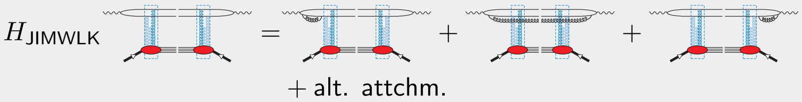
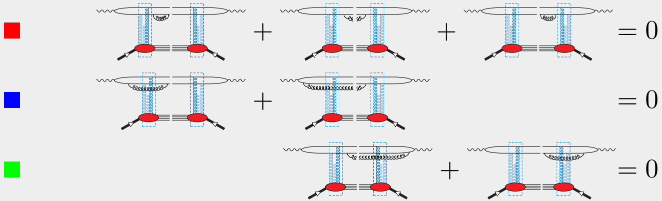
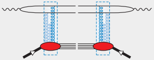
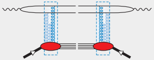


relies on separate final state cancellations



- repeats for alt. attachments of gluon lines
- restrictions on the final state: all diagrams contribute

Extended JIMWLK with final state restrictions

H_{JIMWLK}  $\rightarrow$ $\left[H_{\text{amp}} + H_{\text{fin}} + H_{\overline{\text{amp}}}\right]$ 

Extended JIMWLK with final state restrictions

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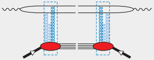
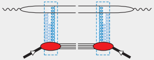
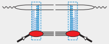
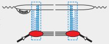
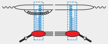
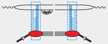
H_{JIMWLK}  → $\left[H_{\text{amp}} + H_{\text{fin}} + H_{\overline{\text{amp}}}\right]$ 

H_{amp}  $=$  $+$  $+$ 

$+$ alt. attachments

Extended JIMWLK with final state restrictions

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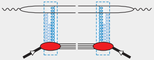
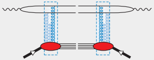
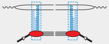
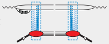
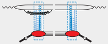
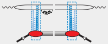
H_{JIMWLK}  → $\left[H_{\text{amp}} + H_{\text{fin}} + H_{\overline{\text{amp}}}\right]$ 

H_{amp}  $=$  $+$  $+$ 

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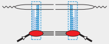
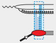
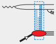
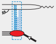
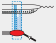
H_{fin}  $= \Big[$  $+$  $\Big] \boldsymbol{M} \Big[$  $+$  $\Big]$

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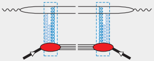
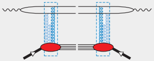
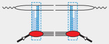
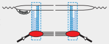
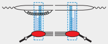
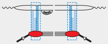
Extended JIMWLK with final state restrictions

H_{JIMWLK}  $\longrightarrow$ $\Big[H_{\text{amp}} + H_{\text{fin}} + H_{\overline{\text{amp}}}\Big]$ 

H_{amp}  $=$  $+$  $+$ 

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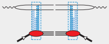
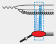
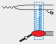
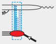
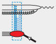
H_{fin}  $= \Big[$  $+$  $\Big]\, \boldsymbol{M} \Big[$  $+$  $\Big]$

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- modified kernel in H_{fin}:

$$\mathcal{K}_{\boldsymbol{xzy}} = \frac{\boldsymbol{x}-\boldsymbol{z}}{(\boldsymbol{x}-\boldsymbol{z})^2} \cdot \frac{\boldsymbol{z}-\boldsymbol{y}}{(\boldsymbol{z}-\boldsymbol{y})^2} \longrightarrow \frac{\boldsymbol{x}-\boldsymbol{z}}{(\boldsymbol{x}-\boldsymbol{z})^2} \cdot \boldsymbol{M}_{\boldsymbol{z}\boldsymbol{z}';Y} \cdot \frac{\boldsymbol{z}'-\boldsymbol{y}}{(\boldsymbol{z}'-\boldsymbol{y})^2}$$

Extended JIMWLK with final state restrictions

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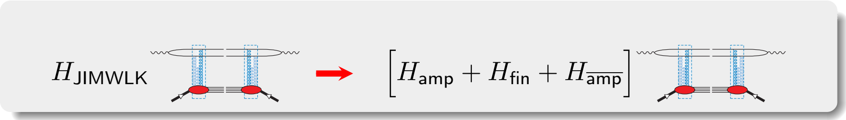

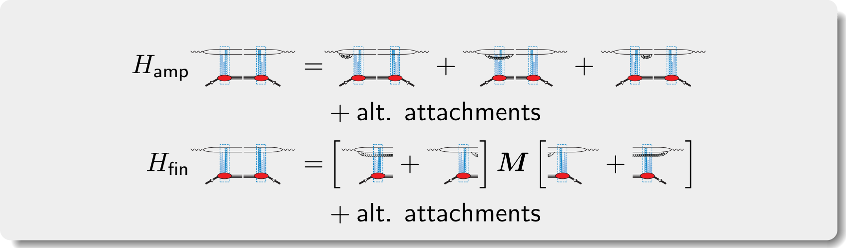

- Target side gap: $\boldsymbol{M}_{\boldsymbol{z}\boldsymbol{z}';Y} = \theta(Y - Y_{\text{gap}})\delta^{(2)}(\boldsymbol{z} - \boldsymbol{z}')$

[Kovchegov, Levin; Hentschinski, Weigert, Schäfer]

- finite momentum transfer $\boldsymbol{q}$: $\boldsymbol{M}_{\boldsymbol{z}\boldsymbol{z}';Y} = \delta^{(2)}(\boldsymbol{z} - \boldsymbol{z}' - \boldsymbol{v})$
- more observables accessible

[Kovner, Lublinsky, Weigert hep-ph/0608258]

Outline

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NLO-corrections and evolution speed

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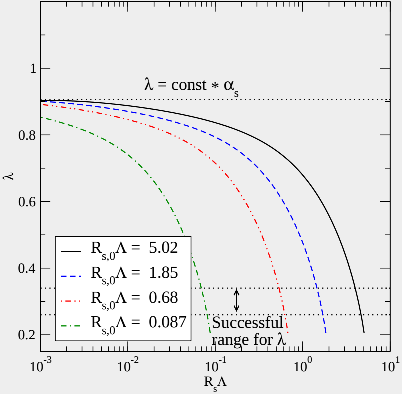
- speed @ fixed coupling:

$$\lambda(Y) := \frac{d}{dY} \ln Q_s^2(Y) \longrightarrow const.$$



NLO-corrections and evolution speed

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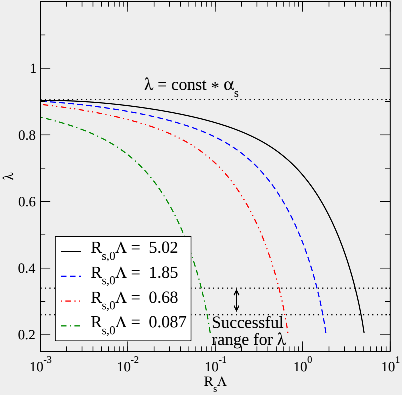
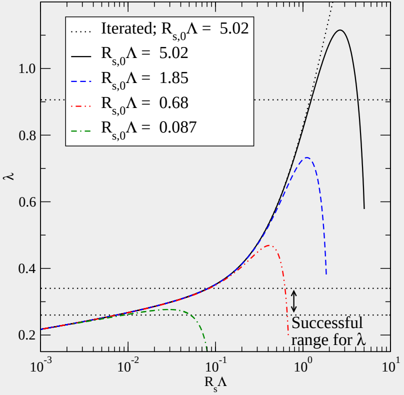


- speed @ running coupling

$$\lambda(Y) := \frac{d}{dY} \ln Q_s^2(Y)$$



NLO-corrections and evolution speed

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- include energy conservation

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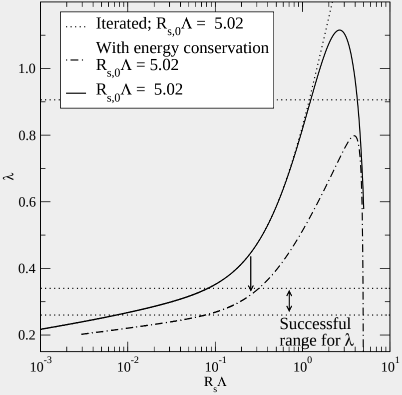

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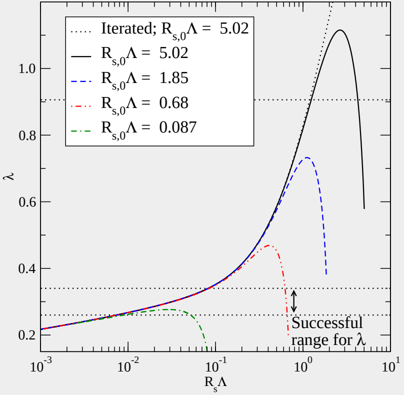

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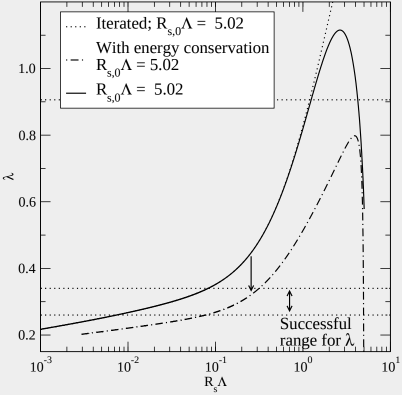
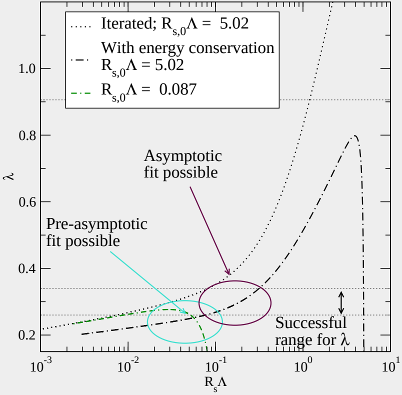


- fit strategies

$$\lambda(Y) := \frac{d}{dY} \ln Q_s^2(Y)$$



Consistent fits to HERA cross sections:

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Kuokkanan, Rummukainen, Weigert, in prep.

Total cross section

- reproduce geometric scaling:
  → scaling region of evolution

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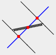
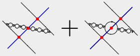
$$\alpha_s(\mu^2) \rightarrow \frac{\alpha_s\left(\boldsymbol{q}^2\right)\,\alpha_s\left(\boldsymbol{q}'^2\right)}{\alpha_s\left(\tilde{Q}^2\right)}$$





  [0609087 (Gardi, Kuokkanen, Rummukainen, Weigert); 0609090,0612071 (Kovchegov, Weigert)]

  2. remaining NLO:
     collinear corrections
     ("energy conservation")

  [hep-ph/0411242 (Gotsman, Levin, Maor, Naftali)]

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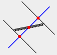

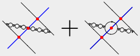

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plots

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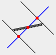

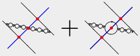

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Diffractive cross sections

- rapidity gap events

$$Y = \ln(1/x) = \underbrace{\ln(1/x_{\mathbb{P}})}_{\text{gap}} + \underbrace{\ln(1/\beta)}_{\text{proj. fragm.}}$$

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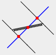

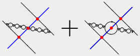

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Consistent fits to HERA cross sections:

Kuokkanan, Rummukainen, Weigert, in prep.

Total cross section

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▸ plots

Diffractive cross sections

- rapidity gap events

$$Y = \ln(1/x) = \underbrace{\ln(1/x_{\mathbb{P}})}_{\text{gap}} + \underbrace{\ln(1/\beta)}_{\text{proj. fragm.}}$$

- reuse parameters from total cross section
- NO NLO evolution equations:
  - ➡ follow GB-W
    Golec-Biernat, Wüsthoff [help-ph/9903358]
  1. evolution used for gaps only
  2. projectile frag. via

$$L_{q\bar{q}} + T_{q\bar{q}} + T_{q\bar{q}g}$$

β not too small, not yet NLO

➡ recycle evo for total x-section

Consistent fits to HERA cross sections:

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Kuokkanan, Rummukainen, Weigert, in prep.

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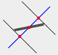

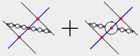

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  ("energy conservation")
  [hep-ph/0411242 (Gotsman, Levin, Maor, Naftali)]
- $Q^2 < 45$: $\chi^2/160 \sim .8$

plots

Diffractive cross sections

- rapidity gap events

$$Y = \ln(1/x) = \underbrace{\ln(1/x_{\mathbb{P}})}_{\text{gap}} + \underbrace{\ln(1/\beta)}_{\text{proj. fragm.}}$$

- reuse parameters from total cross section
- NO NLO evolution equations:
  → follow GB-W
  Golec-Biernat, Wüsthoff [help-ph/9903358]
  1. evolution used for gaps only
  2. projectile frag. via

  $$L_{q\bar{q}} + T_{q\bar{q}} + T_{q\bar{q}g}$$

  β not too small, not yet NLO
  → recycle evo for total x-section
- diffractive slope, α_s for $q\bar{q}g$

Consistent fits to HERA cross sections:

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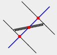
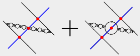
Kuokkanan, Rummukainen, Weigert, in prep.

Total cross section

- reproduce geometric scaling:
  → scaling region of evolution
- NLO corrections: evol. speed ↓
  1. running coupling

$$\alpha_s(\mu^2) \rightarrow \frac{\alpha_s\left(\boldsymbol{q}^2\right)\,\alpha_s\left(\boldsymbol{q}'^2\right)}{\alpha_s\left(\tilde{Q}^2\right)}$$





[0609087 (Gardi, Kuokkanen, Rummukainen, Weigert); 0609090,0612071 (Kovchegov, Weigert)]

  2. remaining NLO: collinear corrections (“energy conservation”)

[hep-ph/0411242 (Gotsman, Levin, Maor, Naftali)]

- $Q^2 < 45$: $\chi^2/160 \sim .8$

plots

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  β not too small, not yet NLO

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- diffractive slope, α_s for $q\bar{q}g$
- $\chi^2/326 \sim 1.5$

plots

Outline

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Gold plated observables: correlators

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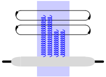
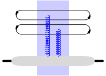
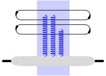
$$\frac{1}{N_c^2}\left[\langle \mathrm{tr}(U_{\boldsymbol{x}} U^\dagger_{\boldsymbol{y}}) \mathrm{tr}(U_{\boldsymbol{y}'} U^\dagger_{\boldsymbol{x}'})\rangle(Y) - \langle \mathrm{tr}(U_{\boldsymbol{x}} U^\dagger_{\boldsymbol{y}})\rangle(Y)\langle \mathrm{tr}(U_{\boldsymbol{y}'} U^\dagger_{\boldsymbol{x}'})\rangle(Y)\right]$$

Gold plated observables: correlators

$$\frac{1}{N_c^2}\left[\langle \mathrm{tr}(U_{\boldsymbol{x}}U^\dagger_{\boldsymbol{y}})\mathrm{tr}(U_{\boldsymbol{y}'}U^\dagger_{\boldsymbol{x}'})\rangle(Y) - \langle \mathrm{tr}(U_{\boldsymbol{x}}U^\dagger_{\boldsymbol{y}})\rangle(Y)\langle \mathrm{tr}(U_{\boldsymbol{y}'}U^\dagger_{\boldsymbol{x}'})\rangle(Y)\right]$$

- $\mathcal{O}(A^4)$: first contribution from 

 ,  $\xrightarrow{\mathrm{tr}(t^a)=0} 0$

beyond DGLAP

Gold plated observables: correlators

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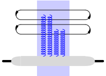
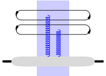
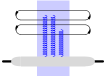
$$\frac{1}{N_c^2}\Big[\langle \text{tr}(U_{\boldsymbol{x}}U^\dagger_{\boldsymbol{y}})\text{tr}(U_{\boldsymbol{y}'}U^\dagger_{\boldsymbol{x}'})\rangle(Y) - \langle \text{tr}(U_{\boldsymbol{x}}U^\dagger_{\boldsymbol{y}})\rangle(Y)\langle \text{tr}(U_{\boldsymbol{y}'}U^\dagger_{\boldsymbol{x}'})\rangle(Y)\Big]$$

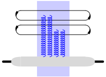
- $\mathcal{O}(A^4)$: first contribution from 

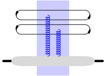
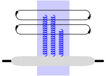
,  $\xrightarrow{\text{tr}(t^a)=0} 0$ beyond DGLAP

- $\mathcal{O}(\frac{1}{N_c^2})$: beyond BK

Gold plated observables: correlators

$$\frac{1}{N_c^2}\Big[\langle \mathrm{tr}(U_{\boldsymbol{x}}U_{\boldsymbol{y}}^\dagger)\mathrm{tr}(U_{\boldsymbol{y}'}U_{\boldsymbol{x}'}^\dagger)\rangle(Y) - \langle \mathrm{tr}(U_{\boldsymbol{x}}U_{\boldsymbol{y}}^\dagger)\rangle(Y)\langle \mathrm{tr}(U_{\boldsymbol{y}'}U_{\boldsymbol{x}'}^\dagger)\rangle(Y)\Big]$$

- $\mathcal{O}(A^4)$: first contribution from 

 ,  $\xrightarrow{\mathrm{tr}(t^a)=0} 0$ beyond DGLAP

- $\mathcal{O}(\frac{1}{N_c^2})$: beyond BK
- present in full JIMWLK

Gold plated observables: correlators

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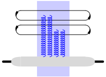
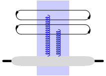
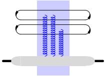
$$\frac{1}{N_c^2}\Big[\langle \text{tr}(U_{\boldsymbol{x}}U^\dagger_{\boldsymbol{y}})\text{tr}(U_{\boldsymbol{y}'}U^\dagger_{\boldsymbol{x}'})\rangle(Y) - \langle \text{tr}(U_{\boldsymbol{x}}U^\dagger_{\boldsymbol{y}})\rangle(Y)\langle \text{tr}(U_{\boldsymbol{y}'}U^\dagger_{\boldsymbol{x}'})\rangle(Y)\Big]$$

- $\mathcal{O}(A^4)$: first contribution from 

,  $\xrightarrow{\text{tr}(t^a)=0} 0$ beyond DGLAP

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Questions:

- How to measure?
- How to calculate at NLO? (in a practicable fashion)

Gold plated observables: what to measure

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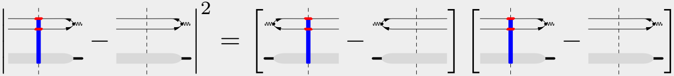
- Total cross section:



Gold plated observables: what to measure

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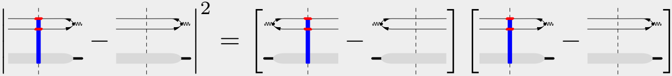
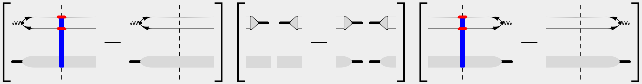


- "Golden" restrictions of final state:



Gold plated observables: what to measure

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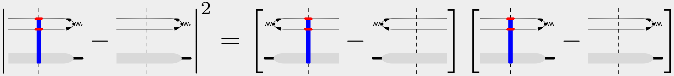

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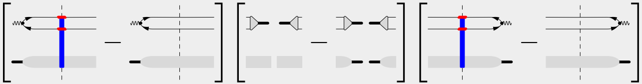

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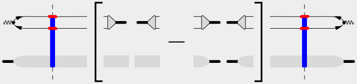

Gold plated observables: what to measure

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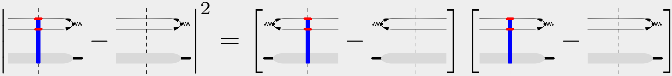

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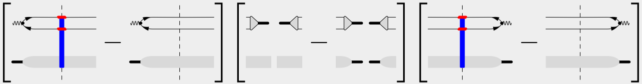

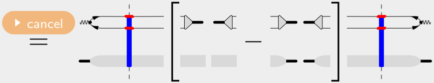

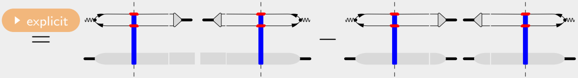

Gold plated observables: how to calculate?

- JIMWLK @ NLO — impractical

Gold plated observables: how to calculate?

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- BK — insufficicent

Gold plated observables: how to calculate?

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- JIMWLK @ NLO — impractical
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- Gaussian truncation:

$$\langle \ldots \rangle(Y) = \exp\left\{ -\frac{1}{2} \int^{Y} dY' \int d^2x\, d^2y\; G_{Y',\boldsymbol{xy}} \; \frac{\delta}{\delta A^{a+}_{\boldsymbol{x},Y'}} \frac{\delta}{\delta A^{a+}_{\boldsymbol{y},Y'}} \right\} \ldots$$

$$\mathcal{G}_{Y,\boldsymbol{xy}} := \int^{Y} dY' \Big(G_{Y',\boldsymbol{xy}} - \frac{1}{2} \left(G_{Y',\boldsymbol{xx}} + G_{Y',\boldsymbol{yy}} \right) \Big)$$

Gold plated observables: how to calculate?

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$$\langle \overset{\mathcal{R}}{\mathrm{tr}}(\overset{\mathcal{R}}{U}_{\boldsymbol{x}} \overset{\mathcal{R}}{U}{}^{\dagger}_{\boldsymbol{y}}) \rangle(Y) = d_{\mathcal{R}}\; e^{-C_{\mathcal{R}} \mathcal{G}_{Y,\boldsymbol{xy}}}$$

$$\langle [\tilde{U}_{\boldsymbol{z}}]^{ab} \overset{\mathcal{R}}{\mathrm{tr}}(\overset{\mathcal{R}}{t}{}^{a} \overset{\mathcal{R}}{U}_{\boldsymbol{x}} \overset{\mathcal{R}}{t}{}^{b} \overset{\mathcal{R}}{U}{}^{\dagger}_{\boldsymbol{y}}) \rangle(Y) = C_{\mathcal{R}} d_{\mathcal{R}}\; e^{-\frac{N_c}{2}(\mathcal{G}_{Y,\boldsymbol{xz}} + \mathcal{G}_{Y,\boldsymbol{zy}} - \mathcal{G}_{Y,\boldsymbol{xy}}) - C_{\mathcal{R}} \mathcal{G}_{Y,\boldsymbol{xy}}}$$

$$\mathcal{G}_{Y,\boldsymbol{xy}} := \int^{Y} dY' \Big(G_{Y',\boldsymbol{xy}} - \frac{1}{2}(G_{Y',\boldsymbol{xx}} + G_{Y',\boldsymbol{yy}}) \Big)$$

Gold plated observables: how to calculate?

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- $$\frac{d}{dY}\mathcal{G}_{Y,\boldsymbol{xy}} = \frac{\alpha_s}{\pi^2} \int d^2z\, \mathcal{K}_{\boldsymbol{xzy}} \left(1 - e^{-\frac{N_c}{2}\left(\mathcal{G}_{Y,\boldsymbol{xz}} + \mathcal{G}_{Y,\boldsymbol{yz}} - \mathcal{G}_{Y,\boldsymbol{xy}}\right)} \right)$$

$$\mathcal{G}_{Y,\boldsymbol{xy}} := \int^{Y} dY' \left(G_{Y',\boldsymbol{xy}} - \frac{1}{2}\left(G_{Y',\boldsymbol{xx}} + G_{Y',\boldsymbol{yy}}\right) \right)$$

Outline

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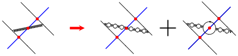
Summary and outlook

- NLO corrections absolutely essential

1. $\alpha_s(\mu^2) \longrightarrow \dfrac{\alpha_s\left(\boldsymbol{q}^2\right)\ \alpha_s\left(\boldsymbol{q}'^2\right)}{\alpha_s\left(\tilde{Q}^2\right)}$ 

   triumvirate coupling

2. new channels (only modelled here: energy conservation corr.)

Summary and outlook

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     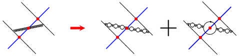

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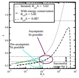

- NLO contributions reduce evolution speed

Summary and outlook

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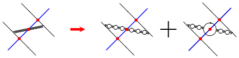
- NLO corrections absolutely essential
  1. $\alpha_s(\mu^2) \longrightarrow \dfrac{\alpha_s\left(\boldsymbol{q}^2\right)\,\alpha_s\left(\boldsymbol{q}'^2\right)}{\alpha_s\left(\tilde{Q}^2\right)}$

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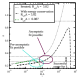
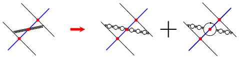
  

- Total cross section and rapidity gaps:
  1. HERA data allow fits in JIMWLK asymptotic range
  2. HERA data do not constrain correlator shapes strongly

Summary and outlook

- NLO corrections absolutely essential

  1. $\alpha_s(\mu^2) \longrightarrow \dfrac{\alpha_s\left(\boldsymbol{q}^2\right)\,\alpha_s\left(\boldsymbol{q}'^2\right)}{\alpha_s\left(\tilde{Q}^2\right)}$

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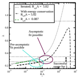

- Total cross section and rapidity gaps:
  1. HERA data allow fits in JIMWLK asymptotic range
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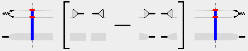
- New "gold plated" observables



  1. beyond DGALP & BK
  2. to calculate: Gaussian truncation

Outline

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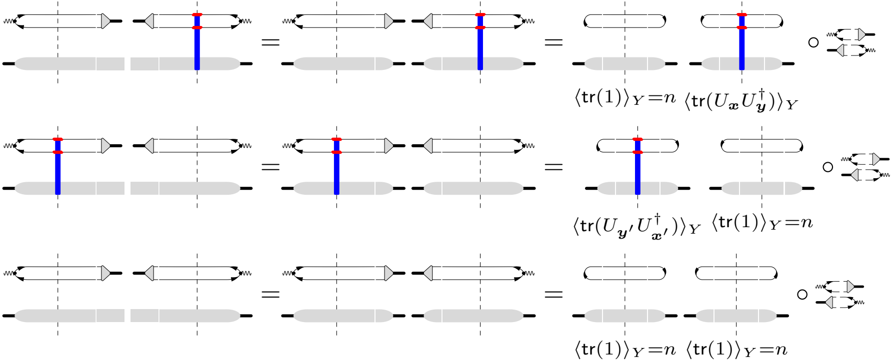
Tagged vs untagged targets: cancellations



observables

Tagged- vs untagged cross sections:

$$4\pi\frac{d\sigma_{T,L}}{dQ^2dt} = \int_0^1 d\alpha d\alpha' \int d^2b d^2b' d^2r d^2r' e^{-i\boldsymbol{l}\cdot[\boldsymbol{b}-\boldsymbol{b}'+(\alpha-\frac{1}{2})\boldsymbol{r}-(\alpha'-\frac{1}{2})\boldsymbol{r}']}$$
$$\times \Psi^*_{T,L}(\alpha',\boldsymbol{r}',Q^2)\Psi_{T,L}(\alpha,\boldsymbol{r},Q^2)$$
$$\times \left[\langle \mathrm{tr}(U_{\boldsymbol{y}'}U^\dagger_{\boldsymbol{x}'})\mathrm{tr}(U_{\boldsymbol{x}}U^\dagger_{\boldsymbol{y}})\rangle_Y - \langle \mathrm{tr}(U_{\boldsymbol{y}'}U^\dagger_{\boldsymbol{x}'})\rangle_Y \langle \mathrm{tr}(U_{\boldsymbol{x}}U^\dagger_{\boldsymbol{y}})\rangle_Y\right]$$

observables

Outline

HERA total cross sections

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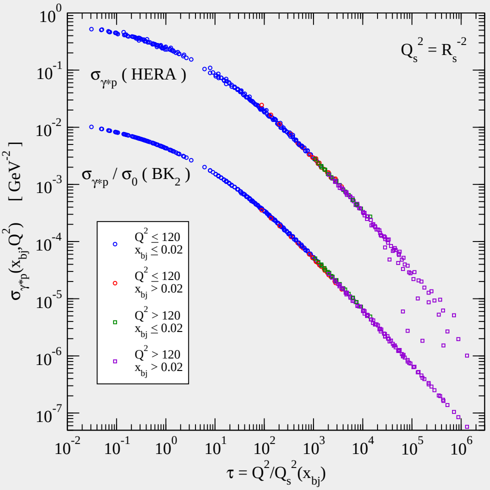
Kuokkanan, Rummukainen, Weigert, in prep.

scaling fit strategy

- Reproduce total cross section & scaling:



HERA total cross sections

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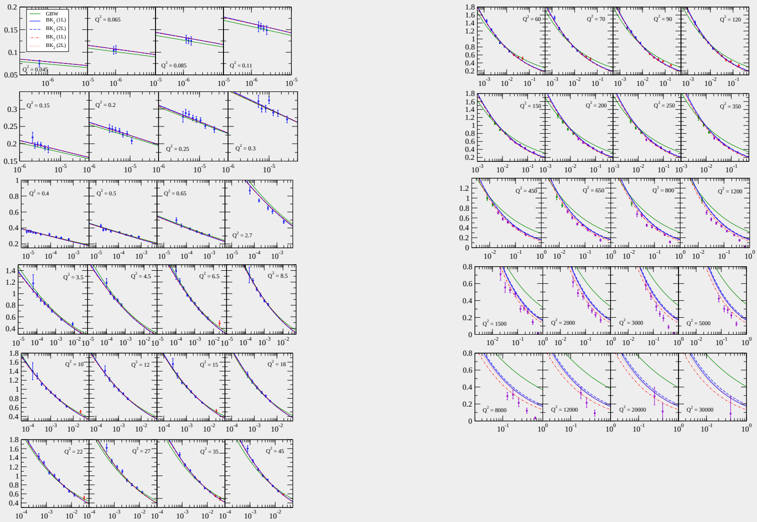
Kuokkanan, Rummukainen, Weigert, in prep.

scaling fit strategy



HERA diffractive cross sections

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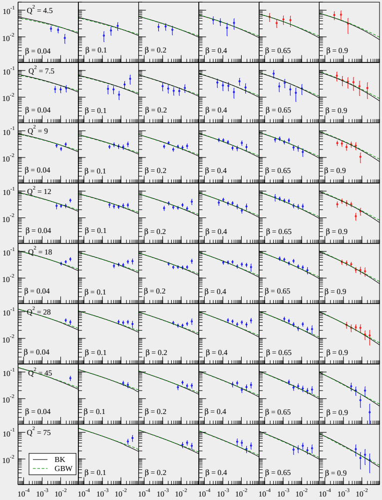
Kuokkanan, Rummukainen, Weigert, in prep.

fit strategy

- $x_{\mathbb{P}} F_2^D(Q^2, \beta, x_{\mathbb{P}})$ examples (H1 data, hep-ph/9708016):



HERA diffractive cross sections

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Kuokkanan, Rummukainen, Weigert, in prep.

fit strategy

- cross section components

$$L_{q\bar{q}} + T_{q\bar{q}} + T_{q\bar{q}g}$$