Diffractive production of $\chi_c(0,1,2)$ mesons at LHC, Tevatron and RHIC

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Diffractive Higgs/meson production: motivation

Search for Higgs – primary task for LHC.

Diffractive production of Higgs – an alternative to inclusive production (background reduction).

QCD mechanism proposed by Kaidalov, Khoze, Martin and Ryskin (ref. as KKMR approach).

AS A SPIN-PARITY ANALYSER
(ref. Valery Khoze's talk)

V.A. Khoze, A.D. Martin and M.G. Ryskin, Phys. Lett. **B401** (1997) 330.
V.A. Khoze, A.D. Martin and M.G. Ryskin, Eur. Phys. J. **C23** (2002) 311.
A.B. Kaidalov, V.A. Khoze, A.D. Martin and M.G. Ryskin, Eur. Phys. J. C **31**, 387 (2003) [arXiv:hep-ph/0307064].
A.B. Kaidalov, V.A. Khoze, A.D. Martin and M.G. Ryskin, Eur. Phys. J. **C33** (2004) 261.

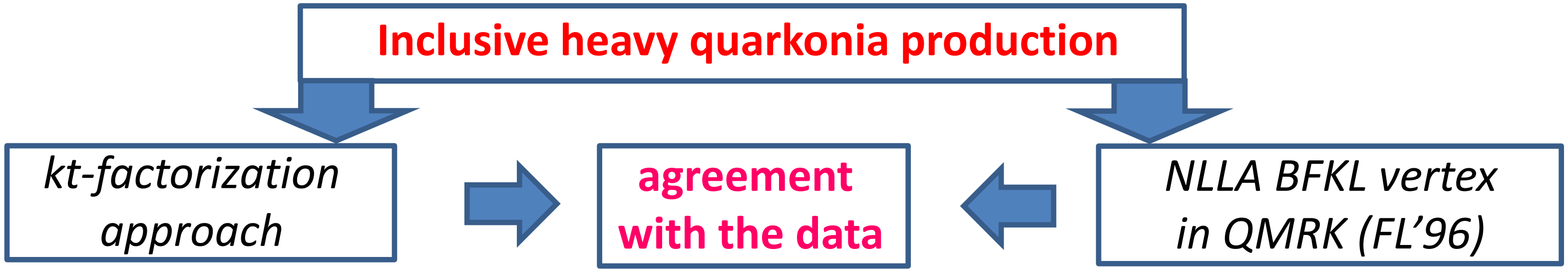
Still not possible to study Higgs at present.
Replace Higgs by a meson (scalar, pseudoscalar, vector, tensor, etc).

Diffractive χ_c production measured recently by CDF collaboration at Tevatron (Ref. James Pinfold's talk).

T. Aaltonen et al, Phys.Rev.Lett.102:242001,2009.

It is interesting to test KKMR approach for diffractive light mesons/heavy quarkonia production at high energies – a good probe of nonperturbative dynamics of partons described by UGDFs and related factorisation concepts.

Diffractive heavy quarkonia production: basic ideas



Inclusive heavy quarkonia production

kt-factorization approach

agreement with the data

NLLA BFKL vertex in QMRK (FL'96)

Based on inclusive production by:

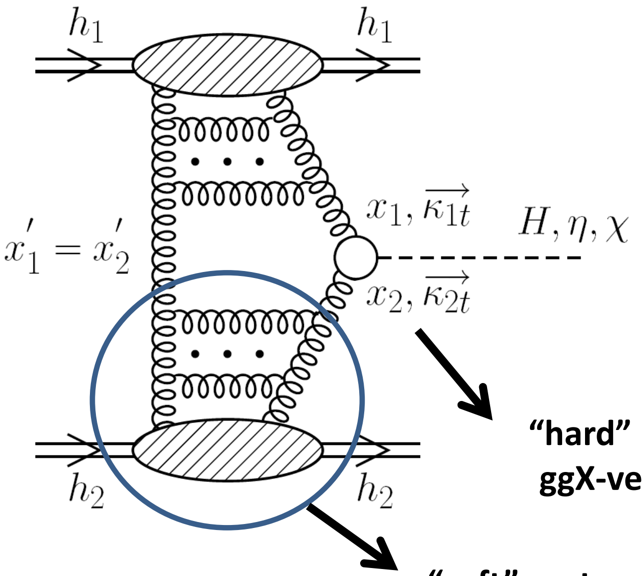
P. Hagler, R. Kirshner, A. Schafer, L. Szymanowski, O. Teryaev, ‘00, ’01
A. Lipatov, V. Saleev, N. Zotov, ‘01, ‘03

Why don't we apply the same ideas for exclusive production processes?

Our goals:

- to apply KKMR QCD mechanism to heavy quarkonia production
- to calculate the off-shell production matrix element
- to explore related uncertainties as indirect check of QCD factorisation principles
- to probe nonperturbative gluon dynamics at small qt by using different models for UGDFs

The QCD mechanism: amplitude



“hard” part:
ggX-vertex

“soft” part:
skewed UGDF

diffractive amplitude

$$\mathcal{M}_{J,\lambda} = const \cdot \delta_{c_1 c_2} \Im \int d^2 q_{0,t} V^{c_1 c_2}_{J,\lambda}(q_1, q_2, P) \frac{f^{\rm off}_{g,1}(x_1, x_1', q^2_{0,t}, q^2_{1,t}, t_1) f^{\rm off}_{g,2}(x_2, x_2', q^2_{0,t}, q^2_{2,t}, t_2)}{q^2_{0,t}\, q^2_{1,t}\, q^2_{2,t}}$$

$f^{off}(\ldots)$ – off-diagonal gluon unintegrated distributions

production vertex

gauge invariance

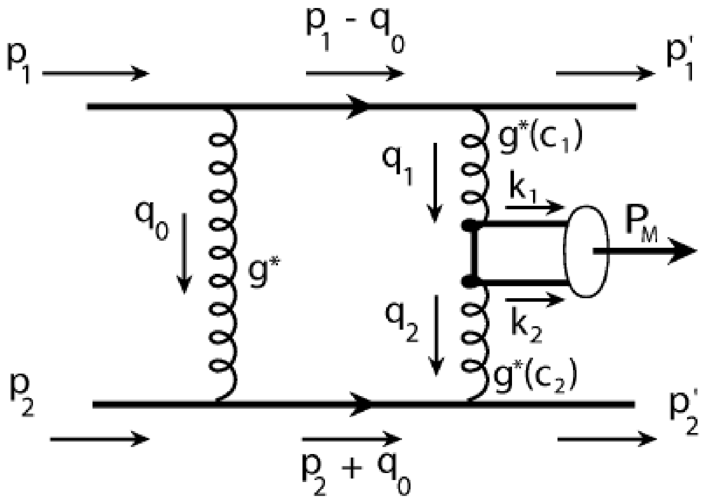
$$V^{c_1 c_2}_J(q_1, q_2) \;=\; n^+_\mu n^-_\nu V^{c_1 c_2}_{J,\mu\nu}(q_1, q_2) = \frac{4}{s} \frac{q^\nu_{1,t}}{x_1} \frac{q^\mu_{2,t}}{x_2} V^{c_1 c_2}_{J,\,\mu\nu}(q_1, q_2)$$

Gribov’s trick

unit light-cone vectors $n^+ = p_2/E_{cms}, \;\; n^- = p_1/E_{cms}$

The QCD mechanism: kinematics

$$\boxed{pp \to pp\chi_{cJ}}$$



$$q_{1,2} = x_{1,2} p_{1,2} + q_{1/2,t}, \quad x'_{1,2} \sim \frac{q_{0,t}}{\sqrt{s}} \ll x_{1,2}$$

- *We goes beyond the forward limit*

$$\mathrm{p}'^{\,2}_{1/2,\mathrm{t}} \simeq -(1-\mathrm{x}_{1,2})\mathrm{t}_{1,2}, \quad |\mathrm{t}_{1,2}| \le 1\,\mathrm{GeV}^2$$

- *Original KKMR approach does not account for x' dependence, just the limit x'<<x; It is hard to do → kinematics of double diffraction does not predict the exact value of x'! (ref. [z,kt]-factorisation approach by Watt, Martin, Ryskin, never been applied to exclusive processes)*

In terms of the meson rapidity

$$x_{1,2} = \frac{M_\perp^2}{\sqrt{s}} \exp(\pm y)$$

- *We probe x' to be small enough w.r.t. x but finite setting up naively:*

$$\boxed{x' = \xi \frac{q_{0,t}}{\sqrt{s}}, \qquad 0.1 < \xi < 2}$$

Due to x'<<x we have: $\boxed{q_{0,t} \ll M_\perp, \quad \text{for} \quad \xi \sim 1}$

Thus, the reliability of KKMR approach is justified if:

1) The most contribution to the diffractive amplitude comes from nonperturbative q0t < 1 GeV!

2) The results change very slowly when ξ → 0! If it is strong, then unknown x' may produce extra theoretical uncertainties in the KKMR approach.

Production vertex: scalar charmonium

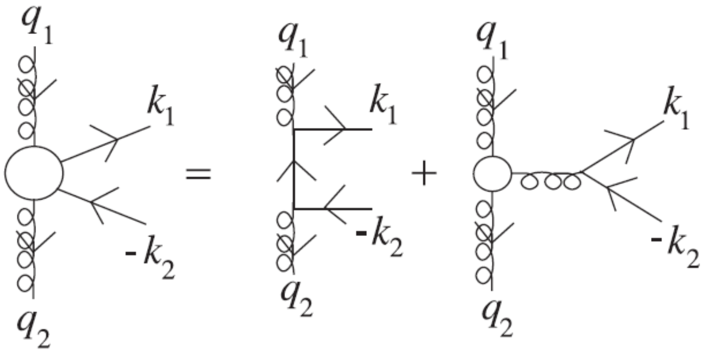
$gg \longrightarrow \chi_c$ ***vertex***

$$\boxed{V_J^{c_1 c_2} = \mathcal{P}(q\bar{q} \to \chi_{cJ}) \bullet \Psi_{ik}^{c_1 c_2}(k_1, k_2)}$$

***pNRQCD projector* to color singlet bound state**

$$\boxed{gg \longrightarrow q\bar{q}}$$

Fadin, Lipatov '96
(for mq=0)



when projecting to colour singlet state

$$\Psi(c_1, c_2; i, k; k_1, k_2) = -g^2 (t_{ij}^{c_1} t_{jk}^{c_2} b(k_1, k_2) - t_{kj}^{c_2} t_{ji}^{c_1} \bar{b}(k_2, k_1)), \quad \alpha_s = \frac{g^2}{4\pi}$$

$$\boxed{\text{Feynman rules of QMRK}}$$

$$b(k_1, k_2) = \gamma^- \frac{\hat{q}_1 - \hat{k}_1 - m}{(q_1 - k_1)^2 - m^2} \gamma^+ - \cancel{\frac{\gamma_\beta \Gamma^{+-\beta}(q_2, q_1)}{(k_1 + k_2)^2}}$$

$$\bar{b}(k_1, k_2) = \gamma^+ \frac{\hat{q}_1 - \hat{k}_1 + m}{(q_1 - k_1)^2 - m^2} \gamma^- - \cancel{\frac{\gamma_\beta \Gamma^{+-\beta}(q_2, q_1)}{(k_1 + k_2)^2}}$$

turn to the standard Feynman rules for the colour singlet state
(see, Kuhn et al Nucl.Phys'79 for $\gamma\gamma$-vertex)

$$\boxed{V_{J=0}^{c_1 c_2}(q_1, q_2) = 8ig^2 \frac{\delta^{c_1 c_2}}{M} \frac{\mathcal{R}'(0)}{\sqrt{\pi M N_c}} \frac{3M^2 (q_{1,t} q_{2,t}) + 2q_{1,t}^2 q_{2,t}^2 - (q_{1,t} q_{2,t})(q_{1,t}^2 + q_{2,t}^2)}{(M^2 - q_{1,t}^2 - q_{2,t}^2)^2}}$$

gluon virtualities are explicitly taken into account!

Production vertex: axial-vector charmonium

Bose-symmetric w.r.t. interchange of gluon polarisation vectors and transverse momenta

In the Lorentz-covariant form

$$V^{c_1c_2}_{J=1} = 2g^2\delta^{c_1c_2}\sqrt{\frac{6}{M\pi N_c}}\frac{\mathcal{R}'(0)}{M^2(q_1q_2)^2}\,\varepsilon_{\sigma\rho\alpha\beta}\epsilon^{\beta}(J_z)\Big[q^{\sigma}_{1,t}q^{\rho}_{2,t}(x_1p^{\alpha}_1 - x_2p^{\alpha}_2)(q^2_{1,t}+q^2_{2,t}) -$$

$$\frac{2}{s}\,p^{\sigma}_1p^{\rho}_2\left(q^{\alpha}_{1,t}(2q^2_{2,t}(q_1q_2) - (q_{1,t}q_{2,t})(q^2_{1,t}+q^2_{2,t})) - q^{\alpha}_{2,t}(2q^2_{1,t}(q_1q_2) - (q_{1,t}q_{2,t})(q^2_{1,t}+q^2_{2,t}))\right)\Big]$$

meson polarisation vector with definite helicity

$$\boxed{\lambda = 0, \pm 1}$$

$$\epsilon^{\beta}(P,\lambda) = (1-|\lambda|)n^{\beta}_3 - \frac{1}{\sqrt{2}}(\lambda n^{\beta}_1 + i|\lambda|n^{\beta}_2),\quad n^{\mu}_0 = \frac{P_{\mu}}{M},\quad n^{\mu}_{\alpha}n^{\nu}_{\beta}g_{\mu\nu} = g_{\alpha\beta},\quad \epsilon^{\mu}(\lambda)\epsilon^{*}_{\mu}(\lambda') = -\delta^{\lambda\lambda'}$$

vertex in the c.m.s. in coordinates with z-axis collinear to meson momentum P

$$V^{c_1c_2}_{J=1,\lambda} = -8g^2\delta^{c_1c_2}\sqrt{\frac{6}{M\pi N_c}}\frac{\mathcal{R}'(0)}{|\mathbf{P}_t|(M^2-q^2_{1,t}-q^2_{2,t})^2}\Bigg\{\frac{1}{\sqrt{2}}\Big[i|\lambda|(q^2_{1,t}-q^2_{2,t})(q_{1,t}q_{2,t})\mathrm{sign}(\sin\psi) +$$

$$\lambda(q^2_{1,t}+q^2_{2,t})\,|[\mathbf{q}_{1,t}\times\mathbf{q}_{2,t}]\times\mathbf{n}_1|\,\mathrm{sign}(Q^y_t)\,\mathrm{sign}(\cos\psi)\Big] +$$

$$(1-|\lambda|)(q^2_{1,t}+q^2_{2,t})\,|[\mathbf{q}_{1,t}\times\mathbf{q}_{2,t}]\times\mathbf{n}_3|\,\mathrm{sign}(Q^y_t)\,\mathrm{sign}(\sin\psi)\Bigg\}$$

simplest form!

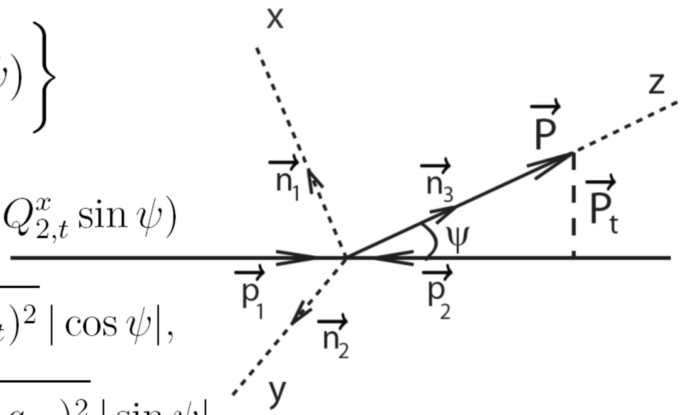
gluon transverse momenta in considered coordinates

$$q_{1,t} = (0,\ Q^x_{1,t}\cos\psi,\ Q^y_t,\ Q^x_{1,t}\sin\psi),\qquad q_{2,t} = (0,\ Q^x_{2,t}\cos\psi,\ -Q^y_t,\ Q^x_{2,t}\sin\psi)$$

Double vector products

$$|[\mathbf{q}_{1,t}\times\mathbf{q}_{2,t}]\times\mathbf{n}_1| = \sqrt{q^2_{1,t}q^2_{2,t} - (q_{1,t}q_{2,t})^2}\,|\cos\psi|,$$

$$|[\mathbf{q}_{1,t}\times\mathbf{q}_{2,t}]\times\mathbf{n}_3| = \frac{E}{M}\sqrt{q^2_{1,t}q^2_{2,t} - (q_{1,t}q_{2,t})^2}\,|\sin\psi|.$$



Production vertex: tensor charmonium

$$V^{c_1c_2}_{J=2} = 2ig^2\sqrt{\frac{3}{M\pi N_c}}\frac{\delta^{c_1c_2}\mathcal{R}'(0)\epsilon^{(\lambda)}_{\rho\sigma}}{MM_\perp^2(q_1q_2)^2}\Big[(q_{1,t}q_{2,t})(q_1^\sigma - q_2^\sigma)\Big\{P^\rho(q_{1,t}^2 - q_{2,t}^2) + (x_1p_1^\rho - x_2p_2^\rho)M^2 -$$
$$(q^\rho_{1,t} - q^\rho_{2,t})M^2\Big\} - 2(q_1q_2)\Big\{M^2(q^\rho_{1,t}q^\sigma_{2,t} + q^\sigma_{1,t}q^\rho_{2,t}) - q^2_{1,t}(q^\rho_{1,t}q^\sigma_{2,t} + q^\sigma_{2,t}q^\rho_{2,t}) -$$
$$q^2_{2,t}(q^\sigma_{1,t}q^\rho_{2,t} + q^\sigma_{1,t}q^\rho_{1,t}) + (x_1p_1^\sigma - x_2p_2^\sigma)(q^2_{1,t}q^\rho_{2,t} - q^2_{2,t}q^\rho_{1,t}) + (q_{1,t}q_{2,t})(x_1p_1^\rho - x_2p_2^\rho)(q^\sigma_{1,t} - q^\sigma_{2,t}) -$$
$$2q^2_{1,t}x_1p_1^\rho q^\sigma_{2,t} - 2q^2_{2,t}x_2p_2^\rho q^\sigma_{1,t} + 2(q_{1,t}q_{2,t})(x_1p_1^\sigma q^\rho_{2,t} + x_2p_2^\sigma q^\rho_{1,t}) + \frac{M_\perp^2}{s}(q_{1,t}q_{2,t})(p_1^\rho p_2^\sigma + p_2^\rho p_1^\sigma)\Big\}$$

meson polarisation tensor with definite helicity λ

$$\boxed{\lambda = 0, \pm 1, \pm 2}$$

$$\begin{aligned}\epsilon_{\mu\nu}(\lambda) \;&=\; \frac{\sqrt{6}}{12}(2-|\lambda|)(1-|\lambda|)\left[g_{\mu\nu} - \frac{P_\mu P_\nu}{M^2}\right] + \frac{\sqrt{6}}{4}(2-|\lambda|)(1-|\lambda|)n_3^\mu n_3^\nu + \\ &+\frac{1}{4}\lambda(1-|\lambda|)[n_1^\mu n_1^\nu - n_2^\mu n_2^\nu] + \frac{1}{4}i|\lambda|(1-|\lambda|)[n_1^\mu n_2^\nu + n_2^\mu n_1^\nu] + \\ &+\frac{1}{2}\lambda(2-|\lambda|)[n_1^\mu n_3^\nu + n_3^\mu n_1^\nu] + \frac{1}{2}i|\lambda|(2-|\lambda|)[n_2^\mu n_3^\nu + n_3^\mu n_2^\nu]\end{aligned}$$

finally, in the same coordinates as for axial-vector case

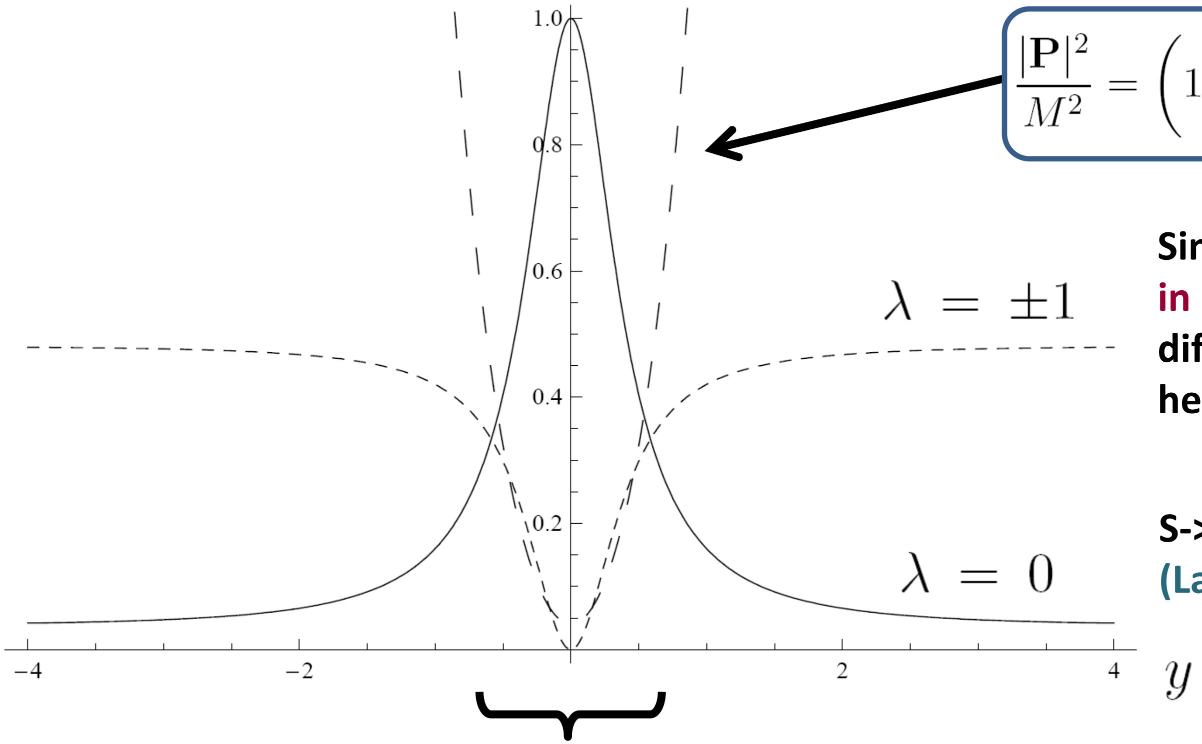
$$V^{c_1c_2}_{J=2,\lambda} = 2ig^2\delta^{c_1c_2}\sqrt{\frac{1}{3M\pi N_c}}\frac{\mathcal{R}'(0)}{M|\mathbf{P}_t|^2(M^2 - q^2_{1,t} - q^2_{2,t})^2}\times$$
$$\Big[6M^2\,i|\lambda|(q^2_{1,t} - q^2_{2,t})\,\mathrm{sign}(Q_t^y)\Big\{|[\mathbf{q}_{1,t}\times\mathbf{q}_{2,t}]\times\mathbf{n}_1|\,(1-|\lambda|)\,\mathrm{sign}(\sin\psi)\,\mathrm{sign}(\cos\psi) +$$
$$2|[\mathbf{q}_{1,t}\times\mathbf{q}_{2,t}]\times\mathbf{n}_3|\,(2-|\lambda|)\Big\} - \left[2q^2_{1,t}q^2_{2,t} + (q^2_{1,t} + q^2_{2,t})(q_{1,t}q_{2,t})\right]\Big\{3M^2(\cos^2\psi + 1)\lambda(1-|\lambda|) +$$
$$6ME\sin(2\psi)\,\lambda(2-|\lambda|)\,\mathrm{sign}(\sin\psi)\,\mathrm{sign}(\cos\psi) + \sqrt{6}\,(M^2 + 2E^2)\sin^2\psi\,(1-|\lambda|)(2-|\lambda|)\Big\}\Big]$$

Properties of helicity amplitudes: maximal helicity enhancement

Helicity amplitudes squared as functions of meson rapidity for $\phi=\pi/2$ (angle between gluon qt's)

$$|V|^2_{\lambda=0} = S\frac{|\mathbf{P}_t|^2(\cosh y+1)}{M^2(\cosh y-1)+|\mathbf{P}_t|^2(\cosh y+1)}, \quad |V|^2_{\lambda=\pm1} = \frac{S}{2}\frac{M^2(\cosh y-1)}{M^2(\cosh y-1)+|\mathbf{P}_t|^2(\cosh y+1)}$$

Kinematical "maximal helicity enhancement" (similar effect observed by WA102 for f1(1285), f1(1420) –production; initially predicted by Boreskov'69 and revived in diffraction in KKMR'03)



$$\frac{|\mathbf{P}|^2}{M^2} = \left(1+\frac{|\mathbf{P}_t|^2}{M^2}\right)\cosh^2 y - 1$$

Similar peaked structure observed in rapidity distributions of the diffractive cross section for different helicities

S->0 in the limit of on-shell gluons (Landau-Yang theorem)

Nonrelativistic (heavy) meson is dominated by $\lambda=0$ contribution.
relativistic (almost massless) meson → by maximal λ contribution.

KMR UGDF: role of nonperturbative transverse momenta

two gluons are replaced by one "effective" gluon with Qt=min(q0t,q1/2t) and x:

$$f_g^{KMR}(x,x',Q_t^2,\mu^2) = R_g \frac{\partial}{\partial \ln Q_t^2}\left[\sqrt{T(Q_t^2,\mu^2)}\,xg(x,Q_t^2)\right]$$

"hard" scale

$$\mu = M_\chi/2$$

depends on only one "effective" gluon transverse momentum

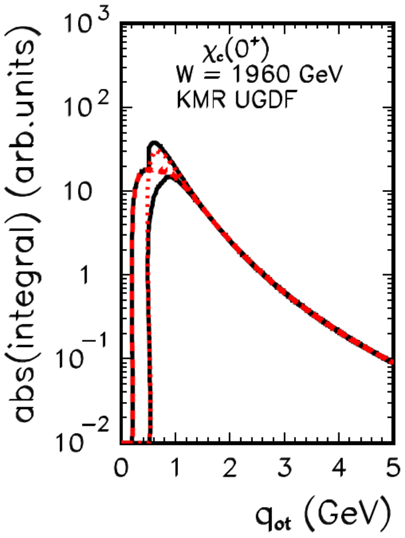
accounts for off-diagonal effect

Sudakov f.f. (ensures the purity of rapidity gaps)

Integrated density, defined at $Q_t > Q_0$

$$\mathcal{M}(y,t_1,t_2,\phi) = \int dq_{0,t}\; I(q_{0,t};y,t_1,t_2,\phi)$$

$$|I(q_{0t};t_1 = \text{-0.1 GeV}^2,\, t_2 = \text{-0.1 GeV}^2,\, \phi = \pi)|$$



main contribution to the amplitude comes from very small gluon transverse momenta q0t

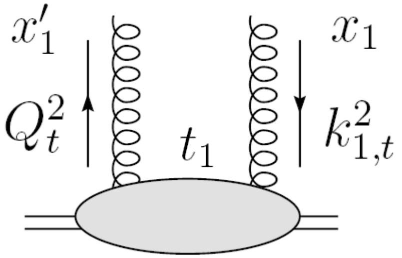
huge sensitivity to details in the nonperturbative domain $Q_t^2 < Q_0^2$

- ***scale effect***
- ***cut-off effect***
- ***Qt-prescription dependence***

KMR UGDF may not be reliable since it is not defined for $Q_t < Q_0$

Off-diagonal (skewed) UGDFs: general properties

Currently unknown; we model the skewedness effect using positivity constraints (Pire,Soffer,Teryaev'99) as



$$f^{\rm off}_{g,1}(x_1, x'_1, k^2_{0,t}, k^2_{1,t}, t_1) \; = \; \sqrt{f_g^{(1)}(x'_1, k^2_{0,t}, \mu_0^2) \cdot f_g^{(1)}(x_1, k^2_{1,t}, \mu^2)} \cdot F_1(t_1)$$

$$f^{\rm off}_{g,2}(x_2, x'_2, k^2_{0,t}, k^2_{2,t}, t_2) \; = \; \sqrt{f_g^{(2)}(x'_2, k^2_{0,t}, \mu_0^2) \cdot f_g^{(2)}(x_2, k^2_{2,t}, \mu^2)} \cdot F_1(t_2)$$

motivated by positivity of density matrix (saturation of Cauchy-Schwarz inequality)

t-dependence -> **isoscalar nucleon f.f.**

$$F_1(t_{1,2}) = \frac{4m_p^2 - 2.79\, t_{1,2}}{(4m_p^2 - t_{1,2})(1 - t_{1,2}/071)^2}$$

describe well t-dependence of the elastic pp-scattering at high energies (Donnachie,Landshoff PL'87)

***factorisation scale* choice – three basic options:**

$$\boxed{\begin{array}{lll} (1) \;\; \mu_0^2 = M^2, & \mu^2 = M^2, & \textit{(KKMR choice)} \\ (2) \;\; \mu_0^2 = Q_0^2, & \mu^2 = M^2, & \\ (3) \;\; \mu_0^2 = q_{0,t}^2 \; (+\text{freezing at } q_{0,t}^2 < Q_0^2), & \mu^2 = M^2 & \end{array}}$$

non-perturbative input
for QCD evolution: $Q_0^2 = 0.26 \;\, \mathrm{GeV}^2$

Gluck,Reya,Vogt '95, '98

kt-dependence: $k_t^2 \to 0 \qquad f(x, k_t^2) \to 0\,, \qquad \dfrac{f(x,k_t^2)}{k_t^2} = \mathcal{F}(x, k_t^2) \to const$

UGDF models

– ***Gaussian smearing*** **–>** ***simplest (nonperturbative) generalisation of collinear distributions***

$$\mathcal{F}_{naive}(x,k_t^2,\mu_F^2) = xg^{coll}(x,\mu_F^2)\cdot f_{Gauss}(k_t^2), \qquad f_{Gauss}(k_t^2) = \frac{1}{2\pi^2\sigma_0^2}e^{-k_t^2/2\sigma_0^2}$$

free parameter

with normalisation $\int \mathcal{F}_{naive}(x,k_t^2,\mu_F^2)\,dk_t^2 = xg^{coll}(x,\mu_F^2)$

factorisation scale

Owens '87; Wong et al '98; Zhang et al '02

– ***Golec-Biernat and Wustoff '99 (GBW) gluon saturation model*** **–>** ***describes well the dipole-nucleon cross section***

$$\alpha_s\mathcal{F}(x,k_t^2) = \frac{3\sigma_0}{4\pi^2}R_0^2(x)k_t^2e^{-R_0^2(x)k_t^2}, \qquad R_0(x) = \left(\frac{x}{x_0}\right)^{\lambda/2}\frac{1}{GeV}$$

with parameters adjusted from HERA data fits on F_2 $\sigma_0 = 29.12\,\mathrm{mb},\ x_0 = 0.41\cdot10^{-4},\ \lambda = 0.277$

– ***Kharzeev and Levin '01 (KL) gluon saturation model*** **–>** ***describes well the inclusive pion production at RHIC***

soft saturation scale

$$\mathcal{F}(x,k_t^2) = \begin{cases} f_0 & \text{if } k_t^2 < Q_s^2, \\ f_0\cdot\frac{Q_s^2(x)}{k_t^2} & \text{if } k_t^2 > Q_s^2. \end{cases} \qquad Q_s^2(x) = 1\,\mathrm{GeV}^2\cdot(x_0/x)^{\lambda}$$

adjusted from HERA data on F_2

UGDF models

– (linear) BFKL UGDF –> parameterization of numerical solution of the (linear) BFKL equation

x –> 0

$$-x\frac{\partial f(x,k_t^2)}{\partial x} = \frac{\alpha_s N_c}{\pi} k_t^2 \int_0^\infty \frac{dq_t^2}{q_t^2}\left[\frac{f(x,q_t^2)-f(x,k_t^2)}{|k_t^2-q_t^2|} + \frac{f(x,k_t^2)}{\sqrt{k_t^4+4q_t^4}}\right]$$

leading logarithmic (LLx) $\alpha_s \ln 1/x$ approximation only!

Parameterization by Askew, Kwiecinsky, Martin, Sutton PRD'94

$$f(x,k_t^2) = \frac{C}{x^\lambda}\left(\frac{k_t^2}{q_0^2}\right)^{1/2} \frac{\bar\phi_0}{\sqrt{2\pi\lambda''\ln(1/x)}} \exp\left[-\frac{\ln^2(k_t^2/\bar q^2)}{2r\lambda''\ln(1/x)}\right]$$

It leads to a very strong power growth of the gluon density with energy

$$\sim s^\lambda \qquad \lambda = 4\ln 2\alpha_s N_c/\pi$$

UGDF models

– (nonlinear) Kutak and Stasto '04 BFKL UGDF –> solution of the modified (nonlinear) BFKL equation

(linear) BFKL part in NLLx approximation within the unified BFKL-DGLAP framework (Kwiecinsky et al '97)

$$
\begin{aligned}
f(x,k^2) \; &= \; \tilde{f}^{(0)}(x,k^2) + \\
&\left\{ \begin{aligned}
&+ \frac{\alpha_s(k^2)N_c}{\pi} k^2 \int_x^1 \frac{dz}{z} \int_{k_0^2} \frac{dk'^2}{k'^2} \left\{ \frac{f(\frac{x}{z},k'^2)\,\Theta(\frac{k^2}{z}-k'^2) \; - \; f(\frac{x}{z},k^2)}{|k'^2-k^2|} + \frac{f(\frac{x}{z},k^2)}{|4k'^4+k^4|^{\frac{1}{2}}} \right\} \\
&\quad + \frac{\alpha_s(k^2)N_c}{\pi} \int_x^1 dz\, \bar{P}_{gg}(z) \int_{k_0^2}^{k^2} \frac{dk'^2}{k'^2} f(\frac{x}{z},k'^2) -
\end{aligned} \right. \\
&- \underbrace{\left(1-k^2\frac{d}{dk^2}\right)^2 \frac{k^2}{R^2} \int_x^1 \frac{dz}{z} \left[\int_{k^2}^{\infty} \frac{dk'^2}{k'^4} \alpha_s(k'^2) \ln\left(\frac{k'^2}{k^2}\right) f(z,k'^2) \right]^2}_{} \; .
\end{aligned}
$$

Nonlinear term from Balitsky-Kovchegov (BK) equation

Initial density $k_0^2 \; = \; 1\mathrm{GeV}^2$

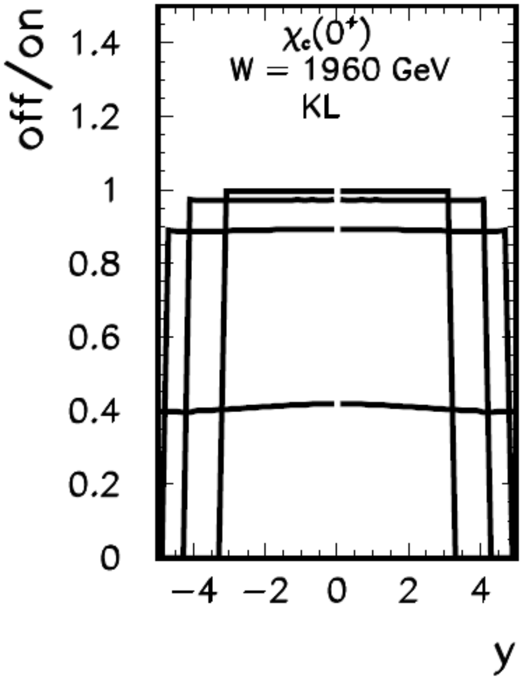
$$
\tilde{f}^{(0)}(x,k^2) = \frac{\alpha_S(k^2)}{2\pi} \int_x^1 dz P_{gg}(z) \frac{x}{z} g\left(\frac{x}{z},k_0^2\right)
$$

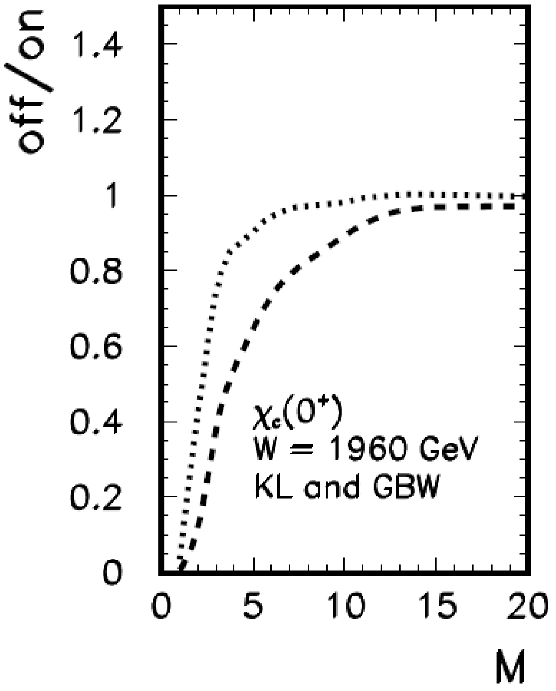
at small k<ko $f(x,k^2) = 4N(1-x)^{\rho} k^4$

The Kutak-Stasto model gives similar results for dipole-nucleon cross section as for GBW model

Off-shell effect

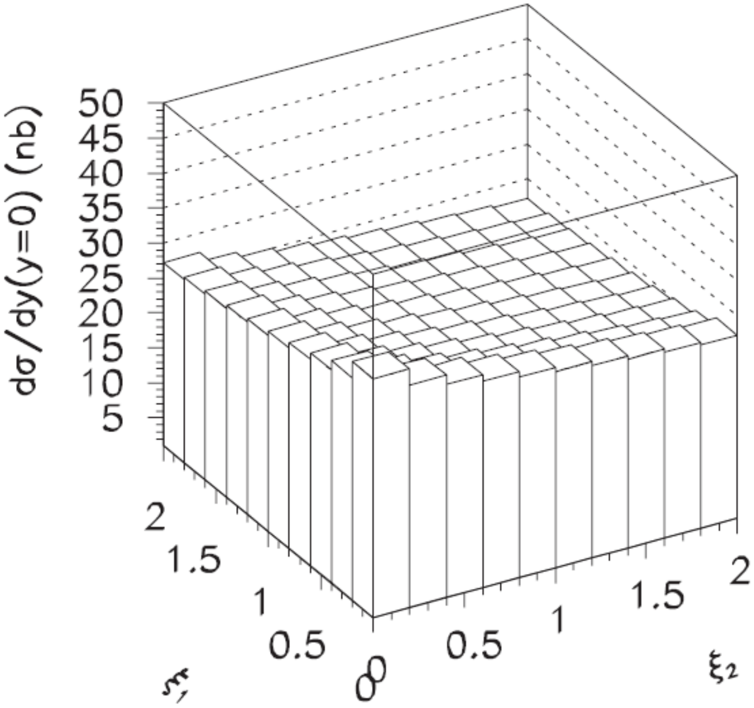
$$M = M_{\chi_c(0)},\ 5,\ 10,\ 20\ \mathrm{GeV}$$



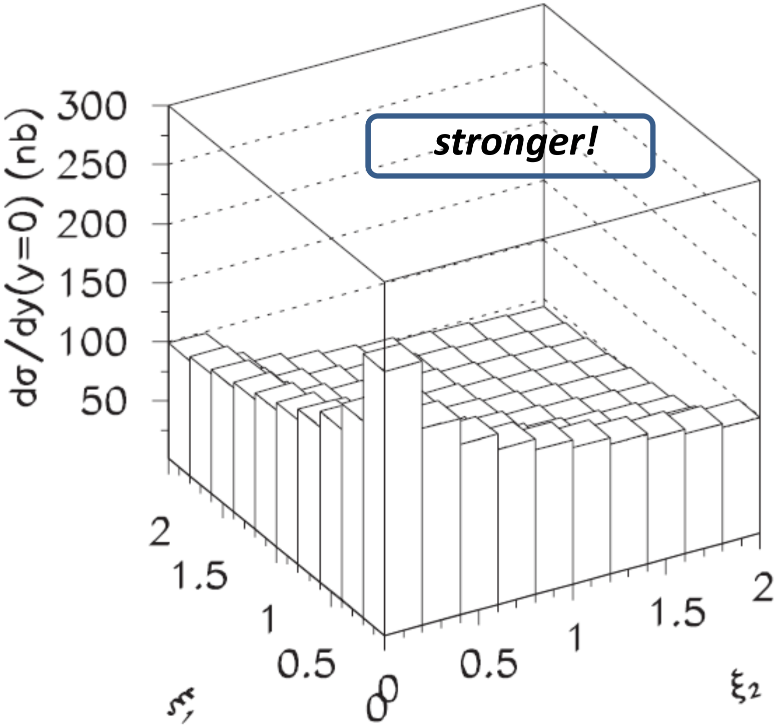


factor 2-5 of reduction in the cross section depending on UGDF

ξ-dependence $\chi_c\,(0+)$



KL UGDF

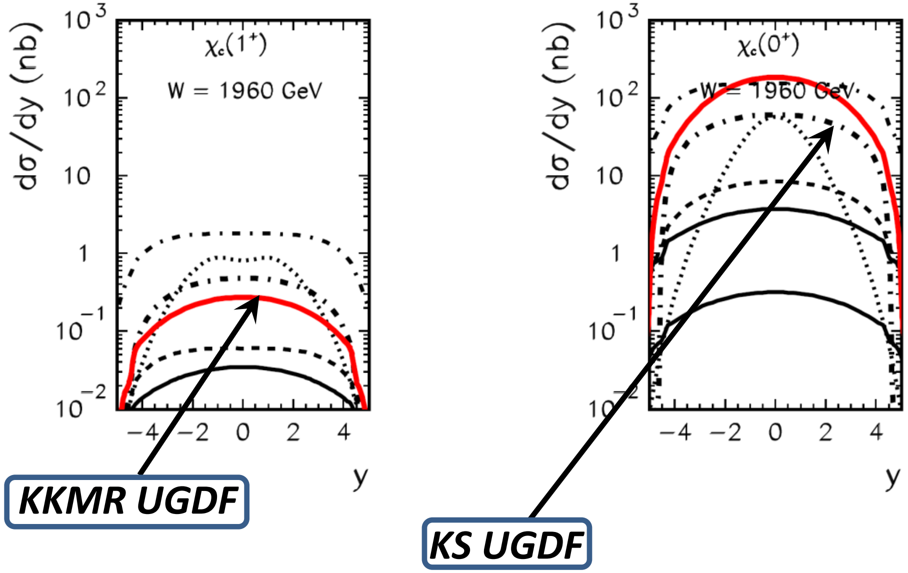


stronger!

Kutak-Stasto UGDF

Results for different UGDFs: scalar and axial-vector charmonia

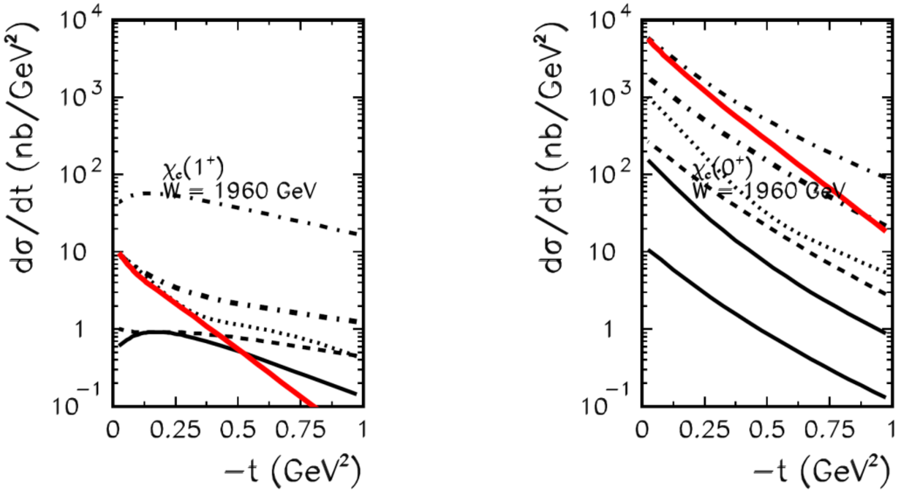
Rapidity distributions



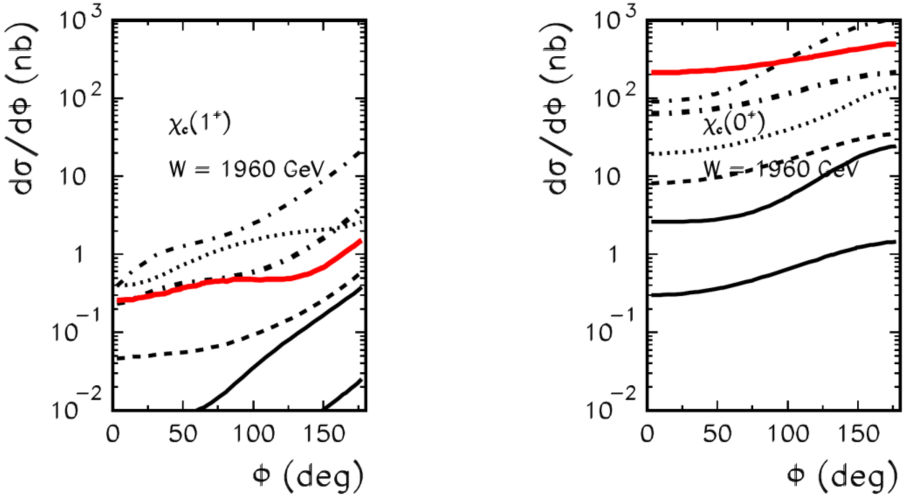
KKMR UGDF

KS UGDF

Momentum transfer distributions



Azimuthal angle correlations



Refs.

PR, Szczurek, Teryaev PRD'08

PR, Szczurek, Teryaev PL'09

Energy dependence of the total cross section (axial-vector case)

UGDF	RHIC	Tevatron	LHC
KL	0.05	0.5	1.7
GBW	0.04	4.2	73.1
BFKL	0.07	14.2	1064
Kutak-Stasto	0.05	3.0	44.8
Gauss, $\sigma_0 = 0.5$ GeV	0.007	0.2	2.5
Gauss, $\sigma_0 = 1.0$ GeV	0.0005	0.02	0.2
KKMR	0.02	1.7	35.2

Energy dependence *is corrected by nonlin BK term*

No absorbtive corrections are included!

Relative contributions of charmonium states

We take the absorbtion factors as known (ref. talks by Alan Martin and Valery Khoze)

$$\langle S^2_{\text{eff}}(\chi_c(0^+))\rangle \simeq 0.02,\ \langle S^2_{\text{eff}}(\chi_c(1^+))\rangle \simeq 0.05 \text{ and } \langle S^2_{\text{eff}}(\chi_c(2^+))\rangle \simeq 0.05.$$

$$\sigma(0^+ \to J/\psi\gamma) : \sigma(1^+ \to J/\psi\gamma) : \sigma(2^+ \to J/\psi\gamma) = \left\{ \begin{array}{l|l|l} \cline{2-2} 1 : 0.71 : & 4.64, & \qquad \text{KL} \\ 1 : 1.88 : & 12.5, & \qquad \text{GBW} \\ 1 : 0.57 : & 4.74, & \qquad \text{KS BFKL} \\ 1 : 0.12 : & 0.64, & \qquad \text{KMR} \\ \cline{2-2} \end{array} \right.$$

we predict dominance of the tensor charmonium state
for any UGDF except KMR one.

Measurement of 1+ and 2+ contributions separately would allow to put strict constraints on UGDF models

Expected signal at CDF

$\boxed{\mathrm{K}_{\mathrm{NLO}} = 1}$

UGDF	$\chi_c(0^+)$		$\chi_c(1^+)$		$\chi_c(2^+)$		ratio	expected signal	
	σ_{χ_c}	$\mathrm{BR}\,\sigma_{\chi_c}$	σ_{χ_c}	$\mathrm{BR}\,\sigma_{\chi_c}$	σ_{χ_c}	$\mathrm{BR}\,\sigma_{\chi_c}$	$\frac{\mathrm{BR}\,\sigma(\chi_c(2^+))}{\mathrm{BR}\,\sigma(\chi_c(0^+))}$	$\sum_{\chi_c} \langle S^2_{\mathrm{eff}} \rangle \cdot \mathrm{BR}\,\sigma_{\chi_c}$	
KL	55.2	0.7	0.5	0.2	6.7	1.3	1.9	0.09	$\xi = 1$
GBW	160	2	4.2	1.5	50.2	10.0	5.0	0.62	
KS BFKL	376	4.8	3.0	1.1	45.6	9.1	1.9	0.61	
KMR, $R_g = 1$	978	12.5	1.7	0.6	16.4	3.2	0.3	0.44	

$R_g = 1.4$

$\mathrm{K}_{\mathrm{NLO}} = 1.5$ ⇨

$$\boxed{\mathrm{KMR} \quad \left.\frac{d\sigma}{dy}\right|_{y=0} (pp \to pp(J/\psi\gamma)) = 0.7 - 0.8 \text{ nb}}$$

somewhat underestimated, but not strongly, w.r.t.

$$\boxed{\mathrm{CDF} \quad \to \quad 0.97 \text{ nb}}$$

KMRS'04 result **90 nb x 0.0128 = 1.15 nb**

Other UGDFs predict even smaller signal at CDF → about 0.3 nb (GBW and Kutak-Stasto), underestimated by a factor of 3!

possible sources of the problem:

1) *Absorbtive corrections may be different for various UGDFs;*
2) *x' may be smaller (i.e. $\xi<1$);*
3) *NNLO corrections may add up to the result.*

Conclusion and discussions

1 . ***Total and differential cross sections*** of exclusive diffractive production of heavy scalar, axial-vector and tensor charmonia are calculated. ***The maximal helicity*** dominance is confirmed.

2. ***Off-shellness*** of the intermediate gluons is estimated ***to be important*** in the case of diffractive charmonium production ***(factor 2-5 in the cross section).*** Strong dependence on ***factorisation scale*** and on ***UGDFs choice*** is also observed.

3. ***Significant contribution*** to the diffractive cross section comes from ***non-perturbative Qt region*** (order of fraction of GeV), so we apply ***a sort of continuation*** of perturbative result to the region where its applicability ***cannot be rigorously proven*****,** and ***is questionable*** for light mesons.