Critical Fluctuations in QCD Matter

N. G. Antoniou

Faculty of Physics, University of Athens, Greece

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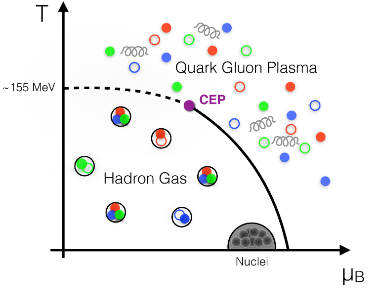
The QCD critical point



from H-T. Ding et al,

Int. J. Mod. Phys. E 24, 1530007 (2015)

- Critical Point (CP) separates **first order line** $(\mu > \mu_c)$ from **analytical crossover** $(\mu < \mu_c)$
- $T_c \approx 150\ MeV$, Lattice QCD (chiral susceptibility)
- $\mu_c \approx 1.8 \cdot T_c$ (sign problem)

(R. V. Gavai, Contemporary Physics 57, 350 (2016))

Search for **critical fluctuations** at BNL-RHIC (BES) and CERN-SPS (NA61) $\Rightarrow$ **Universality** and **scaling**

Universality class; order parameter

- Second order phase transition at **QCD-CP** (if exists) in the 3d Ising universality class
- Critical exponents: $\alpha \approx 0$, $\beta \approx \frac{1}{3}$, $\gamma \approx \frac{4}{3}$, $\nu = \frac{2}{3}$ and $\delta \approx 5$
- Order parameter $\Rightarrow$ **mixing** between **chiral condensate** (σ-field) and **baryon-number density** n_b
- Baryon-number density $=$ slow component of the order parameter

$$\Downarrow$$

appropriate thermodynamic quantity for description of **critical fluctuations** in QCD matter

3d Ising effective action

Partition function for 3d-Ising system in an external field h:

$$\mathcal{Z}(\text{Ising}) = \sum_{\{\phi_i\}} \exp\left[\beta\sum_{i,j}\phi_i\phi_j + h\sum_i \phi_i\right] \quad ; \quad \phi_i = \pm 1$$

$$\Downarrow$$

Effective action for 3d scalar field theory from Monte-Carlo simulations:

$$S_{eff}(\text{scalar}) = \int d^3\mathbf{x}\left[\frac{1}{2}|\nabla\phi|^2 + \frac{1}{2}m^2\phi^2 + g_4 m\phi^4 + g_6\phi^{\delta+1} - h\phi\right]$$

M. M. Tsypin, Phys. Rev. Lett. 73, 2015 (1994)

- S_{eff} is **universal** ($g_4 \approx 1$, $g_6 \approx 2$) describing **any** system in 3d Ising universality class close to the CP
- $\delta \approx 5$ (isothermal critical exponent)

Ising-QCD effective action

3d Ising Universality $\Rightarrow$ QCD critical point

$S_{eff}(\text{Ising-QCD}) = S_{eff}(\text{scalar})$ with:

order parameter $\phi = \beta_c^3 n_b$

external field $h = (\mu - \mu_c)\beta_c$

Length scale: $\beta_c = \frac{1}{k_B T_c}$

Mass parameter (dimensionless) $m = \beta_c \xi^{-1}$, $\xi =$ **correlation length**

$m = m_\pm |t|^\nu$ with $\frac{m_+}{m_-} = \frac{1}{2}$ and $t = \frac{T - T_c}{T_c}$ (ν=critical exponent)

Ising-QCD partition function

The Ising-QCD partition function $\mathcal{Z}_{QCD}^{Ising}$ is determined by the effective action $S_{eff}(\mathrm{Ising-QCD})$:

$$\mathcal{Z}_{QCD}^{Ising} = \sum_{\{\phi\}} \exp\left(-S_{eff}(\mathrm{Ising-QCD})\right)$$

Summing over the zero modes (constant ϕ-configurations) we obtain:

$$\mathcal{Z}_{QCD}^{Ising} = \sum_{N_b} \zeta^{N_b} \exp\left[-\frac{1}{2} m_{\pm}^2 |t|^{2\nu} \frac{N_b^2}{V} - m_{\pm} g_4 |t|^{\nu} \frac{N_b^4}{V^3} - g_6 \frac{N_b^{\delta+1}}{V^{\delta}}\right]$$

with V: volume in units β_c^3, $\zeta = \exp\left(\frac{\mu-\mu_c}{T_c}\right)$, N_b the **total baryon-number** multiplicity and

$$m_+ = 1 \;\; \mathrm{for} \;\; T \gtrsim T_c \quad ; \quad m_- = 2 \;\; \mathrm{for} \;\; T \lesssim T_c$$

Ising-QCD partition function (continued)

The partition function $\mathcal{Z}_{QCD}^{Ising}$:

- Describes dynamical fluctuations near the **QCD CP** on the basis of scaling theory and universality
- Involves two fundamental indices $(\nu,\ q = \frac{d_F}{d})$
- These two determine the **complete set** of **critical exponents**:

$$\alpha = 2 - \nu d \quad ; \quad \beta = \nu d(1-q) \quad ; \quad \gamma = \nu d(2q-1) \quad ; \quad \delta = \frac{q}{1-q}$$

  occurring in the associated scaling laws

- The relevant scaling laws are linked to Ising-QCD universality class if:

$$\nu \approx \frac{2}{3} \qquad ; \qquad q \approx \frac{5}{6}$$

Scaling behaviour of QCD matter

General finite-size scaling theory (FSST) predictions for QCD matter at the **critical point** ($t = 0$, $\zeta = 1$):

- **Baryon-number density** $\rightarrow n_b(L) \propto L^{-\beta/\nu}$
- **Baryon-number susceptibility** $\rightarrow \chi_b(L) \propto L^{\gamma/\nu}$

Ising-QCD predictions (compatible with FSST):

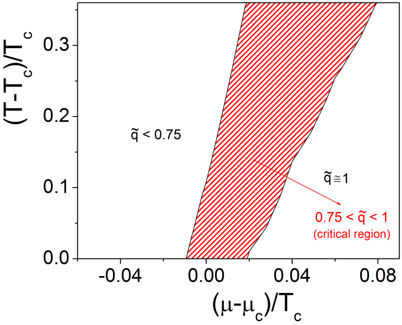
- $\mathcal{Z}_{QCD}^{Ising} = \sum_{N_b} \exp\left(-g_6 \frac{N_b^{\delta+1}}{V^\delta}\right) \rightarrow \langle n_b \rangle \propto L^{d_F - 1} \;\; ; \;\; d_F = 1 - \frac{\beta}{\nu}$
- $\chi_b(L) \sim \frac{1}{V}\left[\langle N_b^2 \rangle - \langle N_b \rangle^2\right] \rightarrow \chi_b(L) \propto L^{d(2q-1)} \;\; ; \;\; 2q - 1 = \frac{\gamma}{\nu d}$

with $L = V^{1/d}$ the **linear size**

Scaling behaviour of QCD matter (continued)

The results of Ising-QCD reveal the nature of global critical fluctuations in the form of a **monofractal** structure:

$$\langle N^k \rangle \propto L^{\kappa d_F} \quad ; \quad d_F = 1 - \frac{\beta}{\nu} \;, \quad \kappa = 1, 2, \ldots$$



N.G. Antoniou et al, hep-ph:1705.09124v1

Away from CP ($t \neq 0$, $\zeta \neq 1$):
$\langle N \rangle \sim V^{\tilde{q}}$

Critical region: $\frac{3}{4} < \tilde{q} < 1$

$\tilde{q} = \frac{3}{4} \Rightarrow \phi^4$-dominance (mean field) over ϕ^6 (3d Ising)

Critical region **narrow** along μ_b:
for $t = 0$ is $\frac{\delta\mu_b}{T_c} \approx 0.03$

Intermittency in nuclear collisions

In the finite-size scaling regime the fractal structure of critical fluctuations

$$\langle N_b \rangle \propto L^{d_F}$$

implies correlations at scales close to the correlation length $\xi > L$:

$$\langle n_b(\mathbf{x}) n_b(\mathbf{x}') \rangle \sim |\mathbf{x} - \mathbf{x}'|^{-(3-d_F)} \quad ; \quad d_F = \frac{5}{2}$$

$$\Downarrow$$

Scaling is transferred to **momentum space** for **small momentum differences** (Fourier transform):

$$\lim_{\mathbf{k} \to \mathbf{k}'} \langle n_b(\mathbf{k}) n_b(\mathbf{k}') \rangle \propto |\mathbf{k} - \mathbf{k}'|^{-d_F}$$

N.G. Antoniou et al, Phys. Rev. C 93, 014908 (2016)

A fractal structure in momentum space with $\hat{d}_F = d - d_F$ is **locally** formed!

At **midrapidity** region the momentum space fractal becomes a cartesian product ($d = 3$):

$$\text{Transverse momentum} \otimes \text{Longitudinal momentum}$$

leading to the **transverse momentum** scaling law:

$$\lim_{\mathbf{k}_\perp \to \mathbf{k}'_\perp} \langle n_b(\mathbf{k}_\perp) n_b(\mathbf{k}'_\perp) \rangle \propto |\mathbf{k}_\perp - \mathbf{k}'_\perp|^{-\frac{2d_F}{3}}$$

$$\Downarrow$$

$2d$-fractal in transverse momentum space with $\hat{d}_{F,\perp} = 2 - \frac{2}{3} d_F$

$$\Downarrow$$

Local, power-law distributed, fluctuations in transverse momentum space!

Measuring $\tilde{q}$ - Intermittency

Experimental observation of local, power-law distributed fluctuations

$\Downarrow$

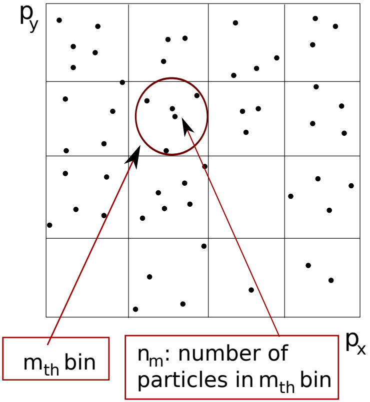
Intermittency in transverse momentum space (net protons at mid-rapidity)

(Critical opalescence in ion collisions)

- Transverse momentum space is partitioned into M^2 cells
- Calculate second factorial moments $F_2(M)$ as a function of cell size $\Leftrightarrow$ number of cells M:

$$F_2(M) \equiv \frac{\sum_m \langle n_m(n_m-1)\rangle}{\sum_m \langle n_m\rangle^2},$$

where $\langle \ldots \rangle$ denotes averaging over events.



For local power-law fluctuations:

$$F_2(M) \propto (M^2)^{\phi_2} \quad \text{for} \quad M^2 \gg 1$$

with $\phi_2 = \frac{1}{2}(2 - \hat{d}_{F,\perp}) \rightarrow$ **Intermittency index**

$$\Downarrow$$

Critical fluctuations linked to the **QCD critical point** imply:

$$\phi_2 = q = \frac{d_F}{3} = \frac{5}{6} \quad ; \quad \text{since } d_F = \frac{5}{2} \text{ for 3d Ising}$$

Critical Intermittency

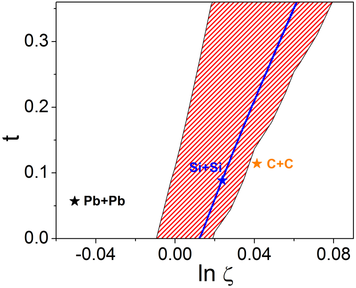
For $t \neq 0$ and/or $\zeta \neq 1$ the intermittency index is $\phi_2 = \tilde{q}$

$$\Downarrow$$

Measurement of $\phi_2 \equiv$ Measurement of $\tilde{q}$!

Roadmap towards critical fluctuations

- Measurement of $\tilde{q}$ (transverse momentum intermittency analysis)
- For $\frac{3}{4} < \tilde{q} < 1 \rightarrow$ indication for critical fluctuations (CP: $\tilde{q} = q = \frac{5}{6}$)
- Measurement of **chemical freeze-out states** $\Rightarrow$ location in the critical region
- Pilot measurement in Si+Si at SPS (NA49 experiment): $\tilde{q} \approx 0.96$ (compatible with $T_c \approx 150\ MeV$, $\mu_c \approx 256\ MeV$)



T. Anticic et al, (NA49), Eur. Phys. J. C 75, 587 (2015)

Perspectives in current experiments

The proposed method is **directly applicable** to current experiments at CERN and BNL (SPS-NA61, RHIC-BES)

(I) **Pb + Pb, Au + Au :** The crucial energy range for these processes at CERN (Pb + Pb) and BNL (Au + Au) is:

- **Pb + Pb** at $12.3\ GeV < \sqrt{s_{NN}} < 17.2\ GeV$ (SPS-NA61, beyond 2020)
- **Au + Au** at $14.5\ GeV < \sqrt{s_{NN}} < 19.6\ GeV$ (RHIC-BES)

(II) **Be + Be, Ar + Sc, Xe + La** at $\sqrt{s_{NN}} = 17.2\ GeV$ (SPS-NA61, highest energy)

Summary and conclusions

- A construction of Ising-QCD effective action was proposed, for studying critical fluctuations in **QCD matter**.
- A description of the **QCD critical point** in terms of baryon-number density, as the appropriate order parameter, was presented, compatible with universality and scaling theory.
- In particular, the phenomenon of finite-size scaling was revealed, linked to the fundamental critical exponents $D_h = qd$, $D_t = \frac{1}{\nu}$, which are incorporated in the Ising-QCD effective action.
- It was emphasized that finite-size scaling leads to global and local critical fluctuations in QCD matter, accessible to measurements (critical intermittency in proton transverse momenta).

Summary and conclusions (continued)

- A critical region in the phase diagram (μ_b, T), extremely narrow in the μ_b-direction was found $\left(\frac{\delta\mu_b}{T_c} \approx 0.03\right)$ providing an **efficient instrument** for the **location of the QCD critical point.**

- Finally, a route to observation was proposed, based on crucial intermittency measurements and chemical freeze-out studies, linked to current experiments at CERN (SPS-NA61) and BNL (RHIC-BES).

More about the QCD critical point
in CPOD-CORFU 2018

THANK YOU!