# General considerations on lepton mass matrices

Andrea Romanino, SISSA

Reyimuaji, R  to appear
Domcke, R   JHEP 1604.08879
Marzocca, R JHEP 1409.3760

# Are lepton masses and mixings determined by symmetries or symmetry breaking?

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# Introduction and definition of the problem

# Origin of lepton masses

$$\lambda^E_{ij} e^c_i l_j h \quad \rightarrow \quad \boxed{m^E_{ij} e^c_i e_j} \qquad m_E = v\lambda_E$$

$$\frac{c_{ij}}{2\Lambda} l_i l_j h h \quad \rightarrow \quad \boxed{\frac{m^\nu_{ij}}{2} \nu_i \nu_j} \qquad m_\nu = c\frac{v^2}{\Lambda}$$

assuming
high scale origin
of neutrino masses

$$\begin{pmatrix} e \leftrightarrow e_L \\ e^c \leftrightarrow \overline{e_R} \end{pmatrix}$$

# Flavour symmetry

- G flavour symmetry group (*any*, commutes with Poincaré and $G_{SM}$)

$$g \in G : \quad \begin{cases} l_i & \rightarrow \quad U_l(g)_{ij}\; l_j \\ e^c_i & \rightarrow \quad U_{e^c}(g)_{ij}\; e^c_j \end{cases}$$

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- Invariance of the lagrangian under G

$$\begin{aligned} U_{e^c}(g)^T m^0_E\ U_l(g) &= m^0_E \\ U_l(g)^T\ m^0_\nu\ U_l(g) &= m^0_\nu \end{aligned} \qquad \text{(symmetric limit)}$$

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(symmetric limit)

# Flavour symmetry breaking: $G \xrightarrow{\langle \phi \rangle} H$ $\quad \langle \phi \rangle \gg \langle h \rangle$

$$m_E = m_E^0 + m_E^1$$
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$$m_E = m_E^0 + m_E^1$$
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$m^0$: invariant under G

$m^1$:
not invariant under G
invariant under H
generated by ϕ
subleading if $m_0 \neq 0$

$$\text{e.g.} \quad \frac{\lambda_{ijk}}{\Lambda}\phi_k e_i^c l_j h \quad \text{(invariant)} \quad \rightarrow \quad (m_E^1)_{ij} = \lambda_{ijk}\frac{\langle\phi\rangle}{\Lambda}v$$

# What accounts for the LO pattern of lepton observables?

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moderate correction necessary for an accurate description

$$\text{e.g.} \quad m_D^0 = \begin{pmatrix} 0 & & \\ & 0 & \\ & & m_b \end{pmatrix} \quad m_U^0 = \begin{pmatrix} 0 & & \\ & 0 & \\ & & m_t \end{pmatrix}$$

$$m_E^0 = \begin{pmatrix} 0 & & \\ & 0 & \\ & & m_\tau \end{pmatrix} \quad V_{\text{CKM}}^0 = \mathbf{1}$$

$(\theta_C$ undetermined$)$

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most general invariant under H

  - G → H only

# What accounts for the LO pattern of lepton observables?

- The flavour symmetry G
- Symmetry breaking effects
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  - Flavon dynamics: quantum numbers, vevs, potential

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flavon-dependent

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# What accounts for the LO pattern of lepton observables?

- The flavour symmetry G
- Symmetry breaking effects

$$m_E = m_E^0 + m_E^1$$
$$m_\nu = m_\nu^0 + m_\nu^1$$

$m_E^0$, $m_\nu^0$: $m_E = 0$ or $m_\nu = 0$ in the symmetric limit

$m_E^1$, $m_\nu^1$: flavon-dependent

  - G → H only
  - Flavon dynamics: quantum numbers, vevs, potential

$$\text{e.g.} \quad G = A_4 \qquad m_\nu^0 = \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & a \\ 0 & a & 0 \end{pmatrix} \quad m_E^0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$m_\nu^1\text{: } H_1 \text{ invariant} \qquad m_E^1\text{: } H_2 \text{ invariant}$$

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systematic analysis of all G and representations ρ on leptons realising these cases

# What accounts for the LO pattern of lepton observables?

- The flavour symmetry G
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  - G → H only

systematic analysis of all G and representations ρ on leptons realising these cases

- in full generality
- complete and compact solution
- only some features of G and ρ are relevant
- no need of explicit construction of $m_E$ or $m_\nu$

G and $\rho$ leading, in the symmetric limit, to lepton masses and mixings close to what observed

Solution - in 2 steps: masses first, then mixings

# Step 1: find G and ρ leading, in the symmetric limit, to lepton masses close to what observed

- Definition of "close to what observed": lepton masses

| | non zero entries of the same order | allow hierarchy among non-zero entries | (fully undetermined in the symmetric limit) |
|---|---|---|---|
| charged leptons | $(0,0,A)$ | $(0,B,A)$<br>$(C,B,A)$ | $(0,0,0)$ |
| neutrinos NH | $(0,0,a)$ | | |
| neutrinos NH or IH | $(a,a,a)$ | $(0,b,a)$<br>$(c,b,a)$ | $(0,0,0)$ |
| neutrinos IH | $(a,a,0)$<br>$(b,b,a)$ | | |

# Step 1: precise formulation of the problem

- Given each of the previous 3x6=18 mass patterns, find all G, ρ s.t.
  - ∀ invariant $m_E$, $m_\nu$ the mass eigenvalues are in that form
  - ∃ invariant $m_E$, $m_\nu$ with mass eigenvalues in that form and generic

# Step 1: results charged lepton entries of the same order

| lepton masses | | decompositions of $U_l$ and $U_{e^c}$ | | | | |
|---|---|---|---|---|---|---|
| $(00A)$ | $(aaa)$ | none | | | | |
| $(00A)$ | $(aab)$ | $\mathbf{1}\quad\overline{\mathbf{1}}\quad 1$<br>$\overline{\mathbf{1}}\quad r \not\supseteq 1, \mathbf{1}$ | $1\quad\mathbf{1}\quad\overline{\mathbf{1}}$<br>$1\quad r \not\supseteq \overline{\mathbf{1}}, \mathbf{1}$ | $1\quad 2$<br>$1\quad r \neq 2$ | | |
| $(00A)$ | $(aa0)$ | $\mathbf{1}\quad\mathbf{1}'\quad\overline{\mathbf{1}}$<br>$\overline{\mathbf{1}}\quad r \not\supseteq \mathbf{1}, \overline{\mathbf{1}'}$ | $\mathbf{1}'\quad\mathbf{1}\quad\overline{\mathbf{1}}$<br>$\overline{\mathbf{1}}'\quad r \not\supseteq \mathbf{1}, \overline{\mathbf{1}}$ | $\mathbf{1}\quad\mathbf{1}\quad\overline{\mathbf{1}}$<br>$\overline{\mathbf{1}}\quad r \not\supseteq \mathbf{1}, \overline{\mathbf{1}}$ | $\overline{\mathbf{1}}\quad\mathbf{1}\quad\mathbf{1}$<br>$\mathbf{1}\quad r \not\supseteq \overline{\mathbf{1}}$ | $\mathbf{1}\quad 2$<br>$\overline{\mathbf{1}}\quad r \neq 2$ |
| $(00A)$ | $(00a)$ | $1\quad\mathbf{1}\quad\mathbf{1}'$<br>$1\quad r \not\supseteq \overline{\mathbf{1}}, \overline{\mathbf{1}'}$ | $\mathbf{1}\quad\mathbf{1}'\quad 1$<br>$\overline{\mathbf{1}}\quad r \not\supseteq 1, \overline{\mathbf{1}'}$ | $1\quad\mathbf{1}\quad\mathbf{1}$<br>$1\quad r \not\supseteq \overline{\mathbf{1}}$ | $\mathbf{1}\quad\mathbf{1}\quad 1$<br>$\overline{\mathbf{1}}\quad r \not\supseteq 1, \overline{\mathbf{1}}$ | $1\quad\mathbf{2}$<br>$1\quad r \not\supseteq \overline{\mathbf{2}}$ |
| $(00A)$ | $(cba)$ | $1\quad 1'\quad 1''$<br>$1\quad r \not\supseteq 1', 1''$ | $1\quad 1\quad 1'$<br>$1\quad r \not\supseteq 1, 1'$ | $1'\quad 1\quad 1$<br>$1'\quad r \not\supseteq 1$ | $1\quad 1\quad 1$<br>$1\quad r \not\supseteq 1$ | |
| $(00A)$ | $(0ba)$ | $1\quad 1'\quad\mathbf{1}$<br>$1\quad r \not\supseteq 1', \overline{\mathbf{1}}$ | $\mathbf{1}\quad 1'\quad 1$<br>$\overline{\mathbf{1}}\quad r \not\supseteq 1, 1'$ | $\mathbf{1}\quad 1\quad 1$<br>$\overline{\mathbf{1}}\quad r \not\supseteq 1$ | $1\quad 1\quad\mathbf{1}$<br>$1\quad r \not\supseteq 1, \overline{\mathbf{1}}$ | |

The results only depend on few features of the irreducible components:

- dimension of the irreps
- nature of the irreps (complex or real)
- equivalence of the irreps

# Step 1: results    hierarchies allowed

| lepton masses | | decompositions of $U_l$ and $U_{e^c}$ | | | |
|---|---|---|---|---|---|
| $(0BA)$ | $(aaa)$ | none | | | |
| $(0BA)$ | $(aab)$ | $\begin{matrix} \mathbf{1} & \overline{\mathbf{1}} & 1 \\ \overline{\mathbf{1}} & \mathbf{1} & r \neq 1 \end{matrix}$ | $\begin{matrix} 1 & \mathbf{1} & \overline{\mathbf{1}} \\ 1 & \overline{\mathbf{1}} & r \neq \mathbf{1} \end{matrix}$ | | |
| $(0BA)$ | $(aa0)$ | $\begin{matrix} \overline{\mathbf{1}} & \mathbf{1} & \mathbf{1}' \\ \mathbf{1} & \overline{\mathbf{1}} & r \neq \overline{\mathbf{1}'} \end{matrix}$ | $\begin{matrix} \mathbf{1} & \mathbf{1}' & \overline{\mathbf{1}} \\ \overline{\mathbf{1}} & \overline{\mathbf{1}'} & r \neq \mathbf{1} \end{matrix}$ | $\begin{matrix} \mathbf{1} & \mathbf{1} & \overline{\mathbf{1}} \\ \overline{\mathbf{1}} & \overline{\mathbf{1}} & r \neq \mathbf{1} \end{matrix}$ | $\begin{matrix} \overline{\mathbf{1}} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \overline{\mathbf{1}} & r \neq \overline{\mathbf{1}} \end{matrix}$ |
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| $(0BA)$ | $(cba)$ | $\begin{matrix} 1 & 1' & 1'' \\ 1 & 1' & r \neq 1'' \end{matrix}$ | $\begin{matrix} 1 & 1 & 1' \\ 1 & 1 & r \neq 1' \end{matrix}$ | $\begin{matrix} 1' & 1 & 1 \\ 1' & 1 & r \neq 1 \end{matrix}$ | $\begin{matrix} 1 & 1 & 1 \\ 1 & 1 & r \neq 1 \end{matrix}$ |
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## Comments

- 3 degenerate neutrinos in the symmetric limit cannot be obtained
- no $d = 3$ irreps
- $d = 2$ irreps can only appear if $m_e = m_\mu = 0$ in the symmetric limit

## Sketch of the proof (valid for n families)

- subspaces in flavour space associated to (zero or non-zero) degenerate $m_E$ masses are invariant under both $U_l$ and $U_{ec}$
- the $U_l$ and $U_{ec}$ sub-representations corresponding to non-zero charged lepton masses are conjugated to each other and irreducible.
- the $U_l$ and $U_{ec}$ sub-representations, nor any of their irreducible components, are conjugated to each other
- each set of degenerate non-zero neutrino masses corresponds to either a real irrep or to a pair of conjugated complex irreps
- none of the remaining irreps (correspond to vanishing neutrino masses) should be real, nor any of them should be conjugated to any other

# Step 2: select the cases also leading, in the symmetric limit, to a PMNS matrix close to what observed

- Definition of “close to what observed”: PMNS matrix
    - $|U_{13}| \lesssim 0.16$

$$|U| = \begin{pmatrix} 0.798 \to 0.843 & 0.517 \to 0.584 & 0.137 \to 0.158 \\ 0.232 \to 0.520 & 0.445 \to 0.697 & 0.617 \to 0.789 \\ 0.249 \to 0.529 & 0.462 \to 0.708 & 0.597 \to 0.773 \end{pmatrix}$$

    - $|U_{21}|$, $|U_{31}|$ can be as small as 0.25
    - all other entries larger than 0.45
- Allowed zero entries: $U_{13}$ or (less appealing) $U_{21}$, $U_{31}$ (not both)

# Results

- $U_{13} = 0$: tension IH
- $U_{ij} \neq 0$: tension $U_{13}$ ν masses solution trivial
- $U_{31,32} \neq 0$: tension $U_{13}$, $U_{31,32}$, ν masses

| lepton masses | $\nu$ hierarchy | irrep decomposition | zeros |
|---|---|---|---|
| $(00A)$<br>$(aa0)$ | IH | $\mathbf{1}\quad\mathbf{1}\quad\overline{\mathbf{1}}$<br>$\overline{\mathbf{1}}\quad r\not\supset\mathbf{1},\overline{\mathbf{1}}$ | none $(\mathbf{13})$ |
| $(00A)$<br>$(0ba)$ | **NH** or IH | $1\quad1\quad\mathbf{1}$<br>$1\quad r\not\supset 1,\overline{\mathbf{1}}$ | $\mathbf{31}, 33$ |
| $(00A)$<br>$(cba)$ | NH or IH | $1\quad1\quad1'$<br>$1\quad r\not\supset 1,1'$ | $\mathbf{31}, 32, 33$ |
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| $(0BA)$<br>$(cba)$ | NH or IH | $1\quad1\quad1$<br>$1\quad1\quad r\neq 1$ | none |
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| $(0BA)$<br>$(cba)$ | NH or IH | $1\quad 1\quad 1$<br>$1\quad 1\quad r \neq 1$ | none |
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  IH
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| $(0BA)$<br>$(cba)$ | NH or IH | $1\quad 1\quad 1$<br>$1\quad 1\quad r \neq 1$ | none |
| $(CBA)$<br>$(aa0)$ | IH | $\mathbf{1}\quad \mathbf{1}\quad \mathbf{1}$<br>$\mathbf{1}\quad \mathbf{1}\quad \bar{\mathbf{1}}$ | $\mathbf{13}, 23, 33$ |
| $(CBA)$<br>$(cba)$ | NH or IH | $1\quad 1\quad 1$<br>$1\quad 1\quad 1$ | none |

# Results

- $U_{13} = 0$: tension IH
- $U_{ij} \neq 0$: tension $U_{13}$ ν masses solution trivial
- $U_{31,32} \neq 0$: tension $U_{13}$, $U_{31,32}$, ν masses

| lepton masses | $\nu$ hierarchy | irrep decomposition | zeros |
|---|---|---|---|
| $(00A)$ $(aa0)$ | IH | $\mathbf{1}\ \ \mathbf{1}\ \ \mathbf{1}$ $\bar{\mathbf{1}}\ \ r \not\supset \mathbf{1}, \bar{\mathbf{1}}$ | none $(\mathbf{13})$ |
| $(00A)$ $(0ba)$ | **NH** or IH | $1\ \ 1\ \ \mathbf{1}$ $1\ \ r \not\supset 1, \bar{\mathbf{1}}$ | $\mathbf{31}, 33$ |
| $(00A)$ $(cba)$ | NH or IH | $1\ \ 1\ \ 1'$ $1\ \ r \not\supset 1, 1'$ | $\mathbf{31}, 32, 33$ |
| $(00A)$ $(cba)$ | NH or IH | $1\ \ 1\ \ 1$ $1\ \ r \not\supset 1$ | none |
| $(0BA)$ $(aa0)$ | IH | $\mathbf{1}\ \ \mathbf{1}\ \ \mathbf{1}$ $\mathbf{1}\ \ \mathbf{1}\ \ r \neq \bar{\mathbf{1}}$ | $\mathbf{13}$ |
| $(0BA)$ $(cba)$ | NH or IH | $1\ \ 1\ \ 1$ $1\ \ 1\ \ r \neq 1$ | none |
| $(CBA)$ $(aa0)$ | IH | $\mathbf{1}\ \ \mathbf{1}\ \ \mathbf{1}$ $\mathbf{1}\ \ \mathbf{1}\ \ \bar{\mathbf{1}}$ | $\mathbf{13}, 23, 33$ |
| $(CBA)$ $(cba)$ | NH or IH | $1\ \ 1\ \ 1$ $1\ \ 1\ \ 1$ | none |

## Sketch of the proof (valid for n families)

- $U = H_E P_E V D^{-1} P_\nu^{-1} H_\nu^{-1}$
  - V commutes with $U_l$
  - D maximal rotation, if $U_l$ contains conjugated complex irreps (Dirac substructure)
  - P permutations possibly needed to bring mass eigenvalues in standard ordering
  - H rotations up to which U is defined in the symmetric limit
    - (…) $H_\nu = H^*_\nu$(…) for generic neutrino mass pattern (…)
    - (…) $H_E = H_E$ (…) for generic charged lepton mass pattern (…)
- No need to write mass matrices explicitly

LO pattern of lepton masses and mixing determined by symmetry breaking effects from $G \to H$

$m_E = 0$ or $m_\nu = 0$ in the symmetric limit
symmetry breaking corrections generic

# Results

- The results obtained above easily extend
  - The allowed cases only depend on the lepton mass pattern and irrep decomposition *after* G breaking
  - An hierarchy between non zero masses can be understood in terms of $G \to H$

# Results (as before)

- $U_{13} = 0$: tension
  IH
- $U_{ij} \neq 0$: tension
  $U_{13}$
  ν masses
  solution trivial
- $U_{31,32} \neq 0$: tension
  $U_{13}$, $U_{31,32}$,
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| lepton masses | $\nu$ hierarchy | irrep decomposition | zeros |
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| $(00A)$<br>$(0ba)$ | **NH** or IH | $1\quad 1\quad \mathbf{1}$<br>$1\quad r \not\supset 1, \overline{\mathbf{1}}$ | $\mathbf{31}, 33$ |
| $(00A)$<br>$(cba)$ | NH or IH | $1\quad 1\quad 1'$<br>$1\quad r \not\supset 1, 1'$ | $\mathbf{31}, 32, 33$ |
| $(00A)$<br>$(cba)$ | NH or IH | $1\quad 1\quad 1$<br>$1\quad r \not\supset 1$ | none |
| $(0BA)$<br>$(aa0)$ | IH | $\overline{\mathbf{1}}\quad\overline{\mathbf{1}}\quad\mathbf{1}$<br>$\mathbf{1}\quad\mathbf{1}\quad r \neq \overline{\mathbf{1}}$ | $\mathbf{13}$ |
| $(0BA)$<br>$(cba)$ | NH or IH | $1\quad 1\quad 1$<br>$1\quad 1\quad r \neq 1$ | none |
| $(CBA)$<br>$(aa0)$ | IH | $\overline{\mathbf{1}}\quad\overline{\mathbf{1}}\quad\mathbf{1}$<br>$\mathbf{1}\quad\mathbf{1}\quad\overline{\mathbf{1}}$ | $\mathbf{13}, 23, 33$ |
| $(CBA)$<br>$(cba)$ | NH or IH | $1\quad 1\quad 1$<br>$1\quad 1\quad 1$ | none |

# Results (as before)

- $U_{13} = 0$: tension<br>IH
- $U_{ij} \neq 0$: tension<br>$U_{13}$<br>ν masses<br>solution trivial
- $U_{31,32} \neq 0$: tension<br>$U_{13}$, $U_{31,32}$,<br>ν masses

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| $(00A)$<br>$(0ba)$ | **NH** or IH | $1\ \ 1\ \ \mathbf{1}$<br>$1\ \ r \not\supset 1, \bar{\mathbf{1}}$ | $\mathbf{31}, 33$ |
| $(00A)$<br>$(cba)$ | NH or IH | $1\ \ 1\ \ 1'$<br>$1\ \ r \not\supset 1, 1'$ | $\mathbf{31}, 32, 33$ |
| $(00A)$<br>$(cba)$ | NH or IH | $1\ \ 1\ \ 1$<br>$1\ \ r \not\supset 1$ | none |
| $(0BA)$<br>$(aa0)$ | IH | $\mathbf{1}\ \ \mathbf{1}\ \ \mathbf{1}$<br>$\mathbf{1}\ \ \mathbf{1}\ \ r \neq \bar{\mathbf{1}}$ | $\mathbf{13}$ |
| $(0BA)$<br>$(cba)$ | NH or IH | $1\ \ 1\ \ 1$<br>$1\ \ 1\ \ r \neq 1$ | none |
| $(CBA)$<br>$(aa0)$ | IH | $\mathbf{1}\ \ \mathbf{1}\ \ \mathbf{1}$<br>$\mathbf{1}\ \ \mathbf{1}\ \ \bar{\mathbf{1}}$ | $\mathbf{13}, 23, 33$ |
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# Results (as before)

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| $(00A)$ <br> $(cba)$ | NH or IH | $1\ \ 1\ \ 1'$ <br> $1\ \ r \not\supset 1, 1'$ | $\mathbf{31}, 32, 33$ |
| $(00A)$ <br> $(cba)$ | NH or IH | $1\ \ 1\ \ 1$ <br> $1\ \ r \not\supset 1$ | none |
| $(0BA)$ <br> $(aa0)$ | IH | $\mathbf{1}\ \ \mathbf{1}\ \ \mathbf{1}$ <br> $\mathbf{1}\ \ \mathbf{1}\ \ r \neq \bar{\mathbf{1}}$ | $\mathbf{13}$ |
| $(0BA)$ <br> $(cba)$ | NH or IH | $1\ \ 1\ \ 1$ <br> $1\ \ 1\ \ r \neq 1$ | none |
| $(CBA)$ <br> $(aa0)$ | IH | $\mathbf{1}\ \ \mathbf{1}\ \ \mathbf{1}$ <br> $\mathbf{1}\ \ \mathbf{1}\ \ \bar{\mathbf{1}}$ | $\mathbf{13}, 23, 33$ |
| $(CBA)$ <br> $(cba)$ | NH or IH | $1\ \ 1\ \ 1$ <br> $1\ \ 1\ \ 1$ | none |

# Caveat (in progress)

- We assumed
  - existence of symmetric limit for $m_\nu$
  - high scale origin of $m_\nu$
- Consider for example the case of the see-saw: $m_\nu = -(m_D)^T M^{-1} m_D$
  - M non-singular in the symmetric limit
    → low-energy discussion is complete (symmetric limit for $m_\nu$)
  - M singular in the symmetric limit
    → low-energy discussion is incomplete (no symmetric limit for $m_\nu$)

# Conclusions

- It is possible to perform a systematic analysis of all G and ρ leading, in the symmetric limit, to lepton masses and mixings close to what observed
- In the case in which symmetry breaking effects are essential, it is possible to perform a systematic analysis of all $G \to H$ breaking patterns leading to lepton masses and mixings close to what observed, in the assumption that all breaking effects that are H invariant are allowed
- We characterise solutions in terms of the decomposition of ρ into irreducible components
- If the NH pattern will be confirmed, non-trivial solutions lead to tensions in the PMNS matrix, due to the prediction that $U_{13} = O(1)$ and that $U_{21} = 0$ or $U_{31} = 0$
- In order to avoid the tension one needs to consider models in which
    - neutrino masses do not originate at high scales
    - neutrino masses originate at high scale, but some heavy masses vanish in the symmetric limit
    - symmetry breaking effects and the flavon spectrum are essential