Confinement/Deconfinement Phase Transitions in Strongly Coupled Anisotropic Theories

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Outline

Intro of AdS/CFT– The Road to 'Reality'

- The initial AdS/CFT correspondence is the harmonic oscillator of the gauge/gravity dualities: $\mathcal{N} = 4$ sYM on flat space $\Leftrightarrow AdS_5 \times S^5$.
- The theory is very simple: Conformal, Maximally Supersymmetric, No Temperature...
- Since the discovery of the initial correspondence, there is an extensive research aiming to construct gauge/gravity dualities that can be thought as toy models to describe realistic systems and theories. Hope for universal behaviors!

  Have been constructed Gauge/Gravity Dualities with: Less/No Supersymmetry; Broken conformal symmetry, confinement; fundamental matter(probe and backreacting Dq branes); etc.

✓ We study Anisotropic theories → New Gauge/Gravity dualities.

Why? An Argument for 'applications'

The existence of strongly coupled anisotropic systems.

- The expansion of the plasma along the longitudinal beam axis at the earliest times after the collision results to momentum anisotropic plasmas.
- Strong Magnetic Fields in strongly coupled theories.
- Anisotropic low dimensional materials in condensed matter.

Why? More:

- Weakly coupled vs strongly coupled anisotropic theories.
- Consistent top-down models. Properties of the supergravity solutions, that are dual to the anisotropic theories.
- Black hole solutions that are AdS in UV to Lifhitz-like in IR **flows**:
  ★ Why there is a fixed scaling parameter z for such solutions?
  ★ Other systems that have fixed scaling IR solution (e.g. in Heavy quark density). Why?
  (Kumar 2012; Faedo, Kundu, Mateos, Tarrio 2014)
- Ⓢtriking Features! Several Universality Relations predicted for the isotropic theories are violated!
  ★ Shear viscosity over entropy density ratio takes parametrically low values $\frac{\eta}{s} < \frac{1}{4\pi}$!
  ★ Implications to QGP hydrodynamic simulations.

Reminding Slide: 1

The Lifshitz-like space, where some spatial dimensions scale in a different way with the rest:

$$ds^2 = r^{2z}\left(-dt^2 + dy_i^2\right) + r^2 dx_j^2 + \frac{dr^2}{r^2} ,$$

where z is a scaling parameter and $i + j = 1, ..., d$. The metric is invariant under

$$t \to \lambda^z t, \qquad y \to \lambda^z y, \qquad x \to \lambda x, \qquad r \to \frac{r}{\lambda} .$$

Equivalently the coordinate transformation $r \to r^{1/z}$, gives

$$ds^2 = r^2\left(-dt^2 + dy_i^2\right) + r^{2/z} dx_j^2 + \frac{dr^2}{r^2} ,$$

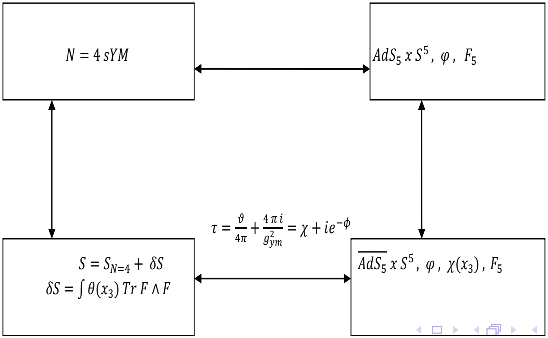
where the x_j directions are the anisotropic ones.

How is Anisotropy introduced? An example:

- Introduction of additional branes: Lifhsitz-like Supergravity solutions
  $ds^2 = r^{2z}(dx_0^2 + dx_i^2) + u^2 dx_3^2 + \frac{du^2}{u^2} + ds^2_{S^5}$. *(Azeyanagi, Li, Takayanagi, 2009)*

	x_0	x_1	x_2	x_3	u	S^5
D3	x	x	x	x		
D7	x	x	x			x

- Which equivalently leads to the following deformation diagram.



New Anisotropic Theories

Consider a generalized (reduced) Einstein-Axion-Dilaton action with a potential for the dilaton and an arbitrary coupling between the axion and the dilaton:

$$S = \frac{1}{2\kappa^2}\int d^5x\,\sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 + V(\phi) - \frac{1}{2}Z(\phi)(\partial\chi)^2\right].$$

The eoms read

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{1}{2}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}Z(\phi)\partial_\mu\chi\partial_\nu\chi - \frac{1}{4}g_{\mu\nu}(\partial\phi)^2 - \frac{1}{4}g_{\mu\nu}Z(\partial\chi)^2 + \frac{1}{2}g_{\mu\nu}V(\phi)\ ,$$
$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi\right) = \frac{1}{2}\partial_\phi Z(\phi)(\partial\chi)^2 - V'(\phi)\ ,$$
$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\chi\right) = 0\ .$$

Where the functions

$$V(\phi) = 12\cosh(\sigma\phi) - 6\sigma^2\,\phi^2, \qquad Z(\phi) = e^{2\gamma\phi},$$

((Gubser, Nellore), Pufu, Rocha 2008a,b)

Note: For $\sigma = 0, \gamma = 1$ the action and the solution of eoms, are of IIB supergravity. *(Mateos, Trancanelli, 2011)*

Let us apply the black hole background ansatz

$$ds^2 = \frac{e^{-\frac{1}{2}\phi(u)}}{u^2}\left(-\mathcal{F}\mathcal{B}\,dt^2 + dx_1^2 + dx_2^2 + \mathcal{H}dx_3^2 + \frac{du^2}{\mathcal{F}}\right),$$
$$\chi = \alpha x_3\,, \qquad \phi = \phi(u)\,,$$

$\phi(u), \mathcal{B}(u), \mathcal{F}(u), \mathcal{H}(u)$ four functions to be found, and α is the constant anisotropic parameter, u_h is the black hole horizon(related to the temperature of the theory).

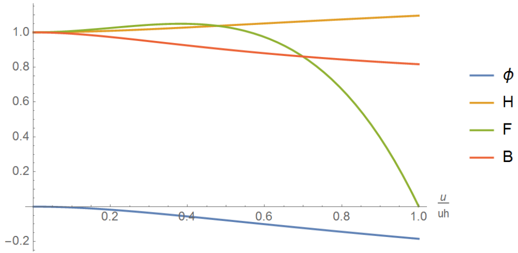
Note:
The linear axion simplifies tremendously the system of equations!

Solve the system...

Solutions: A demonstration

- Fixing (γ, σ) and α and u_h we get the metric flow from boundary to horizon:

$$ds^2 = \frac{e^{-\frac{1}{2}\phi(u)}}{u^2}\left(-\mathcal{F}\mathcal{B}\,dt^2 + dx_1^2 + dx_2^2 + \mathcal{H}dx_3^2 + \frac{du^2}{\mathcal{F}}\right),$$



- In sufficiently high temperatures, $T \gg \alpha$ for $\gamma = 1, \sigma = 0$ the Einstein equations can be solved analytically:

$$\mathcal{F}(u) = 1 - \frac{u^4}{u_h^4} + \alpha^2 \frac{1}{24 u_h^2}\left[8u^2(u_h^2 - u^2) - 10u^4 \log 2 + (3u_h^4 + 7u^4)\log\left(1 + \frac{u^2}{u_h^2}\right)\right]$$

$$\mathcal{B}(u) = 1 - \alpha^2 \frac{u_h^2}{24}\left[\frac{10u^2}{u_h^2 + u^2} + \log\left(1 + \frac{u^2}{u_h^2}\right)\right], \quad \mathcal{H}(u) = \left(1 + \frac{u^2}{u_h^2}\right)^{\frac{\alpha^2 u_h^2}{4}}.$$

We have obtained the theories, are all of them physical and stable?

Null Energy Condition

- The averaged radial acceleration between two null geodesics is

$$A_r = -4\pi T_{\mu\nu} N^\mu N^\nu \ ,$$

if it is negative the null geodesics observe a non-repulsive gravity on nearby particles along them.

- This imposes the Null Energy Condition

$$T_{\mu\nu} N^\mu N^\nu \geq 0 \ , \quad N^\mu N_\mu = 0 \ ,$$

leading to the following constrains:

- For the Lifshitz-like space $z \geq 1$.
- For the Hyperscaling violation anisotropic metric in 3+1-dim spacetime and anisotropic in 1-dim reads

$$(z-1)(1-\theta+3z) \geq 0 \ ,$$
$$\theta^2 - 3 + 3z(1-\theta) \geq 0 \ .$$

Additional conditions from thermodynamics?

Thermodynamics

- The entropy density is proportional to the area of the horizon from the Bekenstein-Hawking formula.
- In the hyperscaling anisotropic metric

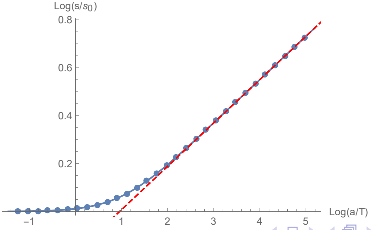
$$s \sim r_h^{c_1} \sim t^{\frac{-c_1}{z}} \sim T^{\frac{c_1}{z}} \ , \qquad c_1 := 1 - \theta + 2z \ .$$

Thermodynamically the theory 'behaves' as being in the $2 + (1-\theta)/z$ isotropic dimensions.

- In UV the AdS black brane has

$$s_0 \sim T^3$$

- The entropy density for the anisotropic flow $(\gamma = 2, \sigma = 1)$:



Local Thermodynamic Stability

- The necessary and sufficient conditions for local thermodynamical stability in the canonical ensemble are

$$c_\alpha = T\left(\frac{\partial S}{\partial T}\right)_\alpha \geq 0\,, \qquad \Phi' = \left(\frac{\partial \Phi}{\partial \alpha}\right)_T \geq 0$$

  c_α is the specific heat: increase of the temperature leads to increase of the entropy.
  Φ' is derivative of the potential: the system is stable under infinitesimal charge fluctuations.

- In the GCE these conditions should be equivalent of having no positive eigenvalues of the Hessian matrix of the entropy with respect to the thermodynamic variables. *(Gubser, Mitra 2001)*

- In the IR the positivity of the specific heat imposes

$$c_\alpha = 1 - \theta + 2z \geq 0$$

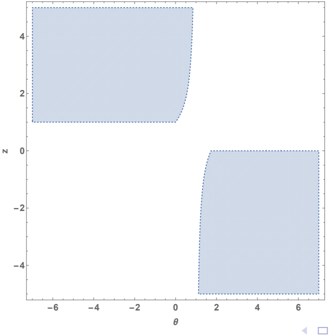
Three conditions that constrain (z, θ) and as a result (γ, σ).

$$(z-1)(1-\theta+3z) \geq 0 \;, \qquad z = \frac{2+4\gamma^2-3\sigma^2}{2\gamma(2\gamma-3\sigma)} \;,$$

$$\theta^2 - 3 + 3z(1-\theta) \geq 0 \;, \qquad \theta = \frac{3\sigma}{2\gamma} \;,$$

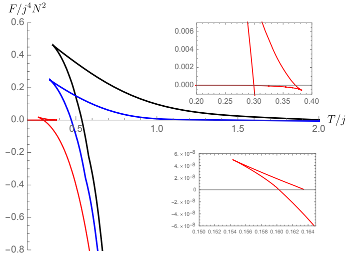
$$1 - \theta + 2z \geq 0 \;.$$

Translated to the following diagram where the blue region is the acceptable for the theory parameters.

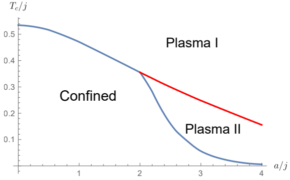


Confinement/Deconfinement Phase transitions

- Let us obtain the theories for the whole range of parameters.
- The free energy of the theories vs the temperature gives(α/j=0,1,3):



- The Critical Temperature of the theories vs the anisotropy gives:



- The T_c is reduced in presence of anisotropies in the theory.

Findings and Proposal:

- The $Tc(\alpha)$ decrease with α, resembling the phenomenon of inverse magnetic catalysis where the confinement-deconfinement temperature decreases with the magnetic field B.
- No charged fermionic degrees of freedom in our case; our plasma is neutral.
- Our findings suggest that the anisotropy by itself could instead be the main cause of lower T_c; e.g. in inverse magnetic catalysis.

Transport and Diffusion

- For the shear viscosity η we need to solve the system of metric fluctuations on the anisotropic theory. The system can be mapped under a toroidal compactification to a Maxwell system with a mass term

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left(-\frac{1}{4g_{eff}^2}F^2 - \frac{1}{4}m^2(u)A^2\right) ,$$

where A is related to metric fluctuations:

$$ds^2 = (g_{\mu\nu} + g_{33}A_\mu A_\nu)dx^\mu dx^\nu + 2g_{33}A_\nu dx^3 dx^\nu + g_{33}(dx^3)^2$$

and

$$m^2(u) = Z(\phi_d + \frac{k}{2})A_\mu A^\mu(\partial_z\chi)^2 , \quad \frac{1}{g_{eff}^2} = (g_{33}(u))^{3/2} ,$$

$$2k := \log(g_{33}) , \qquad \phi_d = \phi - \frac{k}{2} , \qquad \frac{L_3}{2\kappa_5^2} := \frac{1}{2\kappa^2} ,$$

- It can be solved at the zero momentum limit, and AdS asymptotics using the regularity of the horizon.
- The result is

$$\eta_{13} = \frac{g_{11}^2}{2\kappa_5^2 \sqrt{g_{33}}}\Big|_{u=u_h}$$

- The entropy density is

$$s = \frac{2\pi}{\kappa_5^2}\sqrt{g_{11} g_{22} g_{33}}\Big|_{u=u_h}$$

- Therefore

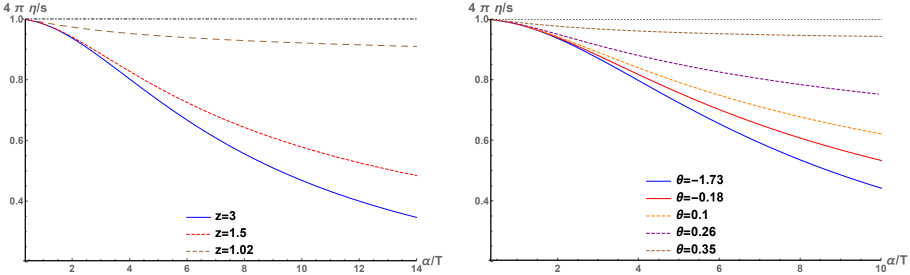
$$\frac{\eta_{13}}{s} = \frac{1}{4\pi}\frac{g_{11}}{g_{33}}\Big|_{u=u_h}$$

For prolate geometries $(g_{33} > g_{11} = g_{22})$ is becoming parametrically less than the KSS prediction$(1/4\pi)$!

(similar to Rebhan,Steineder 2011; D.G. 2013,...)

η/s for our theory: Dependence on the Temperature.

The shear viscosity over entropy ratio for arbitrary (z, θ).



The ratio depends on the temperature as

$$\frac{\eta_{13}}{s} \sim \left(\frac{T}{\tilde{\alpha}|1 + 3z - \theta|} \right)^{2-\frac{2}{z}} .$$

- For Lifshitz-like theories $(\theta = 0)$ the range of the temperature power is $[0, 2)$.
- For hyperscaling violation theories the range of the temperature power is $[0, \infty)$.

Conclusions

- ✓ We have presented new black hole gravity dual solutions that in the UV are AdS isotropic, while in the IR flow to Lifshitz and hyperscaling violation anisotropic solutions with arbitrary exponents.
- ✓ There are certain stability conditions that constrain the parameters of the backgrounds.
- ✓ The Shear viscosity over entropy density ratio, takes values parametrically lower than $1/4\pi$, and depends on the Temperature as $T^{2-2/z}$.
- ✓ The Confinement/Deconfinement phase transitions occur at lower critical Temperature as the anisotropy is increased!
- ✓ We suggest that the anisotropy by itself could instead be the main cause of inverse magnetic catalysis.
- Several ways to probe the theory (Mesons, Energy loss of Quarks, Diffusion of Quarks, Speed of Sound, Entanglement Entropy...).

(D.G 2012,...)

Thank you

Reminding Slide: 2

The anisotropic hyperscaling violation metric

$$ds^2 = r^{-\frac{2\theta}{d}}\left(-r^{2z}(dt^2+dy_i^2)+r^2dx_j^2+\frac{dr^2}{r^2}\right) ,$$

which exhibits a critical exponent z and a hyperscaling violation exponent θ. The metric is not scale invariant as

$$t\to\lambda^z t,\qquad y\to\lambda^z y,\qquad x\to\lambda x,\qquad r\to\frac{r}{\lambda} ,\qquad ds\to\lambda^{\frac{\theta}{d}}ds .$$

The coordinate transformation $r\to r^{1/z}$ bring the metric in the form

$$ds^2 = r^{-\frac{2\theta}{dz}}\left(-r^2(dt^2+dy_i^2)+r^{\frac{2}{z}}dx_i^2+\frac{dr^2}{r^2}\right) .$$

which reveals clearly the anisotropic directions.