Risk Management and Trading Strategies for Financial Derivatives: Foreign Exchange (FX) & Interest Rates (IR)

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Outline: Wednesday 28 October 2009

- **Interest Rates**
  - Risk-free Interest Rate (IR) Curves, Compounding & Continuous IR, Present Value of Money
  - Term Structure of IR
- **Vanilla Foreign Exchange (FX) Options**
  - Spot FX, Forward FX
  - Stochastic Spot FX: Put-Call Parity, Ito/Weiner process for (Domestic=$ and Foreign=Yen) Investor
  - Black-Scholes-Merton (BSM) pricing of vanilla FX options (i.e. constant volatility surface)
  - Heat/Diffusion differential equation and Probability Kernal
  - Results: BSM Prices and Hedging Greeks
  - Ito/Weiner process for (Domestic=Yen/Foreign=$) Investor
  - Martingales: Streamlined BSM pricing of vanilla FX options
Outline: Thursday 29 October 2009

• Market Trading and Risk Management of Vanilla FX Options
• Measures of Market Risk
• Implied Volatility, FX Volatility Surface
• FX Risk Reversals, FX Strangles
• Valuation and Risk Calculations
• Risk Management
• Market Trading Strategies
Outline: Friday 30 October 2009

- **IR Derivatives, Long Term FX Derivatives**
- Stochastic IR curve evolution, Martingales
- Implied Volatility Along the IR Curve
- IR Libor Bonds
- **Long Term FX Derivatives**
- Interaction of Stochastic FX, Stochastic Domestic IR, Stochastic Foreign IR
- $-Yen (Bermudan) Power Reverse Duals
- **Hybrid Stochastic Volatility Models**
- What products are they used for? Why?
- Local volatility and stochastic volatility
- Why “hybrid”. Brief description of StochDetVol
References

• *Options, Futures and Other Derivatives*, J. C. Hull, Prentice Hall Int’l

• *Financial Calculus*, M. Baxter and A. Rennie, Cambridge University Press
Interest Rates (IR)

- Risk-free IR is (maximally) unaffected by (sovereign) issuer credit rating, has units of %/year (i.e. annualized).

- Compound IR, Continuous IR

\[ N_s(t_0 + 2yr) = N_s(t_0) \left(1 + yr \ r_s^{\text{Compound}}\right)^2 \]
\[ = N_s(t_0) e^{2yr \ r_s^{\text{Continuous}}} \]

Present Value of Money

\[ N_s(t_0) = N_s(t_{\text{Maturity}}) e^{-r_s(t_{\text{Maturity}} - t_0)} \]
Zero coupon (LIBOR) IR curves

- Eurodollar (ED) Futures contracts:
  - Traded on Chicago Mercantile Exchange (CME)
  - Contract is on 90 day LIBOR (London Interbank Offer Rate) IR (i.e. not Bond price)
  - Cash settled on International Money Market (IMM) dates: 2nd London business day before 3rd Wednesday of quarter months. Need accurate calendar software
  - $1m Notional

\[
ED_{Future} = \$1m(1 - r_{\text{Compound}}^\frac{1}{4})
\]
## Build IR Yield Curves from Standard Sets of Instruments

<table>
<thead>
<tr>
<th>Instrument</th>
<th>instrument type</th>
</tr>
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<tbody>
<tr>
<td>1m</td>
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</tr>
<tr>
<td>3m</td>
<td>Cash Rate</td>
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<tr>
<td>First ED contract</td>
<td>Future Contract</td>
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<tr>
<td>2\textsuperscript{nd} ED Contract</td>
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<tr>
<td>Third ED Contract</td>
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<td>Fourth ED Contract</td>
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<td>Strip 2 (Greens)</td>
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<tr>
<td>Strip 3 (Blues)</td>
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<tr>
<td>Strip 4 (Golds)</td>
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<tr>
<td>S5_5y</td>
<td>5 yr forward 5y Swap Rate</td>
</tr>
<tr>
<td>S10_10y</td>
<td>10 yr forward 10 yr Swap Rate</td>
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<tr>
<td>S20_10y</td>
<td>20 yr forward 10 yr Swap Rate</td>
</tr>
<tr>
<td>S30_20y</td>
<td>30 yr forward 20 yr Swap Rate</td>
</tr>
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</table>
USD Interest Rate Term Structure on 10 October 2009 (Highly Smoothed)
History of USD 1m 3m Interest Rates

Graph showing the historical trend of USD 1m and 3m interest rates from 14/11/2007 to 22/01/2010.
History of RUB 1m, 2m, 3m Interest Rates
History of Term Structure of RUB Interest Rates
Spot and Forward USDJPY FX Rates

\[ N_{\$} (t_{\text{Maturity}}) = N_{\$} (t_0) e^{r_{\$} (t_{\text{Maturity}} - t_0)} \]

\[ N_{\text{Yen}} (t_{\text{Maturity}}) = N_{\text{Yen}} (t_0) e^{r_{\text{Yen}} (t_{\text{Maturity}} - t_0)} \]

\[ S_{/\text{Yen}} (t_0) = N_{/\text{Yen}} (t_0) / N_{\text{Yen}} (t_0) \]

\[ F_{/\text{Yen}} (t_{\text{Maturity}}) = N_{/\text{Yen}} (t_{\text{Maturity}}) / N_{\text{Yen}} (t_{\text{Maturity}}) \]

\[ = S_{/\text{Yen}} (t_0) e^{(r_{\$} - r_{\text{Yen}})(t_{\text{Maturity}} - t_0)} \]
"USDJPY" Spot FX

\[ S_{Yen/\$}(t) \]
USDRUB Spot FX

\[ S_{RUB/\$}(t) \]
Kernal \( (t_0, x_0; t_{\text{Maturity}}, x_{\text{Maturity}}; \sigma_{$/\text{Yen}}, r_\$, r_{\text{Yen}}) \)

\[ x_0 = \ln(S_{$/\text{Yen}}(t_0)); \quad x_{\text{Maturity}} = \ln(S_{$/\text{Yen}}(t_{\text{Maturity}})) \]

\[ \int_{-\infty}^{\infty} dx_{\text{Maturity}} \text{ Kernal} \ (t_0, x_0; t_{\text{Maturity}}, x_{\text{Maturity}}; \sigma_{$/\text{Yen}}, r_\$, r_{\text{Yen}}) = 1 \]

\[ \int_{-\infty}^{\infty} dx_{\text{Maturity}} \text{ Kernal} \ (t_0, x_0; t_{\text{Maturity}}, x_{\text{Maturity}}; \sigma_{$/\text{Yen}}, r_\$, r_{\text{Yen}}) \]

\[ \bullet S_{$/\text{Yen}}(t_{\text{Maturity}}) \]

\[ = F_{$/\text{Yen}}(t_{\text{Maturity}}) \]

\[ = S_{$/\text{Yen}}(t_0)e^{(r_\$-r_{\text{Yen}})(t_{\text{Maturity}}-t_0)} \]
Vanilla European Call and Put Options for USD Investor

\[ CallYen(t_0, S_{$/Yen}(t_0); \sigma_{$/Yen}, r, r_{Yen}; N, N_{Yen}) \]

\[ = e^{-r_S \tau} \int_{-\infty}^{\infty} dx_{Maturity} \ Kernal(t_0, x_0; t_{Maturity}, x_{Maturity}; \sigma_{$/Yen}, r, r_{Yen}) \]

\[ \bullet \ \text{Maximum}[N_{Yen}S_{$/Yen}(t_{Maturity}) - N, 0] \]

\[ \tau = t_{Maturity} - t_0 \geq 0 \]

\[ PutYen(t_0, S_{$/EUR}(t_0); \sigma_{$/Yen}, r, r_{Yen}; N, N_{Yen}) \]

\[ = e^{-r_S \tau} \int_{-\infty}^{\infty} dx_{Maturity} \ Kernal(t_0, x_0; t_{Maturity}, x_{Maturity}; \sigma_{$/Yen}, r, r_{Yen}) \]

\[ \bullet \ \text{Maximum}[N - N_{Yen}S_{$/Yen}(t_{Maturity}), 0] \]
“Put/Call Parity” for Vanilla European Options  
Doesn’t depend on Details of Probability Distribution!

\[ \text{Maximum}\left[N_s - N_{\text{Yen}} S_{$/\text{Yen}}(t_{\text{Maturity}}), 0\right] \]

\[- \text{Maximum}\left[N_{\text{Yen}} S_{$/\text{Yen}}(t_{\text{Maturity}}) - N_s, 0\right] \]

\[= N_s - N_{\text{Yen}} S_{$/\text{Yen}}(t_{\text{Maturity}}) \]

\[\text{PutYen}(t_0, S_{$/\text{Yen}}(t_0); \sigma_{$/\text{Yen}}, r_s, r_{\text{Yen}}; N_s, N_{\text{Yen}}) \]

\[- \text{CallYen}(t_0, S_{$/\text{Yen}}(t_0); \sigma, r_s, r_{\text{Yen}}; N_s, N_{\text{Yen}}) \]

\[= e^{-r_s \tau} \int_{-\infty}^{\infty} dx_{Maturity} \text{Kernal}(t_0, x_0; t_{\text{Maturity}}, x_{Maturity}; \sigma_{$/\text{Yen}}, r_s, r_{\text{Yen}}) \]

\[\bullet \left(N_s - N_{\text{Yen}} S_{$/\text{Yen}}(t_{\text{Maturity}})\right) \]

\[= N_s e^{-r_s \tau} - N_{\text{Yen}} e^{-r_{\text{Yen}} \tau} S_{$/\text{Yen}}(t_0) \]
Black Scholes Merton (BSM) Vanilla FX Option Pricing

Asset \[ S = S_{$/Yen}(t)N_{Yen}; \]

Ito Process \[ dS / S = \mu(t)dt + \sigma_{$/Yen}dW(t); \]

Continuous Random Walk \[ dW(t) = \varepsilon \sqrt{dt} \]

\[ dS^2 \approx S^2 \sigma^2_{$/Yen}dt; \quad \sigma_{$/Yen} \sim \frac{1}{\sqrt{yr}}; \]

\[ C = \text{CallYen}(t_0, S_{$/Yen}(t_0); \sigma_{$/Yen}, r$, r$_{Yen}; N$, N$_{Yen}) \]

\[ dC \approx \left( dt \frac{\partial}{\partial t} + dS \frac{\partial}{\partial S} + \frac{1}{2} dS^2 \frac{\partial^2}{\partial S^2} \right) C \]

Portfolio \[ P = -C + \Delta S \]

\[ dP = \left( -dt \frac{\partial}{\partial t} - \frac{1}{2} dS^2 \frac{\partial^2}{\partial S^2} \right) C + dS \left( - \frac{\partial C}{\partial S} + \Delta \right) + r_{Yen} dt \Delta S \]
Risk-free Portfolio Earns Risk-free IR

Choose  \[ \Delta = \frac{\partial C}{\partial S} \]

Cancel  \( dS: \ Both \ Random \ Component \ & \ Drift \ \mu(t) \)

\[ dP = r_s dtP \]

\[ 0 = dt \left( \frac{\partial}{\partial t} + \frac{\sigma_{S/Yen}^2}{2} S^2 \frac{\partial^2}{\partial S^2} + (r_s - r_{Yen})S \frac{\partial}{\partial S} - r_s \right) C \]

\[ x = \ln(S_{S/Yen}) \]

\[ 0 = dt \left( \frac{\partial}{\partial t} + \frac{\sigma_{S/Yen}^2}{2} \frac{\partial^2}{\partial x^2} + (r_s - r_{Yen} - \sigma_{S/Yen}^2 / 2) \frac{\partial}{\partial x} - r_s \right) C \]
Solution Maps to
Heat/Diffusion Differential
Equation and
Probability Kernal
(Fokker-Plank Forward, Kolmogorov Backward)

\[
Kernal(t_0, x_0; t_{\text{Maturity}}, x_{\text{Maturity}}, S/Yen, \sigma_{S/Yen}) = \frac{1}{\sqrt{2\pi \sigma_{S/Yen}^2 \tau}} \exp\left(-\frac{(x_0 - x_{\text{Maturity}} - (f_{S/Yen} - S/Yen) \sigma_{S/Yen}^2 / 2 \tau)^2}{2\sigma_{S/Yen}^2 \tau}\right)
\]

\[\tau = t_{\text{Maturity}} - t_0 \geq 0\]

\[x_0 = \ln[S_{S/Yen}(t_0)]; \quad x_{\text{Maturity}} = \ln[S_{S/Yen}(t_{\text{Maturity}})]\]
Option Price, Hedging “Greeks” for $ Investor

\[
\begin{align*}
\text{CallYen} & \quad (t_0, S_{$/Yen}(t_0), \sigma_{$/Yen}, r_{$/Yen}, r_{Yen}, N_{$/}, N_{Yen}) \\
& = S_{$/Yen}(t_0) N_{Yen} e^{-r_{Yen}\tau} N(d_+) - N_{$/} e^{-r_{$/}\tau} N(d_-) \\
\tau &= t_{\text{Maturity}} - t_0 > 0; \quad N(d) = \int_{-\infty}^{d} dx \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \\
d_{\pm} &= \frac{1}{\sigma_{$/Yen} \sqrt{\tau}} \ln \left( \frac{S_{$/Yen}(t_0) N_{Yen} e^{-r_{Yen}\tau}}{N_{$/} e^{-r_{$/}\tau}} \right) \pm \frac{1}{2} \sigma_{$/Yen} \sqrt{\tau} \\
\text{Delta} &= \frac{\partial C}{\partial S_{$/Yen}}; \quad \text{Vega} = \frac{\partial C}{\partial \sigma_{$/Yen}} \\
\text{Gamma} &= \frac{\partial^2 C}{\partial S^2_{$/Yen}}; \quad \text{Vomma} = \frac{\partial^2 C}{\partial \sigma^2_{$/Yen}} \\
\text{Vanna} &= \frac{\partial^2 C}{\partial \sigma_{$/Yen} \partial S_{$/Yen}} \\
\end{align*}
\]
Vanilla European Call Greeks: Delta, Vega, Gamma
Vanilla European Call Greeks: Vomma, Vanna
**Ito Processes & Option Prices**

**for**

$\text{Investor} : \quad \$ & \text{Yen Investors}$

$$d \ln[ S_{\$/\text{Yen}} (t) ] = (r_\$ - r_{\text{Yen}} - \sigma^2_{\$/\text{Yen}} / 2)dt + \sigma_{\$/\text{Yen}} dW_{\$/\text{Yen}} (t)$$

$$\text{CallYen} = e^{-r_\$ \tau} \left\langle \max \left( S_{\$/\text{Yen}} (t_{\text{Maturity}}) N_{\text{Yen}} - N_\$, 0 \right) \right\rangle_{S_{\$/\text{Yen}} \text{ Process}}$$

$$\text{PutYen} = e^{-r_\$ \tau} \left\langle \max \left( N_\$ - S_{\$/\text{Yen}} (t_{\text{Maturity}}) N_{\text{Yen}}, 0 \right) \right\rangle_{S_{\$/\text{Yen}} \text{ Process}}$$

**YenInvestor \ $ : \quad \text{r}$$

$$d \ln[ S_{\text{Yen} / \$} (t) ] = (r_{\text{Yen}} - r_\$ - \sigma^2_{\$/\text{Yen}} / 2)dt + \sigma_{\$/\text{Yen}} dW_{\text{Yen} / \$} (t)$$

$$\text{Call \$} = e^{-r_{\text{Yen}} \tau} \left\langle \max \left( S_{\text{Yen} / \$} (t_{\text{Maturity}}) N_\$ - N_{\text{Yen}}, 0 \right) \right\rangle_{S_{\text{Yen} / \$} \text{ Process}}$$

$$\text{Put \$} = e^{-r_{\text{Yen}} \tau} \left\langle \max \left( N_{\text{Yen}} - S_{\text{Yen} / \$} (t_{\text{Maturity}}) N_\$, 0 \right) \right\rangle_{S_{\text{Yen} / \$} \text{ Process}}$$
$ and Yen Investors

- **Agree** on Underlying Asset:
  \[ S_{$/Yen}(t) = \frac{1}{S_{Yen/\$}(t)} \]

- **Disagree** on Ito Process:
  \[ d \ln[S_{$/Yen}(t)S_{Yen/\$}(t)] \neq 0 \]

**Agree** on Vanilla Option Prices

- **Call**Yen = \( S_{$/Yen}(t_0) \) **Put**
- **Put**Yen = \( S_{$/Yen}(t_0) \) **Call**

Hedge with Greeks easily related to each other’s and will agree on the prices of all other traded derivative and cash instruments.

**Most transparent through Martingales!**
What is “Volatility”

- Vanilla European Implied Volatility is a quoting convention: the standard deviation that is entered into the Black-Scholes - Merton pricing formula to get the quoted market vanilla option price.
- Historical or “realized”: the standard deviation of periodic returns of an asset - daily, weekly, monthly, etc.
- Simulated: the volatility that produces a net zero return for that strike over a predetermined horizon. This is basis for the IVI index.
Origins of the FX Volatility Term Structure

\[ \sigma_{12} = 9.0\% \]
\[ \sigma_{02} = 8.5\% \]
\[ \sigma_{01} = 8.0\% \]

- Using Black-Scholes-Merton we can calculate a forward volatility
- European Options depend only upon the volatility at maturity
- Exotic Options can have a sensitivity to the entire term structure

\[ \sigma_2 = \left( \frac{\sigma_{02}^2 \cdot t_{02} - \sigma_{01}^2 \cdot t_{01}}{t_{12}} \right)^{1/2} \]
Market FX Implied Volatility Surface

- Market Vanilla FX Option volatilities depend on expiry $T$. This itself does not invalidate the Black-Scholes-Merton approach.
- They also depend on the strike level
  - Broadly defined as “smile” or “skew”;
  - “Volatility surface” defined by expiry and strike
  - Inconsistent with Black-Scholes-Merton
- Apart from At The Money (ATM) volatilities, the skew/smile is characterised by “risk reversal” and “strangle” inputs.
- In FX Market, these are quoted in terms of their “deltas” which float with the market.
FX Market USD/JPY
Implied Volatility Surface
[%/Sqrt(yr)]

10% P
25% P
50%
25% C
10% C
18M
9M
3M
1M
1Y
Market for Exotics

• For the last 30 years, we have seen an increase in:

  - Client Types
  - Risk (Volatility)
  - Countries
  - Techniques
  - Volume
  - Products
  - Number of Underlying Assets