Risk Management and Trading Strategies for Financial Derivatives: Foreign Exchange (FX) & Interest Rates (IR)

CERN Academic Training
28-30 October 2009

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Outline: Wednesday 28 October 2009

- **Interest Rates**
- Risk-free Interest Rate (IR) Curves, Compounding & Continuous IR, Present Value of Money
- Term Structure of IR
- **Vanilla Foreign Exchange (FX) Options**
- Spot FX, Forward FX
- Stochastic Spot FX: Put-Call Parity, Ito/Weiner process for (Domestic=$ and Foreign=Yen) Investor
- Black-Scholes-Merton (BSM) pricing of vanilla FX options (i.e. constant volatility surface)
- Heat/Diffusion differential equation and Probability Kernal
- Results: BSM Prices and Hedging Greeks
- Ito/Weiner process for (Domestic=Yen/Foreign=$) Investor
- Martingales: Streamlined BSM pricing of vanilla FX options

Outline: Thursday 29 October 2009

- **Market Trading and Risk Management of Vanilla FX Options**
- Measures of Market Risk
- Implied Volatility, FX Volatility Surface
- FX Risk Reversals, FX Strangles
- Valuation and Risk Calculations
- Risk Management
- Market Trading Strategies

Outline: Friday 30 October 2009

- **IR Derivatives, Long Term FX Derivatives**
- Stochastic IR curve evolution, Martingales
- Implied Volatility Along the IR Curve
- IR Libor Bonds
- **Long Term FX Derivatives**
- Interaction of Stochastic FX, Stochastic Domestic IR, Stochastic Foreign IR
- $-Yen (Bermudan) Power Reverse Duals
- **Hybrid Stochastic Volatility Models**
- What products are they used for? Why?
- Local volatility and stochastic volatility
- Why “hybrid”. Brief description of StochDetVol

References

- *Options, Futures and Other Derivatives*, J. C. Hull, Prentice Hall Int'l
- *Financial Calculus*, M. Baxter and A. Rennie, Cambridge University Press

Interest Rates (IR)

- Risk-free IR $r_{\$}$ is (maximally) unaffected by (sovereign) issuer credit rating, has units of %/year (i.e. annualized).

- Compound IR, Continuous IR

$$N_{\$}(t_0+2yr)=N_{\$}(t_0)\left(1+yr\ r_{\$}^{Compound}\right)^2$$

$$=N_{\$}(t_0)\ e^{2yr\, r_{\$}^{Contiuous}}$$

Present Value of Money

$$N_{\$}(t_0) = N_{\$}(t_{Maturity})e^{-r_{\$}(t_{Maturity}-t_0)}$$

Zero coupon (LIBOR) IR curves

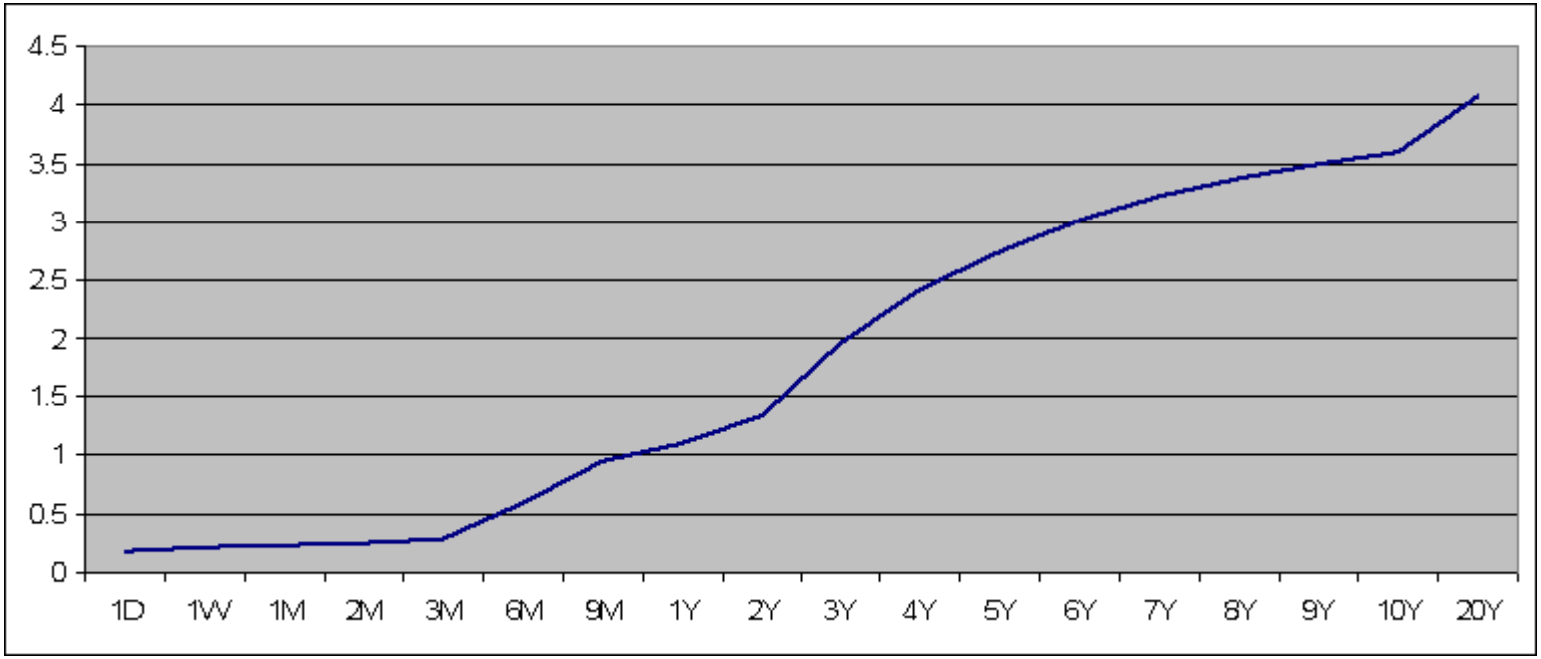
- Eurodollar (ED) Futures contracts:
  - Traded on Chicago Mercantile Exchange (CME)
  - Contract is on 90 day LIBOR (London Interbank Offer Rate) IR (i.e. not Bond price)
  - Cash settled on International Money Market (IMM) dates: 2nd London business day before 3rd Wednesday of quarter months. Need accurate calendar software
  - $1m Notional

$$ED_{Future} = \$1m(1 - r_{\$}^{Compound} / 4)$$

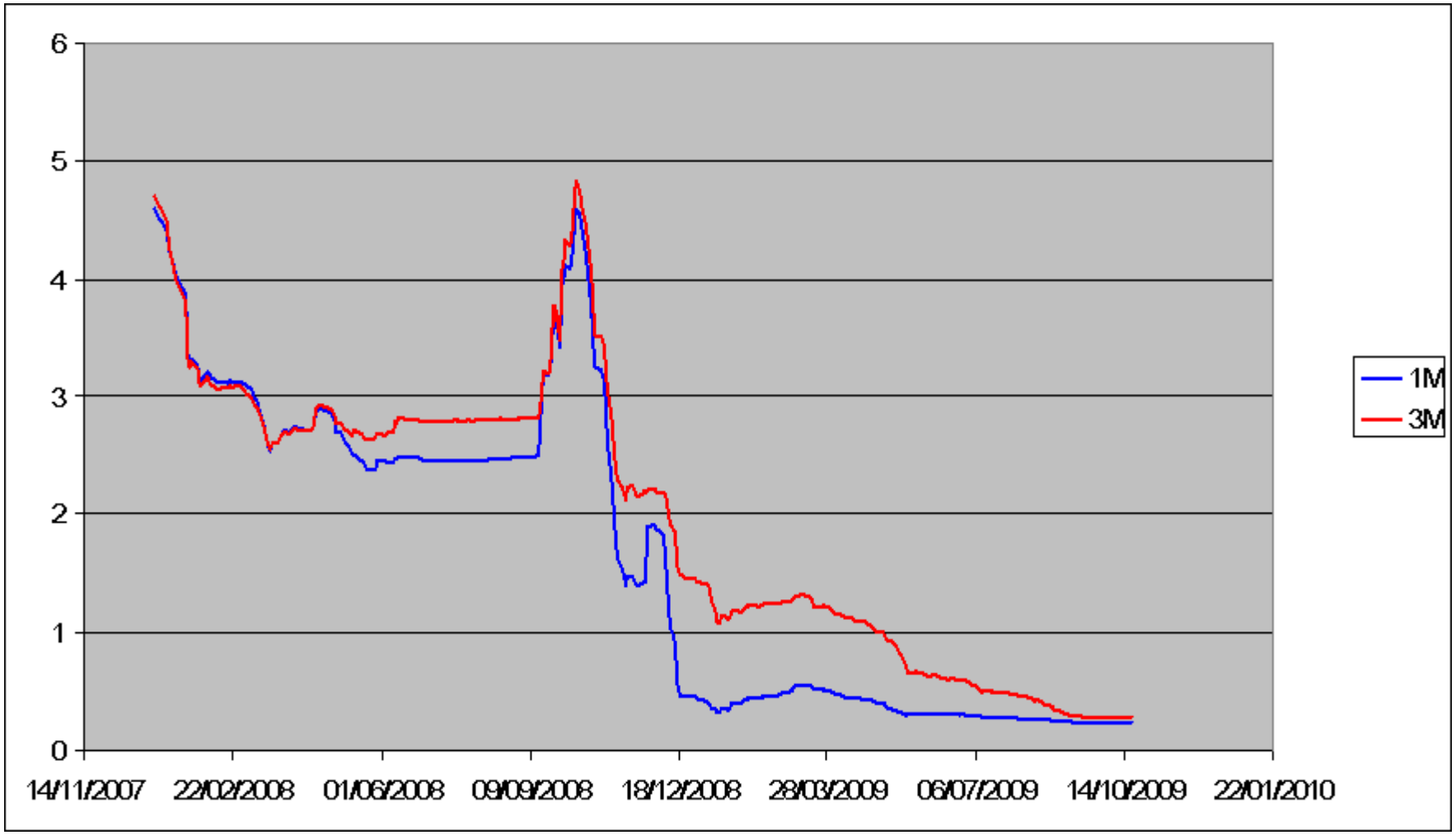
Build IR Yield Curves from Standard Sets of Instruments

Instrument	instrument type
1m 3m	Cash Rate Cash Rate
First ED contract	Future Contract
2nd ED Contract	Future Contract
Third ED Contract Fourth ED Contract	Future Contract Future Contract
Strip 1 (Reds)	Strip
Strip 2 (Greens)	Strip
Strip 3 (Blues)	Strip
Strip 4 (Golds)	Strip
S5_5y	5 yr forward 5y Swap Rate
S10_10y	10 yr forward 10 yr Swap Rate
S20_10y	20 yr forward 10 yr Swap Rate
S30_20y	30 yr forward 20 yr Swap Rate

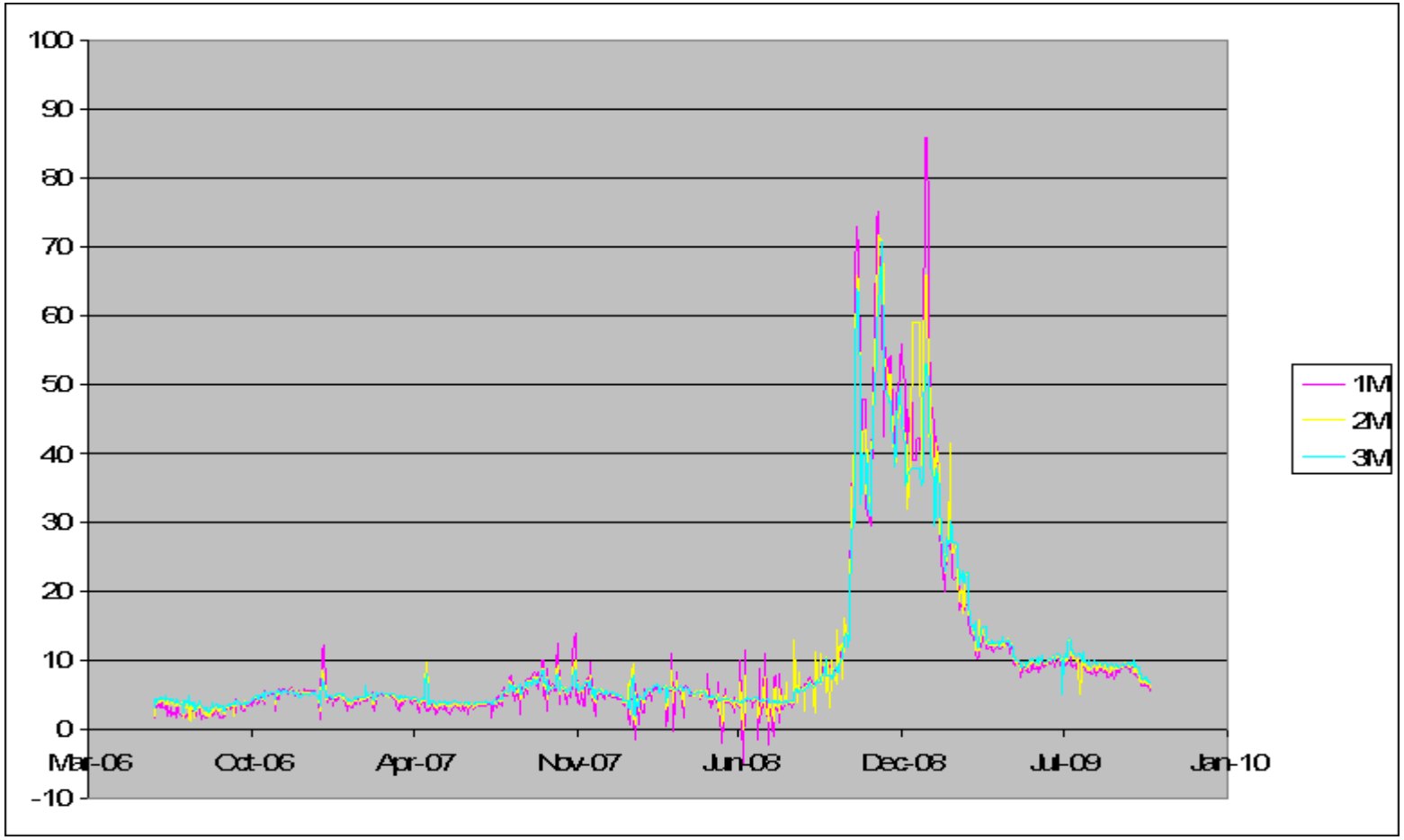
USD Interest Rate Term Structure on 10 October 2009 (Highly Smoothed)



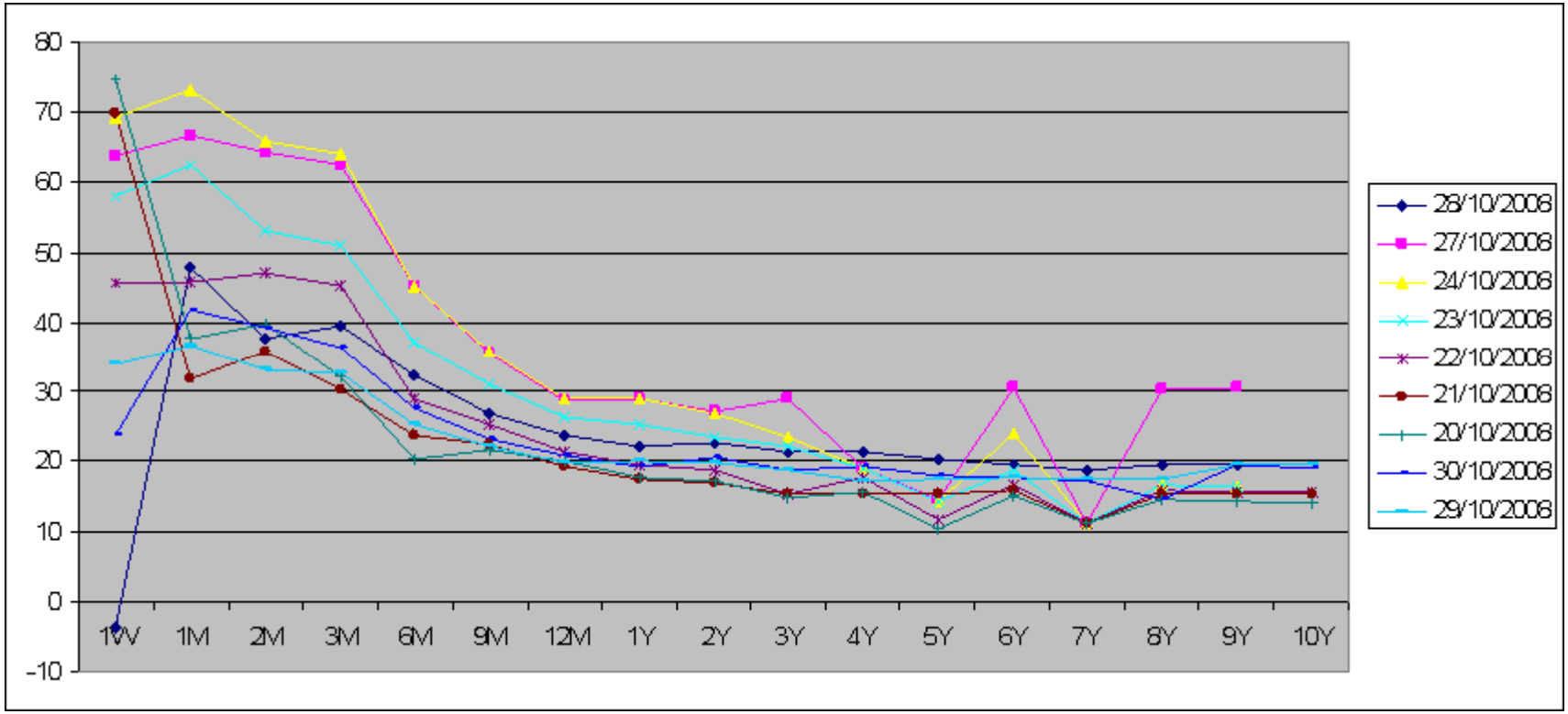
History of USD 1m 3m Interest Rates



History of RUB 1m, 2m, 3m Interest Rates



History of Term Structure of RUB Interest Rates



Spot and Forward USDJPY FX Rates

$$N_{\$}(t_{Maturity}) = N_{\$}(t_0)e^{r_{\$}(t_{Maturity} - t_0)}$$

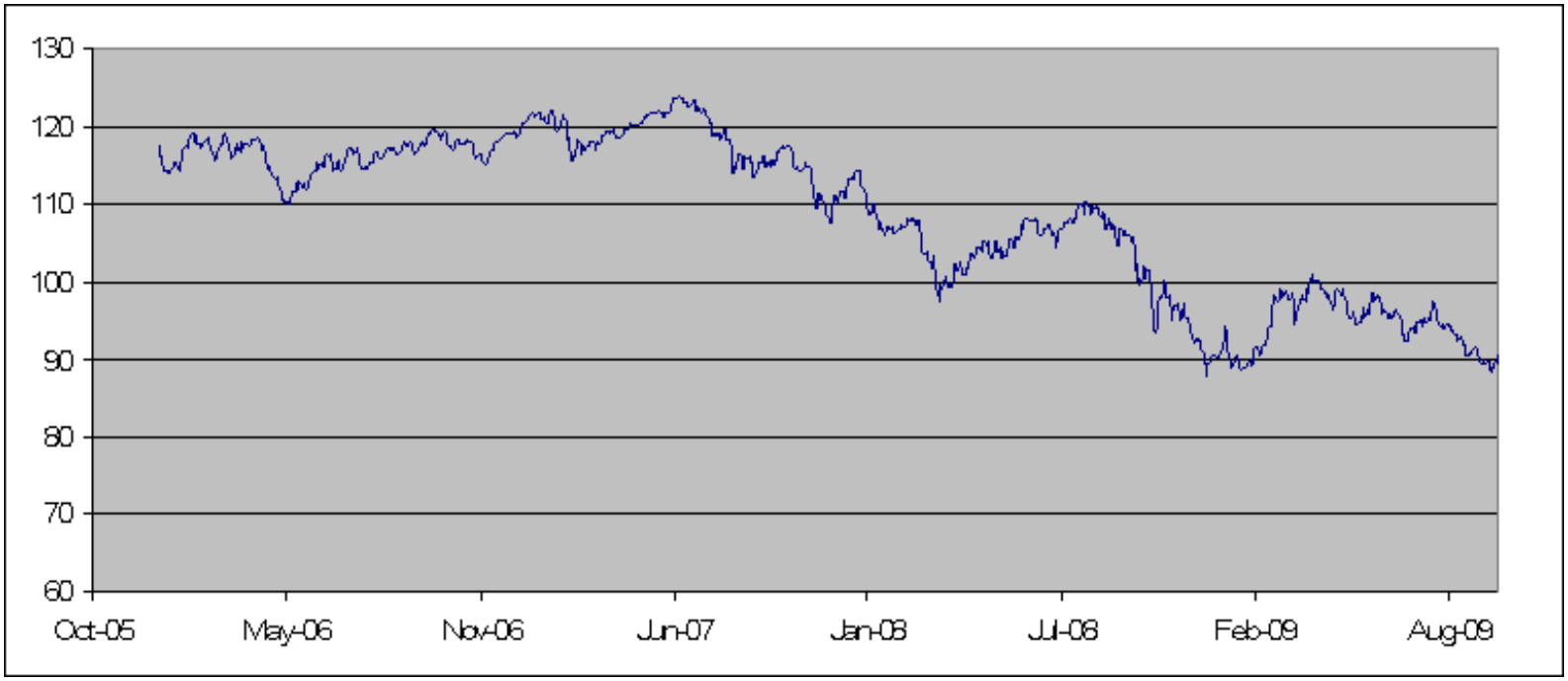
$$N_{Yen}(t_{Maturity}) = N_{Yen}(t_0)e^{r_{Yen}(t_{Maturity} - t_0)}$$

$$S_{\$/Yen}(t_0) = N_{\$}(t_0) / N_{Yen}(t_0)$$

$$F_{\$/Yen}(t_{Maturity}) = N_{\$}(t_{Maturity}) / N_{Yen}(t_{Maturity})$$

$$= S_{\$/Yen}(t_0)e^{(r_{\$} - r_{Yen})(t_{Maturity} - t_0)}$$

“USDJPY” Spot FX



$$S_{Yen/\$}(t)$$

USDRUB Spot FX

$$S_{RUB/\$}(t)$$

Probability Kernal of Future FX Rate

$$Kernal\ (t_0, x_0; t_{Maturity}, x_{Maturity}; \sigma_{\$/Yen}, r_{\$}, r_{Yen})$$

$$x_0 = \ln(S_{\$/Yen}(t_0)); \qquad x_{Maturity} = \ln(S_{\$/Yen}(t_{Maturity}))$$

$$\int_{-\infty}^{\infty} dx_{Maturity}\ Kernal\ (t_0, x_0; t_{Maturity}, x_{Maturity}; \sigma_{\$/Yen}, r_{\$}, r_{Yen}) = 1$$

$$\int_{-\infty}^{\infty} dx_{Maturity}\ Kernal\ (t_0, x_0; t_{Maturity}, x_{Maturity}; \sigma_{\$/Yen}, r_{\$}, r_{Yen})$$

$$\bullet\ S_{\$/Yen}(t_{Maturity})$$

$$= F_{\$/Yen}(t_{Maturity})$$

$$= S_{\$/Yen}(t_0) e^{(r_{\$} - r_{Yen})(t_{Maturity} - t_0)}$$

Vanilla European Call and Put Options for USD Investor

$$CallYen(t_0, S_{\$/Yen}(t_0); \sigma_{\$/Yen}, r_\$, r_{Yen}; N_\$, N_{Yen})$$

$$= e^{-r_\$ \tau} \int_{-\infty}^{\infty} dx_{Maturity} Kernal(t_0, x_0; t_{Maturity}, x_{Maturity}; \sigma_{\$/Yen}, r_\$, r_{Yen})$$

$$\bullet Maximum[N_{Yen} S_{\$/Yen}(t_{Maturity}) - N_\$, 0]$$

$$\tau = t_{Maturity} - t_0 \geq 0$$

$$PutYen(t_0, S_{\$/EUR}(t_0); \sigma_{\$/Yen}, r_\$, r_{Yen}; N_\$, N_{Yen})$$

$$= e^{-r_\$ \tau} \int_{-\infty}^{\infty} dx_{Maturity} Kernal(t_0, x_0; t_{Maturity}, x_{Maturity}; \sigma_{\$/Yen}, r_\$, r_{Yen})$$

$$\bullet Maximum[N_\$ - N_{Yen} S_{\$/Yen}(t_{Maturity}), 0]$$

“Put/Call Parity” for Vanilla European Options Doesn’t depend on Details of Probability Distribution!

$$
\begin{aligned}
& Maximum[N_{\$} - N_{Yen} S_{\$/Yen}(t_{Maturity}),0] \\
& \quad - Maximum[N_{Yen} S_{\$/Yen}(t_{Maturity}) - N_{\$},0] \\
& \quad = N_{\$} - N_{Yen} S_{\$/Yen}(t_{Maturity}) \\
& PutYen(t_0, S_{\$/Yen}(t_0); \sigma_{\$/Yen}, r_{\$}, r_{Yen}; N_{\$}, N_{Yen}) \\
& \quad - CallYen(t_0, S_{\$/Yen}(t_0); \sigma, r_{\$}, r_{Yen}; N_{\$}, N_{Yen}) \\
& \quad = e^{-r_{\$}\tau} \int_{-\infty}^{\infty} dx_{Maturity} Kernal(t_0, x_0; t_{Maturity}, x_{Maturity}; \sigma_{\$/Yen}, r_{\$}, r_{Yen}) \\
& \qquad \bullet (N_{\$} - N_{Yen} S_{\$/Yen}(t_{Maturity})) \\
& \quad = N_{\$} e^{-r_{\$}\tau} - N_{Yen} e^{-r_{Yen}\tau} S_{\$/Yen}(t_0)
\end{aligned}
$$

Black Scholes Merton (BSM) Vanilla FX Option Pricing

$$Asset \qquad S = S_{\$/Yen}(t) N_{Yen};$$

$$Ito\,\mathrm{Pr}\,ocess \quad dS/S = \mu(t)dt + \sigma_{\$/Yen} dW(t);$$

$$ContinuousRandomWalk \qquad dW(t) = \varepsilon\sqrt{dt}$$

$$dS^2 \approx S^2 \sigma_{\$/Yen}^2 dt; \qquad \sigma_{\$/Yen} \sim 1\Big/\sqrt{yr};$$

$$C = CallYen(t_0, S_{\$/Yen}(t_0); \sigma_{\$/Yen}, r_{\$}, r_{Yen}; N_{\$}, N_{Yen})$$

$$dC \approx \left(dt\frac{\partial}{\partial t} + dS\frac{\partial}{\partial S} + \frac{1}{2} dS^2 \frac{\partial^2}{\partial S^2} \right) C$$

$$Portfolio \quad P = -C + \Delta S$$

$$dP = \left(-dt\frac{\partial}{\partial t} - \frac{1}{2} dS^2 \frac{\partial^2}{\partial S^2} \right) C + dS\left(-\frac{\partial C}{\partial S} + \Delta \right) + r_{Yen} dt \Delta S$$

Risk-free Portfolio Earns Risk-free IR

$$Choose \qquad \Delta = \frac{\partial C}{\partial S}$$

$$Cancel \qquad dS: \quad Both\,Random\,Component \quad \& \,Drift\,\mu(t)$$

$$dP = r_{\$} dt P$$

$$0 = dt\left(\frac{\partial}{\partial t} + \frac{\sigma^2_{\$/Yen}}{2} S^2 \frac{\partial^2}{\partial S^2} + (r_{\$} - r_{Yen}) S \frac{\partial}{\partial S} - r_{\$}\right) C$$

$$x = \ln(S_{\$/Yen})$$

$$0 = dt\left(\frac{\partial}{\partial t} + \frac{\sigma^2_{\$/Yen}}{2} \frac{\partial^2}{\partial x^2} + (r_{\$} - r_{Yen} - \sigma^2_{\$/Yen}/2) \frac{\partial}{\partial x} - r_{\$}\right) C$$

Solution Maps to Heat/Diffusion Differential Equation and Probability Kernal (Fokker-Plank Forward, Kolmogorov Backward)

$$Kernal(t_0, x_0, t_{Maturity}, x_{Maturity}, \sigma_{\$/Yen}, r_{\$}, r_{Yen})$$

$$= \frac{1}{\sqrt{2\pi\sigma_{\$/Yen}^2\tau}} \exp\left(-\frac{[x_0 - x_{Maturity} + (r_{\$} - r_{Yen} - \sigma_{\$/Yen}^2/2)\tau]^2}{2\sigma_{\$/Yen}^2\tau} \right)$$

$$\tau = t_{Maturity} - t_0 \geq 0$$

$$x_0 = \ln[S_{\$/Yen}(t_0)]; \quad x_{Maturity} = \ln[S_{\$/Yen}(t_{Maturity})]$$

Option Price, Hedging "Greeks" for $ Investor

$$CallYen\quad(t_0, S_{\$/Yen}(t_0), \sigma_{\$/Yen}, r_\$, r_{Yen}, N_\$, N_{Yen})$$

$$= S_{\$/Yen}(t_0) N_{Yen} e^{-r_{Yen}\tau} N(d_+) - N_\$ e^{-r_\$\tau} N(d_-)$$

$$\tau = t_{Maturity} - t_0 > 0; \qquad N(d) = \int_{-\infty}^{d} dx \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

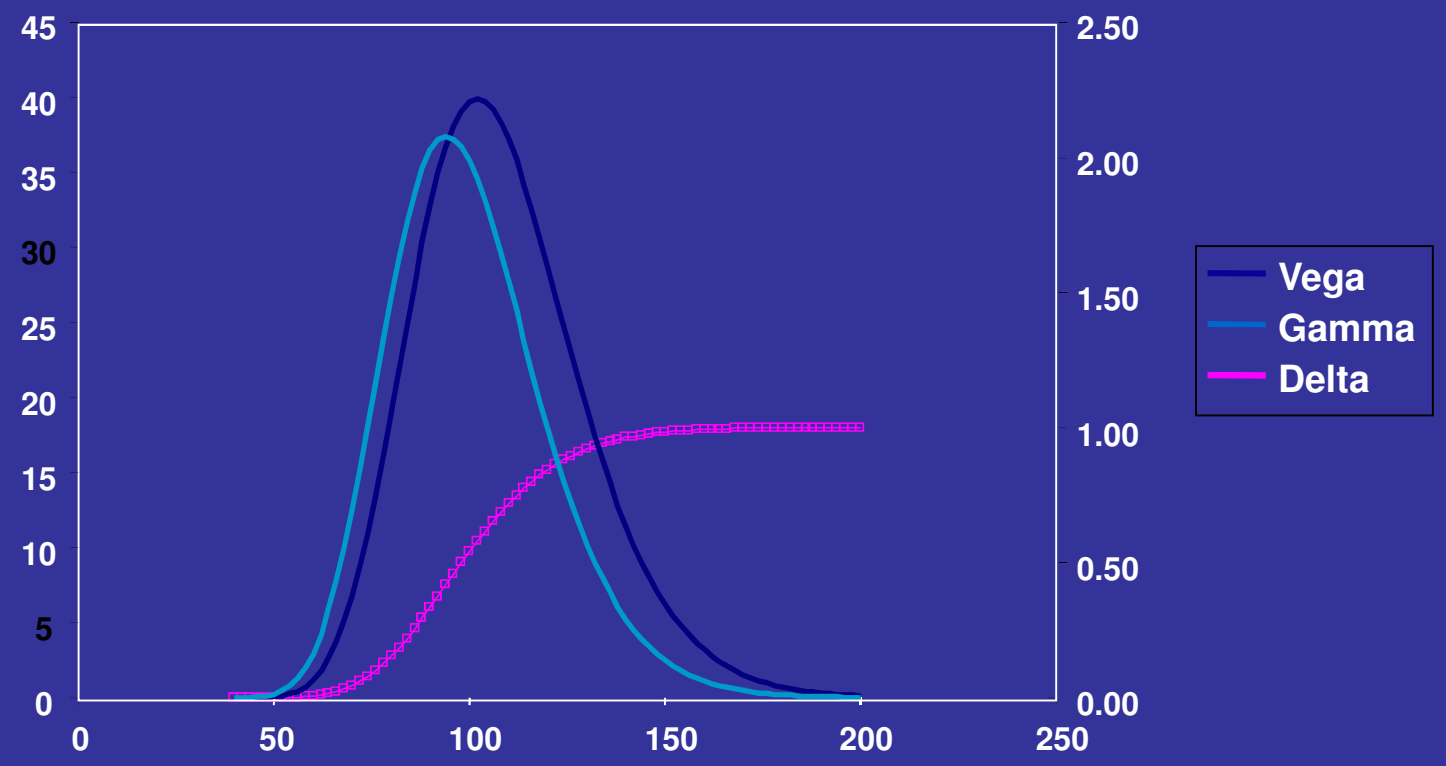
$$d_\pm = \frac{1}{\sigma_{\$/Yen}\sqrt{\tau}} \ln\left(\frac{S_{\$/Yen}(t_0) N_{Yen} e^{-r_{Yen}\tau}}{N_\$ e^{-r_\$\tau}}\right) \pm \frac{1}{2}\sigma_{\$/Yen}\sqrt{\tau}$$

$$Delta \quad \Delta = \frac{\partial C}{\partial S_{\$/Yen}}; \qquad Vega = \frac{\partial C}{\partial \sigma_{\$/Yen}}$$

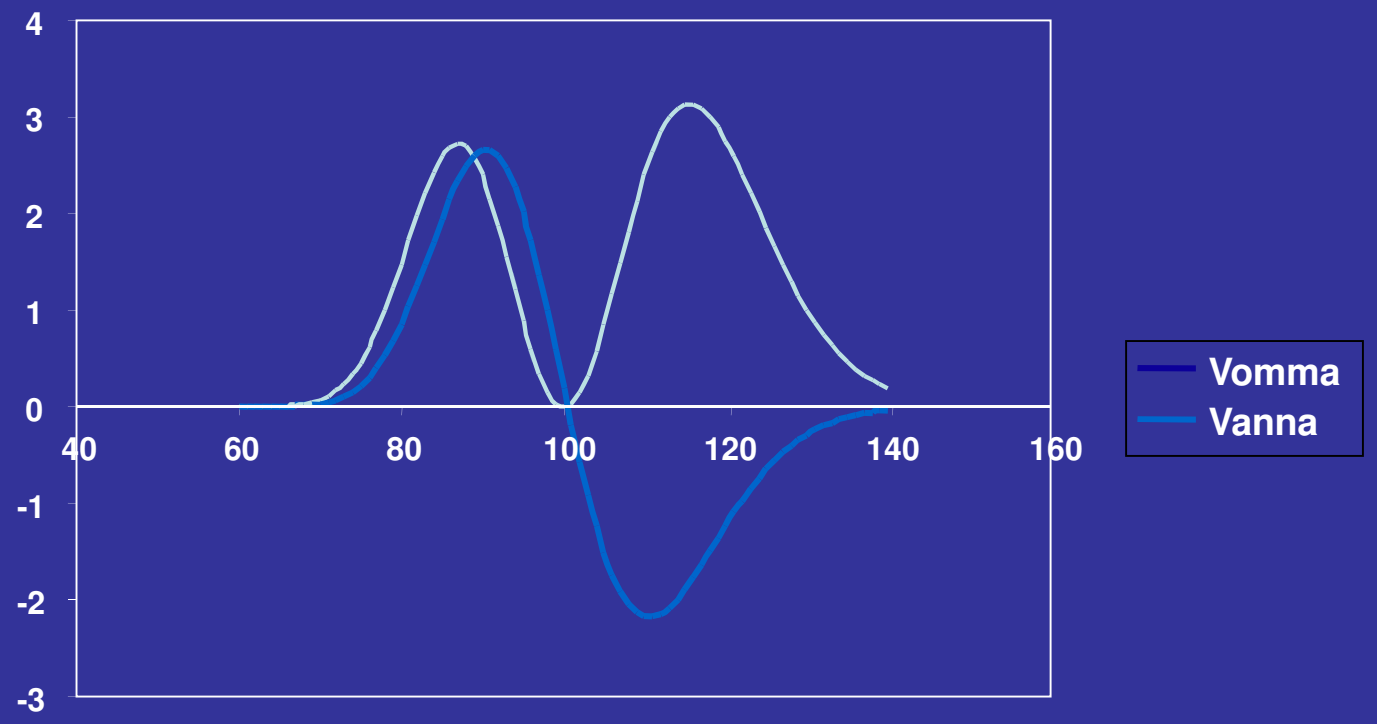
$$Gamma \quad \Gamma = \frac{\partial^2 C}{\partial S^2_{\$/Yen}}; \qquad Vomma = \frac{\partial^2 C}{\partial \sigma^2_{\$/Yen}}$$

$$Vanna = \frac{\partial^2 C}{\partial \sigma_{\$/Yen}\, \partial S_{\$/Yen}}$$

Vanilla European Call Greeks: Delta, Vega, Gamma



Vanilla European Call Greeks: Vomma, Vanna



Ito Processes & Option Prices for $ & Yen Investors

$\$ Investor$:

$$d\ln[S_{\$/Yen}(t)] = (r_{\$} - r_{Yen} - \sigma^2_{\$/Yen}/2)dt + \sigma_{\$/Yen}dW_{\$/Yen}(t)$$

$$CallYen = e^{-r_{\$}\tau}\left\langle \max\left(S_{\$/Yen}(t_{Maturity})N_{Yen} - N_{\$}, 0\right)\right\rangle_{S_{\$/Yen}\,\mathrm{Process}}$$

$$PutYen = e^{-r_{\$}\tau}\left\langle \max\left(N_{\$} - S_{\$/Yen}(t_{Maturity})N_{Yen}, 0\right)\right\rangle_{S_{\$/Yen}\,\mathrm{Process}}$$

$YenInvestor$:

$$d\ln[S_{Yen/\$}(t)] = (r_{Yen} - r_{\$} - \sigma^2_{\$/Yen}/2)dt + \sigma_{\$/Yen}dW_{Yen/\$}(t)$$

$$Call\ \$ = e^{-r_{Yen}\tau}\left\langle \max\left(S_{Yen/\$}(t_{Maturity})N_{\$} - N_{Yen}, 0\right)\right\rangle_{S_{Yen/\$}\,\mathrm{Process}}$$

$$Put\ \$ = e^{-r_{Yen}\tau}\left\langle \max\left(N_{Yen} - S_{Yen/\$}(t_{Maturity})N_{\$}, 0\right)\right\rangle_{S_{Yen/\$}\,\mathrm{Process}}$$

$ and Yen Investors

- <u>Agree</u> on Underlying Asset: $S_{\$/Yen}(t) = 1 \big/ S_{Yen/\$}(t)$

- <u>Disagree</u> on Ito Process: $d\ln[S_{\$/Yen}(t)S_{Yen/\$}(t)] \neq 0$

<u>Agree</u> on Vanilla Option Prices

$$CallYen = S_{\$/Yen}(t_0)Put\$$$

$$PutYen = S_{\$/Yen}(t_0)Call\$$$

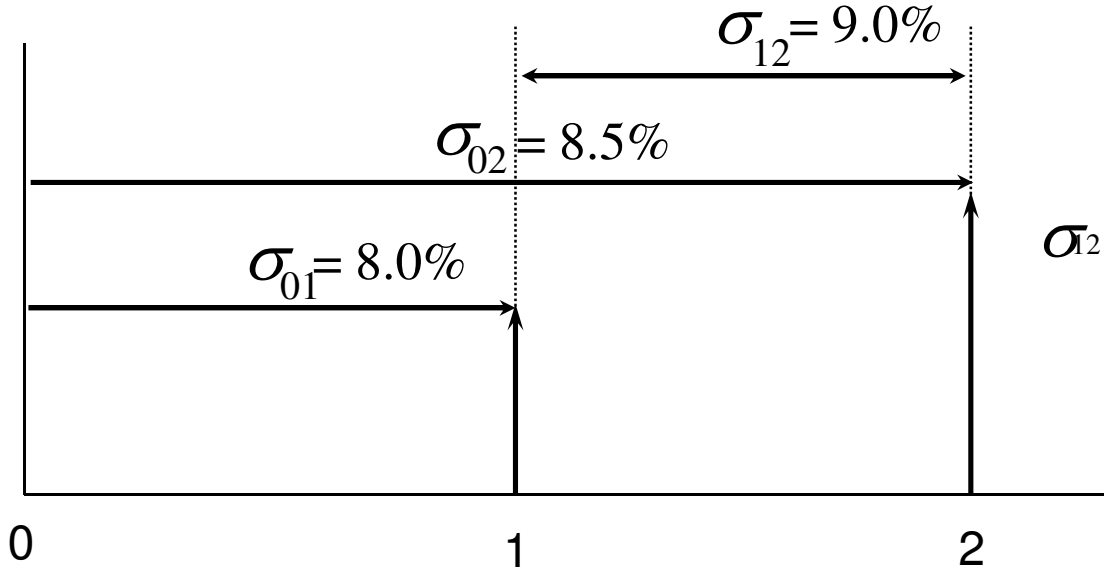
Hedge with Greeks easily related to eachother's and will agree on the prices of all other traded derivative and cash instruments.

Most transparent through Martingales!

What is “Volatility”

- Vanilla European Implied Volatility is a quoting convention: the standard deviation that is entered into the Black-Scholes - Merton pricing formula to get the quoted market vanilla option price
- Historical or “realized”: the standard deviation of periodic returns of an asset - daily, weekly, monthly, etc.
- Simulated: the volatility that produces a net zero return for that strike over a predetermined horizon. This is basis for the IVI index

Origins of the FX Volatility Term Structure



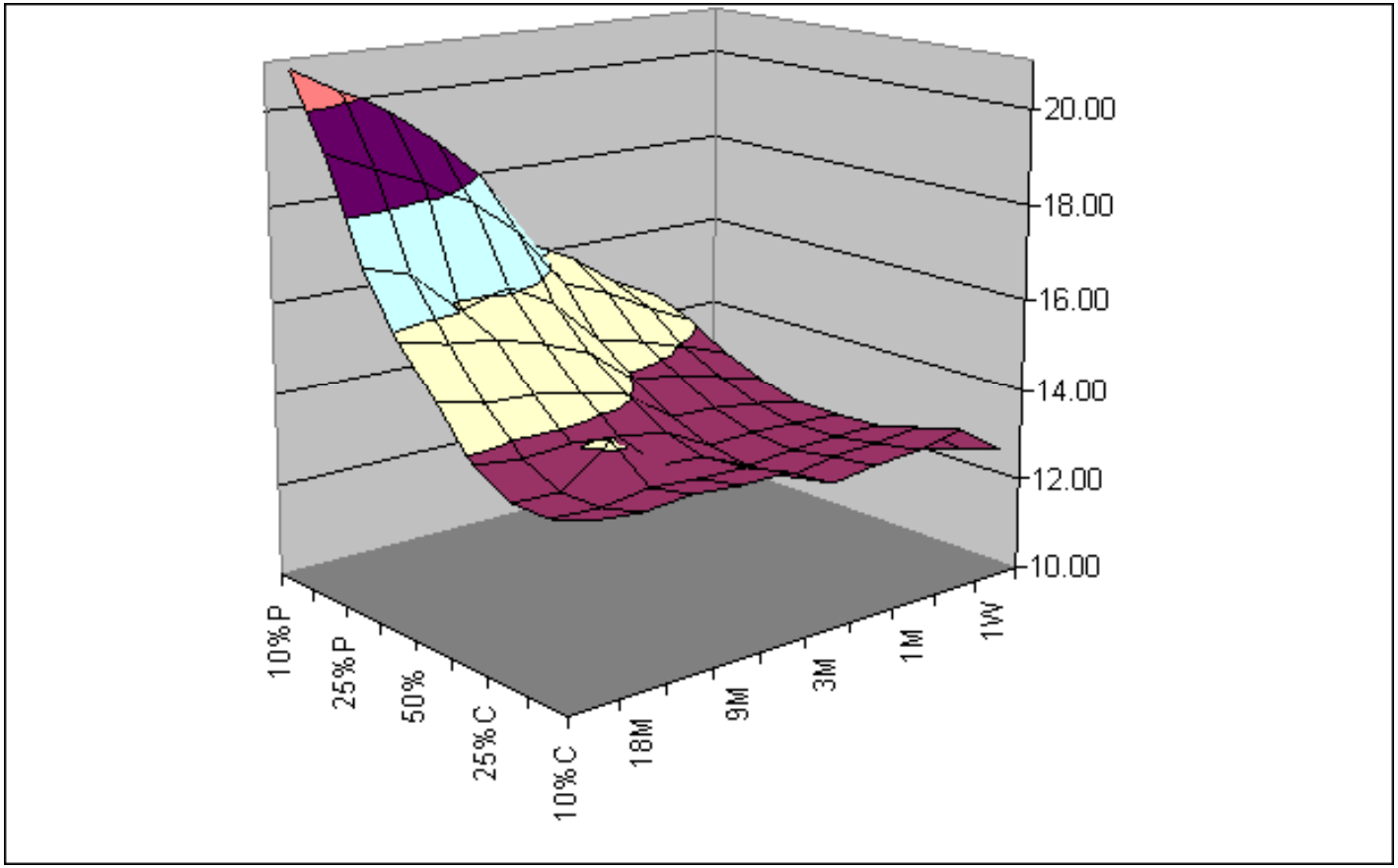
$$\sigma_{12} = \left(\frac{\sigma^2_{02} \cdot t_{02} - \sigma^2_{01} \cdot t_{01}}{t_{12}} \right)^{1/2}$$

- Using Black-Scholes-Merton we can calculate a forward volatility
- European Options depend only upon the volatility at maturity
- Exotic Options can have a sensitivity to the entire term structure

Market FX Implied Volatility Surface

- Market Vanilla FX Option volatilities depend on expiry T. This itself does not invalidate the Black-Scholes-Merton approach
- They also depend on the strike level
  - Broadly defined as “smile” or “skew”;
  - “Volatility surface” defined by expiry and strike
  - Inconsistent with Black-Scholes-Merton
- Apart from At The Money (ATM) volatilities, the skew/smile is characterised by “risk reversal” and “strangle” inputs
- In FX Market, these are quoted in terms of their “deltas” which float with the market

FX Market USD/JPY Implied Volatility Surface [%/Sqrt(yr)]



Market for Exotics

- For the last 30 years, we have seen an increase in: