IWHSS 2018

UNIVERSITÄT BONN

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A global fit of partonic Transverse Momentum Dependent distributions

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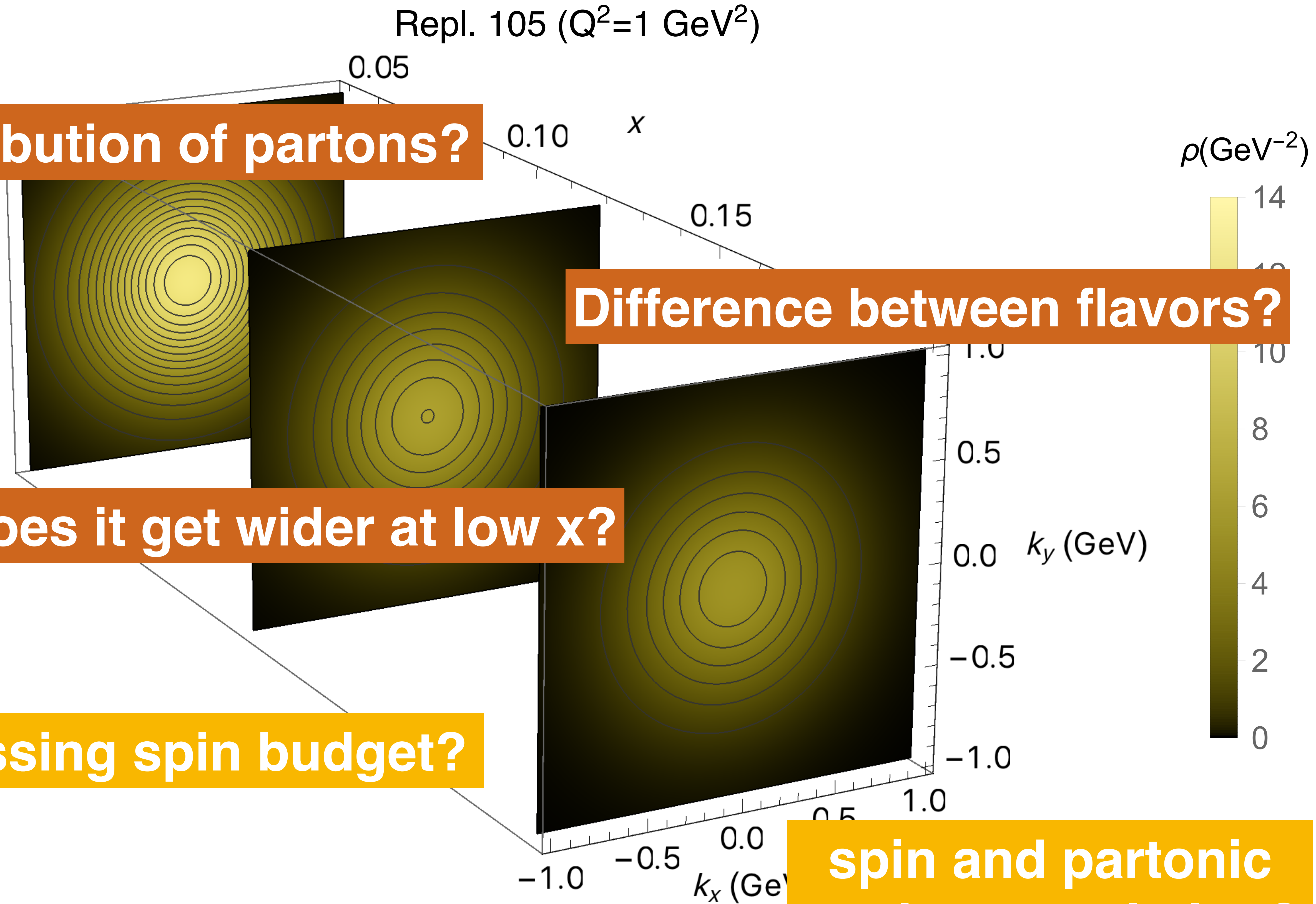
3DSPIN MAPPING THE PROTON IN 3D

3DSPIN: structure of the nucleon



distribution of partons?

Difference between flavors?

Does it get wider at low x?

missing spin budget?

spin and partonic motion correlation?

Transverse Momentum Distributions: TMD PDF

Unpolarized

quark pol.

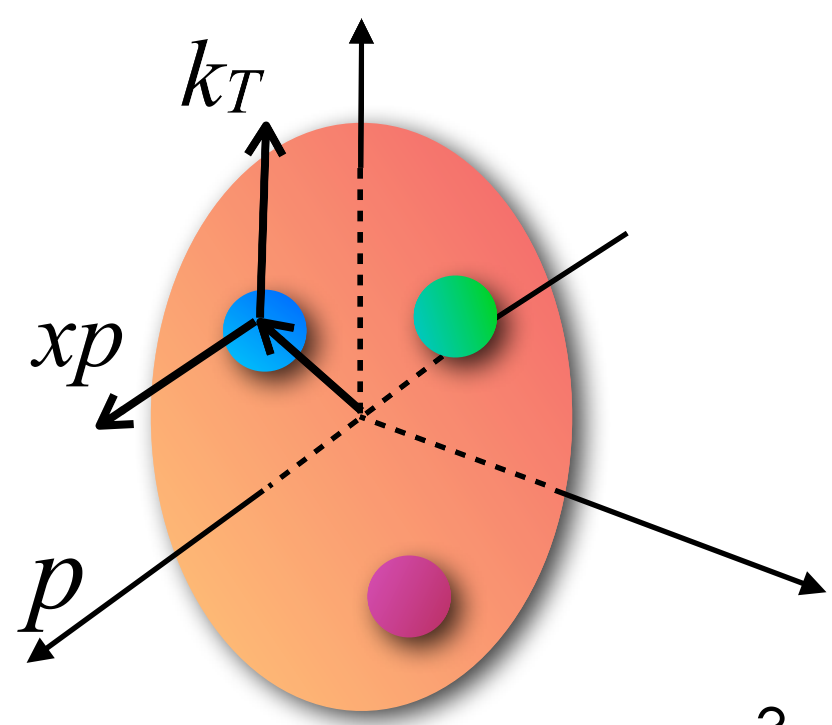
nucleon pol.	U	L	T
U	f_1		$h_1^{\perp}$
L		g_{1L}	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	$h_1, \; h_{1T}^{\perp}$

dependence on:

longitudinal momentum fraction x

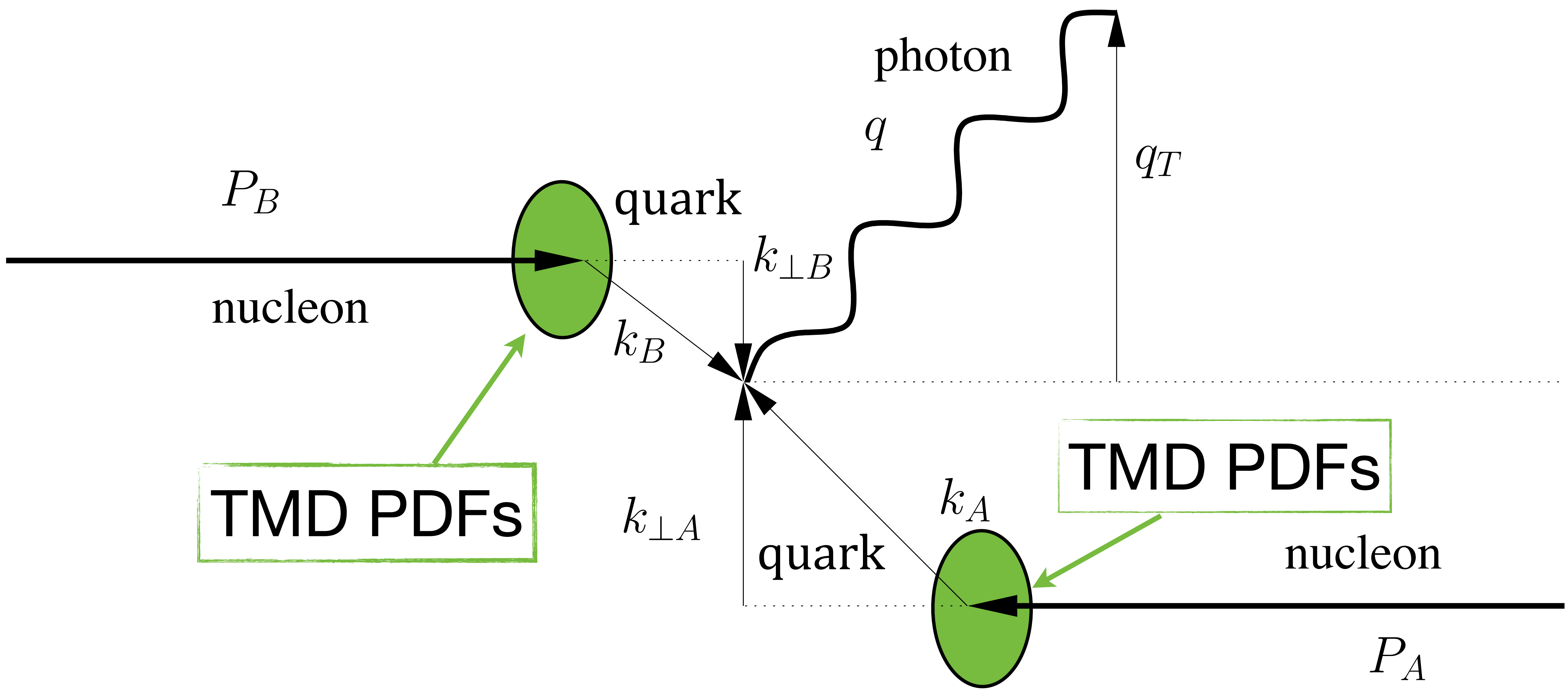
transverse momentum $\boldsymbol{k}_{\perp}$

energy scale



Extraction from SIDIS & Drell-Yan

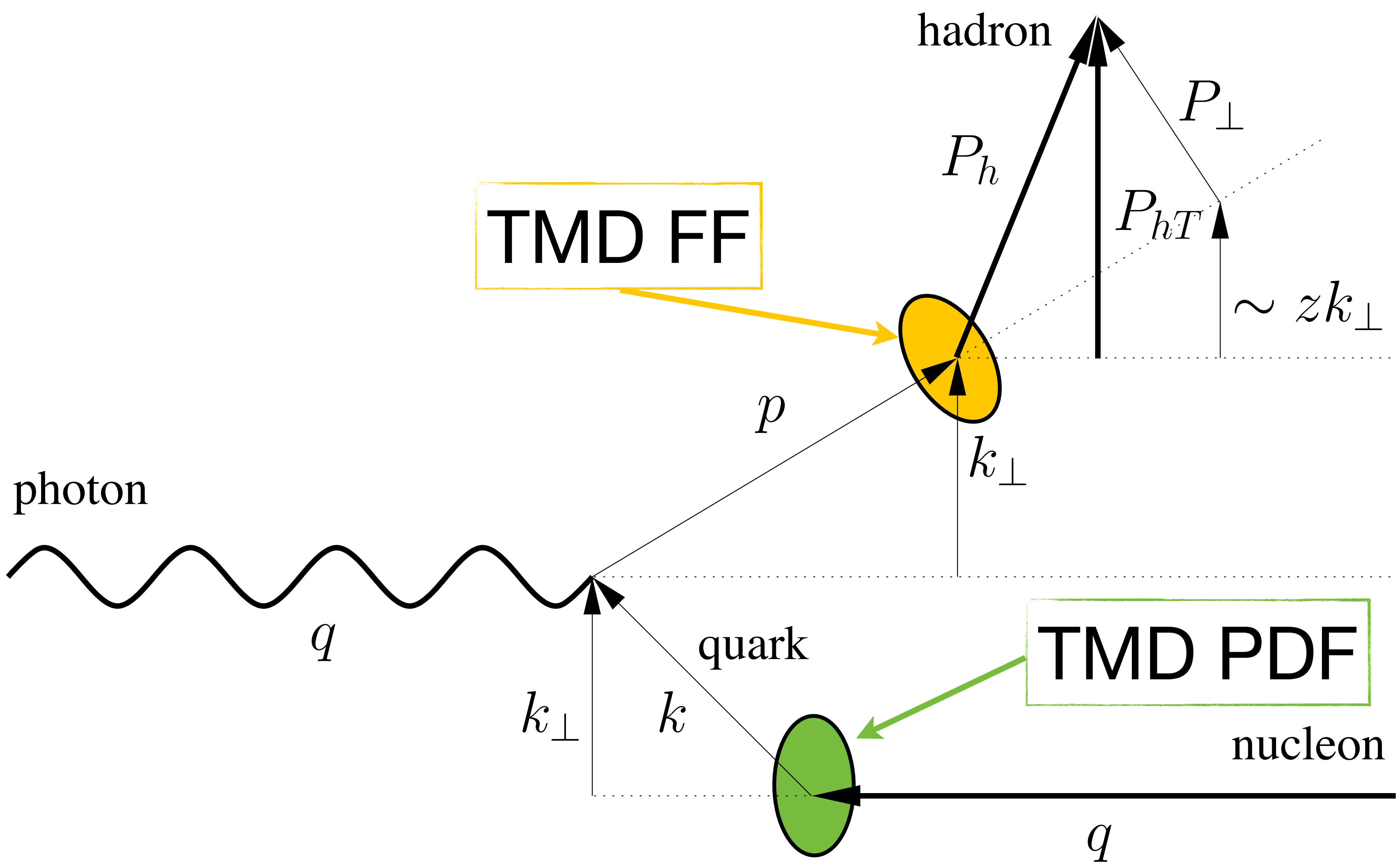
Drell-Yan \ Z production



$$A + B \to \gamma^* \to l^+ l^- \qquad A + B \to Z \to l^+ l^-$$

Extraction from SIDIS & Drell-Yan

Semi-inclusive Deep Inelastic Scattering



universality

$$l(\ell) + N(\mathcal{P}) \to l(\ell') + h(\mathcal{P}_h) + X$$

TMDs: Fragmentation Function

quark pol.

U	L	T
D_1		$H_1^{\perp}$

Unpolarized

TMD Fragmentation Functions
(TMD FFs)

dependence on:

longitudinal momentum fraction z

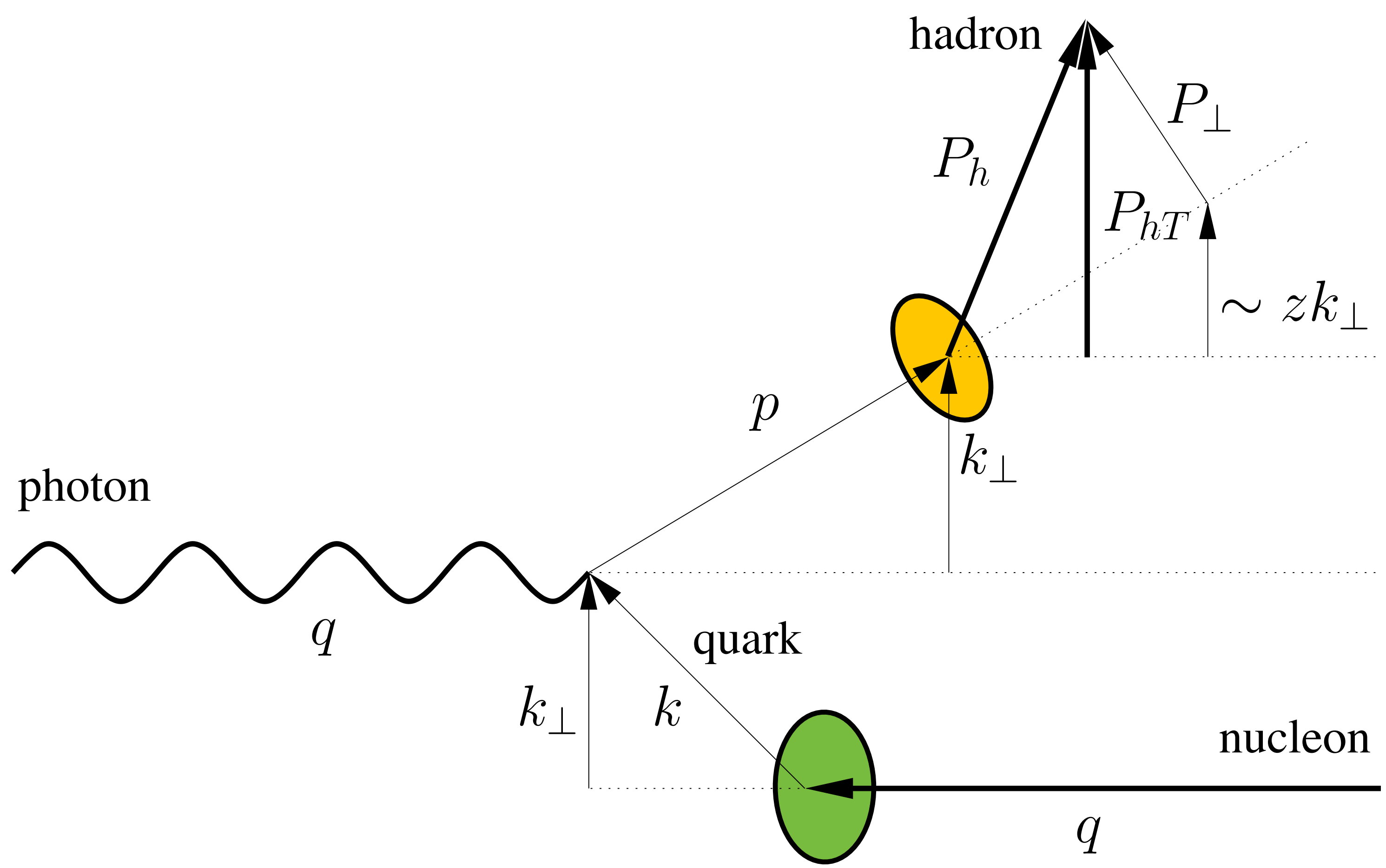
transverse momentum $\boldsymbol{P}_{\perp}$

energy scale

Structure functions and TMDs

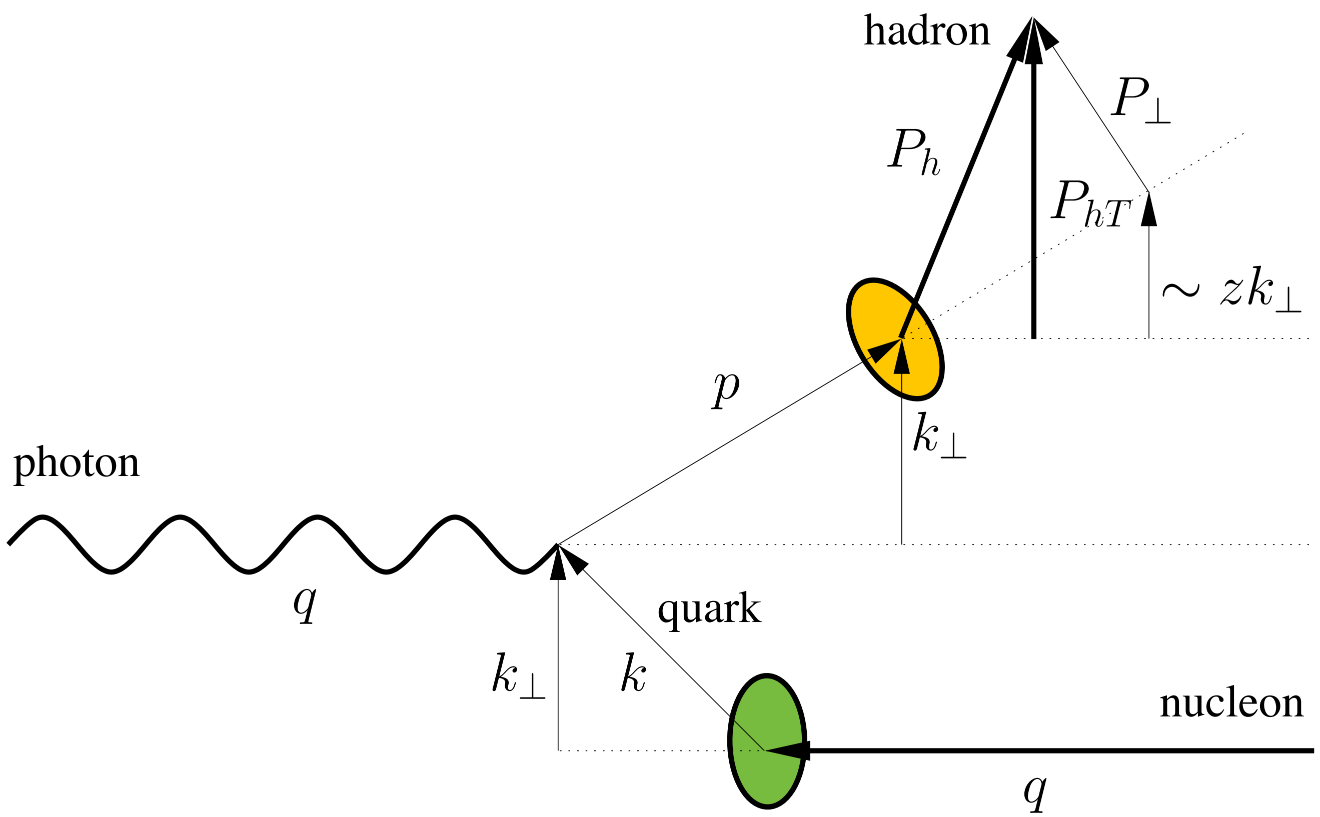
multiplicities

$$m_N^h\left(x,z,\boldsymbol{P}_{hT}^2,Q^2\right)=\frac{d\sigma_N^h/\left(dx\,dz\,d\boldsymbol{P}_{hT}^2dQ^2\right)}{d\sigma_{DIS}/\left(dx\,dQ^2\right)}\approx\frac{\pi F_{UU,T}\left(x,z,\boldsymbol{P}_{hT}^2,Q^2\right)}{F_T(x,Q^2)}$$



$$F_{UU,T}\left(x,z,P_{hT}^2,Q^2\right)=\sum_a \mathcal{H}_{UU,T}^a\left(Q^2;\mu^2\right)\int d^2k_T\,d^2P_T\,f_1^a\left(x,k_T^2;\mu^2\right)D_1^{h/a}\left(z,P_T^2;\mu^2\right)$$
$$\cdot\delta^2(zk_T-P_{hT}+P_T)+Y_{UU,T}(Q^2,P_{hT}^2)+\mathcal{O}\left(M^2/Q^2\right)$$

Structure functions and TMDs



$$F_{UU,T}\left(x,z,P_{hT}^2,Q^2\right)=\sum_a \mathcal{H}^a_{UU,T}\left(Q^2;\mu^2\right)\int d^2k_T\, d^2P_T\, f_1^a\left(x,k_T^2;\mu^2\right) D_1^{h/a}\left(z,P_T^2;\mu^2\right)$$

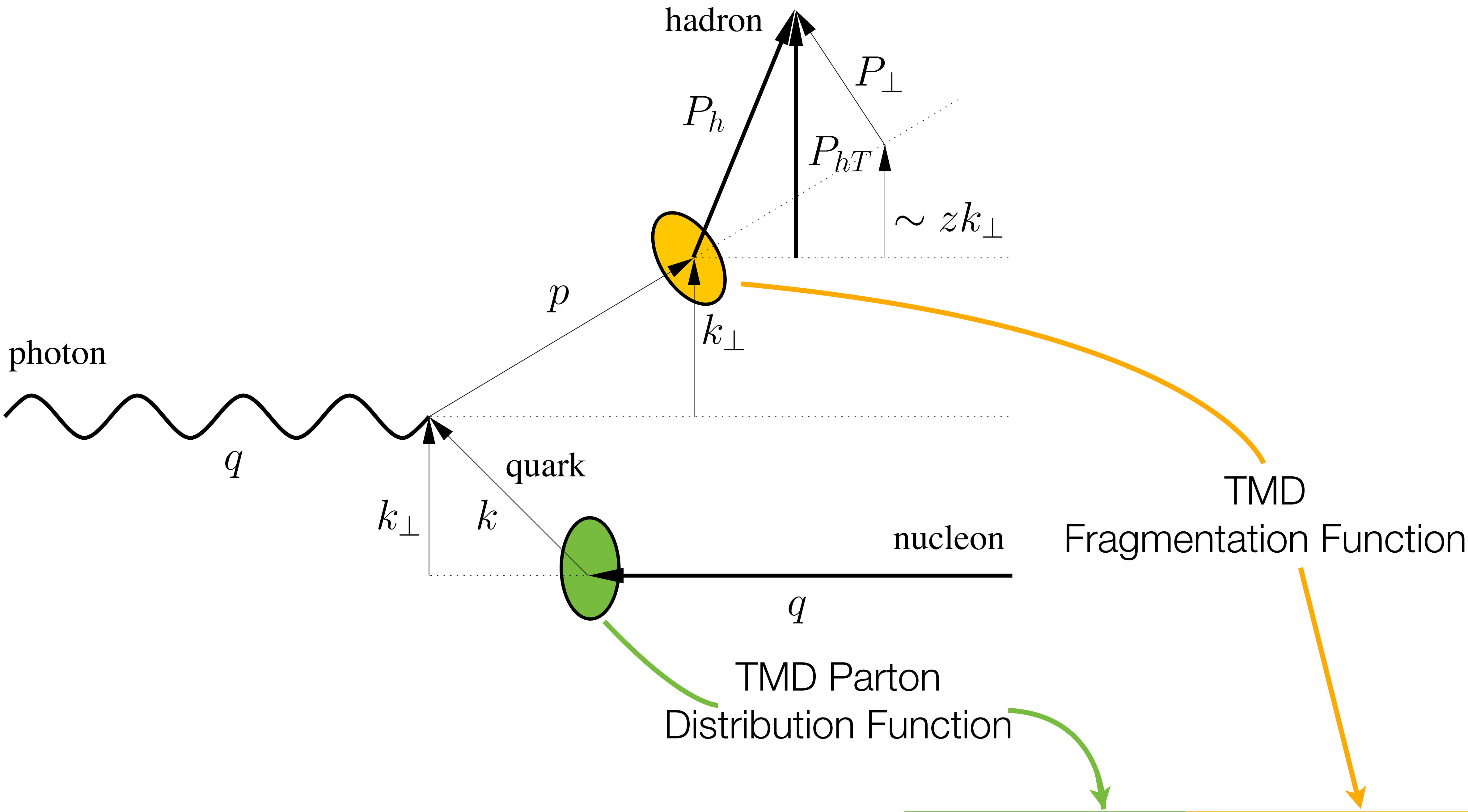
$$\cdot\delta^2(zk_T-P_{hT}+P_T)+Y_{UU,T}(Q^2,P_{hT}^2)+\mathcal{O}\left(M^2/Q^2\right)$$

At our accuracy level (LO-NLL):

$$\mathcal{H}_{UU,T}\simeq\mathcal{O}\left(\alpha_s^0\right)$$

$$Y_{UU,T}\left(Q^2,P_h^2T\right)\simeq 0$$

Structure functions and TMDs

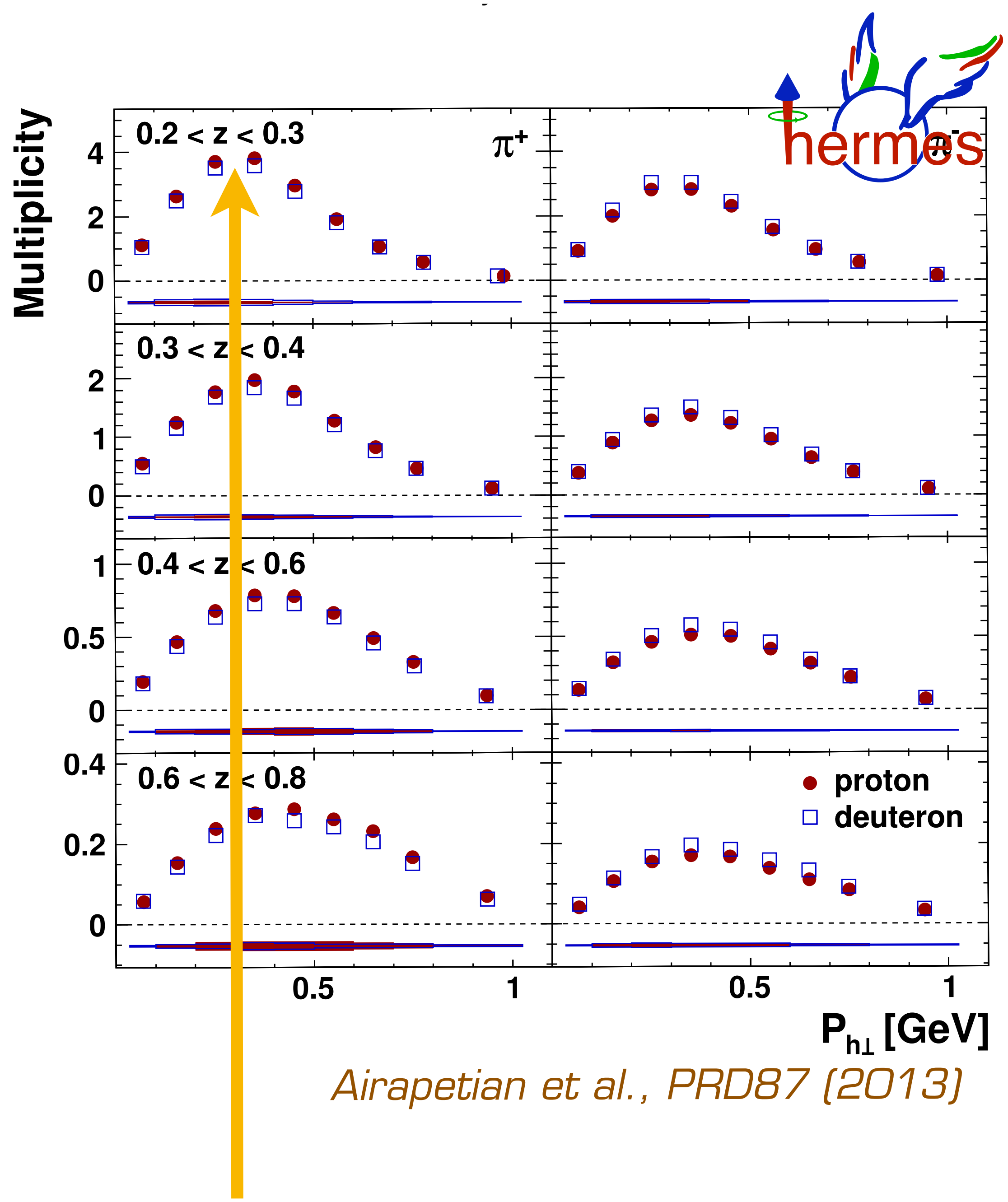


$$F_{UU,T}\left(x, z, P_{hT}^2, Q^2\right) \simeq \sum_a \int d^2 k_T \, d^2 P_T \, f_1^a\left(x, k_T^2; \mu^2\right) D_1^{h/a}\left(z, P_T^2; \mu^2\right)$$

$$\cdot \delta^2 (z k_T - P_{hT} + P_T)$$

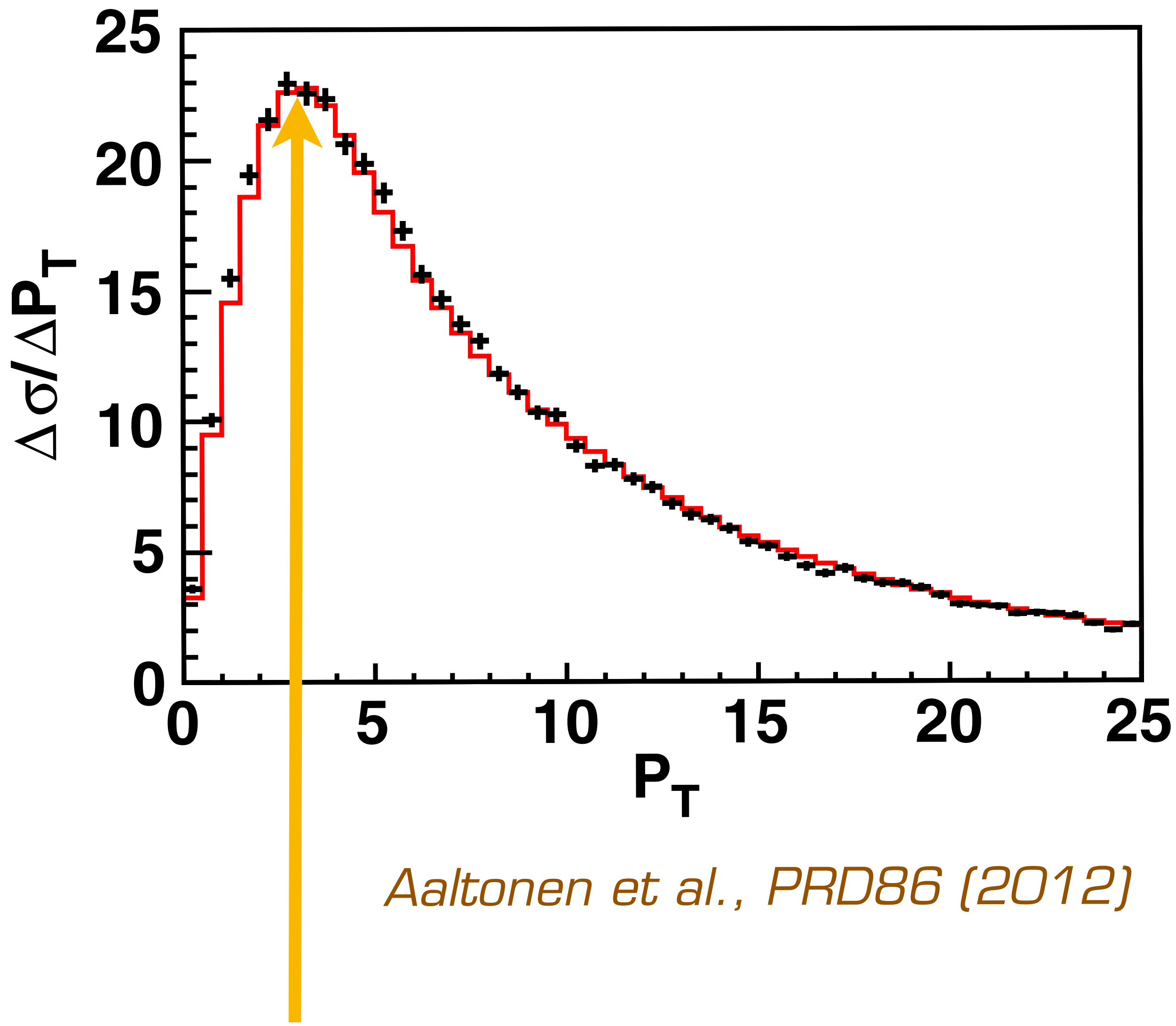
TMD Evolution

HERMES, Q ≈ 1.5 GeV



Airapetian et al., PRD87 (2013)

CDF, Q ≈ 91 GeV



Aaltonen et al., PRD86 (2012)

Width of TMDs changes of one order of magnitude → Evolution

Evolved TMDs

Fourier transform: ξ_T space

$$\tilde{f}_1^a\left(x, \xi_T; \mu^2\right) =$$

$$= \sum_i \left(\tilde{C}_{a/i} \otimes f_1^i\right)\left(x, \bar{\xi}_*; \mu_b\right) e^{\tilde{S}(\bar{\xi}_*;\mu_b,\mu)} e^{g_K(\xi_T)\ln(\mu/\mu_0)} \hat{f}_{NP}^a\left(x, \xi_T\right)$$

nonperturbative part of evolution

(Wilson Coefficient)

(Sudakov form factor)

collinear PDF

pQCD

non-perturbative part of TMD

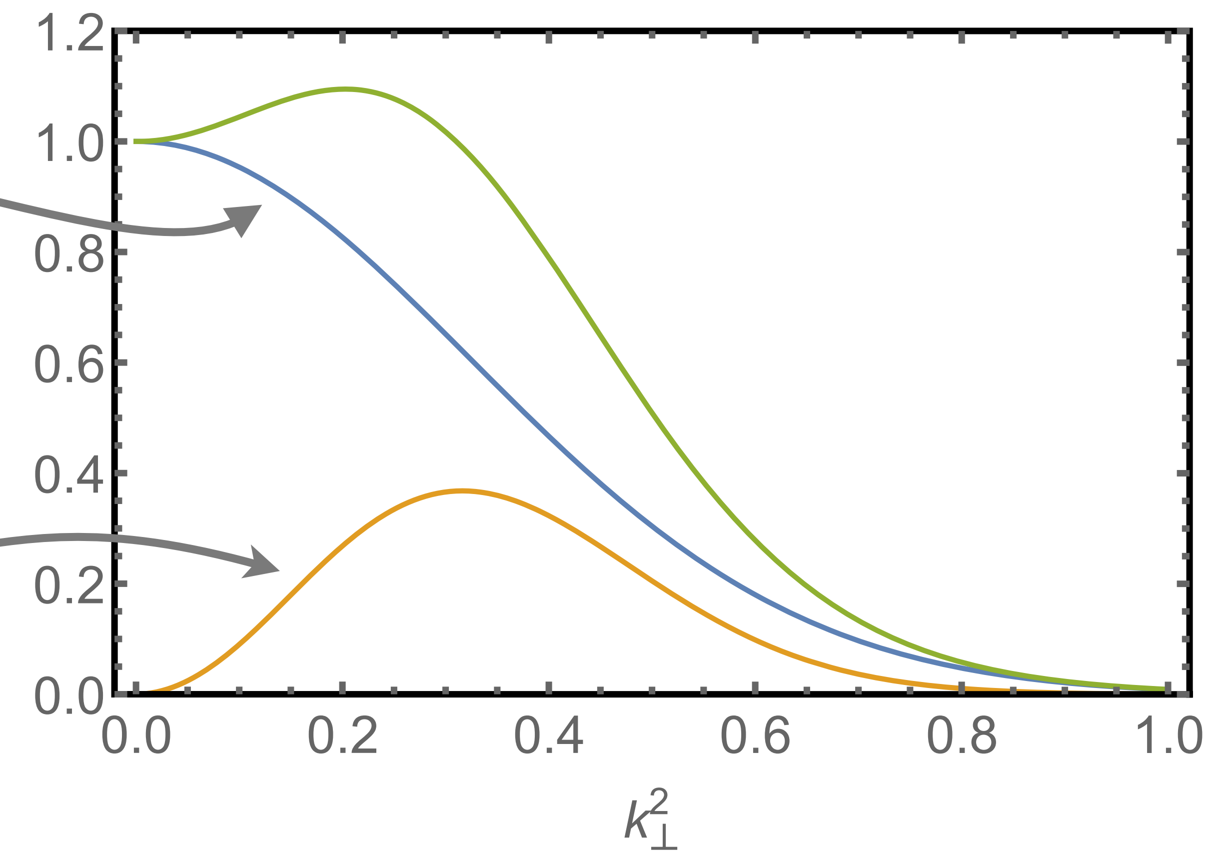
Non-perturbative contributions have to be extracted from experimental data, after parametrization

Model: non perturbative elements

input TMD PDF (Q^2=1GeV^2)

$$\hat{f}^a_{NP} = \mathcal{F}.\mathcal{T}. \text{ of}$$

$$\left(e^{-\frac{k_T^2}{g1a}} + \lambda k_T^2 e^{-\frac{k_T^2}{g1a}} \right)$$



sum of two different gaussians
with kinematic dependence on transverse momenta

width x-dependence

$$g_1(x) = N_1 \frac{(1-x)^\alpha \, x^\sigma}{(1-\hat{x})^\alpha \, \hat{x}^\sigma}$$

where

$$N_1 \equiv g_1(\hat{x})$$
$$\hat{x} = 0.1$$

Model: non perturbative elements

Free parameters

$N_1, \alpha, \sigma, \lambda$ 4 for TMD PDF

$N_3, N_4, \beta, \delta, \gamma, \lambda_F$ 6 for TMD FF

$$g_K = -g_2 \frac{b_T^2}{2}$$

1 for NP contribution to TMD evolution

In total we have 11 parameters, for intrinsic transverse momentum (4 PDFs, 6 FFs) and evolution (g2)

Experimental data

COMPASS

SIDIS μN

6252
data points

E288

E605

Drell-Yan

203
data points

hermes

SIDIS eN

1514
data points

DØ

CDF II

Z Production

90
data points

Data selection and analysis

$Q2 > 1.4$ GeV^2

$0.2 < z < 0.7$

P_{hT} , q_T < Min[0.2Q , 0.7Qz] + 0.5 GeV

Motivations behind kinematical cuts

TMD factorization ($Ph_T/z << Q^2$)

Avoid target fragmentation (low z)

and exclusive contributions (high z)

Experimental data

SIDIS μN

6252
data points

E288

E605

Drell-Yan

203
data points

Total: **8059** data

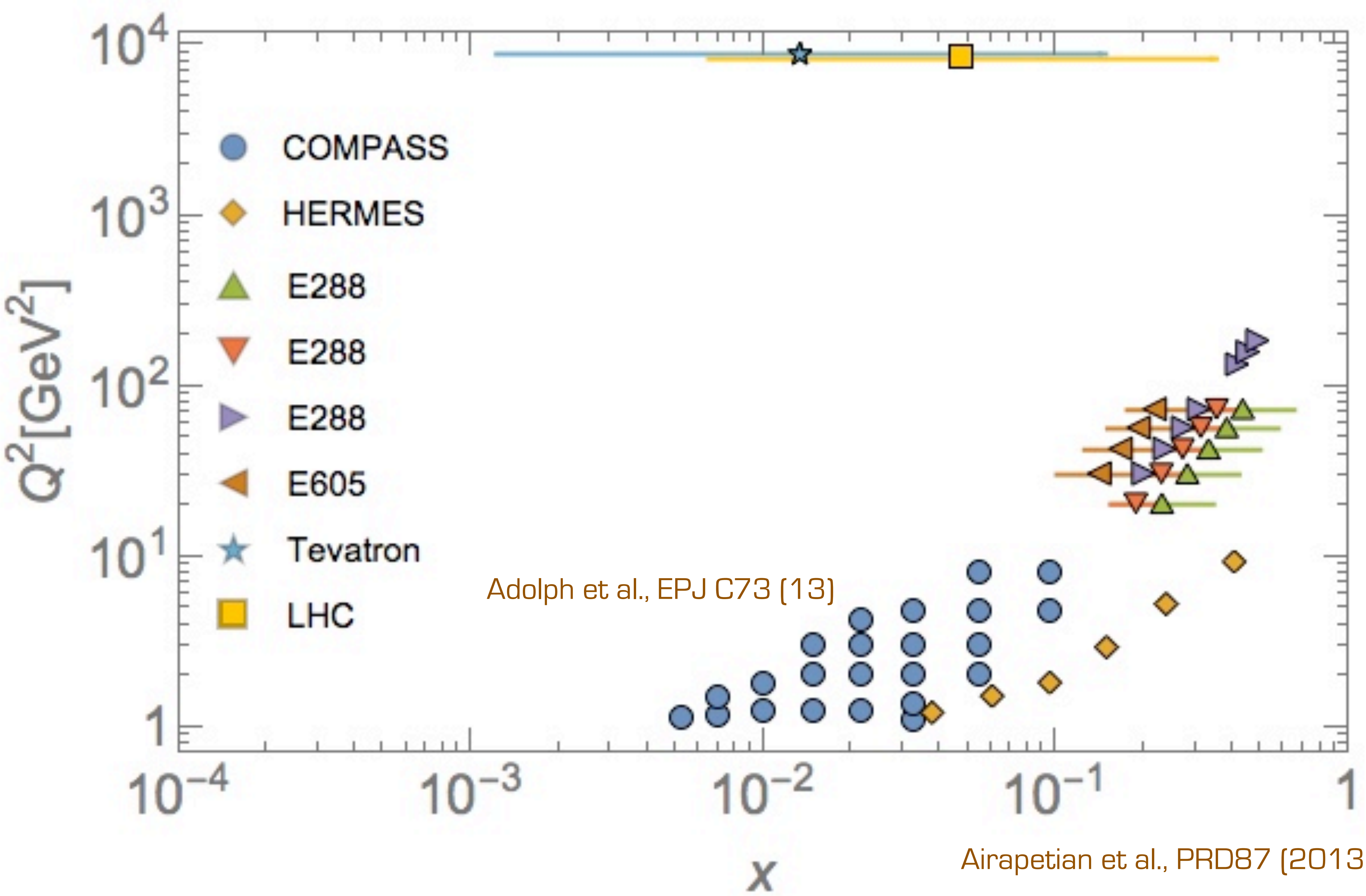
SIDIS eN

1514
data points

Z Production

90
data points

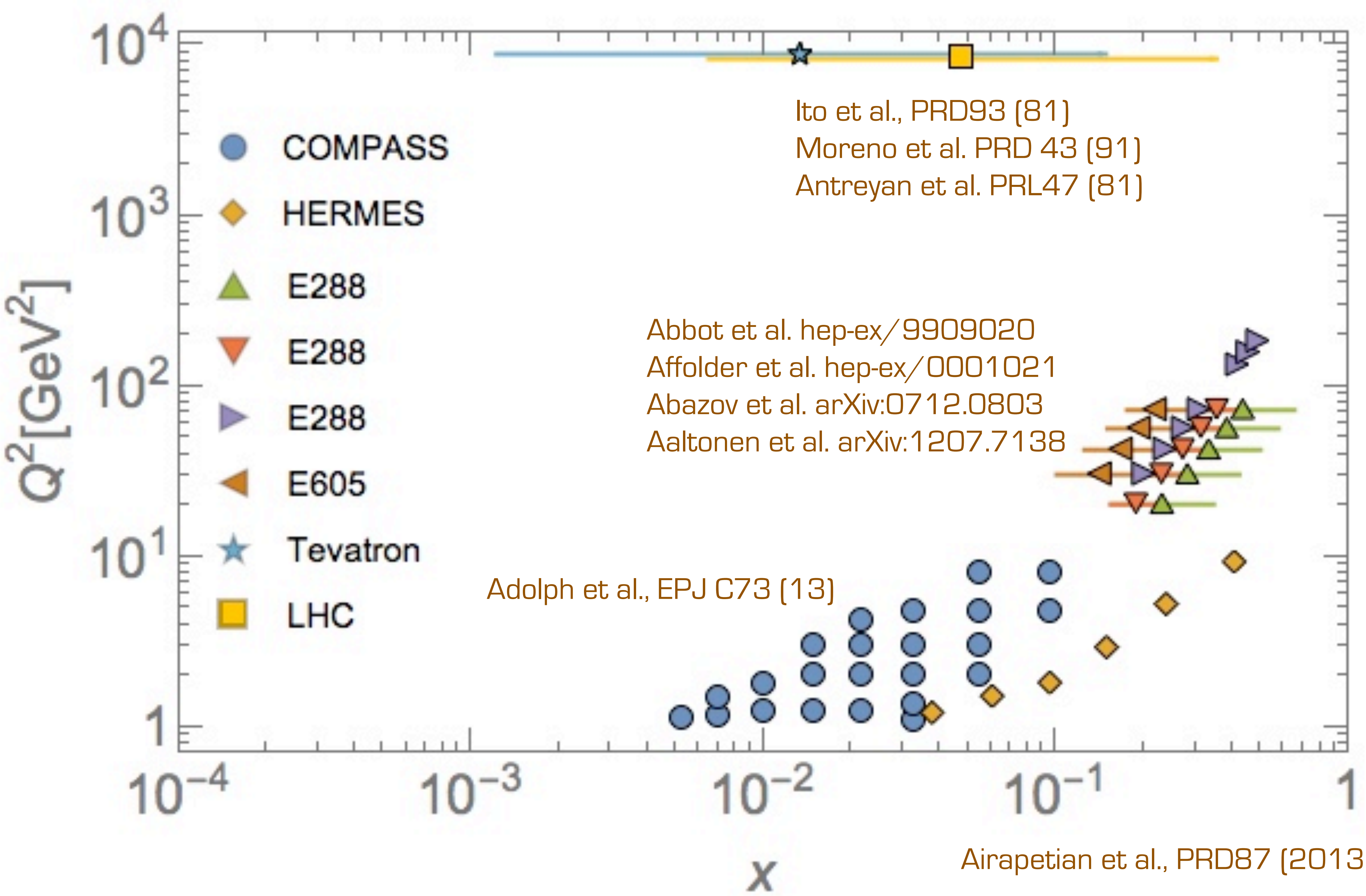
Data region



Adolph et al., EPJ C73 (13)

Airapetian et al., PRD87 (2013)

Data region



10^4
10^3
10^2
10^1
1
Q²[GeV²]
COMPASS
HERMES
E288
E288
E288
E605
Tevatron
LHC
Ito et al., PRD93 (81)
Moreno et al. PRD 43 (91)
Antreyan et al. PRL47 (81)
Abbot et al. hep-ex/9909020
Affolder et al. hep-ex/0001021
Abazov et al. arXiv:0712.0803
Aaltonen et al. arXiv:1207.7138
Adolph et al., EPJ C73 (13)
10^-4
10^-3
10^-2
10^-1
1
x
Airapetian et al., PRD87 (2013)

An almost global fit

	Framework	HERMES	COMPASS	DY	Z production	N of points
Pavia 2017 (+ JLab)	LO-NLL	✔	✔	✔	✔	8059

[JHEP06(2017)081]

Summary of results

Total number of data points: 8059

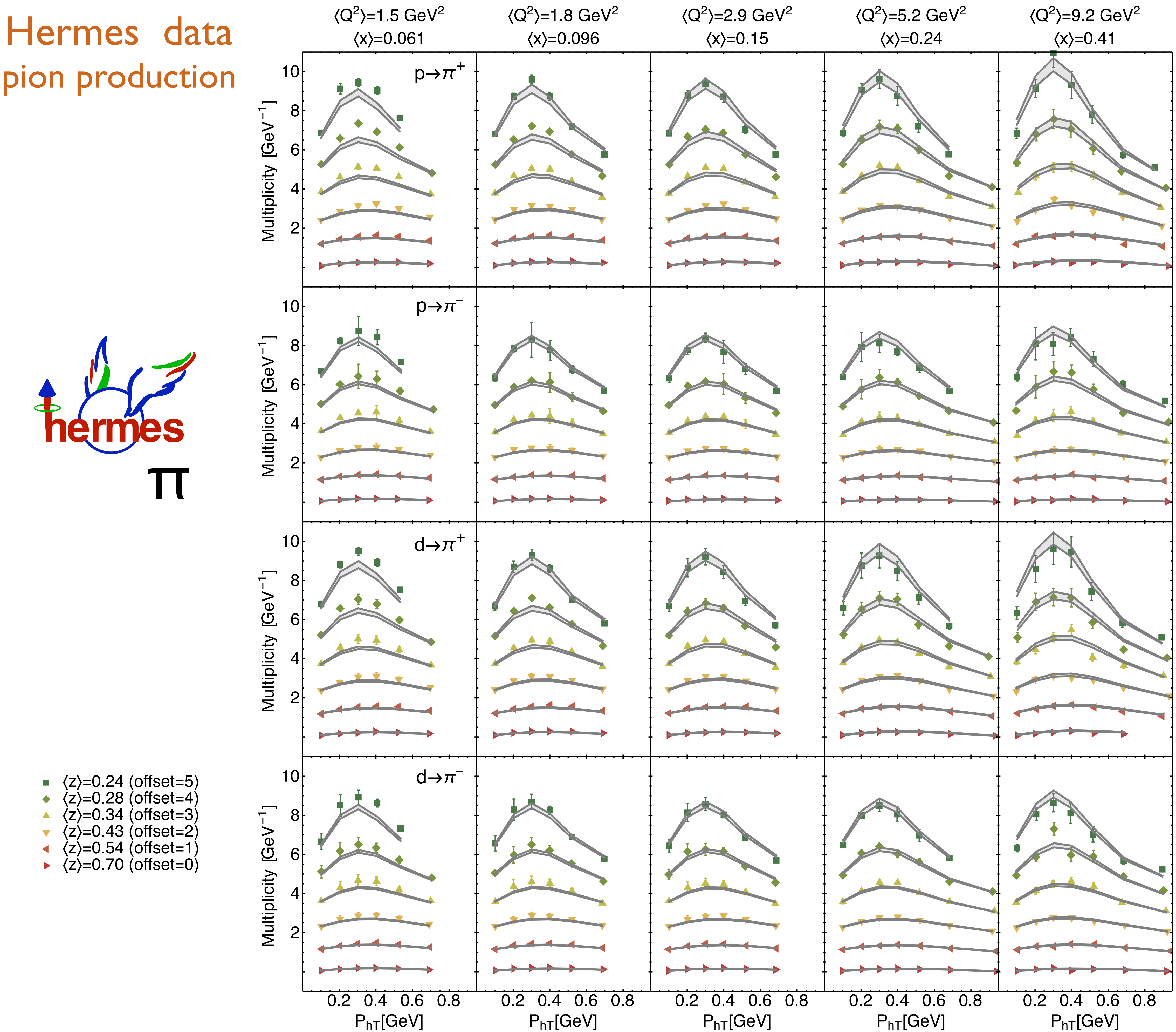
Total number of free parameters: 11

➛ 4 for TMD PDFs ➛ 6 for TMD FFs

➛ 1 for TMD evolution

$$\chi^2/d.o.f. = 1.55 \pm 0.05$$

Hermes data
pion production



π

χ^2/dof

4.83

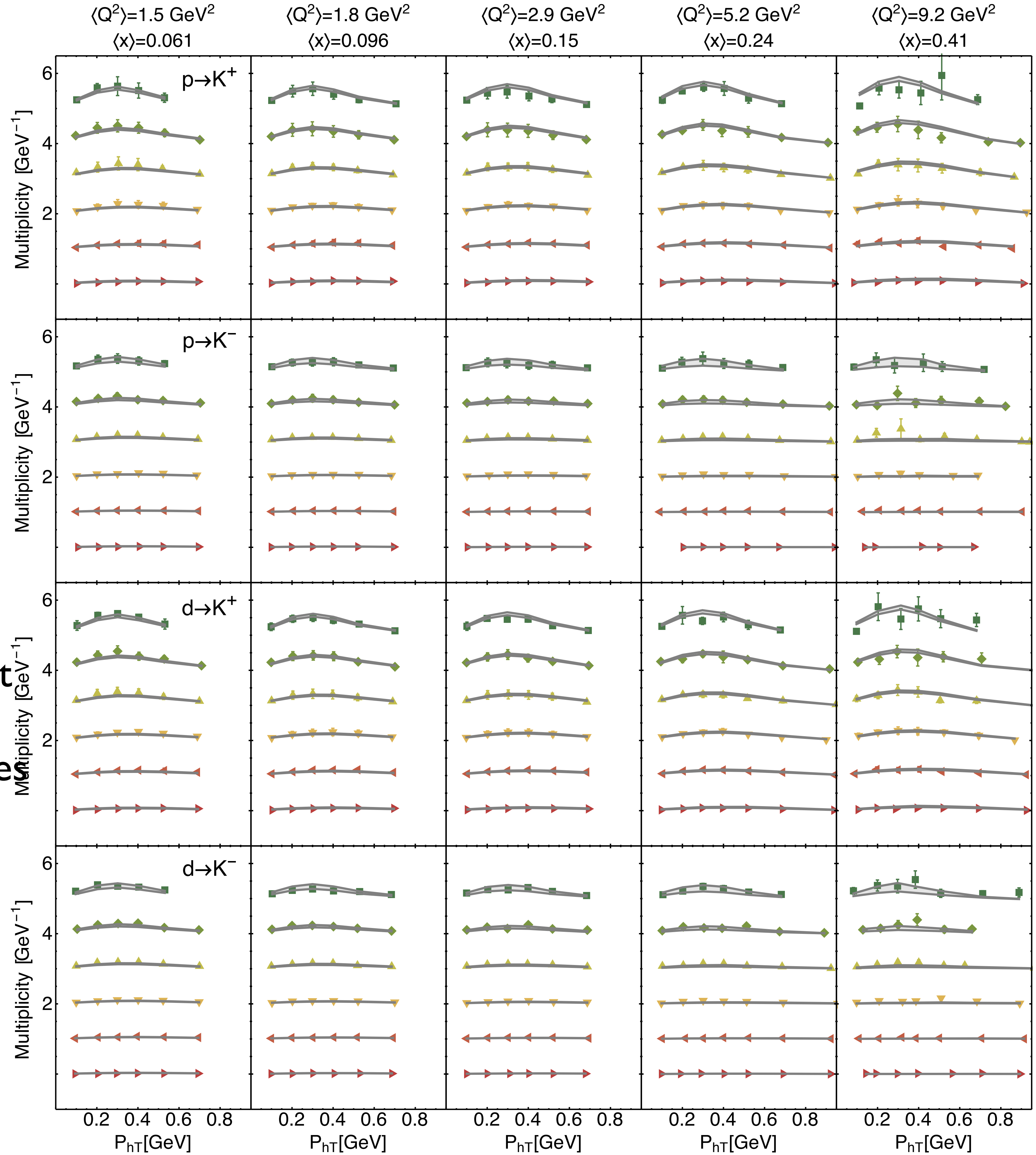
2.47

3.46

2.00

Hermes data
kaon production

K



better agreement than pions: larger uncertainties form FFs

χ^2/dof

0.91

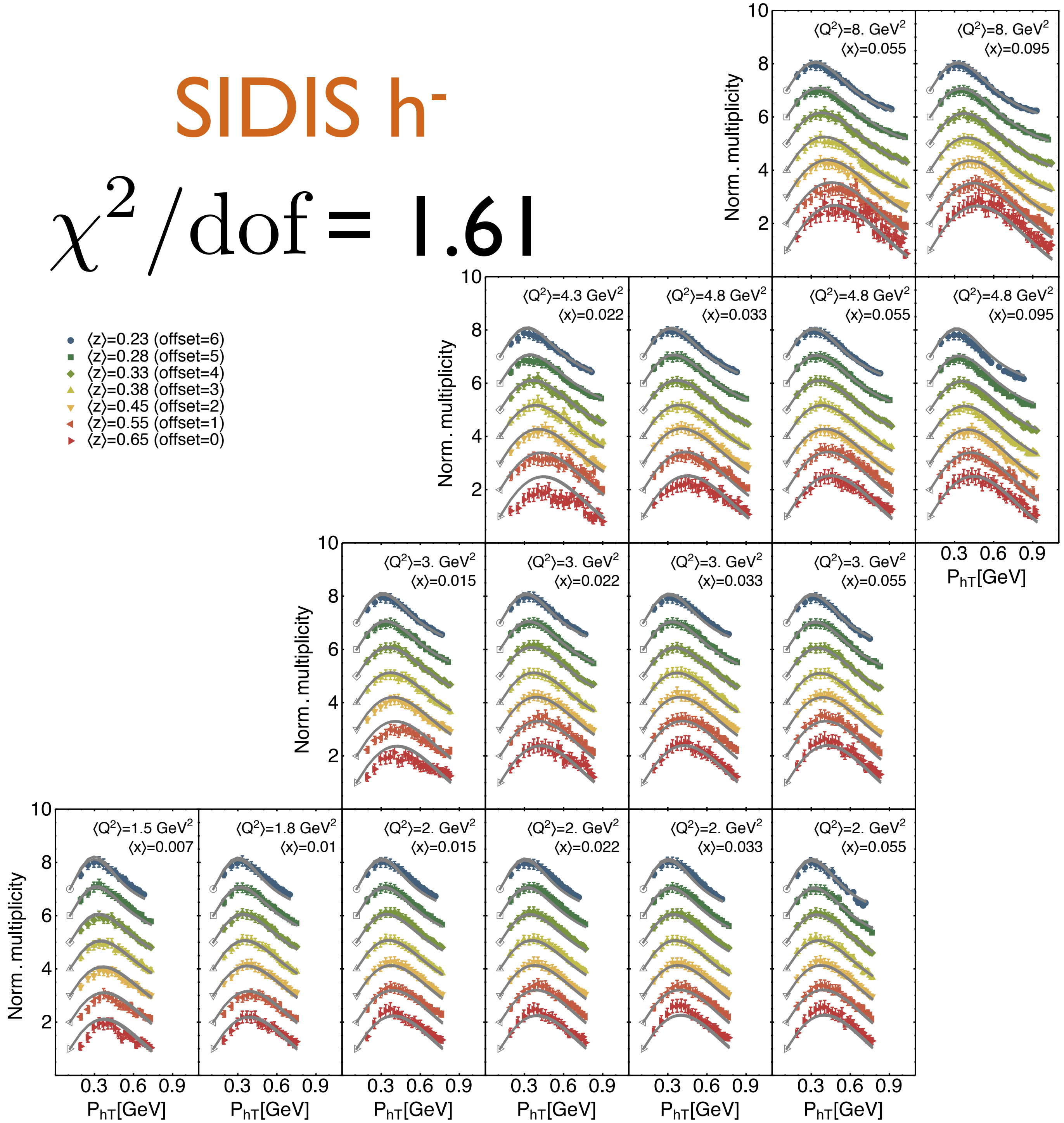
0.82

1.31

2.54

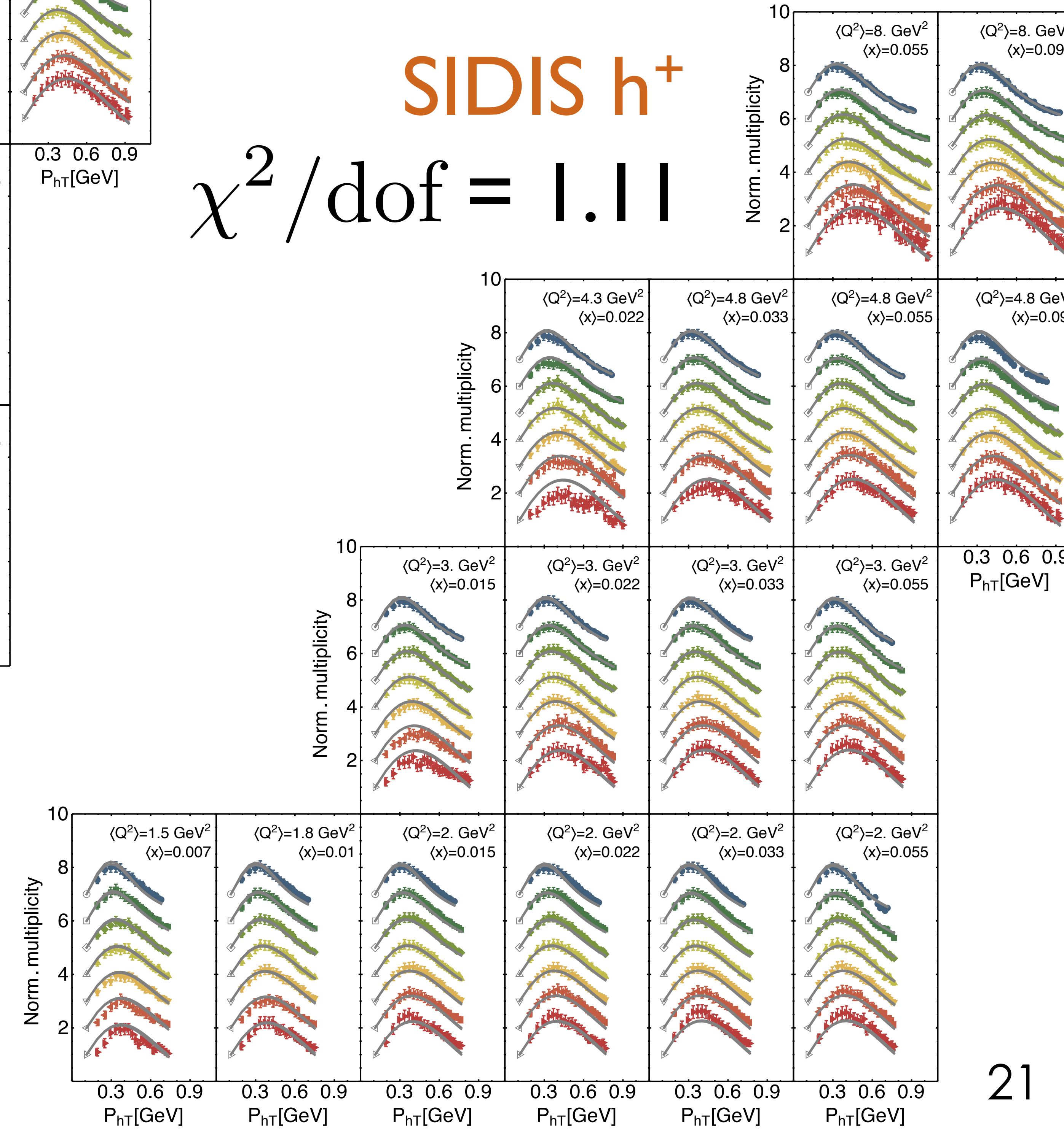
SIDIS h^-

$$\chi^2/\mathrm{dof} = 1.61$$

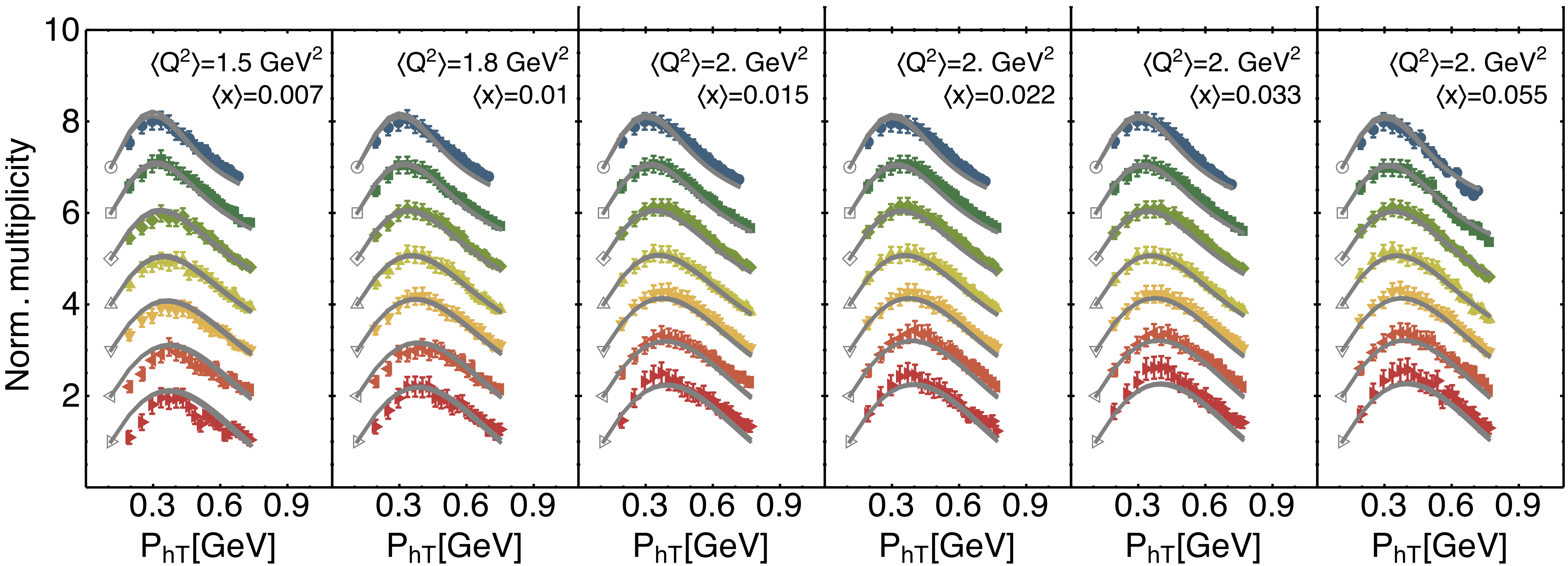


SIDIS h^+

$$\chi^2/\mathrm{dof} = 1.11$$



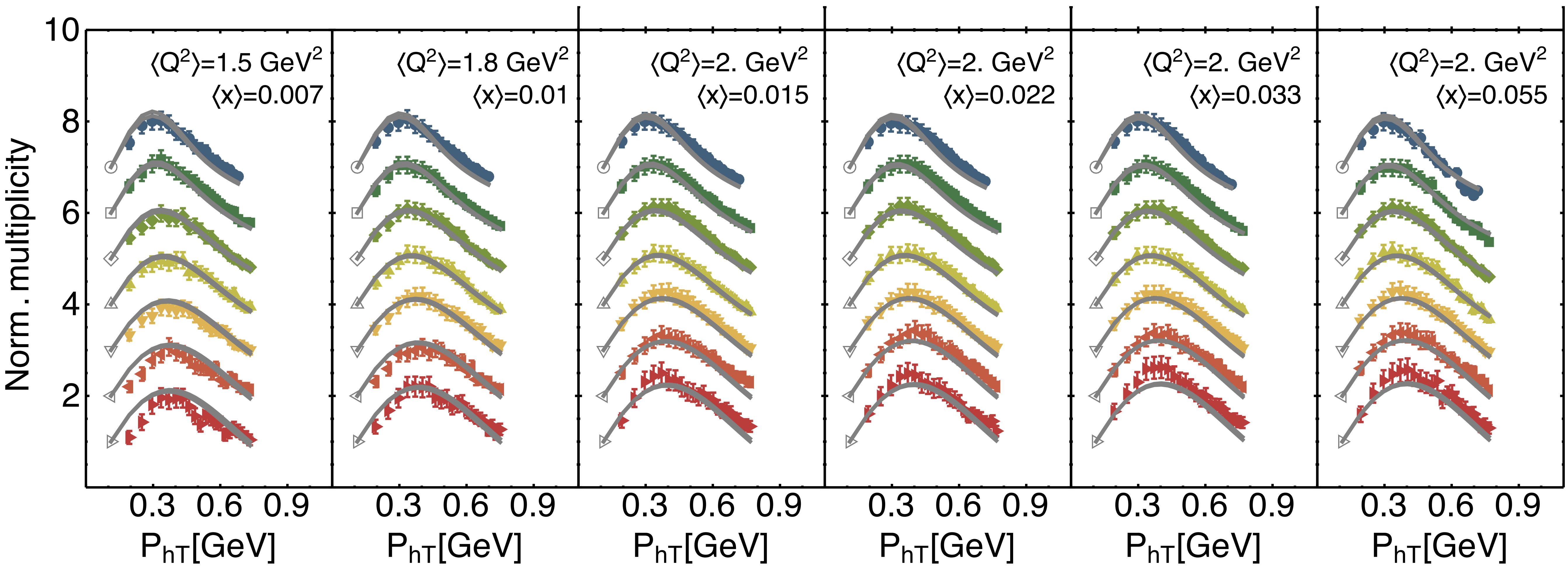
COMPASS data
SIDIS h^+



to avoid known problems
with Compass data normalization:

Observable $\dfrac{m_N^h\left(x, z, \boldsymbol{P}_{hT}^2, Q^2\right)}{m_N^h\left(x, z, \min[\boldsymbol{P}_{hT}^2], Q^2\right)}$

COMPASS data
SIDIS h^+

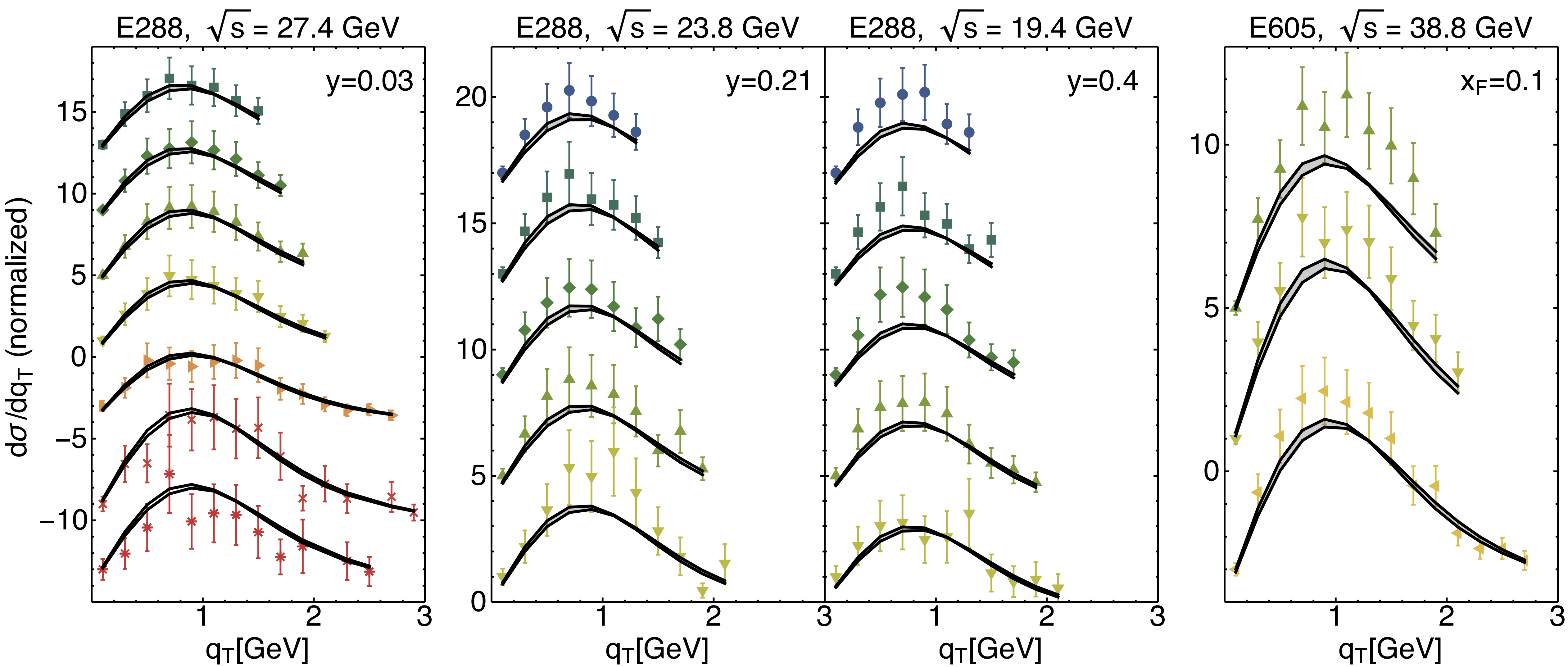


Revised Data:
[Phys.Rev. D97 (2018) no.3, 032006]

Observable: $$\frac{m_N^h\left(x,z,\boldsymbol{P}_{hT}^2,Q^2\right)}{m_N^h\left(x,z,\min[\boldsymbol{P}_{hT}^2],Q^2\right)}$$

Drell-Yan data

χ^2/dof **0.32** **0.84** **0.99** **1.12**



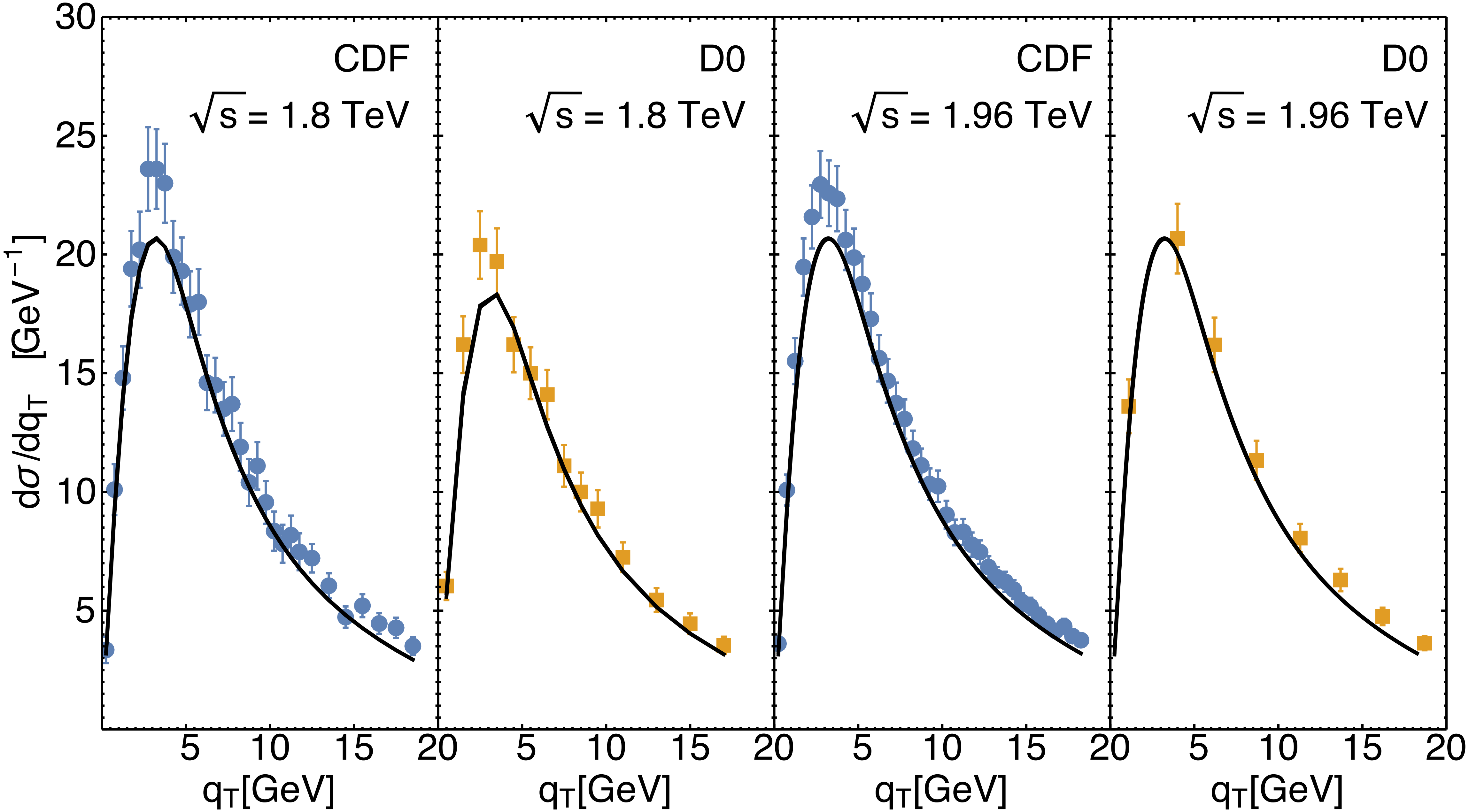
Fermilab

Q² Evolution: The peak is now at about 1 GeV, it was at 0.4 GeV for SIDIS

Z-boson production data

normalization : fixed from DEMS fit, different from exp. (not really relevant for TMD parametrizations)

χ^2/dof 1.36 1.11 2.00 1.73



Q[2] Evolution: The peak is now at about 4 GeV

Stability of our results

Test of our default choices

How does the χ^2 of a single replica change if we modify them?

Original χ^2/dof = 1.51

Normalization of HERMES data as done for COMPASS:
χ^2/dof = 1.27

Parametrizations for collinear PDFs (NLO GJR 2008 default choice):
NLO MSTW 2008 (1.84), NLO CJ12 (1.85)

More stringent cuts (TMD factorization better under control)
χ^2/dof $\rightarrow$ 1
Ex: $Q2 > 1.5$ GeV^2; $0.25 < z < 0.6$; $PhT < 0.2Qz \Rightarrow \chi^2$/dof = 1.02 (477 bins)

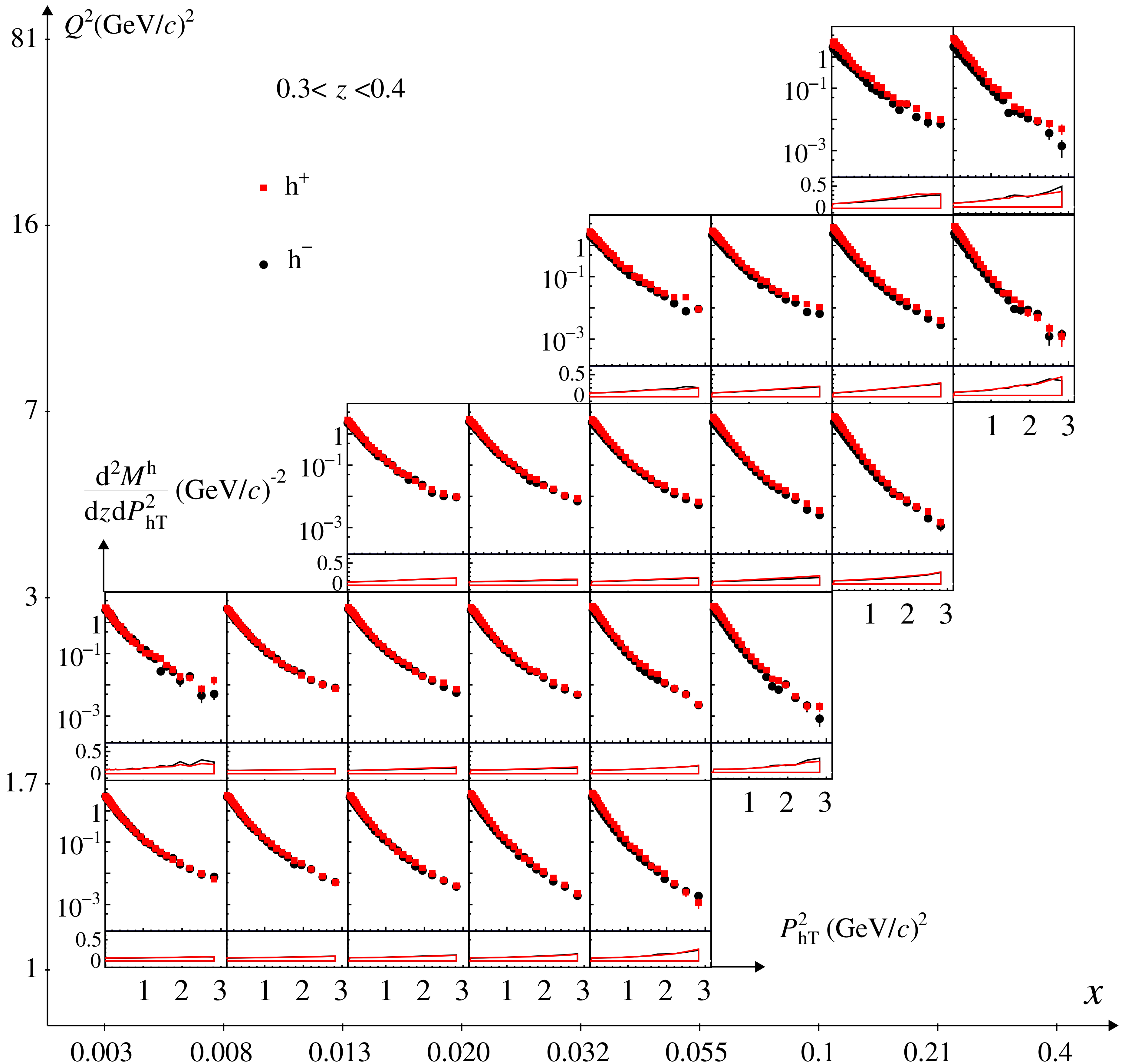
Analysis of revised SIDIS data from COMPASS

[Phys.Rev. D97 (2018) no.3, 032006]

3DSPIN
MAPPING
THE PROTON IN 3D

COMPASS

Revised Compass Data



Revised Compass Data: binning

[Eur. Phys. J. C (2013) 73:2531]

Bin	x_{bj}^{min}	x_{bj}^{max}	$\langle x_{bj} \rangle$	Q^2_{min}	Q^2_{max}	$\langle Q^2 \rangle$
1	0.0045	0.0060	0.0052	1.0	1.25	1.11
2	0.0060	0.0080	0.0070	1.0	1.30	1.14
3	0.0060	0.0080	0.0070	1.3	1.70	1.48
4	0.0080	0.0120	0.0099	1.0	1.50	1.22
5	0.0080	0.0120	0.0099	1.5	2.10	1.76
6	0.0120	0.0180	0.0148	1.0	1.50	1.22
7	0.0120	0.0180	0.0148	1.5	2.50	1.92
8	0.0120	0.0180	0.0150	2.5	3.50	2.90
9	0.0180	0.0250	0.0213	1.0	1.50	1.23
10	0.0180	0.0250	0.0213	1.5	2.50	1.92
11	0.0180	0.0250	0.0213	2.5	3.50	2.94
12	0.0180	0.0250	0.0216	3.5	5.00	4.07
13	0.0250	0.0350	0.0295	1.0	1.20	1.10
14	0.0250	0.0400	0.0316	1.2	1.50	1.34
15	0.0250	0.0400	0.0318	1.5	2.50	1.92
16	0.0250	0.0400	0.0319	2.5	3.50	2.95
17	0.0250	0.0400	0.0323	3.5	6.00	4.47
18	0.0400	0.0500	0.0447	1.5	2.50	1.93
19	0.0400	0.0700	0.0533	2.5	3.50	2.95
20	0.0400	0.0700	0.0536	3.5	6.00	4.57
21	0.0400	0.0700	0.0550	6.0	10.0	7.36
22	0.0700	0.1200	0.0921	3.5	6.00	4.62
23	0.0700	0.1200	0.0932	6.0	10.0	7.57

z bins	$\rightarrow$						
0.2÷0.25	0.25÷0.3	0.3÷0.35	0.35÷0.4	0.4÷0.5	0.5÷0.6	0.6÷0.7	0.7÷0.8

[Phys.Rev. D97 (2018) no.3, 032006]

TABLE I. Bin limits for the four-dimensional binning in x, Q^2, z and P^2_{hT}.

	Bin limits								
x	0.003	0.008	0.013	0.02	0.032	0.055	0.1	0.21	0.4
Q^2 $(\text{GeV}/c)^2$	1.0	1.7	3.0	7.0	16	81			
z	0.2	0.3	0.4	0.6	0.8				
P^2_{hT} $(\text{GeV}/c)^2$	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.17	0.196
	0.23	0.27	0.30	0.35	0.40	0.46	0.52	0.60	0.68
	0.76	0.87	1.00	1.12	1.24	1.38	1.52	1.68	1.85
	2.05	2.35	2.65	3.00					

Number of experimental data

Same kinematical cuts in x, Q^2, z, P_{hT}

Same data for DY	203
Z	90
SIDIS eN	1514

Total: 3931 data

COMPASS

SIDIS μN

2124

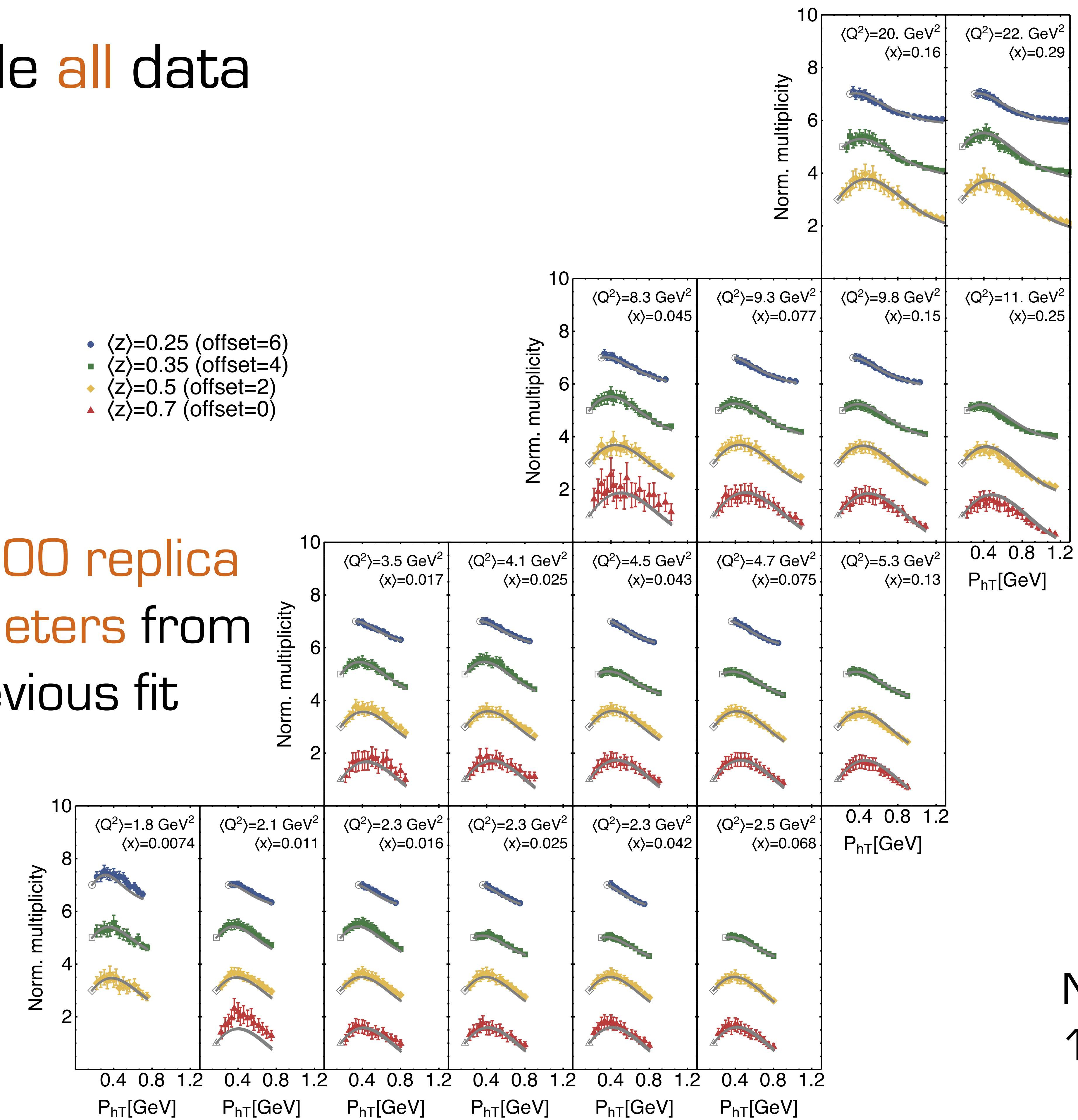
data points

Preliminary results

Include all data

SIDIS h^+

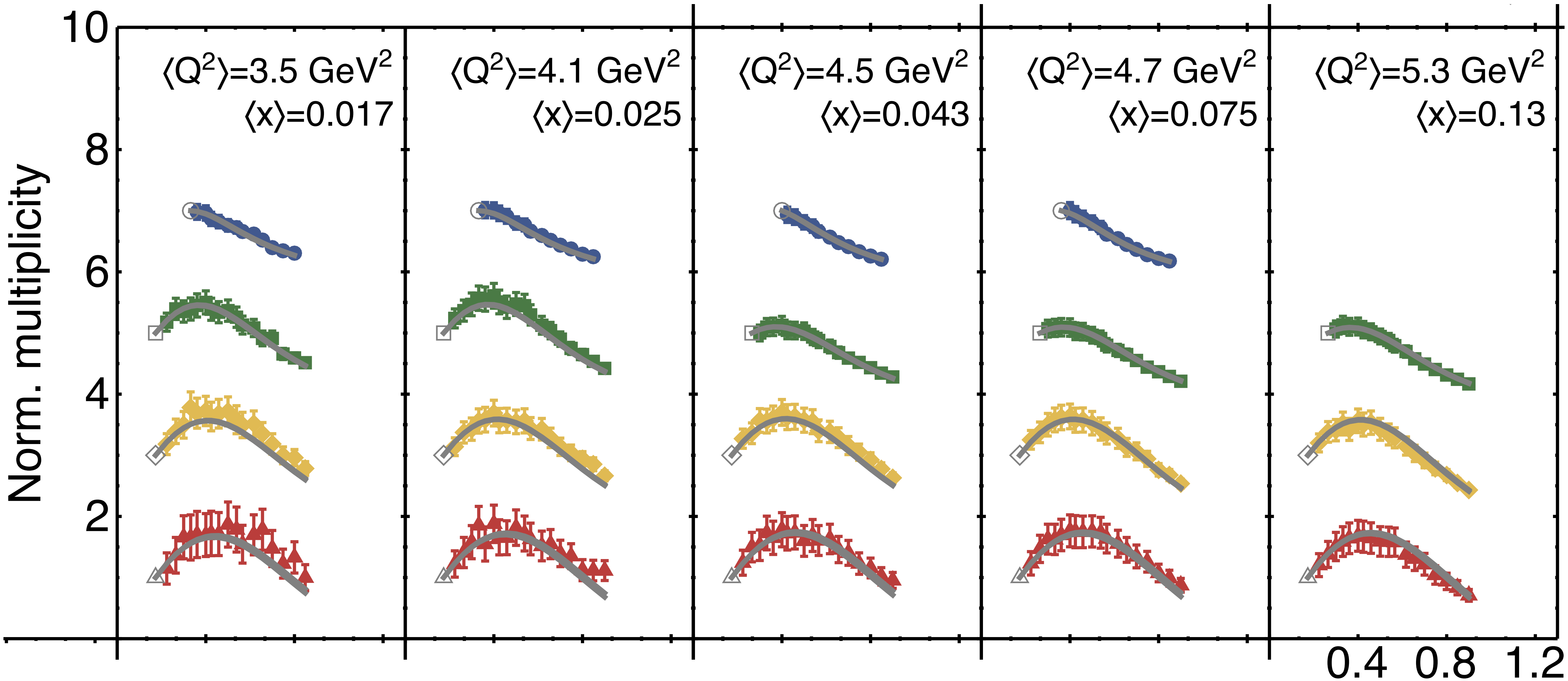
Use 200 replica parameters from previous fit



Normalized at 1st data point of bin

Include all data

SIDIS h^+

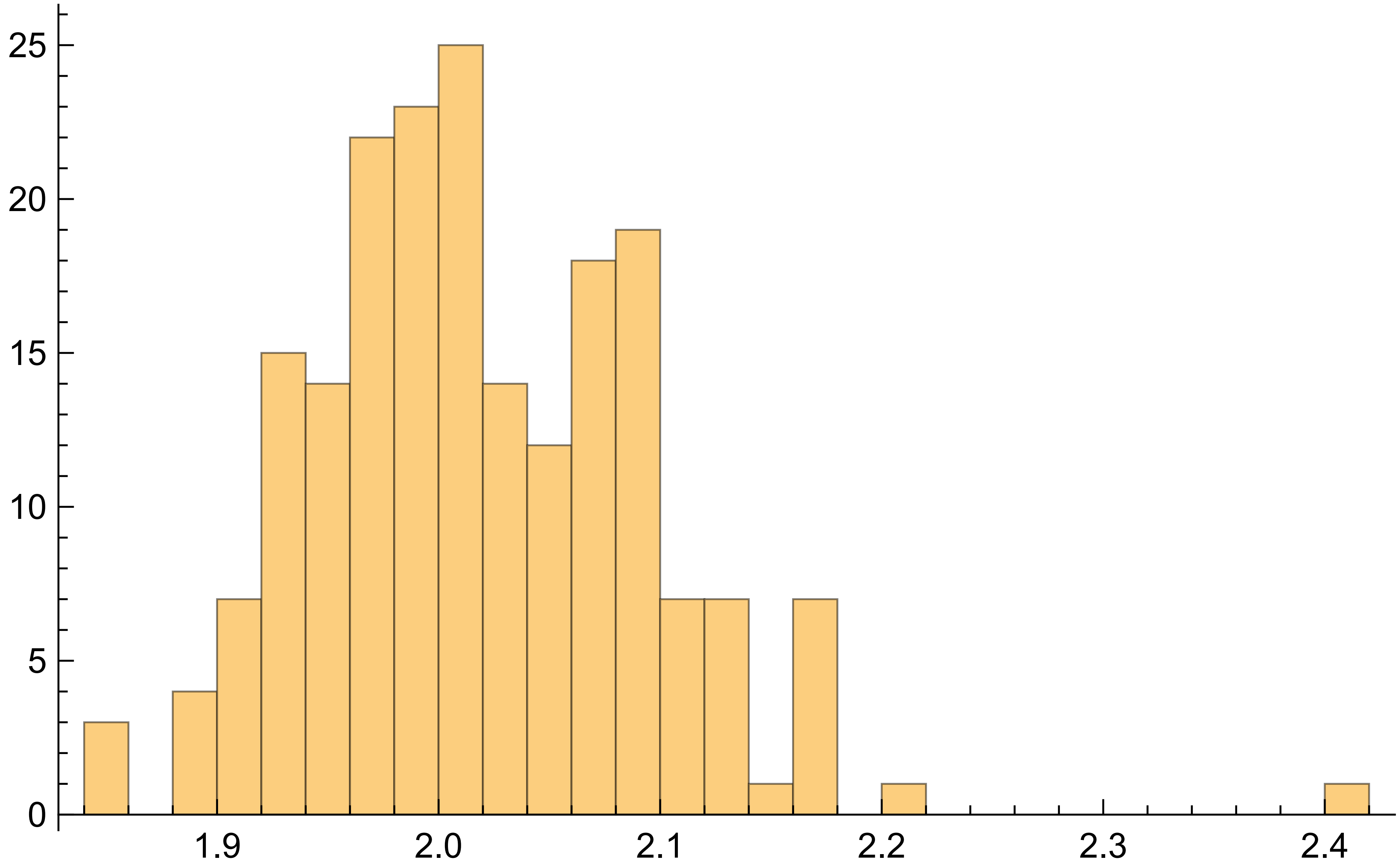


Use 200 replica parameters from previous fit

Normalized at 1st data point of bin

Include all data

SIDIS h^+



Use 200 replica parameters from previous fit

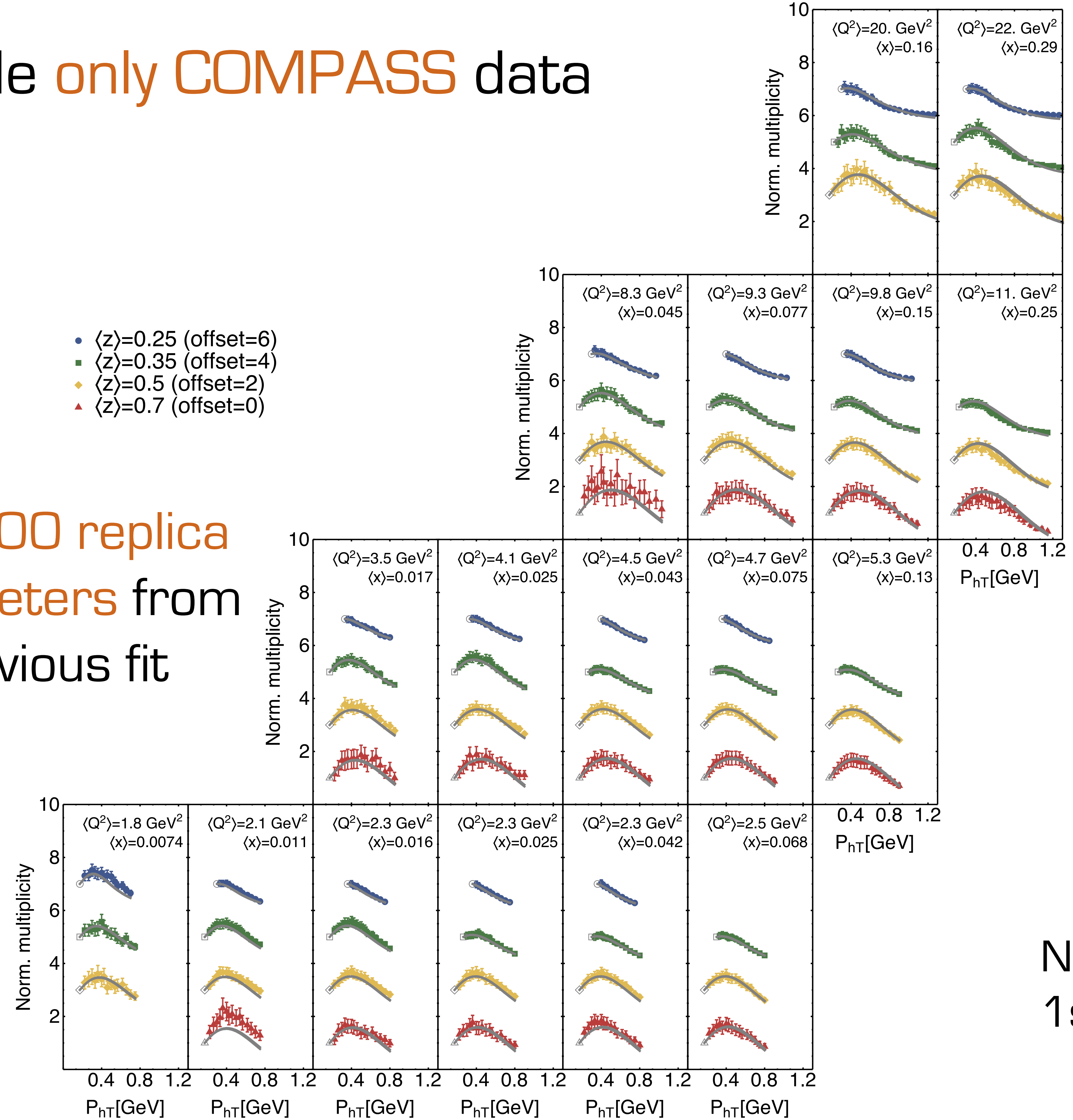
$\chi^2/\text{dof} = 2.01$

Normalized at 1st data point of bin

Include only COMPASS data

SIDIS h^+

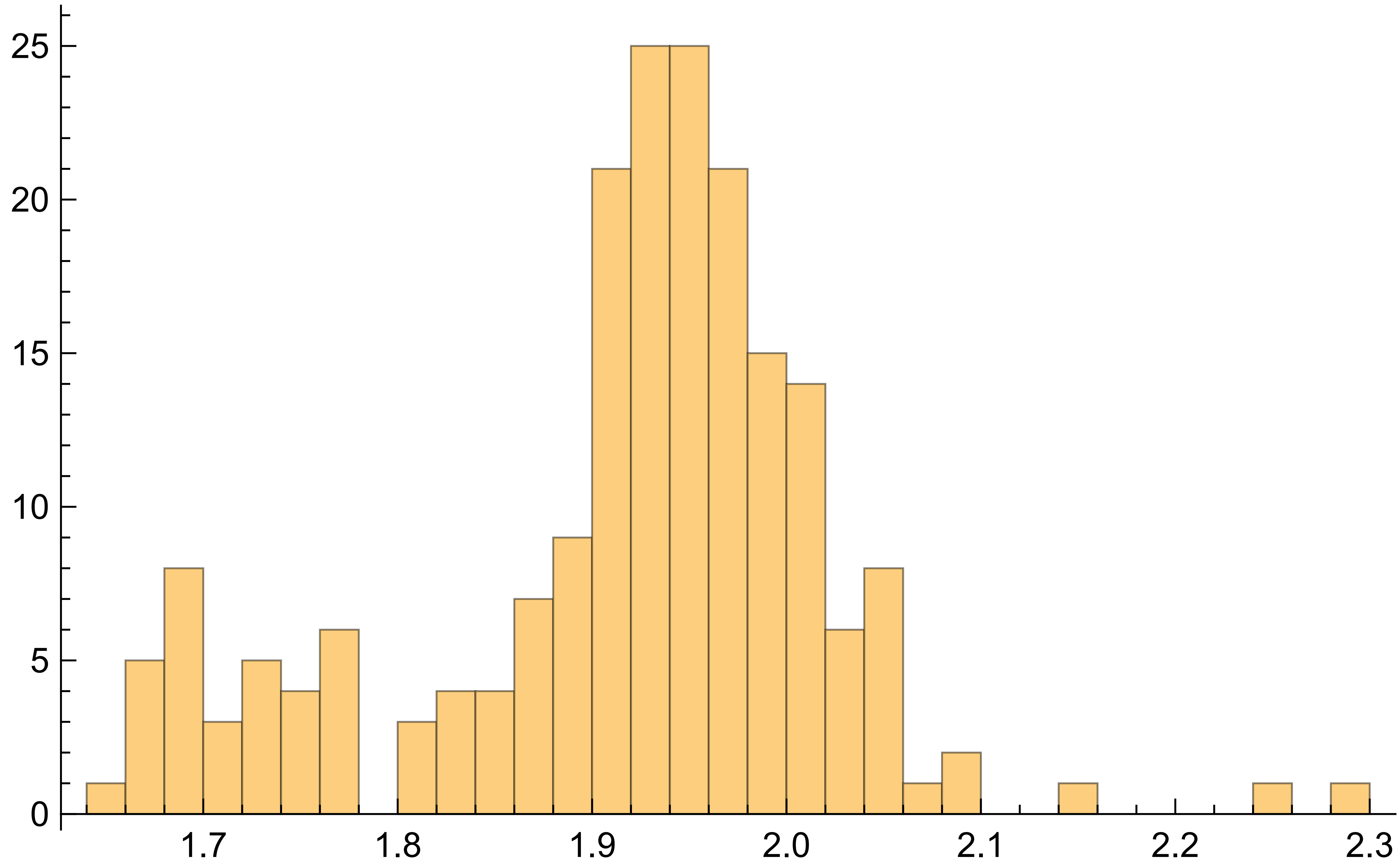
Use 200 replica parameters from previous fit



Normalized at 1st data point of bin

Include only COMPASS data

SIDIS h^+



Use 200 replica parameters from previous fit

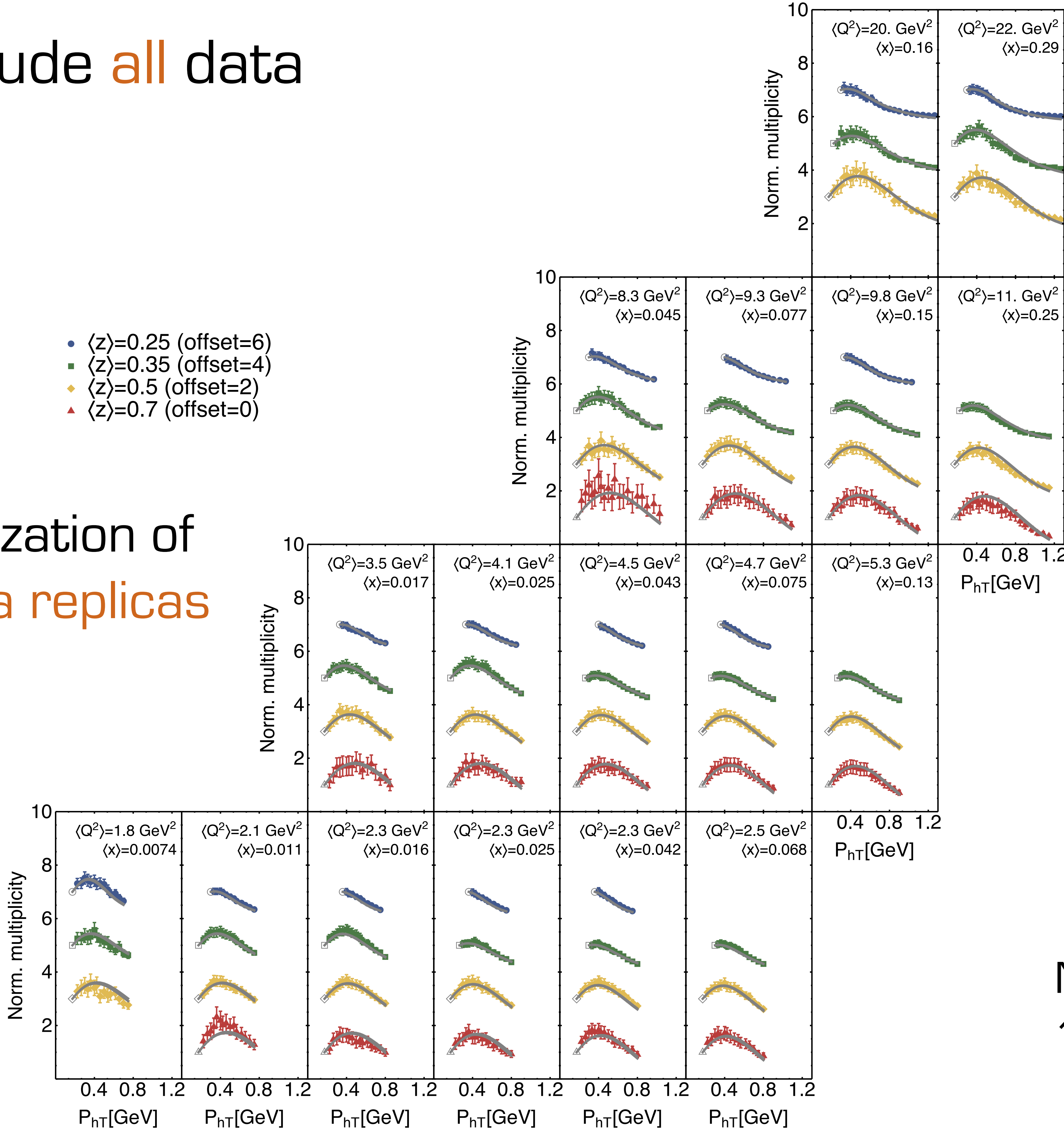
$\chi^2/\mathrm{dof} = 1.91$

Normalized at 1st data point of bin

Include all data

SIDIS h^+

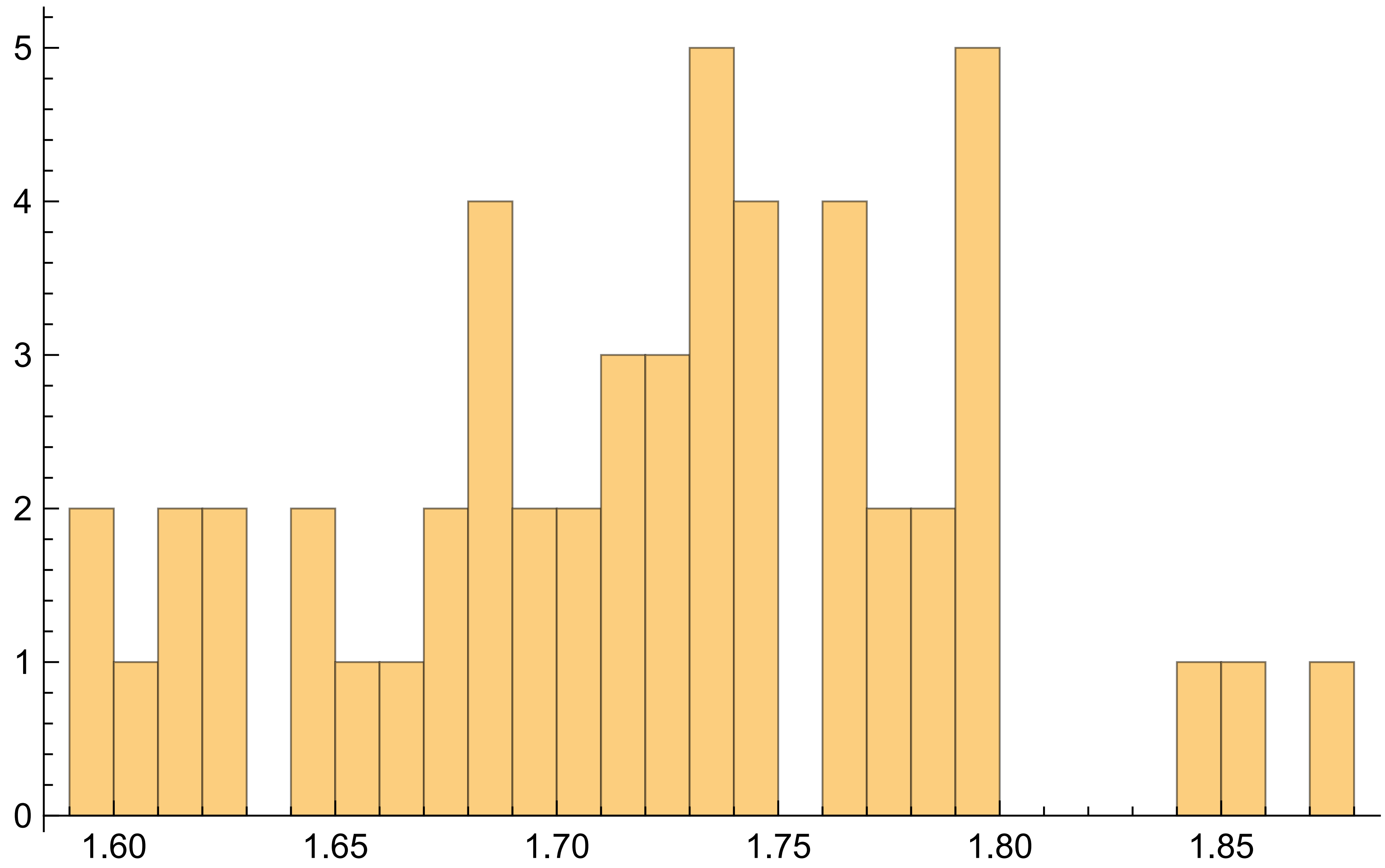
Minimization of
50 data replicas



Normalized at
1st data point
of bin

Include all data

SIDIS h^+



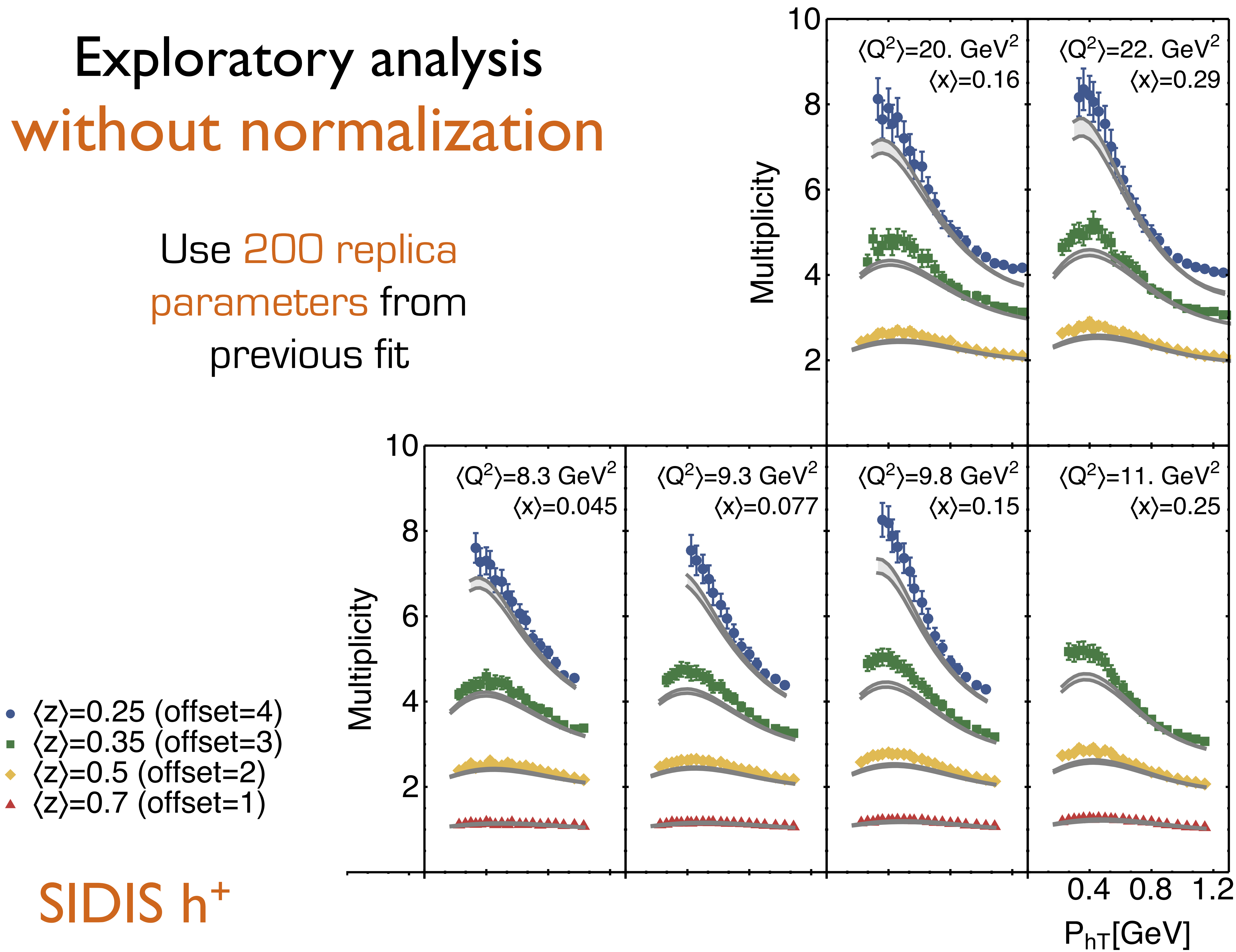
$$\chi^2/\mathrm{dof} = 1.71$$

Minimization of
50 data replicas

Normalized at
1st data point
of bin

Exploratory analysis
without normalization

Use 200 replica parameters from previous fit



SIDIS h^+

Exploratory analysis
without normalization

Use 200 replica parameters from previous fit $\longrightarrow$ χ^2/dof > 4

SIDIS h^+

Exploratory analysis without normalization

Use 200 replica parameters from previous fit $\longrightarrow$ χ^2/dof > 4

Sensitive to z value

Less stable with regards to kinematical cuts

...

SIDIS h^+

Conclusions

For the first time we demonstrated that it is possible to fit simultaneously SIDIS, DY and Z boson

We extracted a reasonable functional form for TMD from more than 8000 data points

We tested the universality and applicability of the TMD framework and it works quite well
(most of the discrepancies come from normalization)

Conclusions

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Conclusions

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Conclusions and open issues

For the first time we demonstrated that it is possible to fit simultaneously SIDIS, DY and Z boson

We extracted TMDs from more than 8000 data points

We tested the universality and applicability of the TMD framework and it works quite well

Revised Compass Data

- Reduced number of data points
- compatible with parameters obtained from previous analysis
- removing normalization requires further considerations

.....

BACKUP

Best fit values

TMD PDFs	N_1 [GeV^2]	α	σ		λ [GeV^{-2}]	
All replicas	0.28 ± 0.06	2.95 ± 0.05	0.17 ± 0.02		0.86 ± 0.78	
Replica 105	0.285	2.98	0.173		0.39	
TMD FFs	N_3 [GeV^2]	β	γ	δ	λ_F [GeV^{-2}]	N_4 [GeV^2]
All replicas	0.21 ± 0.02	1.65 ± 0.49	2.28 ± 0.46	0.14 ± 0.07	5.50 ± 1.23	0.04 ± 0.01
Replica 105	0.212	2.10	2.52	0.094	5.29	0.04

TABLE XI: 68% confidence intervals of best-fit values for parametrizations of TMDs at $Q = 1$ GeV.

Flavor independent scenario:

$$N_1 = 0.28 \pm 0.06 \text{ GeV}^2$$

$$N_3 = 0.21 \pm 0.02 \text{ GeV}^2$$

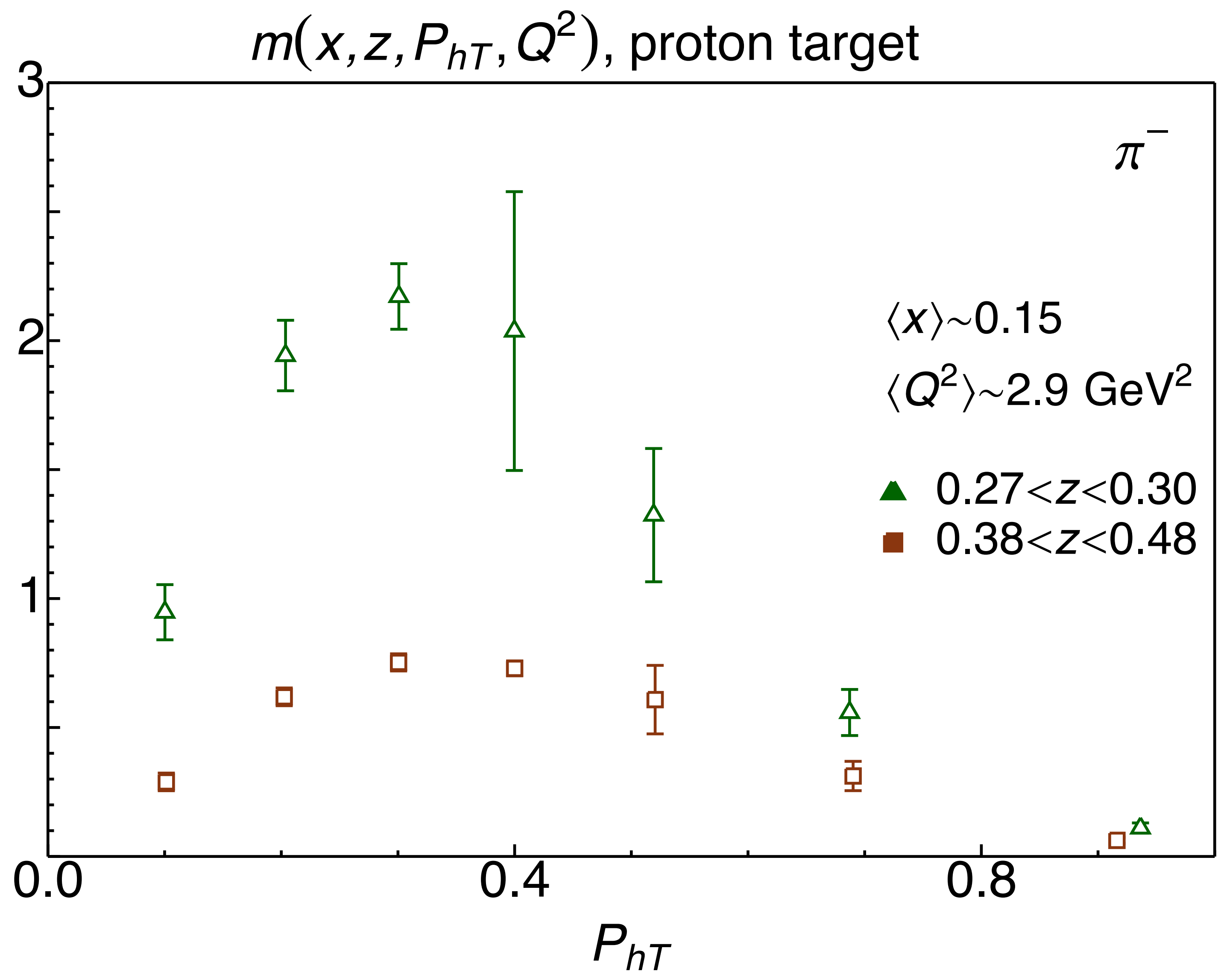
$$N_4 = 0.04 \pm 0.01 \text{ GeV}^2$$

$$g_2 = 0.13 \pm 0.01 \text{ GeV}^2$$

best value from 200 replicas

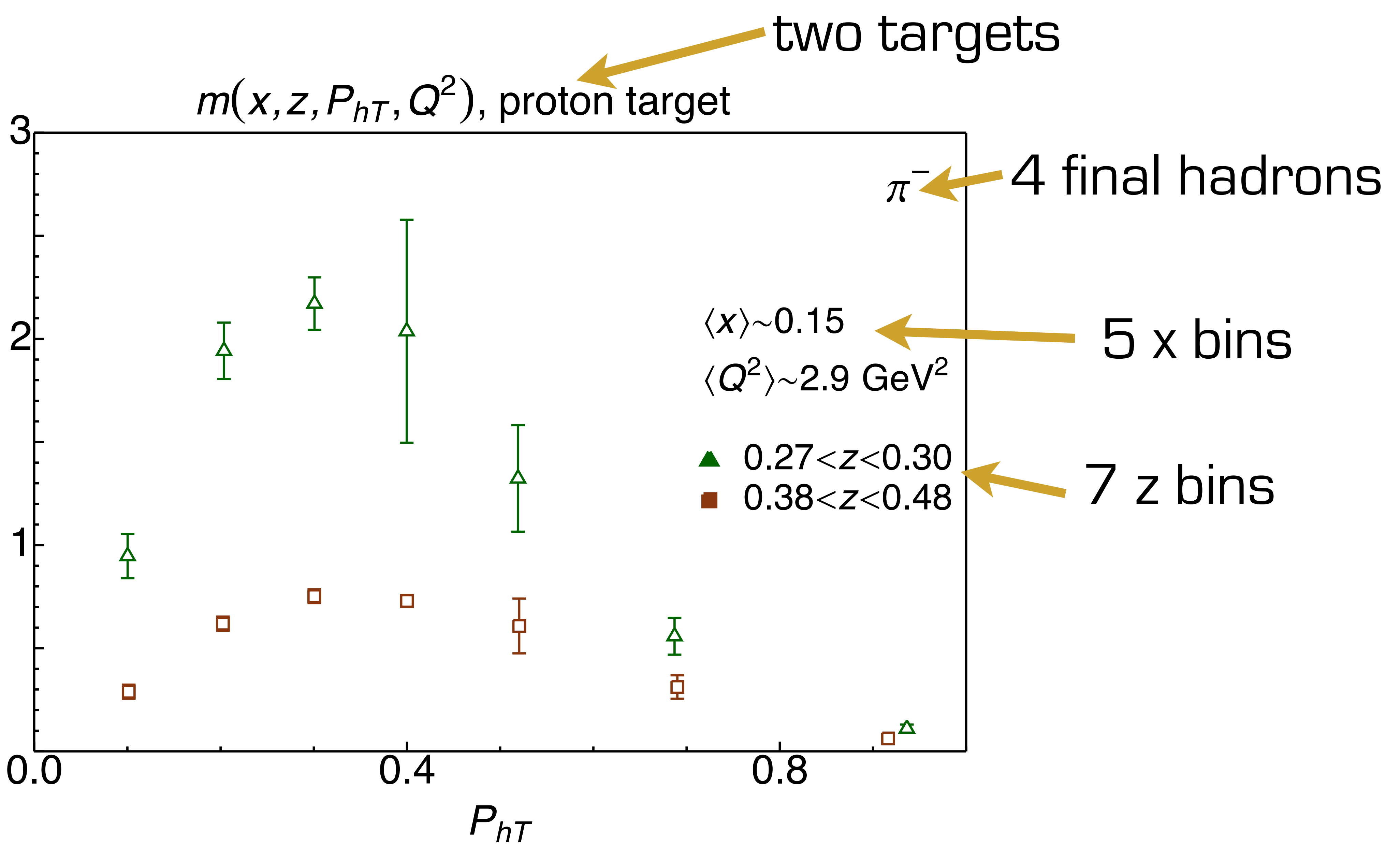
compatible with other extractions

The replica method



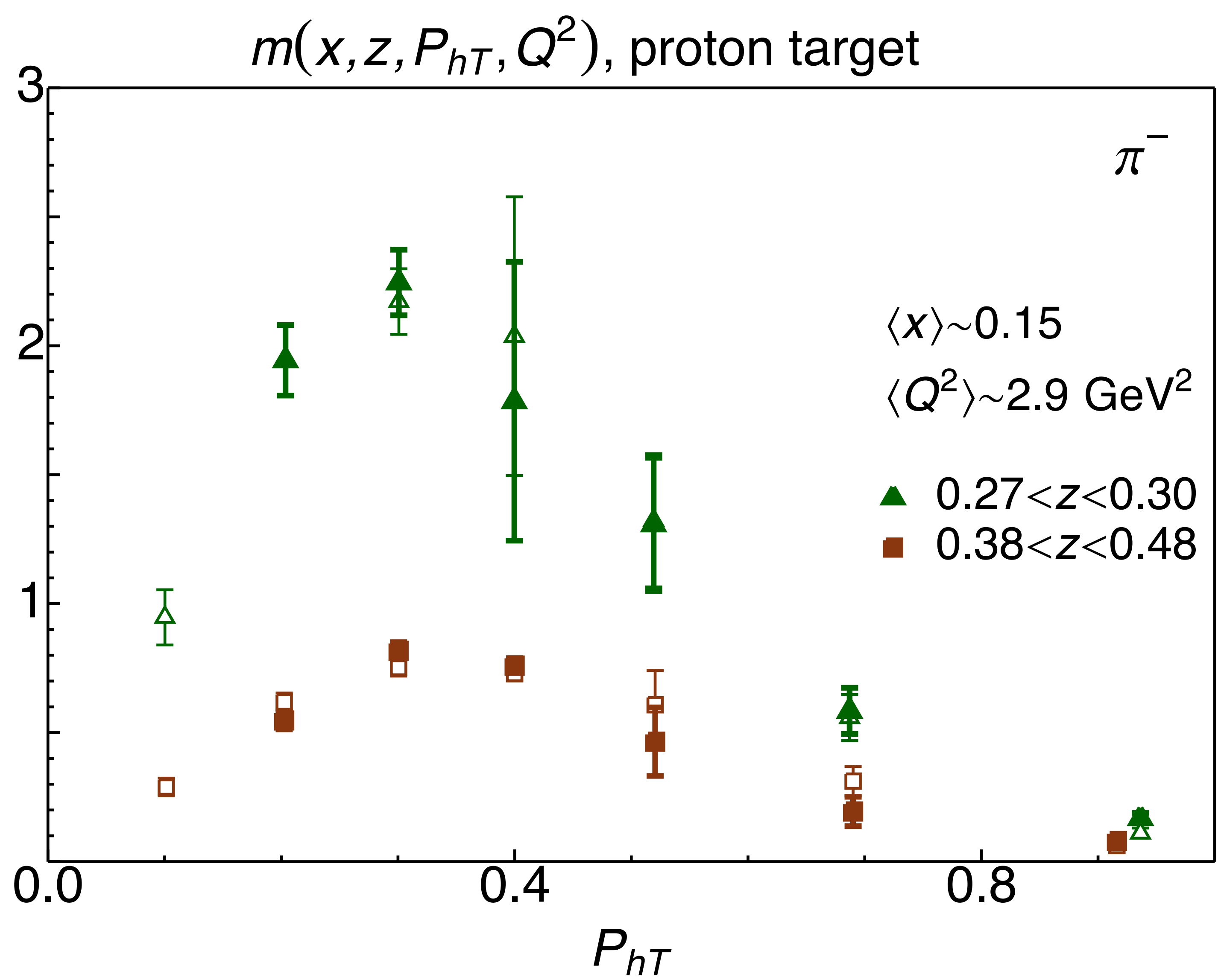
Example of original data

The replica method



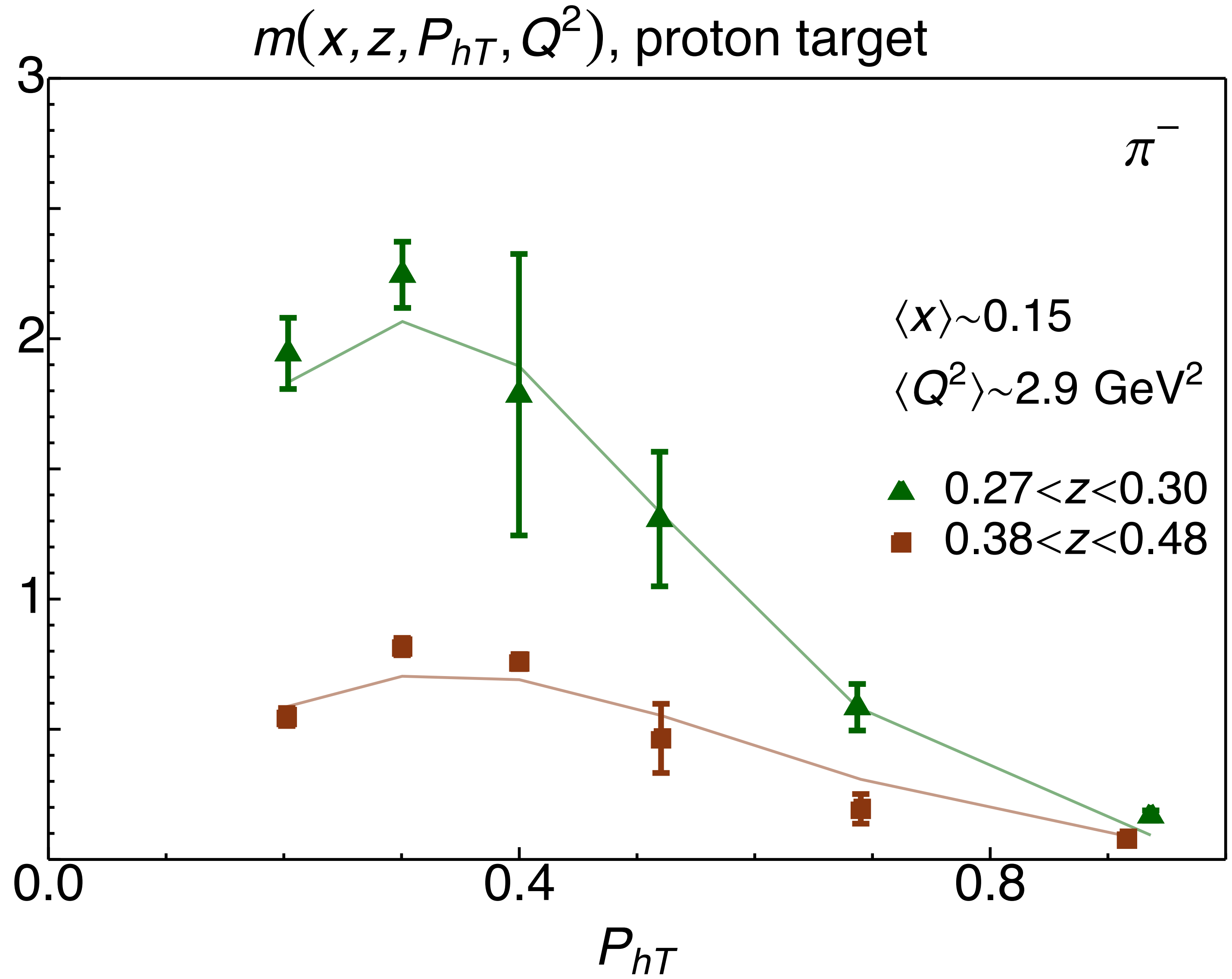
Example of original data

The replica method



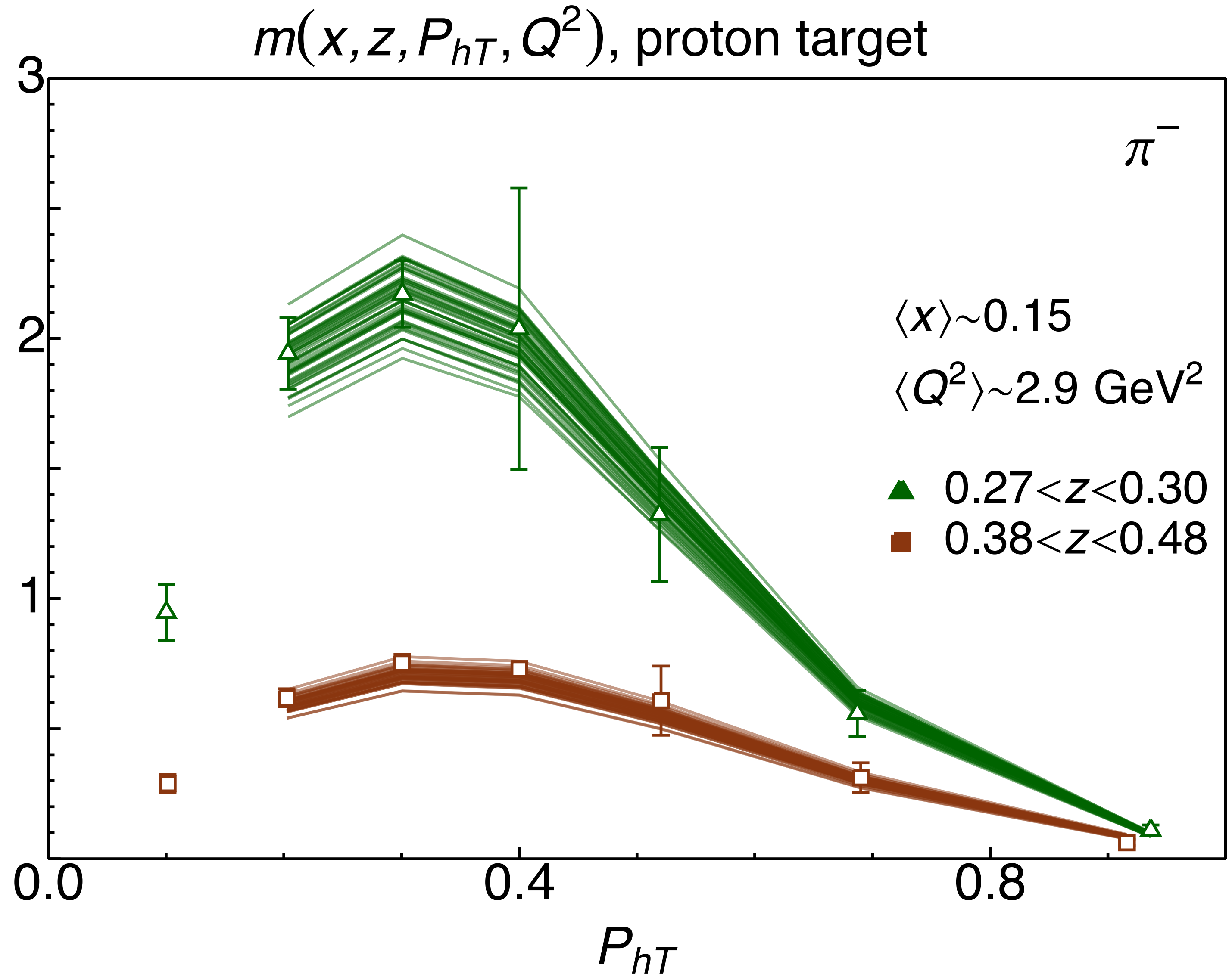
Data are replicated (with Gaussian distribution)

The replica method



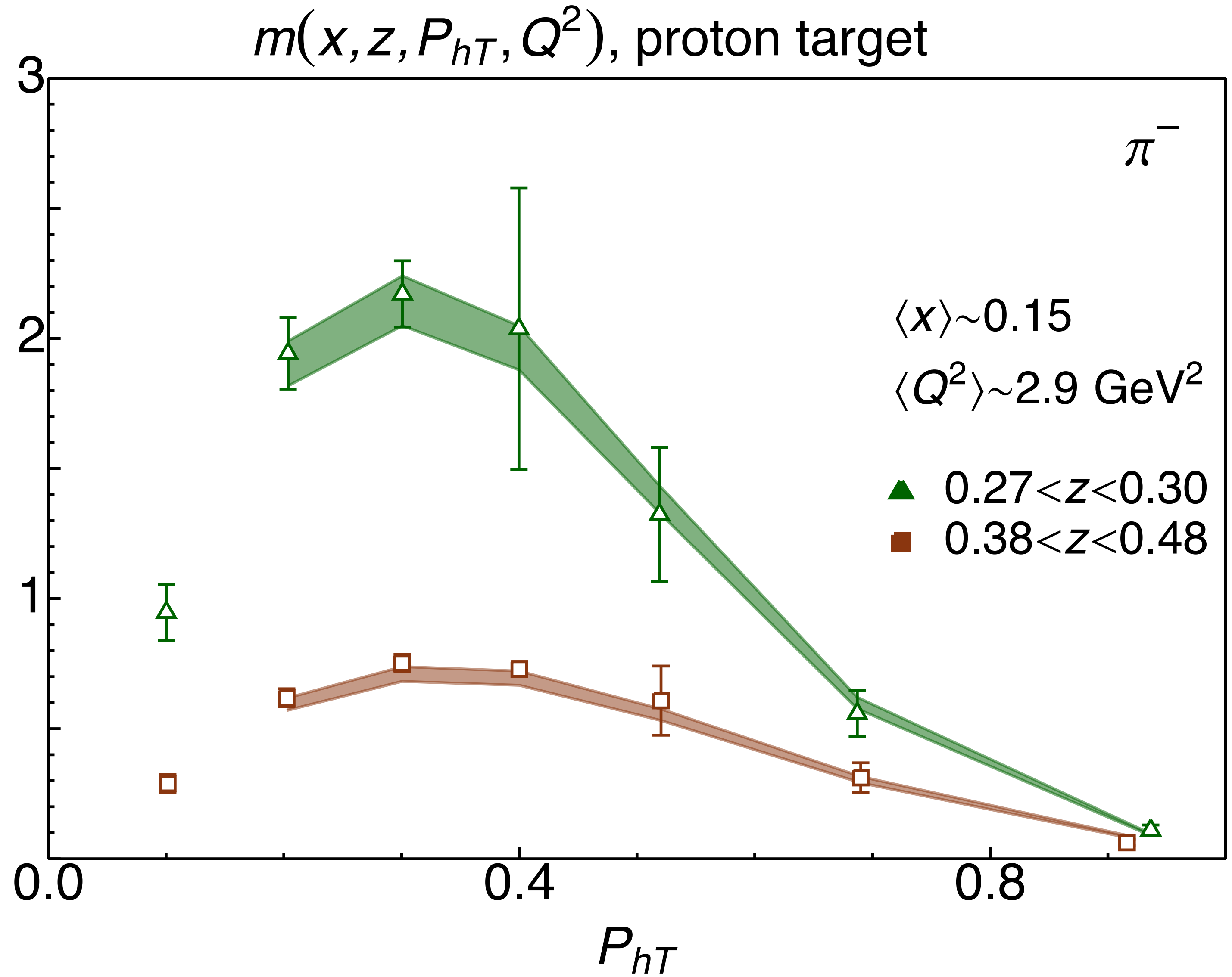
The fit is performed on the replicated data

The replica method



The procedure is repeated 200 times

The replica method



For each point, a central 68% confidence interval is identified

Previous fit studies

	Framework	HERMES	COMPASS	DY	Z production	N of points
KN 2006 hep-ph/0506225	LO-NLL	✘	✘	✔	✔	98
Pavia 2013 (+Amsterdam, Bilbao) arXiv:1309.3507	No evo (QPM)	✔	✘	✘	✘	1538
Torino 2014 (+JLab) arXiv:1312.6261	No evo (QPM)	✔ (separately)	✔ (separately)	✘	✘	576 (H) 6284 (C)
DEMS 2014 arXiv:1407.3311	NLO-NNLL	✘	✘	✔	✔	223
EIKV 2014 arXiv:1401.5078	LO-NLL	1 (x,Q^2) bin	1 (x,Q^2) bin	✔	✔	500 (?)
Pavia 2017 (+ JLab)	LO-NLL	✔	✔	✔	✔	8059

Data selection

SIDIS proton-target data

	HERMES $p \to \pi^+$	HERMES $p \to \pi^-$	HERMES $p \to K^+$	HERMES $p \to K^-$
Reference				
Cuts	$Q^2 > 1.4$ GeV^2 $0.2 < z < 0.7$ $P_{hT} < \text{Min}[0.2\ Q, 0.6\ Qz] + 0.5$ GeV			
Points	188	186	187	185
Max. Q^2	9.2 GeV^2			
x range	$0.06 < x < 0.4$			
Notes				

Motivations behind kinematical cuts

TMD factorization ($Ph_T/z \ll Q^2$)

Avoid target fragmentation (low z)

and exclusive contributions (high z)

Data selection

SIDIS
deuteron-target
data

	HERMES $D \to \pi^+$	HERMES $D \to \pi^-$	HERMES $D \to K^+$	HERMES $D \to K^-$	COMPASS $D \to h^+$	COMPASS $D \to h^-$
Cuts	$Q^2 > 1.4$ GeV^2 $0.2 < z < 0.7$ $P_{hT} < \text{Min}[0.2\ Q, 0.6\ Qz] + 0.5$ GeV					
Points	188	188	186	187	3024	3021
Max. Q^2	9.2 GeV^2				10 GeV^2	
x range	$0.06 < x < 0.4$				$0.006 < x < 0.12$	
Notes					Observable: $\dfrac{m_N^h(x, z, \boldsymbol{P}_{hT}^2, Q^2)}{m_N^h(x, z, \text{Min}[\boldsymbol{P}_{hT}^2], Q^2)}$	

to avoid problems
with Compass data normalization

Data selection

	E288 200	E288 300	E288 400	E605
Cuts	$q_T < 0.2\ Q + 0.5$ GeV			
Points	45	45	78	35
$\sqrt{s}$	19.4 GeV	23.8 GeV	27.4 GeV	38.8 GeV
Q range	4-9 GeV	4-9 GeV	5-9, 11-14 GeV	7-9, 10.5-18 GeV
Kin. var.	y=0.4	y=0.21	y=0.03	$-0.1 < x_F < 0.2$

Drell-Yan data

Z production data

	CDF Run I	D0 Run I	CDF Run II	D0 Run II
Cuts	$q_T < 0.2\ Q + 0.5\ \text{GeV} = 18.7$ GeV			
Points	31	14	37	8
$\sqrt{s}$	1.8 TeV	1.8 TeV	1.96 TeV	1.96 TeV
Normalization	1.114	0.992	1.049	1.048

fixed from DEMS fit, different from exp. (not really relevant for TMD parametrizations)

μ and b$_*$ prescriptions

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i \big(\tilde{C}_{a/i} \otimes f_1^i\big)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\mathrm{NP}}^a(x, b_T)$$

μ and b_* prescriptions

Choice **Choice**

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\rm NP}^a(x, b_T)$$

μ and b_* prescriptions

Choice **Choice**

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\rm NP}^a(x, b_T)$$

$$\mu_b = 2e^{-\gamma_E}/b_* \qquad b_* \equiv \frac{b_T}{\sqrt{1 + b_T^2/b_{\max}^2}}$$

Collins, Soper, Sterman, NPB250 (85)

$$\mu_b = 2e^{-\gamma_E}/b_* \qquad b_* \equiv b_{\max}\left(1 - e^{-\frac{b_T^4}{b_{\max}^4}}\right)^{1/4}$$

Bacchetta, Echevarria, Mulders, Radici, Signori
arXiv:1508.00402

$$\mu_b = Q_0 + q_T \qquad b_* = b_T$$

DEMS 2014

μ and b_* prescriptions

Choice Choice

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\rm NP}^a(x, b_T)$$

$$\mu_b = 2e^{-\gamma_E}/b_* \qquad b_* \equiv \frac{b_T}{\sqrt{1 + b_T^2/b_{\rm max}^2}}$$

Collins, Soper, Sterman, NPB250 (85)

$$\mu_b = 2e^{-\gamma_E}/b_* \qquad b_* \equiv b_{\rm max}\left(1 - e^{-\frac{b_T^4}{b_{\rm max}^4}}\right)^{1/4}$$

Bacchetta, Echevarria, Mulders, Radici, Signori
arXiv:1508.00402

$$\mu_b = Q_0 + q_T \qquad b_* = b_T$$

DEMS 2014

Complex-b prescription

Laenen, Sterman, Vogelsang, PRL 84 (00)

Nonperturbative ingredients 1

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i \big(\tilde{C}_{a/i} \otimes f_1^i\big)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\rm NP}^a(x, b_T)$$

Nonperturbative ingredients 1

Choice

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i \big(\tilde{C}_{a/i} \otimes f_1^i\big)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\rm NP}^a(x, b_T)$$

Nonperturbative ingredients 1

Choice

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i \big(\tilde{C}_{a/i} \otimes f_1^i\big)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T)\ln\frac{\mu}{\mu_0}} \hat{f}_{\rm NP}^a(x, b_T)$$

$$e^{-\frac{b_T^2}{\langle b_T^2 \rangle}}$$

almost everybody

$$e^{-\frac{b_T^2}{\left\langle b_T^2(x)\right\rangle_a}}$$

Pavia 2013, KN 2006

$$e^{-\lambda_1 b_T}\left(1+\lambda_2 b_T^2\right)$$

DEMS 2014

Low-b_T modifications

$$\log\left(Q^2 b_T^2\right) \to \log\left(Q^2 b_T^2 + 1\right)$$

see, e.g., Bozzi, Catani, De Florian, Grazzini
hep-ph/0302104

see talks by Collins, Boglione, (Rogers?)

Low-b_T modifications

$$\log\left(Q^2 b_T^2\right) \rightarrow \log\left(Q^2 b_T^2 + 1\right)$$

see, e.g., Bozzi, Catani, De Florian, Grazzini
hep-ph/0302104

$$b_*(b_c(b_\mathrm{T})) = \sqrt{\frac{b_\mathrm{T}^2 + b_0^2/(C_5^2 Q^2)}{1 + b_\mathrm{T}^2/b_\mathrm{max}^2 + b_0^2/(C_5^2 Q^2 b_\mathrm{max}^2)}}$$

$$b_\mathrm{min} \equiv b_*(b_c(0)) = \frac{b_0}{C_5 Q}\sqrt{\frac{1}{1 + b_0^2/(C_5^2 Q^2 b_\mathrm{max}^2)}}$$

Collins et al.
arXiv:1605.00671

see talks by Collins, Boglione, (Rogers?)

Data selection

$Q^2 > 1.4\ \mathrm{GeV}^2$

$0.2 < z < 0.7$

$P_{hT}, q_T < 0.2\,Q + 0.5\ \mathrm{GeV}$

$P_{hT} < 0.8\ \mathrm{GeV}\ (\mathrm{if}\ z < 0.3)$

Data selection

$Q^2 > 1.4 \text{ GeV}^2$

$0.2 < z < 0.7$

$P_{hT}, q_T < 0.2\, Q + 0.5 \text{ GeV}$ $P_{hT} < 0.8 \text{ GeV (if } z < 0.3)$

Total number of data points: 8156

Total χ^2/dof = 1.45

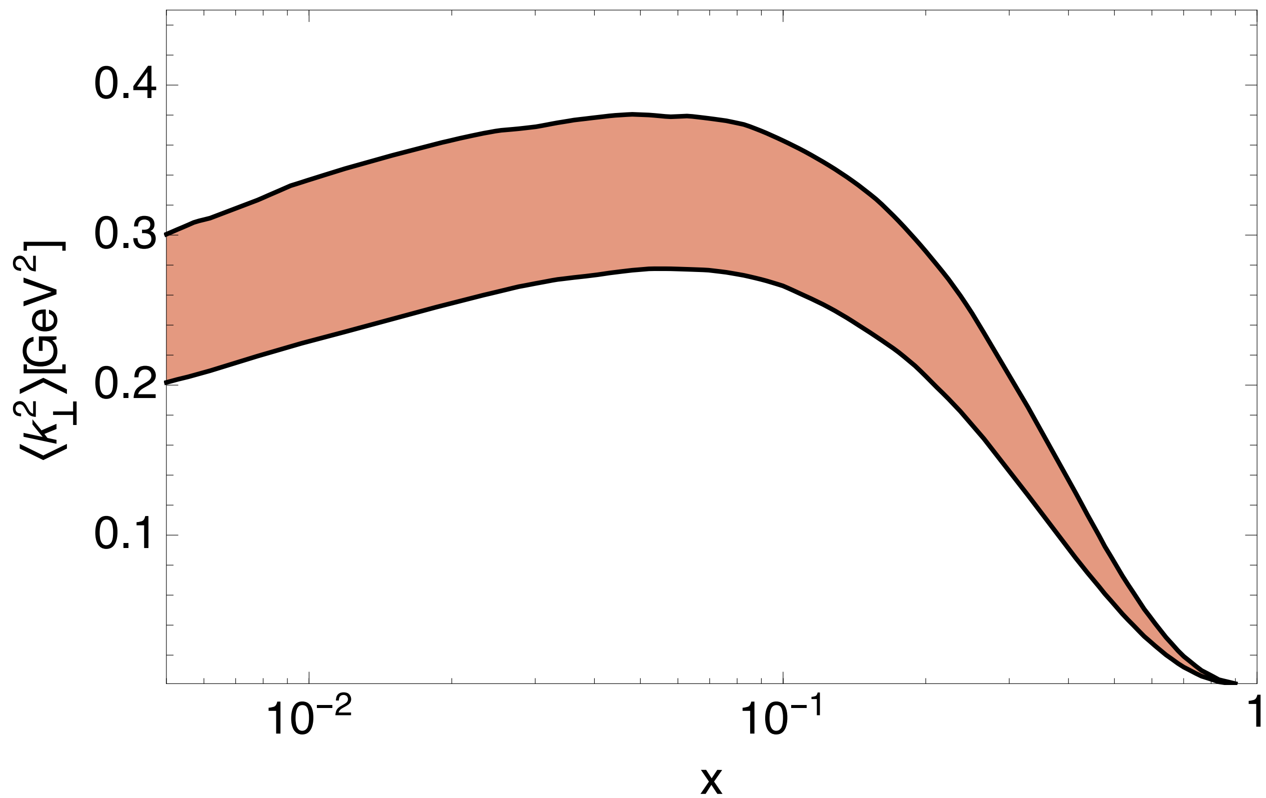
Preliminary

Pavia 2016 perturbative ingredients

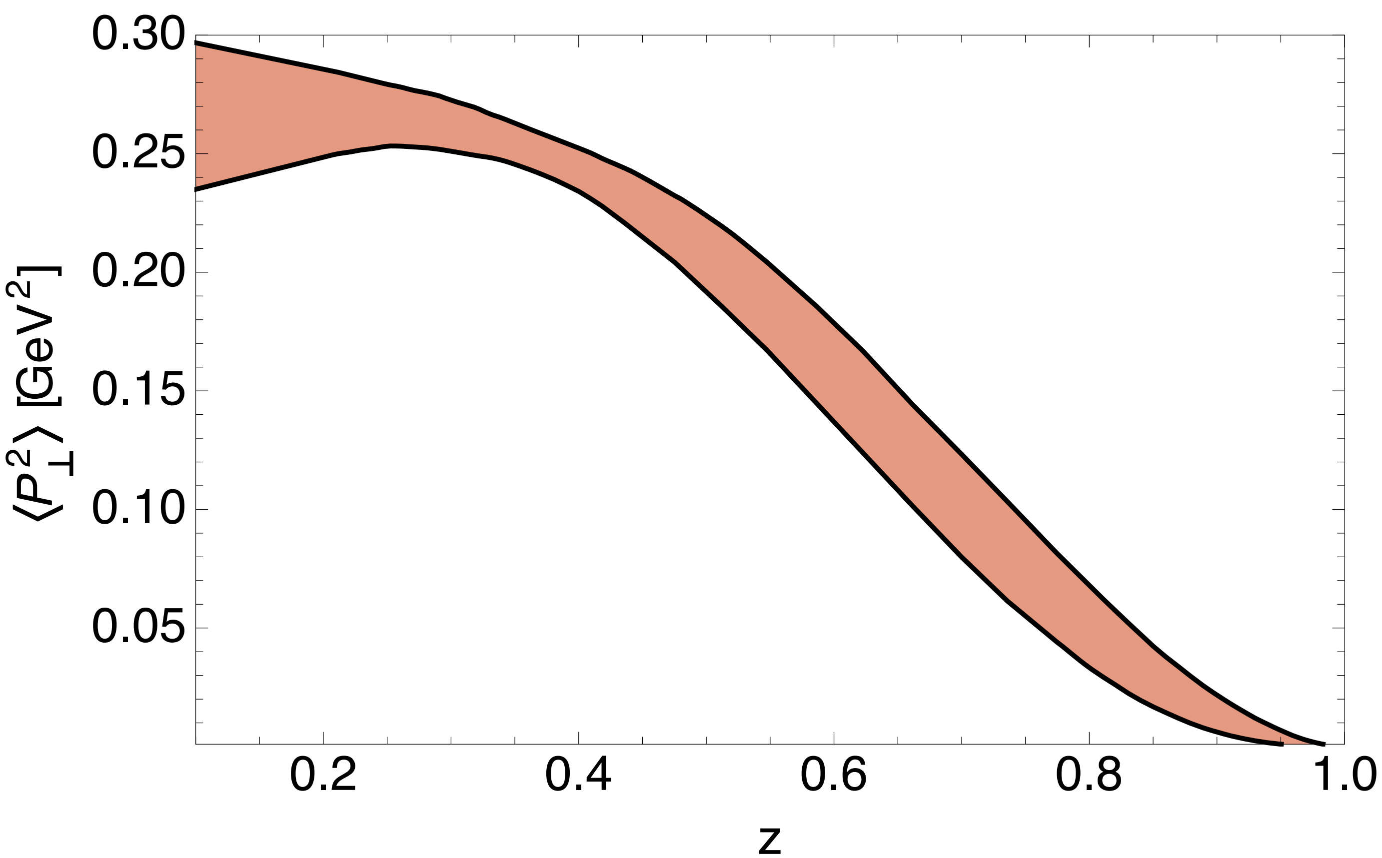
$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i \left(\tilde{C}_{a/i} \otimes f_1^i\right)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\rm NP}^a(x, b_T)$$

$A_1(\mathcal{O}(\alpha_S^1))$ ✔	$A_2(\mathcal{O}(\alpha_S^2))$ ✔	$A_3(\mathcal{O}(\alpha_S^3))$	...
	$B_1(\mathcal{O}(\alpha_S^1))$ ✔	$B_2(\mathcal{O}(\alpha_S^2))$	...
$C_0(\mathcal{O}(\alpha_S^0))$ ✔	$C_1(\mathcal{O}(\alpha_S^1))$	$C_2(\mathcal{O}(\alpha_S^2))$	...
$H_0(\mathcal{O}(\alpha_S^0))$ ✔	$H_1(\mathcal{O}(\alpha_S^1))$	$H_2(\mathcal{O}(\alpha_S^2))$	...
	$Y_1(\mathcal{O}(\alpha_S^1))$	$Y_2(\mathcal{O}(\alpha_S^2))$	...

Mean transverse momentum



In TMD PDF

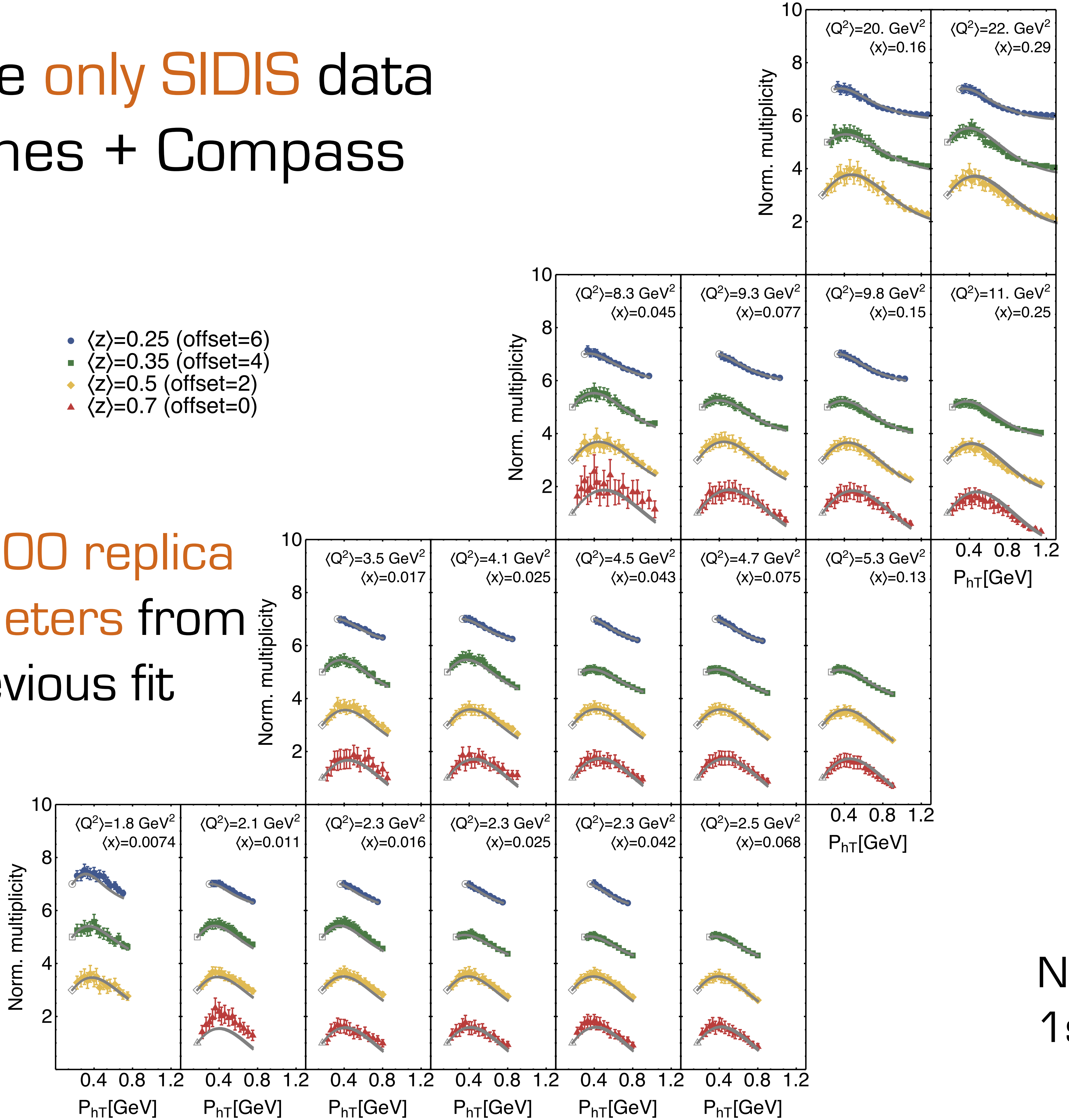


In TMD FF

Q^2=1GeV^2

Include only SIDIS data Hermes + Compass

SIDIS h^+

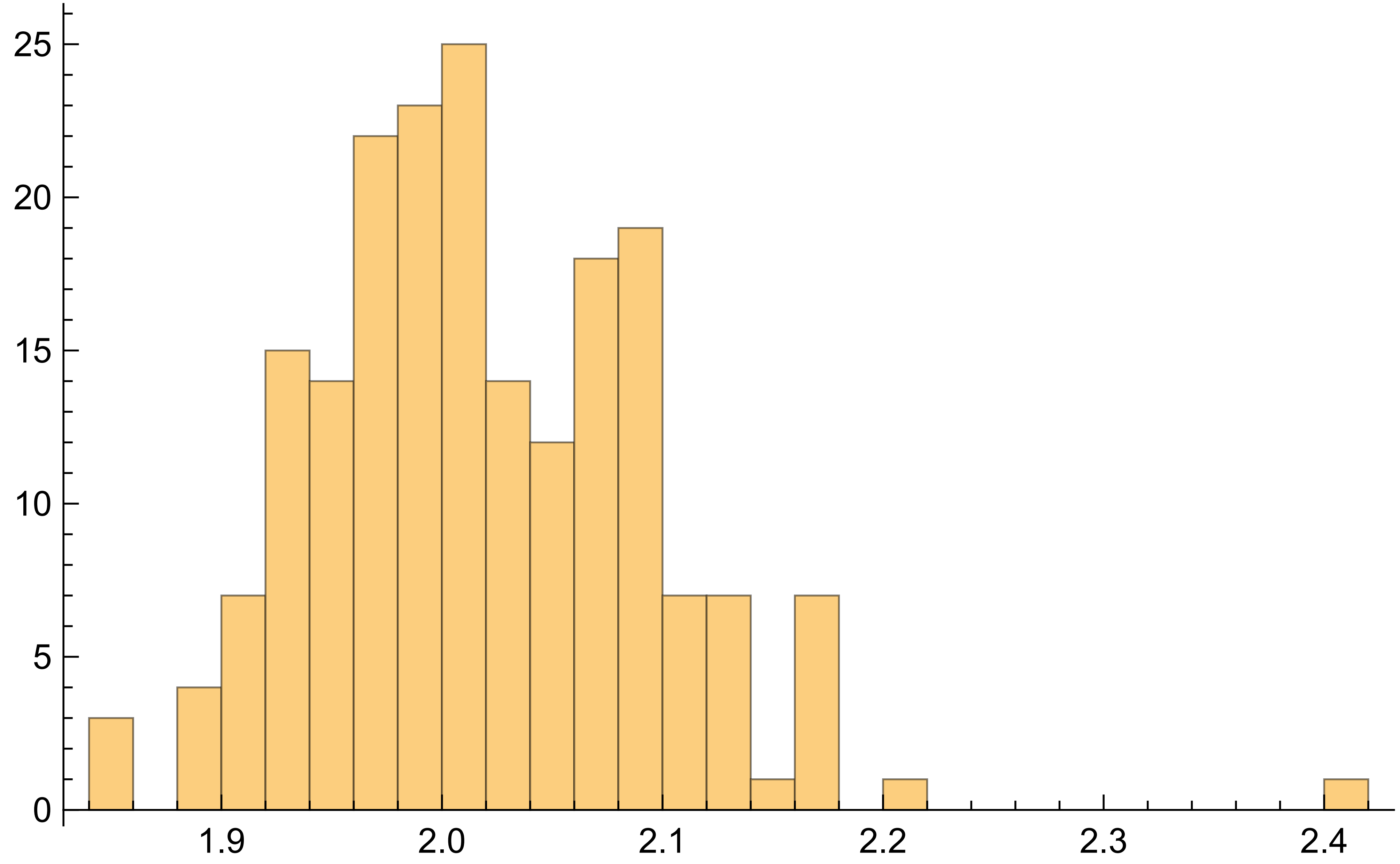


Use 200 replica parameters from previous fit

Normalized at 1st data point of bin

Include only SIDIS data

SIDIS h^+



Use 200 replica parameters from previous fit

χ^2/dof = **2.07**

Normalized at 1st data point of bin