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A new phenomenological extraction of the Sivers distribution function

Mariaelena Boglione



UNIVERSITÀ
DEGLI STUDI
DI TORINO
ALMA UNIVERSITAS
TAURINENSIS



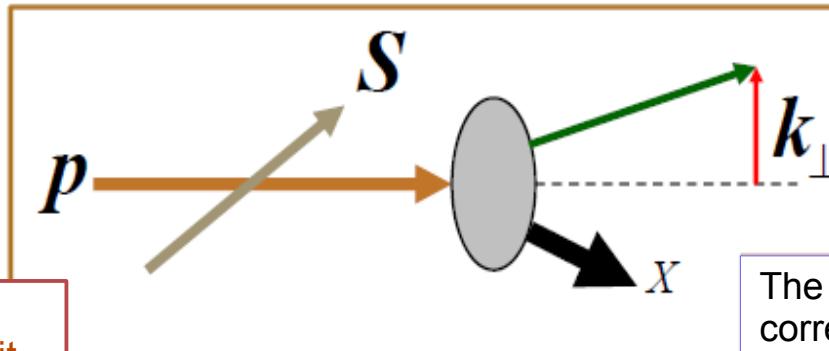
In collaboration with Anselmino, D'alesio, Flore, Gonzalez, Murgia, Prokudin

The Sivers Distribution Function

$$f_{q/p,S}(x, k_\perp) = f_{q/p}(x, k_\perp) + \frac{1}{2} \Delta^N f_{q/p\uparrow}(x, k_\perp) S \cdot (\hat{p} \times \hat{k}_\perp)$$

The Sivers function is related to the probability of finding an unpolarized quark inside a transversely polarized proton

$$= f_{q/p}(x, k_\perp) - \frac{k_\perp}{M} f_{1T}^{\perp q}(x, k_\perp) S \cdot (\hat{p} \times \hat{k}_\perp)$$

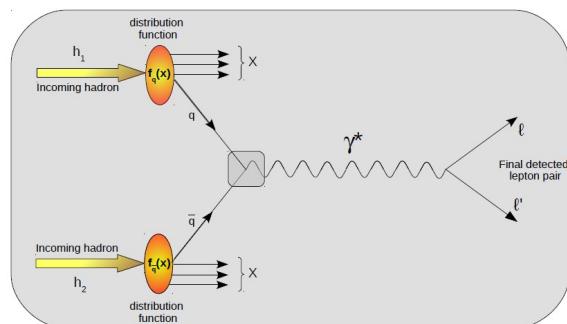


The Sivers function, is particularly interesting, as it provides information on the partonic orbital angular momentum

The Sivers function embeds correlations between proton spin and quark transverse momentum

Where do we learn about the Sivers function ?

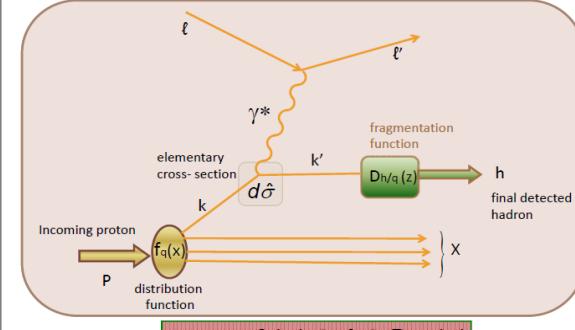
Unpolarized and Polarized Drell-Yan scattering



$$\sigma_{Drell-Yan} = f_q(x, k_\perp) \otimes f_{\bar{q}}(x, k_\perp) \otimes \hat{\sigma}^{q\bar{q} \rightarrow \ell^+\ell^-}$$

Allows extraction of
distribution functions

Unpolarized and Polarized SIDIS scattering



$$\sigma_{SIDIS} = f_q(x) \otimes \hat{\sigma} \otimes D_{h/q}(z)$$

Allows extraction
of **distribution** and
fragmentation
functions



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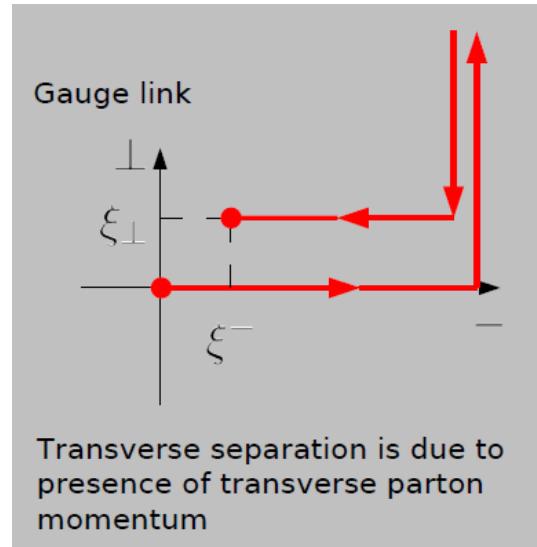


Sivers function sign change

- TMDs have to be defined in a color-gauge invariant way

$$\Phi_{ij}(x, \mathbf{k}_\perp) = \int \frac{d\xi^-}{(2\pi)} \frac{d^2\xi_\perp}{(2\pi)^2} e^{ix\mathbf{P}^+\xi^-} e^{-i\mathbf{k}_\perp\xi_\perp} \langle \mathbf{P}, \mathbf{S}_\mathbf{P} | \bar{\psi}_j(\mathbf{0}) \mathcal{U}(\mathbf{0}, \xi) \psi_i(\xi) | \mathbf{P}, \mathbf{S}_\mathbf{P} \rangle \Big|_{\xi^+=0}$$

- The struck quark propagates in the gauge field of the remnant and forms gauge links

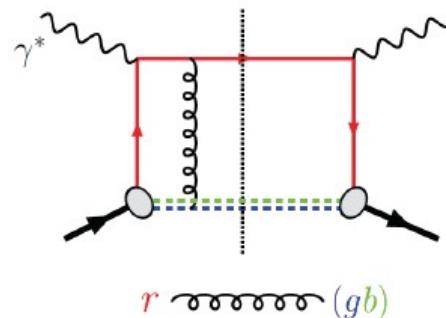


- Gauge links generate initial and final state interactions

Sivers function sign change

SIDIS

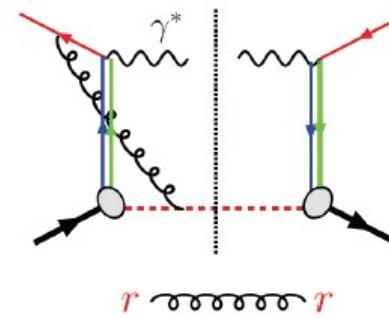
- The gluon couples to the proton remnant after the quark is scattered
- Attractive final state interaction



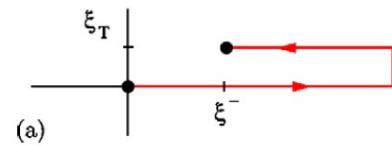
Attractive

DRELL YAN

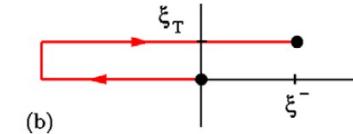
- The gluon couples before the quark annihilates
- Repulsive initial state interaction



Repulsive



(a)



(b)

The Sivers function is process dependent: it reverses its sign when measured in SIDIS w.r.t Drell Yan processes

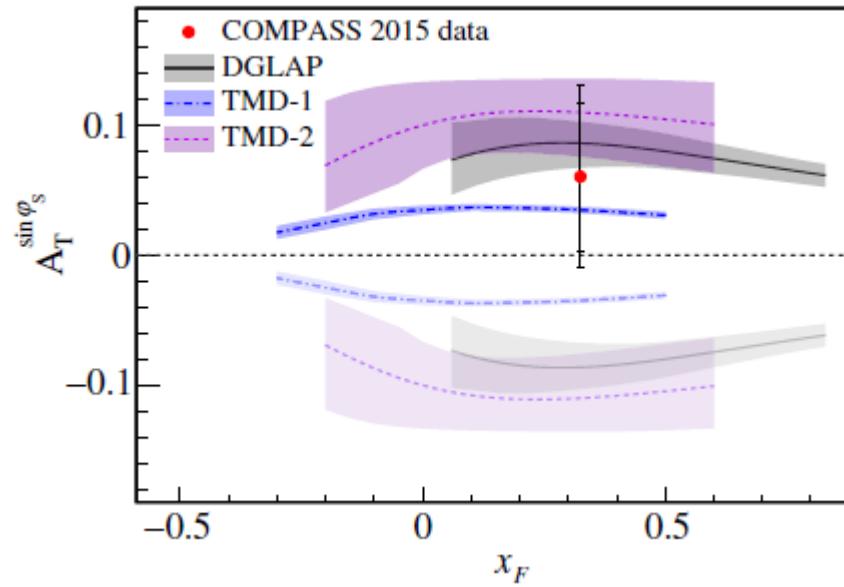
$$[f_{1T}^{q\perp}]_{\text{SIDIS}} = -[f_{1T}^{q\perp}]_{\text{DY}}$$

First hints of sign change

Sivers single spin asymmetry in pion induced Drell Yan @ COMPASS

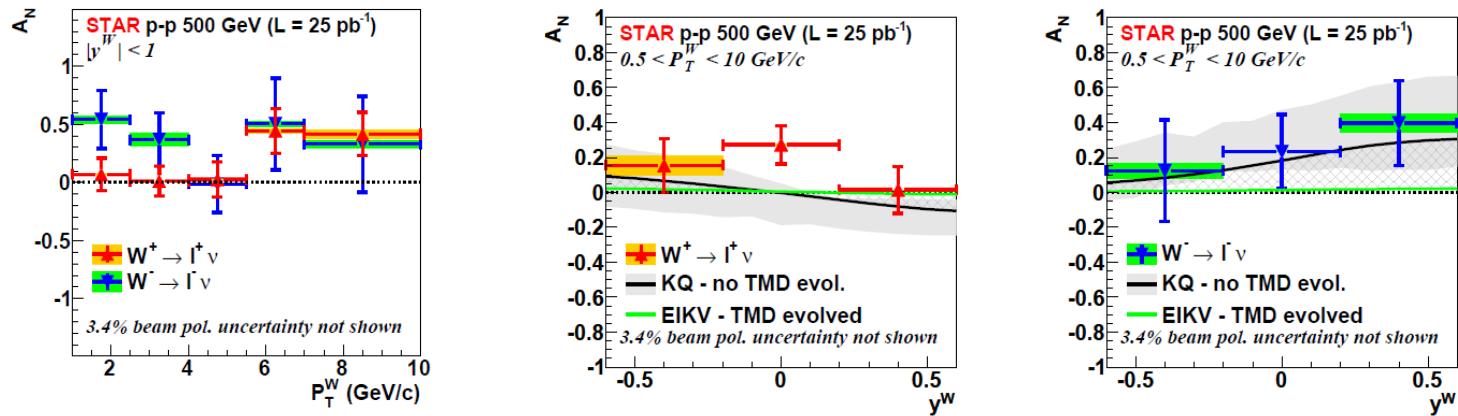
COMPASS Collaboration, Phys. Rev. Lett. 119, 112002 (2017)

190GeV/c π^- beam scattered off a transversely polarized NH₃ target (polarized proton)



Sivers function in $p^\uparrow + p \rightarrow W^\pm/Z$ @ RHIC

STAR Collaboration, Phys. Rev. Lett. 116 132301 (2016)



$$A_N^W = \frac{d\sigma^{p \rightarrow WX} - d\sigma^{p \rightarrow WX}}{d\sigma^{p \rightarrow WX} + d\sigma^{p \rightarrow WX}} \equiv \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

$$= \frac{\sum_{q_1, q_2} |V_{q_1, q_2}|^2 \int d^2 k_{\perp 1} d^2 k_{\perp 2} \delta^2(k_{\perp 1} + k_{\perp 2} - q_T) \mathbf{S} \cdot (\hat{\mathbf{p}}_1 \times \hat{\mathbf{k}}_{\perp 1}) \Delta^N f_{q_1/p^\uparrow}(x_1, k_{\perp 1}) f_{q_2/p}(x_2, k_{\perp 2})}{2 \sum_{q_1, q_2} |V_{q_1, q_2}|^2 \int d^2 k_{\perp 1} d^2 k_{\perp 2} \delta^2(k_{\perp 1} + k_{\perp 2} - q_T) f_{q_1/p}(x_1, k_{\perp 1}) f_{q_2/p}(x_2, k_{\perp 2})}$$

Importance of thorough knowledge of the Sivers function (valence and sea contributions)

Sivers function in $p^\uparrow + p \rightarrow W^\pm/Z$ @ RHIC

Anselmino, Boglione, D'Alesio, Murgia, Prokudin, JHEP 1704 (2017) 046

- The quark flavours involved in W production include **anti-quarks**
- In order to estimate A_N^W , it is important to have a reliable extraction of both quark and **anti-quark Sivers functions**.

Need to measure the **Sivers sea**
(access Sivers asymmetries at low low-x)
→ COMPASS and EIC

$$W+ : |V_{u,d}|^2 \left(\Delta^N f_{u/p} \otimes f_{\bar{d}/p} + \Delta^N f_{\bar{d}/p} \otimes f_{u/p} \right) + |V_{u,s}|^2 \left(\Delta^N f_{u/p} \otimes f_{\bar{s}/p} + \Delta^N f_{\bar{s}/p} \otimes f_{u/p} \right)$$

$$W- : |V_{u,d}|^2 \left(\Delta^N f_{\bar{u}/p} \otimes f_{d/p} + \Delta^N f_{d/p} \otimes f_{\bar{u}/p} \right) + |V_{u,s}|^2 \left(\Delta^N f_{\bar{u}/p} \otimes f_{s/p} + \Delta^N f_{s/p} \otimes f_{\bar{u}/p} \right)$$

dominant

suppressed

This single spin asymmetry is very sensitive to \bar{u} and \bar{d} as well as u_v and d_v

Extraction of Sivers functions from SIDIS data

Anselmino, Boglione, D'Alesio, Murgia, Prokudin, JHEP 1704 (2017) 046

Unpolarized TMD PDF

$$f_{q/p}(x, k_\perp) = f_q(x) \frac{1}{\pi \langle k_\perp^2 \rangle} e^{-k_\perp^2 / \langle k_\perp^2 \rangle}$$

Unpolarized TMD FF

$$D_{h/q}(z, p_\perp) = D_{h/q}(z) \frac{1}{\pi \langle p_\perp^2 \rangle} e^{-p_\perp^2 / \langle p_\perp^2 \rangle}$$

Sivers function

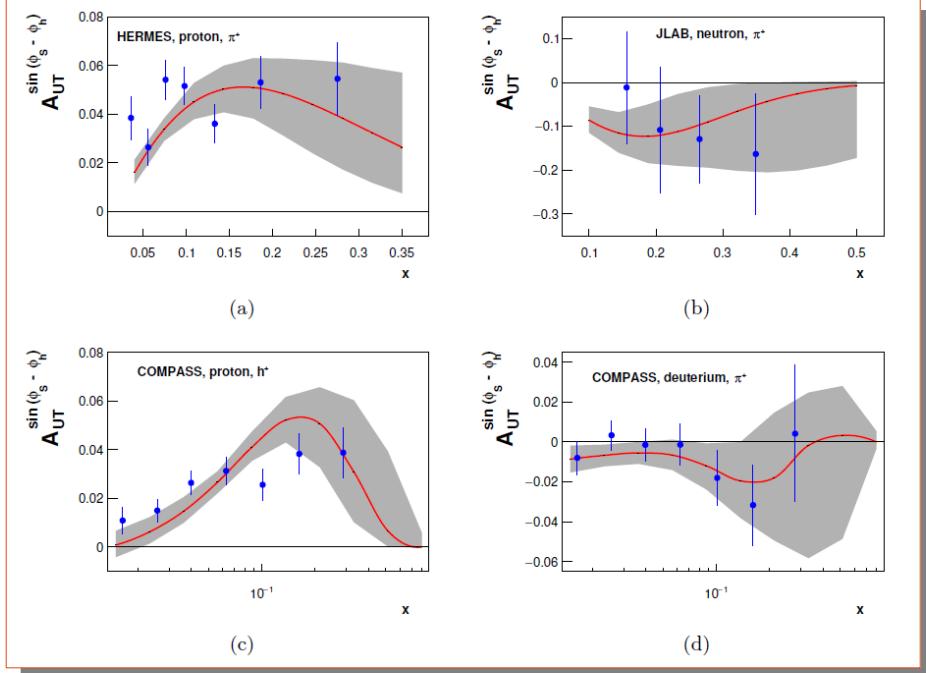
$$\left\{ \begin{array}{l} \Delta^N f_{q/p}(x, k_\perp) = 2 \mathcal{N}_q(x) h(k_\perp) f_{q/p}(x, k_\perp) \\ h(k_\perp) = \sqrt{2e} \frac{k_\perp}{M_1} e^{-k_\perp^2 / M_1^2} \\ \mathcal{N}_q(x) = N_q x^{\alpha_q} (1-x)^{\beta_q} \frac{(\alpha_q + \beta_q)^{(\alpha_q + \beta_q)}}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}} \\ \mathcal{N}_{\bar{q}}(x) = N_{\bar{q}} \end{array} \right.$$

Sivers function
parametrized
starting from
unpolarized PDF

Sivers width
parametrized
starting from
unpolarized width

Extraction of Sivers functions from SIDIS data

Anselmino, Boglione, D'Alesio, Murgia, Prokudin, JHEP 1704 (2017) 046

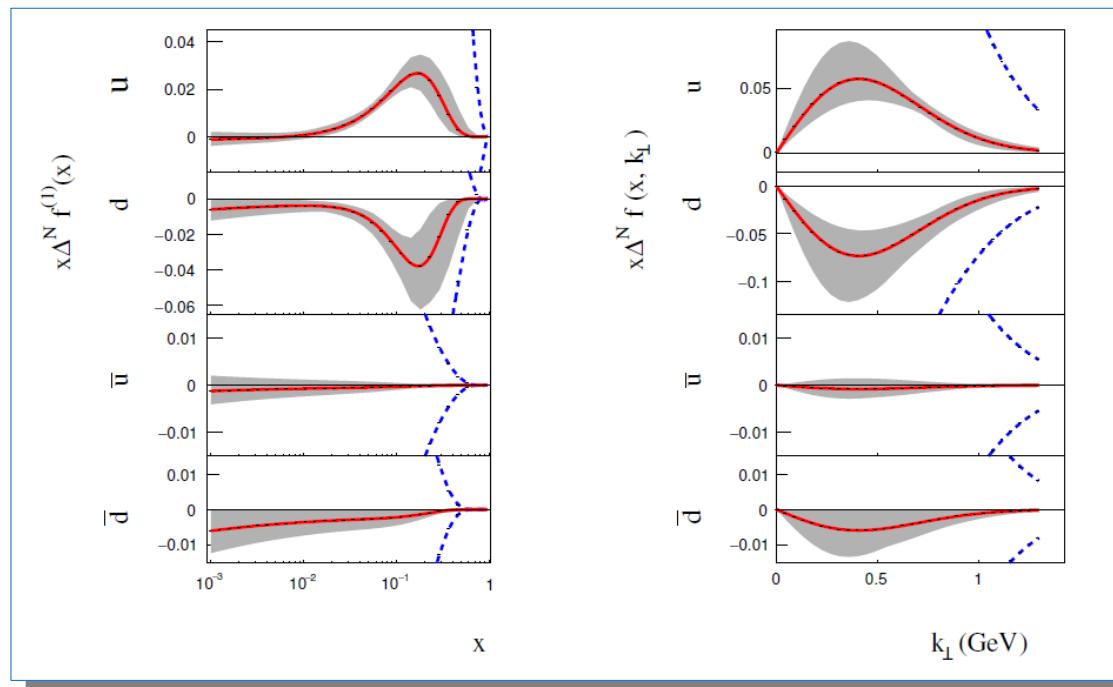


$$\left. \begin{array}{l} \langle k_{\perp}^2 \rangle = 0.57 \text{ GeV}^2 \\ \langle p_{\perp}^2 \rangle = 0.12 \text{ GeV}^2 \end{array} \right\}$$

Extracted from
HERMES multiplicities

$$\chi^2_{\min}/\text{dof} = 1.29$$

$$A_{UT}^{\sin(\phi_h - \phi_S)}(x, y, z, P_T) = \frac{[z^2 \langle k_{\perp}^2 \rangle + \langle p_{\perp}^2 \rangle] \langle k_S^2 \rangle^2}{[z^2 \langle k_S^2 \rangle + \langle p_{\perp}^2 \rangle]^2 \langle k_{\perp}^2 \rangle} \exp \left[-\frac{P_T^2 z^2 (\langle k_{\perp}^2 \rangle - \langle k_S^2 \rangle)}{(z^2 \langle k_S^2 \rangle + \langle p_{\perp}^2 \rangle)(z^2 \langle k_{\perp}^2 \rangle + \langle p_{\perp}^2 \rangle)} \right] \\ \times \frac{\sqrt{2} e z P_T}{M_1} \frac{\sum_q e_q^2 N_q(x) f_q(x) D_{h/q}(z)}{\sum_q e_q^2 f_q(x) D_{h/q}(z)}.$$



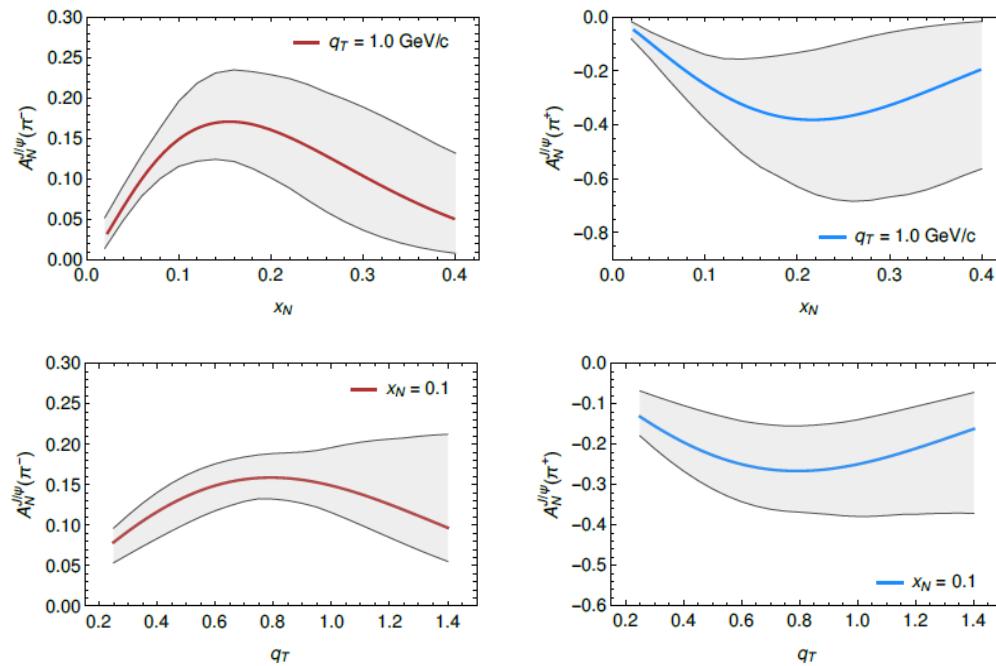
u_v well determined
 d_v and d poorly determined
 u : no information (not even sign)

Sivers function in Drell-Yan at the J/ ψ peak

M. Anselmino, V. Barone, M. Boglione, Phys. Lett. B (2017)

$$A_N^{J/\Psi}(\pi^-; x_1, x_2, q_T) \simeq \frac{\int d^2 k_{\perp 1} d^2 k_{\perp 2} \delta^2(k_{\perp 1} + k_{\perp 2} - q_T) S \cdot (\hat{p}_2 \times \hat{k}_{\perp 2}) f_{\bar{u}/\pi^-}(x_1, k_{\perp 1}) \Delta^N f_{u/p^\uparrow}(x_2, k_{\perp 2})}{2 \int d^2 k_{\perp 1} d^2 k_{\perp 2} \delta^2(k_{\perp 1} + k_{\perp 2} - q_T) f_{\bar{u}/\pi^-}(x_1, k_{\perp 1}) f_{u/p}(x_2, k_{\perp 2})}$$

$$A_N^{J/\Psi}(\pi^+; x_1, x_2, q_T) \simeq \frac{\int d^2 k_{\perp 1} d^2 k_{\perp 2} \delta^2(k_{\perp 1} + k_{\perp 2} - q_T) S \cdot (\hat{p}_2 \times \hat{k}_{\perp 2}) f_{\bar{d}/\pi^+}(x_1, k_{\perp 1}) \Delta^N f_{d/p^\uparrow}(x_2, k_{\perp 2})}{2 \int d^2 k_{\perp 1} d^2 k_{\perp 2} \delta^2(k_{\perp 1} + k_{\perp 2} - q_T) f_{\bar{d}/\pi^+}(x_1, k_{\perp 1}) f_{d/p}(x_2, k_{\perp 2})}$$



Usual DY elementary cross section

$$e_q^2 \hat{\sigma}_0 = e_q^2 \frac{4\pi\alpha^2}{9M^2}$$

With the replacements

$$\left\{ \begin{array}{l} 16\pi^2\alpha^2 e_q^2 \rightarrow (g_q^V)^2 (g_\ell^V)^2 \\ \frac{1}{M^4} \rightarrow \frac{1}{(M^2 - M_V^2)^2 + M_V^2 \Gamma_V^2} \end{array} \right.$$

Measurements from COMPASS
will soon be available

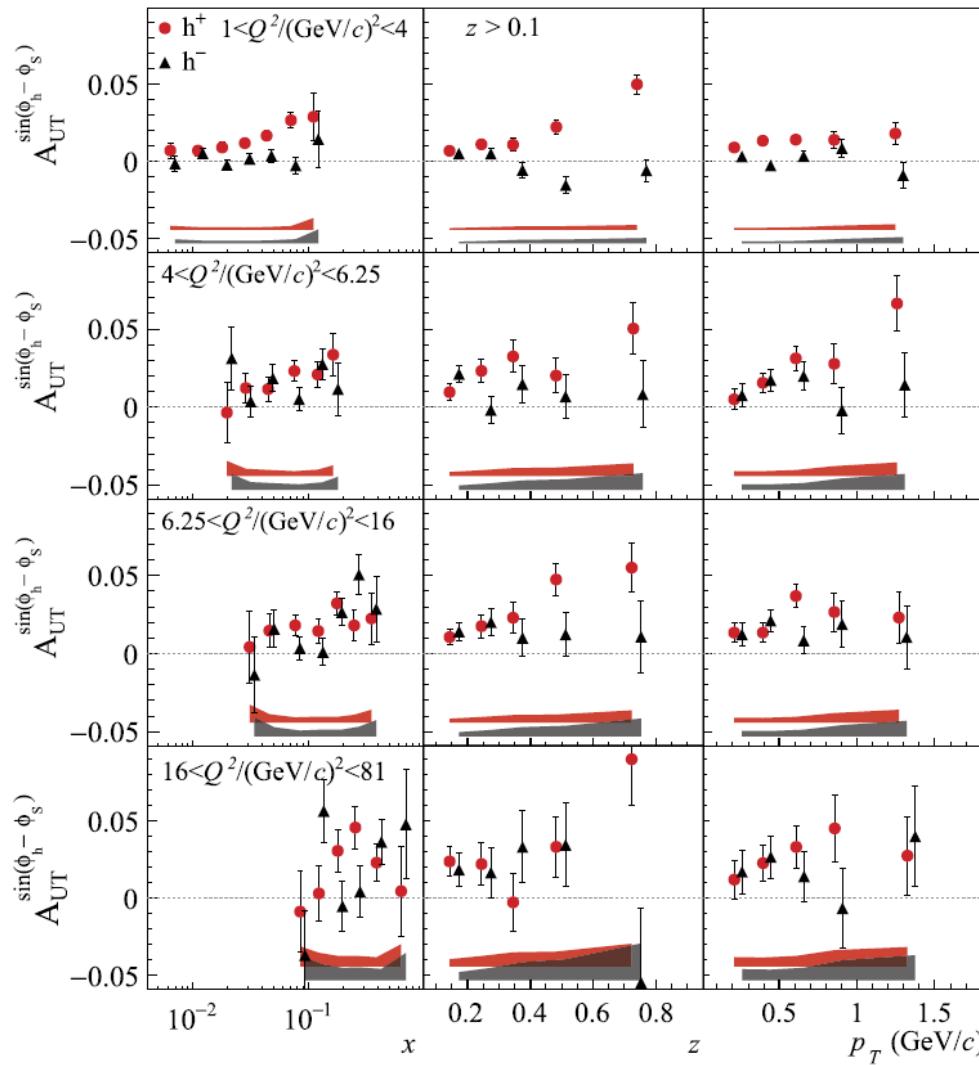


That was history ...

New Sivers data (*higher statistics, higher precision, multidimensional binning*) require a new *phenomenological extraction* (*more detailed estimation of uncertainties, evaluation of the bias induced by parametric form, study of Q^2 evolution*)

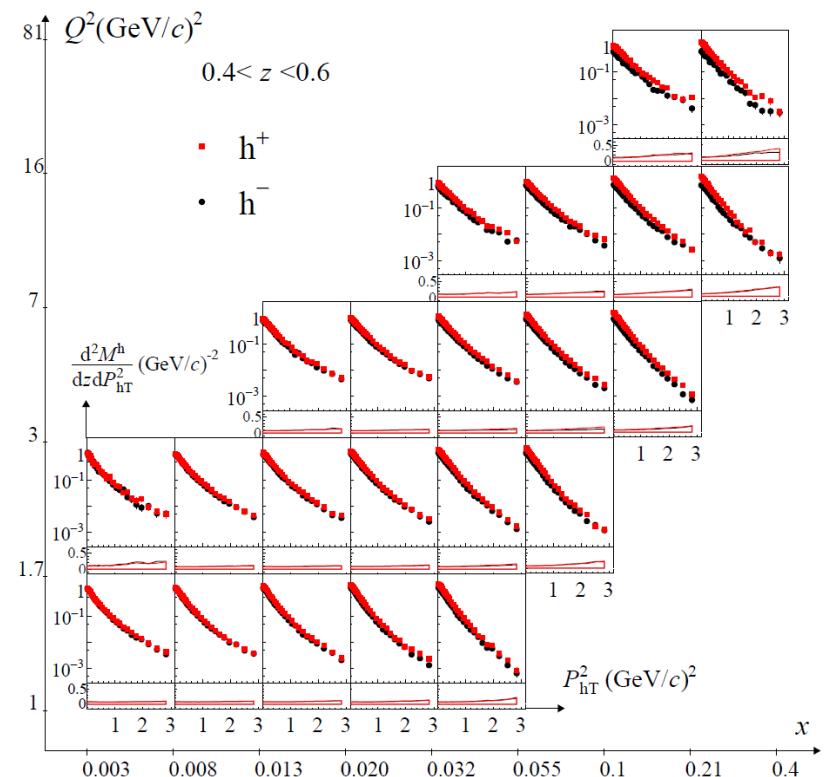
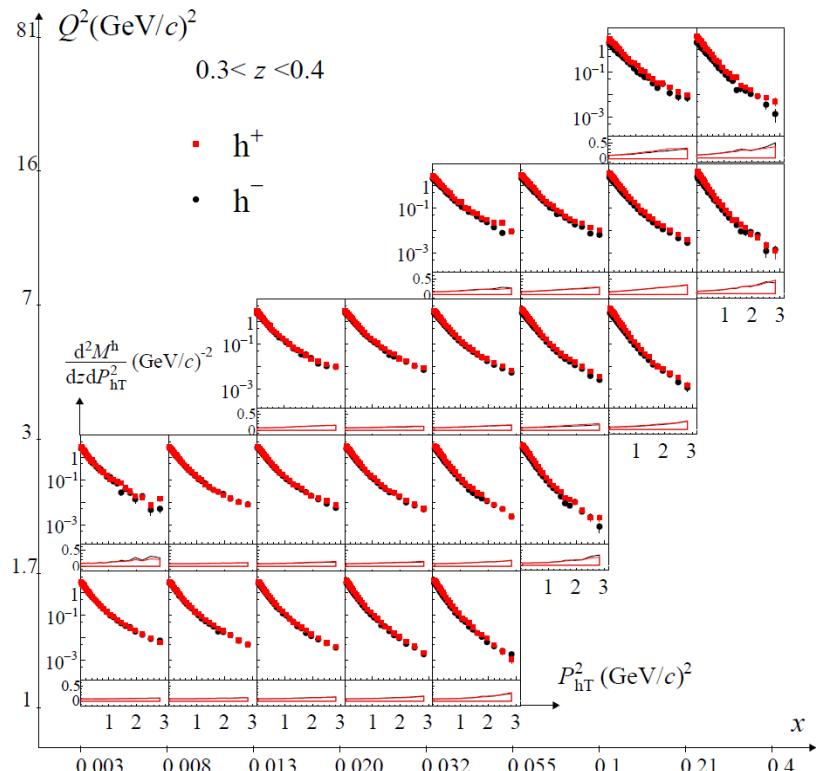
Sivers single spin asymmetry in SIDIS at the hard scales of Drell Yan @ COMPASS

COMPASS Collaboration, Phys. Lett. B 770, 138 (2017)



Multidimensional TMD multiplicities @ COMPASS

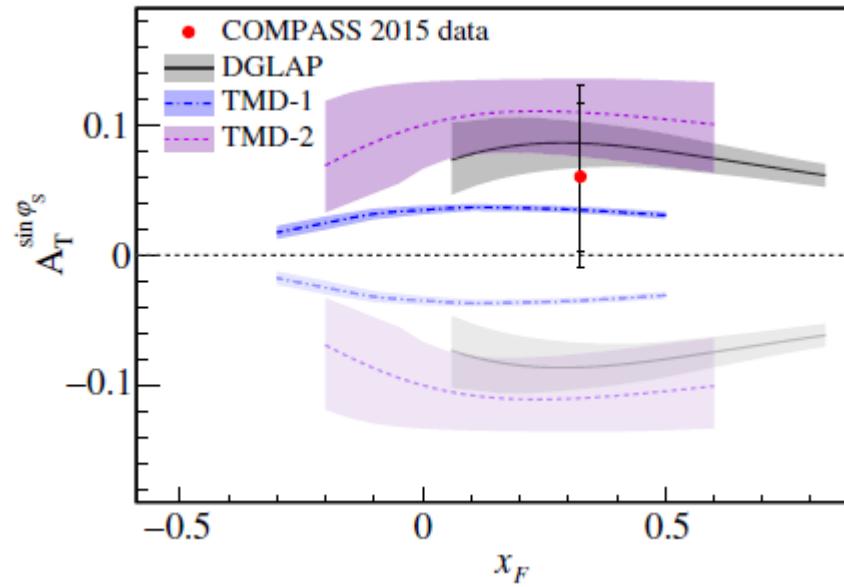
COMPASS Collaboration, arXiv:1709.07374 [hep-ex]



Sivers single spin asymmetry in pion induced Drell Yan @ COMPASS

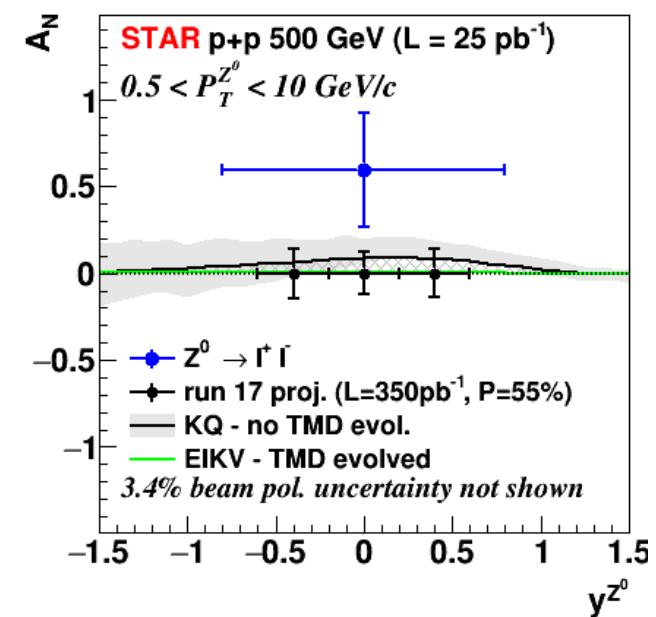
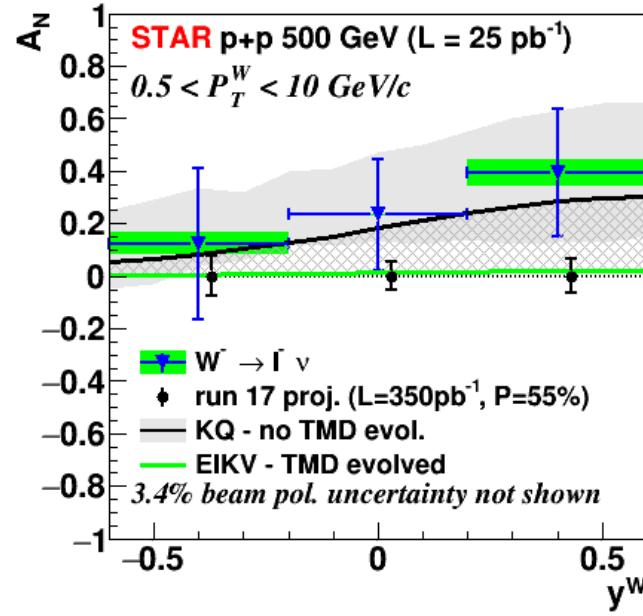
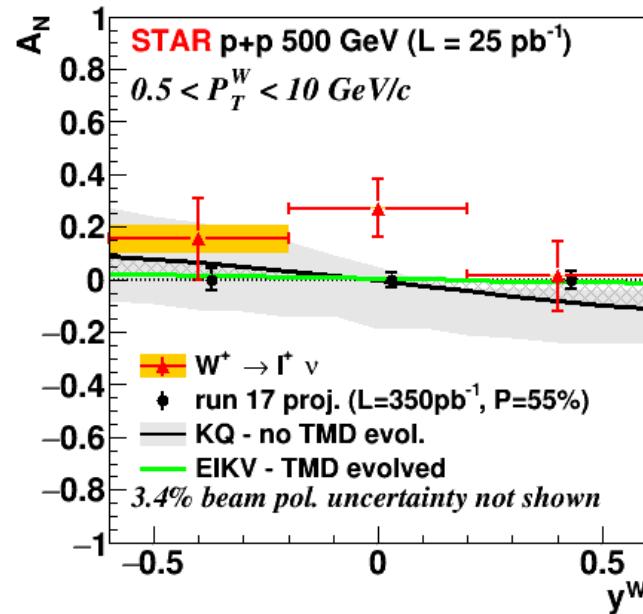
COMPASS Collaboration, Phys. Rev. Lett. 119, 112002 (2017)

190GeV/c π^- beam scattered off a transversely polarized NH₃ target (polarized proton)



Sivers function in $p^\uparrow + p \rightarrow W^\pm/Z$ @ RHIC RUN 2017

STAR Collaboration, Phys. Rev. Lett. 116 132301 (2016)



***Need for a new,
comprehensive
study of the
Sivers effect***

New extraction of the Sivers function

New parametrization of the Sivers function

$$\Delta^N f_{q/p^\uparrow} = 4N_q x^{\alpha_q} (1-x)^{\beta_q} \frac{M_p}{\langle k_\perp^2 \rangle_S} k_\perp \frac{e^{-k_\perp^2 / \langle k_\perp^2 \rangle_S}}{\pi \langle k_\perp^2 \rangle_S}$$

First moment of the Sivers fn.
Flavour dependent (u_v, d_v)

k_\perp dependence of the Sivers fn.
Flavour independent

Sivers functions
not proportional
to TMD PDFs

No direct control on
the positivity bound

M_p is a fixed parameter to
give the right dimensions.
It is fixed to 0.938 GeV

- In perspective: parametrization in terms of momentum better suited for the study of TMD evolution
- It makes the expression of the actual Sivers asymmetry as simple as possible (within this model)

Sivers Asymmetry (numerator)

$$F_{UT}^{\sin(\phi_S - \phi_h)} = 2 \frac{z P_T}{\langle P_T^2 \rangle_S} \frac{e^{-P_T^2 / \langle P_T^2 \rangle_S}}{\pi \langle P_T^2 \rangle_S} \sum_q e_q^2 \left(N_q x^{\alpha_q} (1-x)^{\beta_q} \right) D_{h/q}(z)$$

New extraction of the Sivers function

New parametrization of the Sivers function

$$\Delta^N f_{q/p^\uparrow} = 4N_q x^{\alpha_q} (1-x)^{\beta_q} \frac{M_p}{\langle k_\perp^2 \rangle_S} k_\perp \frac{e^{-k_\perp^2 / \langle k_\perp^2 \rangle_S}}{\pi \langle k_\perp^2 \rangle_S}$$

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- In perspective: parametrization in terms of momentum better suited for the study of TMD evolution
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First moment of the Sivers function

$$\Delta^N f_{q/p^\uparrow}^{(1)}(x) = \int d^2 k_\perp \frac{k_\perp}{4M_p} \Delta^N f_{q/p^\uparrow}(x, k_\perp)$$

$$\Delta^N f_{q/p^\uparrow}^{(1)} = N_q x^{\alpha_q} (1-x)^{\beta_q}$$

New extraction of the Sivers function

Consistent data selection:

- Exclude negative kaons for valence dominance assumption
- u_v seems well constrained
- d_v is not constrained: it can be replaced by sea contributions with equally good fits:
hard to distinguish where this contribution comes from
- Sivers sea is totally unconstrained

**It is of vital importance to gain information
on the d content of the Sivers function**

**We strongly rely on SIDIS
measurements of the Sivers
asymmetry on deuterium target
@ COMPASS !**

TMD Factorization approach and Collinear twist-three factorization approach

TMD factorization approach

- Spin asymmetries are generated by spin and transverse momentum correlations between the identified hadron and the active parton.
- This correlations are embedded in the TMD parton distribution or fragmentation functions, which can be interpreted as probability densities.

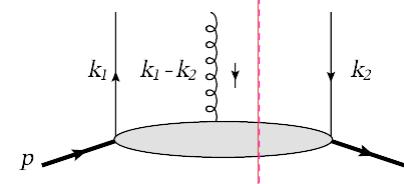
Q^2 evolution affects both x and k_\perp

Collinear twist-three factorization approach

- The correlation between spin and transverse momentum is included into the **high twist** collinear parton distributions or fragmentation functions.
- Twist-3 collinear parton distributions or fragmentation functions have no probability interpretation. They are interpreted as the quantum interference between a collinear active quark state in the scattering amplitude and a collinear quark-gluon composite state in its complex conjugate amplitude.

Q^2 evolution occurs only through x

- TMDs and quark-gluon correlation functions are closely related to each other.
- The first k_\perp -moment of the Sivers function is equal to the twist-3 quark-gluon correlation functions $T_{q,F}(x, x)$
- Evolution kernels for $T_{q,F}(x, x)$ are known; we can exploit them in our study (off diagonal terms are not included)**



$$\begin{aligned} \frac{\partial T_{q,F}(x, x, \mu)}{\partial \ln \mu^2} = & \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} \left\{ P_{qq}(z) T_{q,F}(\xi, \xi, \mu) \right. \\ & + \frac{N_c}{2} \left[\frac{1+z^2}{1-z} (T_{q,F}(\xi, x, \mu) - T_{q,F}(\xi, \xi, \mu)) + z T_{q,F}(\xi, x, \mu) + T_{\Delta q,F}(x, \xi, \mu) \right] \\ & \left. - N_c \delta(1-z) T_{q,F}(x, x, \mu) + \frac{1}{2N_c} [(1-2z) T_{q,F}(x, x - \xi, \mu) + T_{\Delta q,F}(x, x - \xi, \mu)] \right\} \end{aligned}$$

J.B. Kang and J.W. Qiu, Phys. Rev. D 79 (2009) 016003

W. Vogelsang and F. Yuan, Phys. Rev. D 79 (2009) 094010

V.M. Braun, A.N. Manashov, B. Pirnay, Phys. Rev. D 80 (2009) 114002

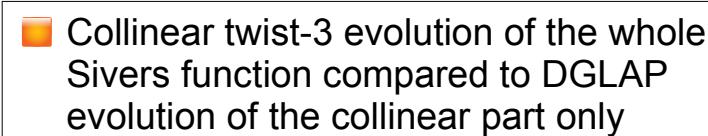
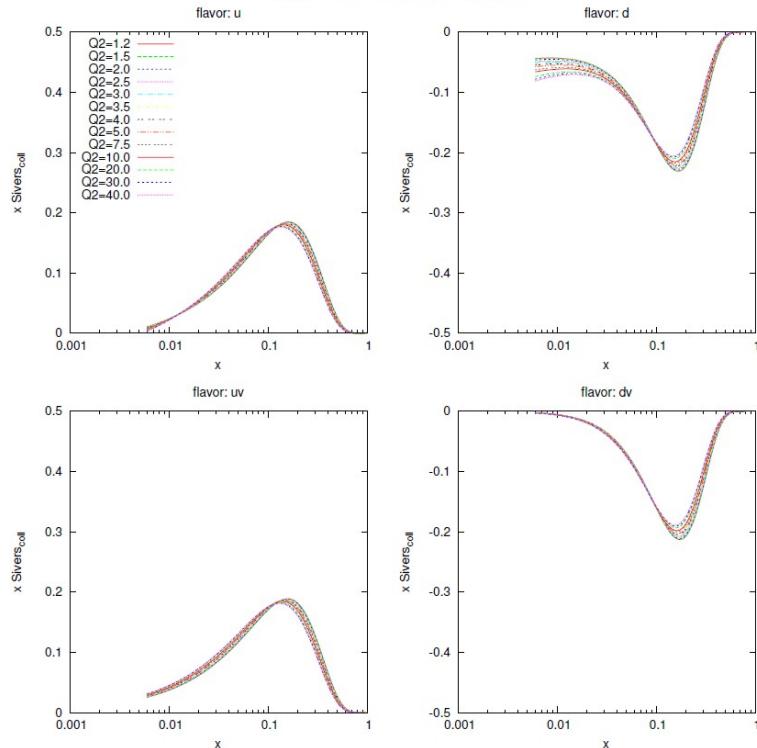
Z.B Kang and J.W. Qiu, Phys. Lett. B713 (2012) 273-276

New extraction of the Sivers function

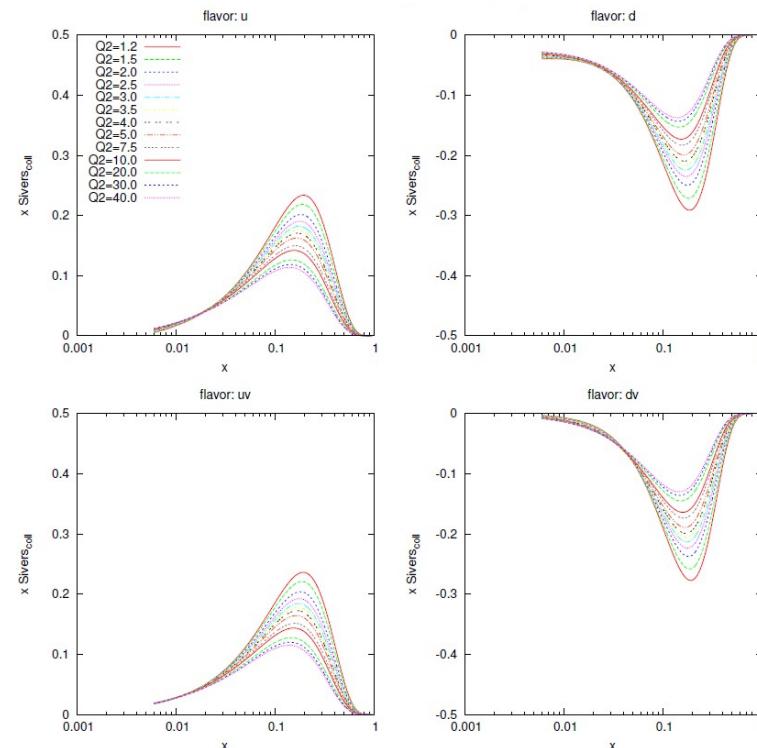
Anselmino, Boglione, D'Alesio, Flore, Gonzalez, Murgia, Prokudin



Q dependence of the first moment of the Sivers function
DGLAP evolution of the collinear part only (PDF)



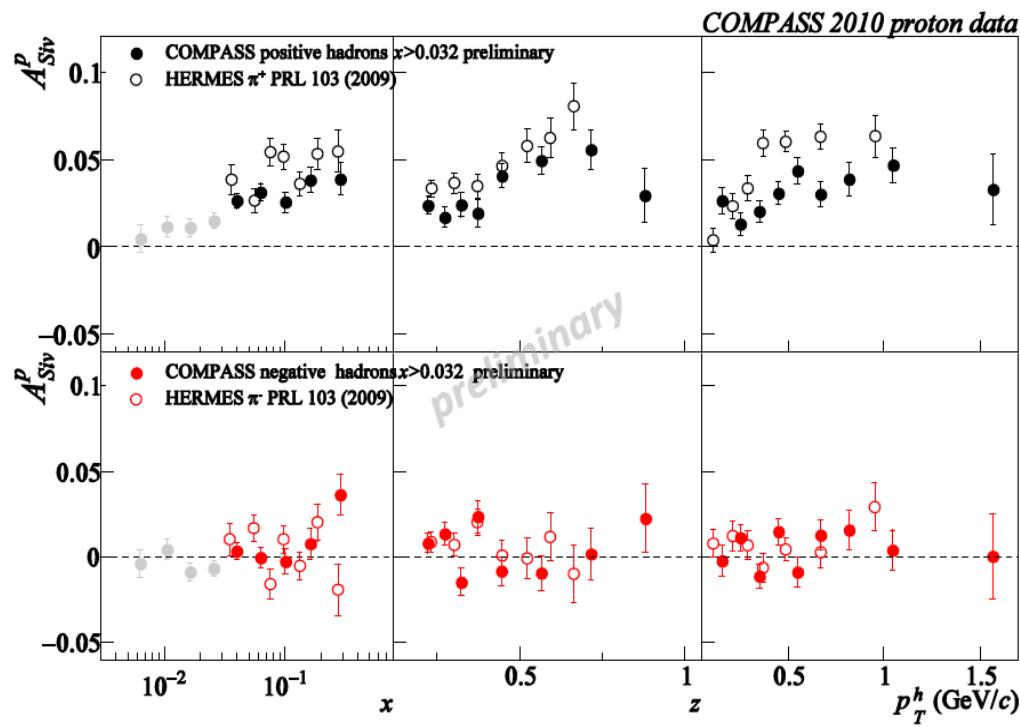
Q dependence of the first moment of the Sivers function
Collinear Twist-3 HOPPET evolution of the full Sivers function



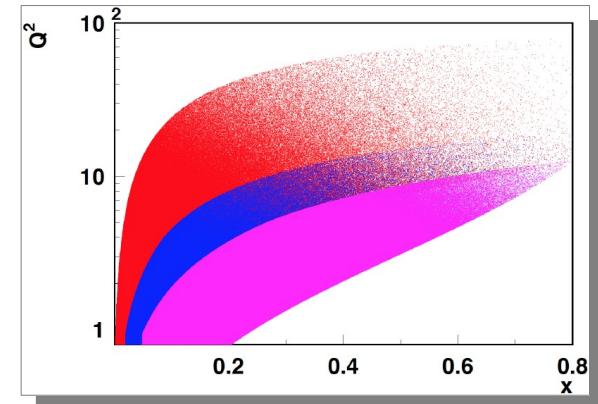
Sivers effect: COMPASS vs. HERMES

Anselmino, Boglione, D'Alesio, Flore, Gonzalez, Murgia, Prokudin

Apparently ...
some tension between
COMPASS and HERMES data



However, COMPASS and HERMES span different ranges in Q^2 and have different $\langle Q^2 \rangle$.



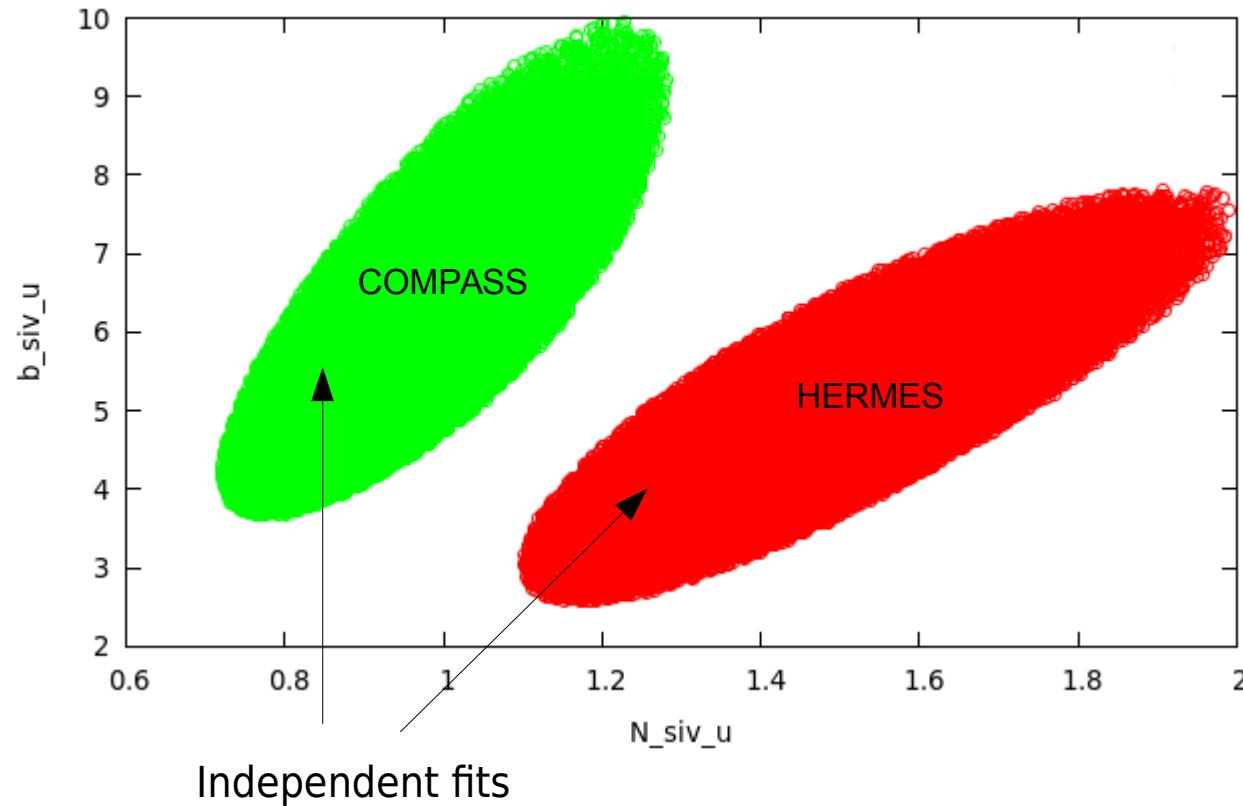
Possible signal of
TMD evolution?

New extraction of the Sivers function

Anselmino, Boglione, D'Alesio, Flore, Gonzalez, Murgia, Prokudin

Signal of some tension between independent fit solutions for COMPASS and HERMES data

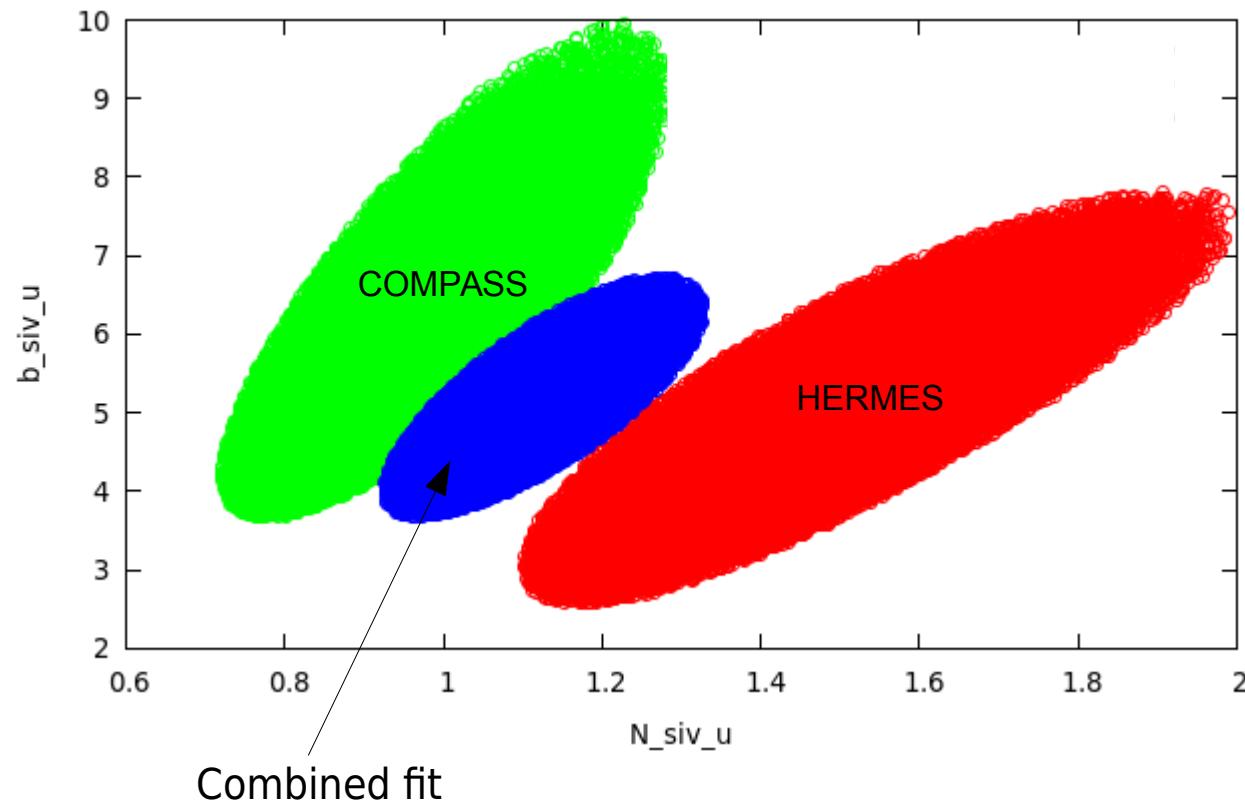
Start by using a very simple model only 5 parameters and no Q evolution



New extraction of the Sivers function

Anselmino, Boglione, D'Alesio, Flore, Gonzalez, Murgia, Prokudin

Signal of some tension between independent fit solutions for COMPASS and HERMES data



Relevance of unpolarized p_T distributions

To calculate any spin asymmetry it is crucial to use the appropriate denominator, i.e. the appropriate unpolarized cross section

$$F_{UU} = \sum_q e_q^2 f_{q/p}(x_B) D_{h/q}(z_h) \frac{e^{-P_T^2/\langle P_T^2 \rangle}}{\pi \langle P_T^2 \rangle}$$

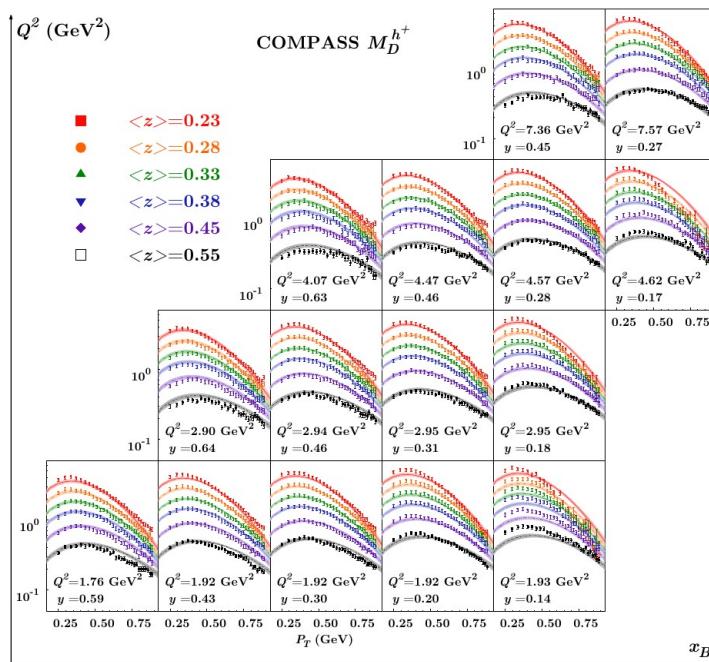
$$\text{with } \langle P_T^2 \rangle = \langle p_\perp^2 \rangle + z_h^2 \langle k_\perp^2 \rangle$$

- It is very important to measure **p_T distributions of unpolarized cross sections** in SIDIS, Drell-Yan, e+e- processes

Relevance of unpolarized p_T distributions

M. Anselmino, M. Boglione, O. Gonzalez, S. Melis, A. Prokudin, JHEP 1404 (2014) 005

$$F_{UU} = \sum_q e_q^2 f_{q/p}(x_B) D_{h/q}(z_h) \frac{e^{-P_T^2/\langle P_T^2 \rangle}}{\pi \langle P_T^2 \rangle} \quad \text{with} \quad \langle P_T^2 \rangle = \langle p_\perp^2 \rangle + z_h^2 \langle k_\perp^2 \rangle$$

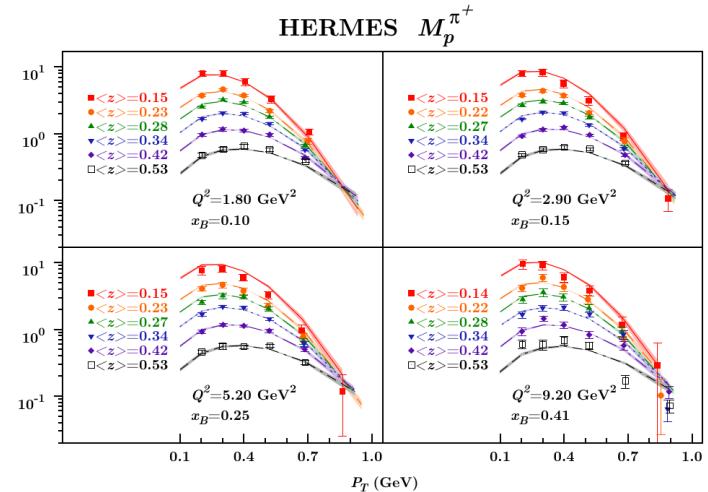


COMPASS, Adolph et al., Eur. Phys. J. C 73 (2013) 2531

$$\langle k_\perp^2 \rangle = 0.60 \pm 0.14 \text{ GeV}^2$$

$$\langle p_\perp^2 \rangle = 0.20 \pm 0.02 \text{ GeV}^2$$

$$\chi_{\text{dof}}^2 = 3.42$$



Airapetian et al, Phys. Rev. D 87 (2013) 074029

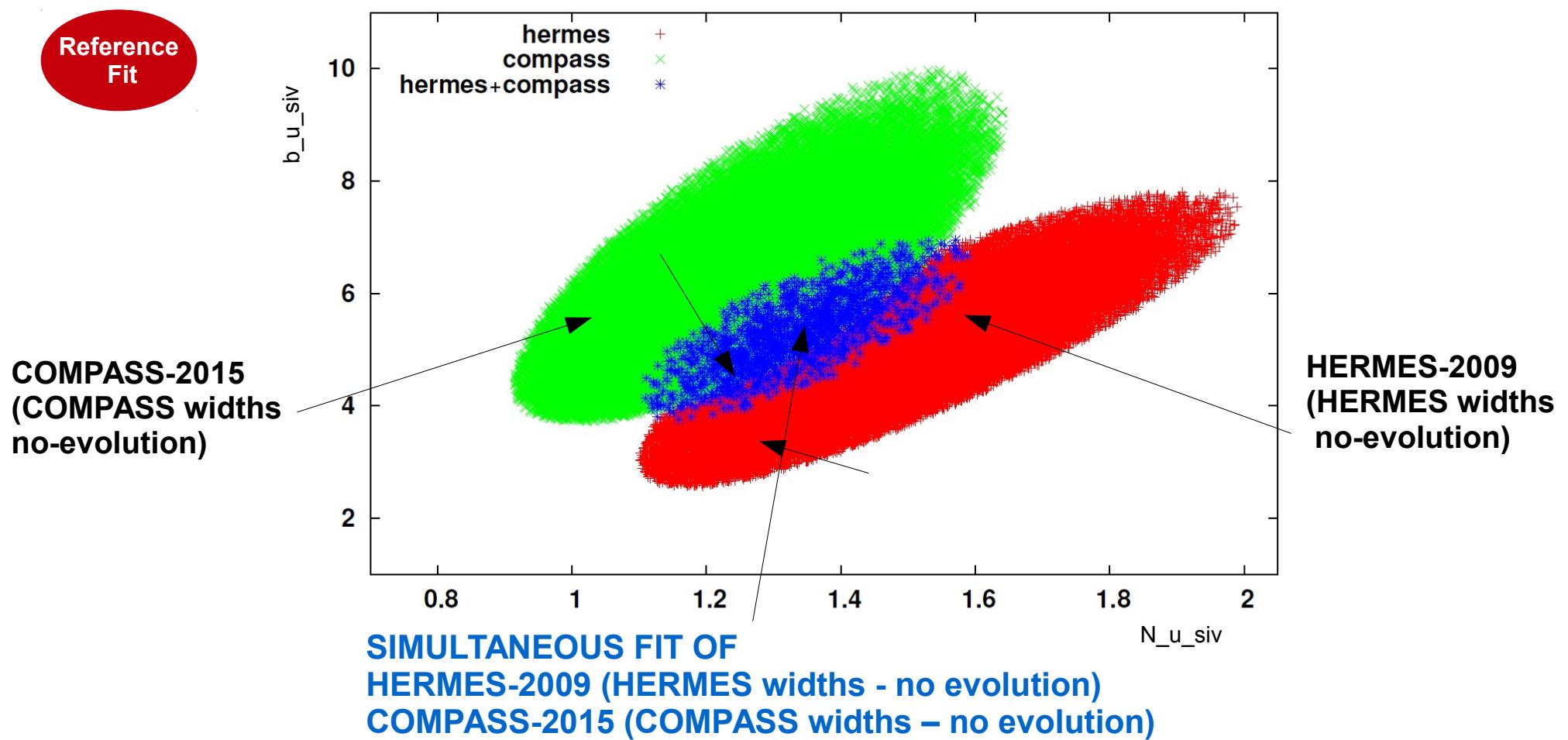
$$\langle k_\perp^2 \rangle = 0.57 \pm 0.08 \text{ GeV}^2$$

$$\langle p_\perp^2 \rangle = 0.12 \pm 0.01 \text{ GeV}^2$$

$$\chi_{\text{dof}}^2 = 1.69$$

New extraction of the Sivers function

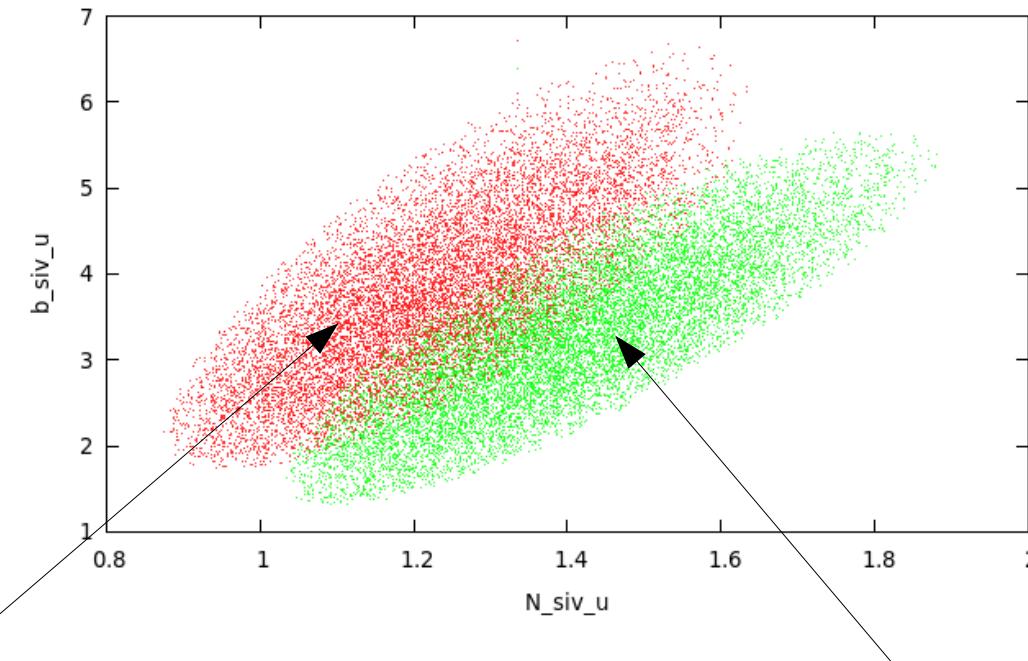
Tension relaxes when the asymmetry is computed using the appropriate unpolarized widths for each data set



New extraction of the Sivers function

Anselmino, Boglione, D'Alesio, Flore, Gonzalez, Murgia, Prokudin

Tension relaxes when the asymmetry is computed using the appropriate unpolarized widths for each data set



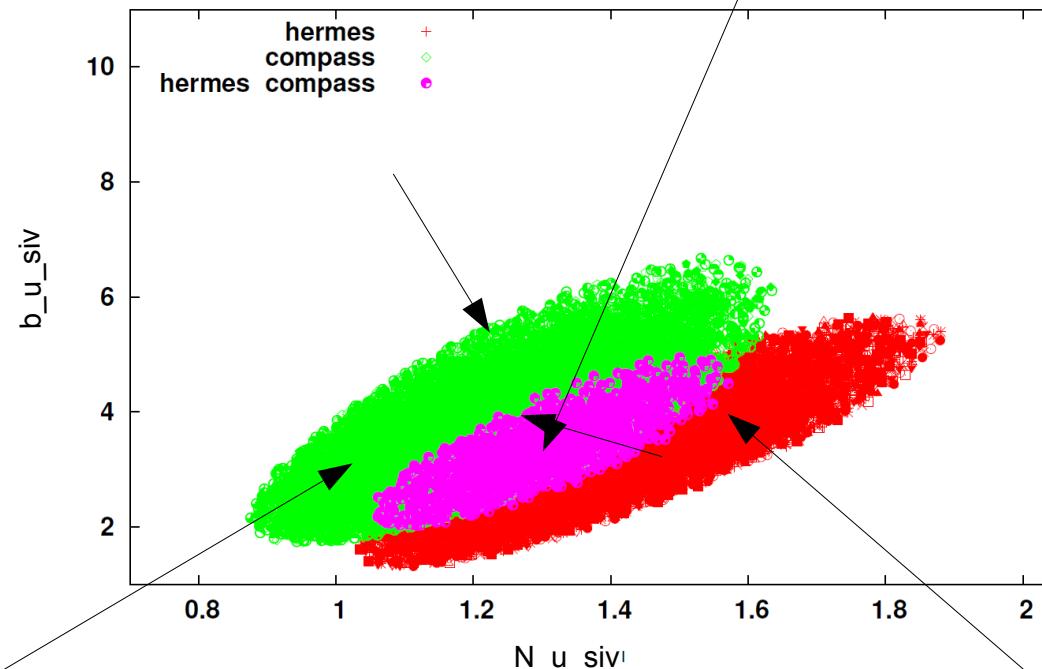
COMPASS-2017
(COMPASS widths + Twist-3 evolution)

HERMES-2009
(HERMES widths + Twist-3 evolution)

New extraction of the Sivers function

Anselmino, Boglione, D'Alesio, Flore, Gonzalez, Murgia, Prokudin

Simultaneous fit of
HERMES-2009 (HERMES widths + Twist-3 evolution)
COMPASS-2017 (COMPASS widths + Twist-3 evolution)



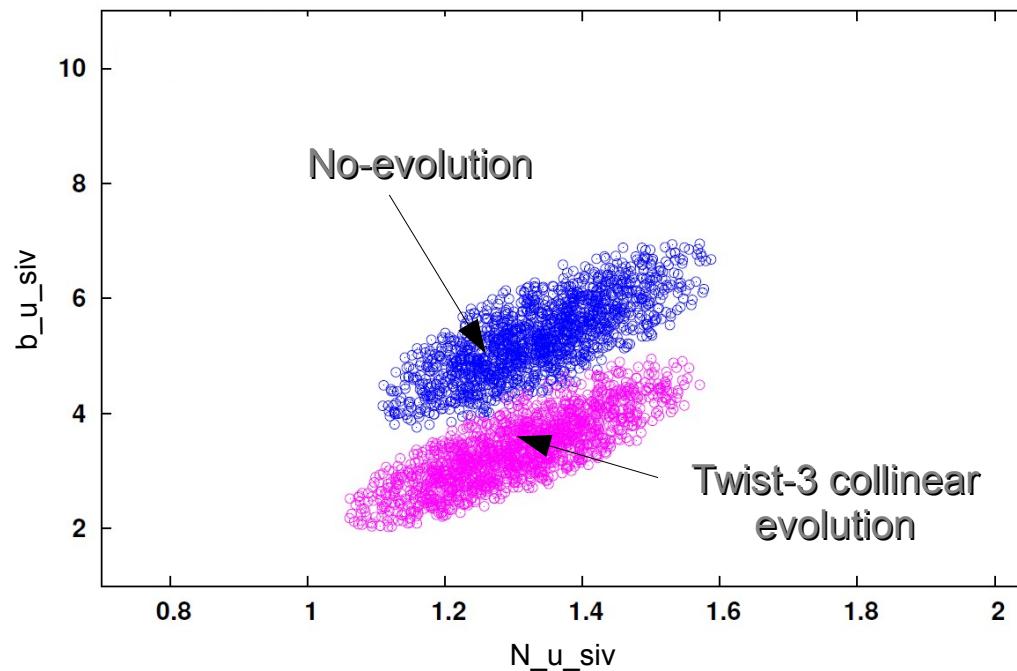
COMPASS-2017
(COMPASS widths + Twist-3 evolution)

HERMES-2009
(HERMES widths + Twist-3 evolution)

New extraction of the Sivers function

Anselmino, Boglione, D'Alesio, Flore, Gonzalez, Murgia, Prokudin

Simultaneous fit of
HERMES-2009 (HERMES widths)
COMPASS-2017 (COMPASS widths)

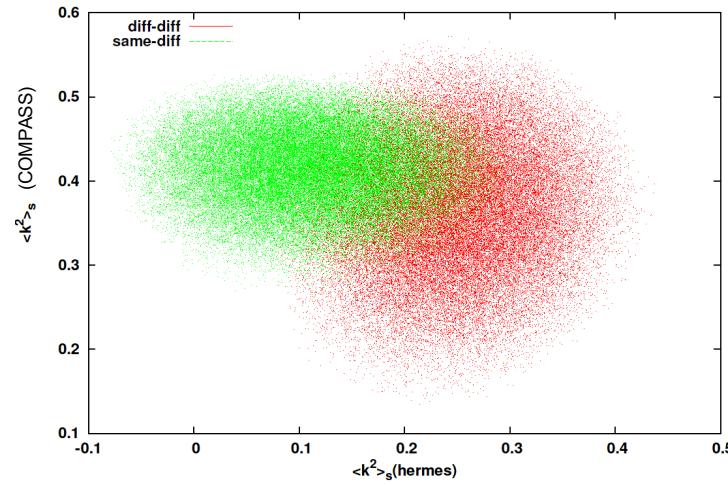


New extraction of the Sivers function

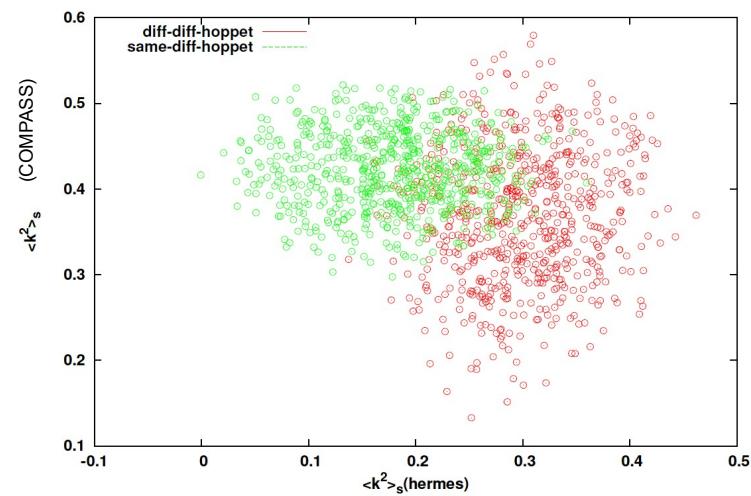
Sivers widths: HERMES vs. COMPASS

Allowing for different **Sivers** widths for each experiments, does not improve the quality of the fit, and the extracted values are very similar

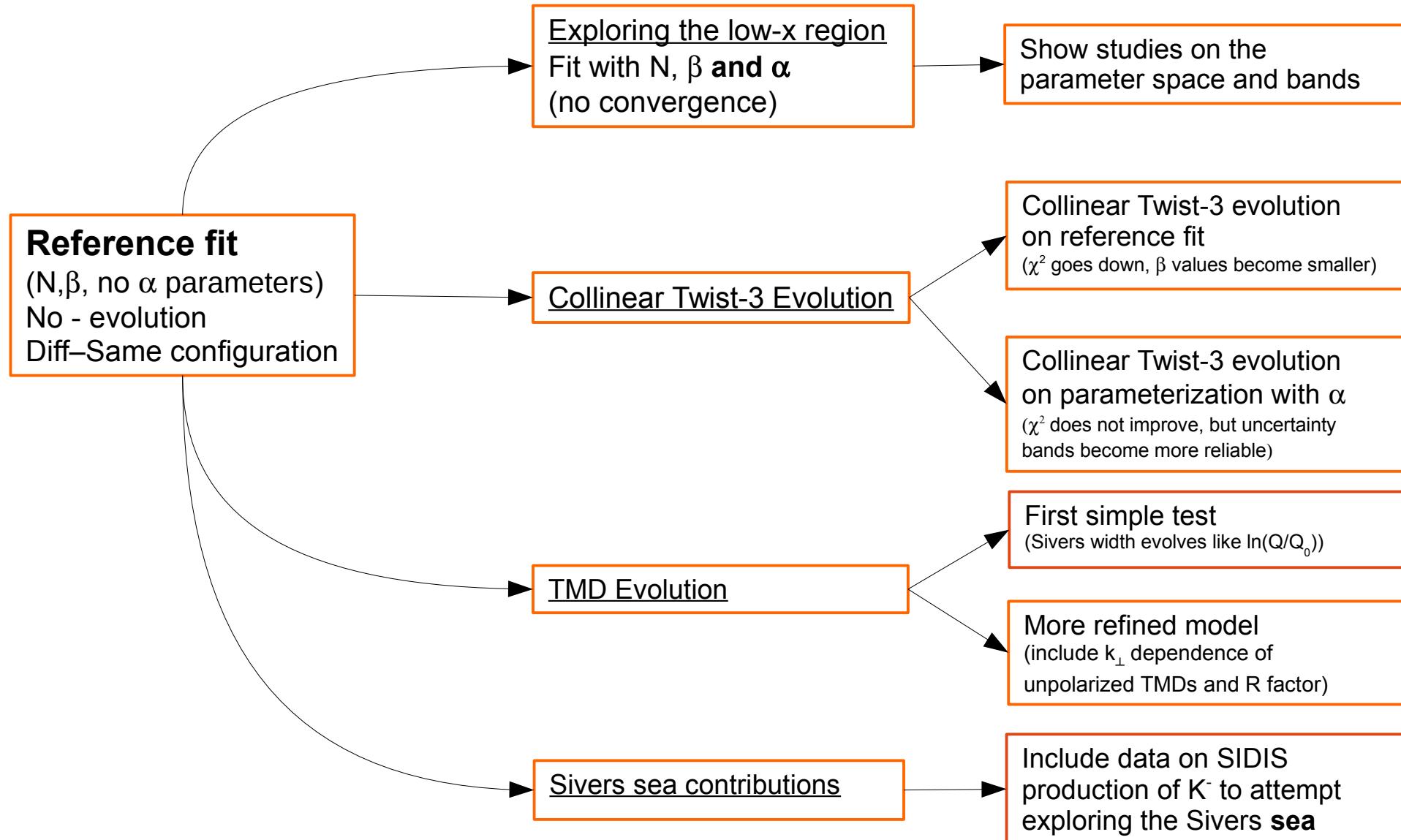
No-evolution



Twist-3 Collinear evolution



Study of the uncertainties in the extraction of the Sivers function



Study of the uncertainties in the extraction of the Sivers function

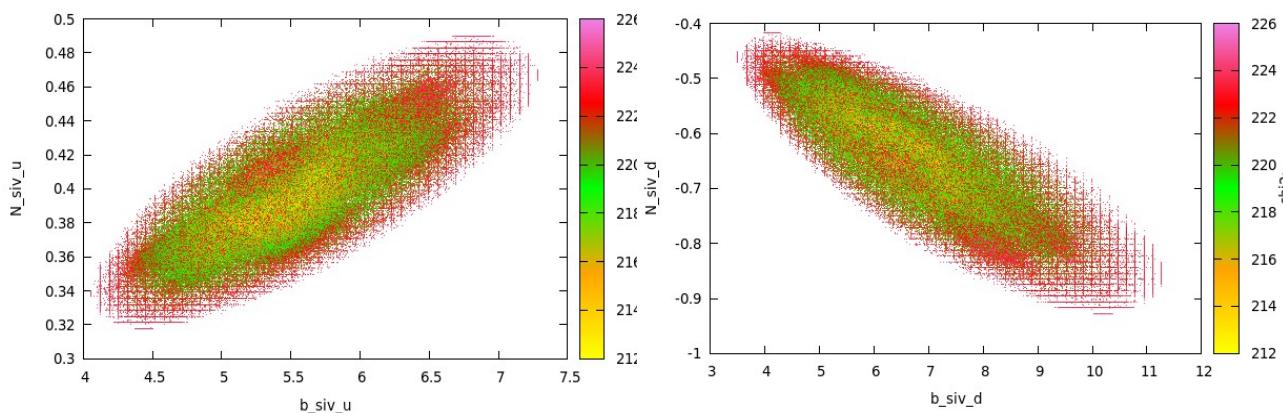
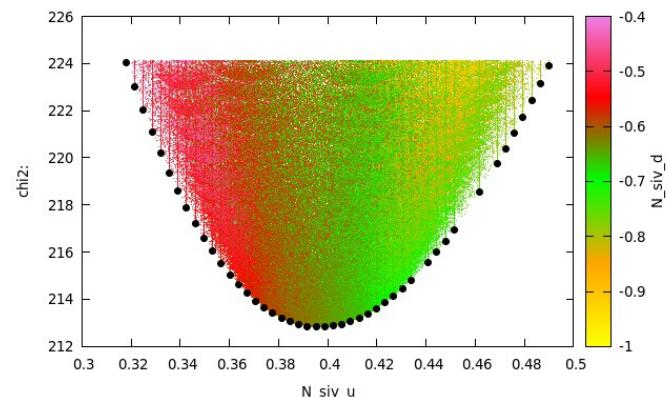
Reference fit – no evolution

$\chi^2_{pt} = 0.96$	$COMPASS \langle k_\perp^2 \rangle = 0.6 \pm$
$pts = 220$	$COMPASS \langle p_\perp^2 \rangle = 0.2 \pm$
	$HERMES \langle k_\perp^2 \rangle = 0.57 \pm$
	$HERMES \langle p_\perp^2 \rangle = 0.12 \pm$
	$\alpha_{u_v} = 0 \pm$
	$\alpha_{d_v} = 0 \pm$
	$\beta_{u_v} = 5.427 \pm 1.588$
	$\beta_{d_v} = 6.448 \pm 3.643$
	$N_{u_v} = 0.396 \pm 0.085$
	$N_{d_v} = -0.633 \pm 0.234$
	$\langle k_\perp^2 \rangle_s = 0.296 \pm 0.148$

Reference Fit

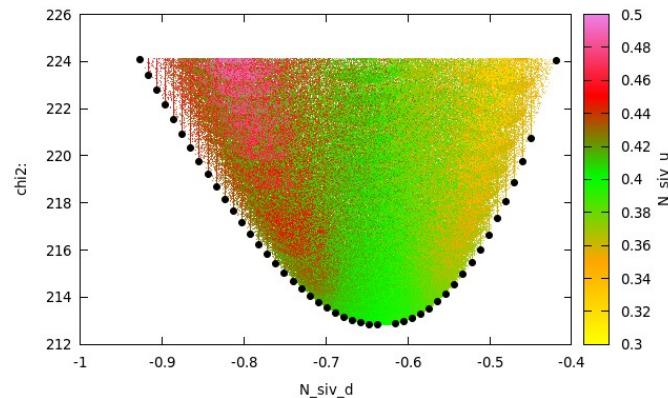
220 data points
5 free-parameters
 $\Delta\chi^2 = 11.31$

χ^2 scans: some examples



Parameter correlations

$$P = \int_0^{\Delta\chi^2} \frac{1}{2\Gamma(M/2)} \left(\frac{\chi^2}{2}\right)^{(M/2)-1} \exp\left(-\frac{\chi^2}{2}\right) d\chi^2.$$



The χ^2 profile can reasonably be approximated with a quadratic, Hessian approx. works well, MINUIT errors give reliable estimates of the uncertainty on the free parameters.

Comparing reference fits: no-evolution vs. collinear twist-3 evolution

Reference fit – no evolution

$$\begin{aligned}
 \chi_{pt}^2 &= 0.96 \\
 pts &= 220 \\
 \langle k_\perp^2 \rangle &= 0.6 \pm 0.2 \\
 \langle p_\perp^2 \rangle &= 0.2 \pm 0.1 \\
 (hermes) \langle k_\perp^2 \rangle &= 0.57 \pm 0.12 \\
 (hermes) \langle p_\perp^2 \rangle &= 0.12 \pm 0.05 \\
 \alpha_{u_v} &= 0 \pm 0 \\
 \alpha_{d_v} &= 0 \pm 0 \\
 \beta_{u_v} &= 5.427 \pm 1.588 \\
 \beta_{d_v} &= 6.448 \pm 3.643 \\
 N_{u_v} &= 0.396 \pm 0.085 \\
 N_{d_v} &= -0.633 \pm 0.234 \\
 \langle k_\perp^2 \rangle_s &= 0.296 \pm 0.148
 \end{aligned}$$

Reference fit – collinear twist-3 evolution

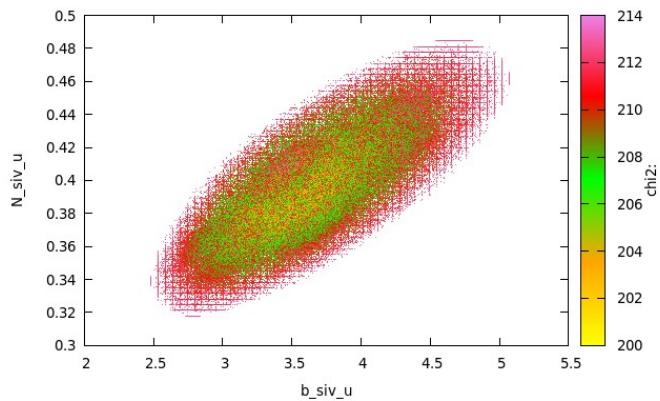
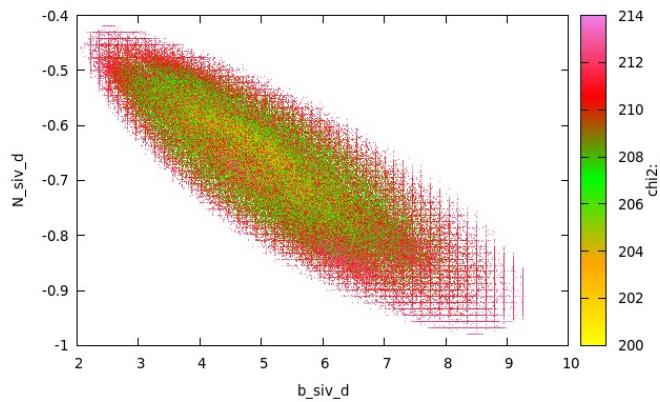
$$\begin{aligned}
 \chi_{pt}^2 &= 0.91 \\
 pts &= 220 \\
 \langle k_\perp^2 \rangle &= 0.6 \pm 0.2 \\
 \langle p_\perp^2 \rangle &= 0.2 \pm 0.1 \\
 (hermes) \langle k_\perp^2 \rangle &= 0.57 \pm 0.12 \\
 (hermes) \langle p_\perp^2 \rangle &= 0.12 \pm 0.05 \\
 \alpha_{u_v} &= 0 \pm 0 \\
 \alpha_{d_v} &= 0 \pm 0 \\
 \beta_{u_v} &= 3.548 \pm 1.262 \\
 \beta_{d_v} &= 4.772 \pm 3.405 \\
 N_{u_v} &= 0.394 \pm 0.082 \\
 N_{d_v} &= -0.653 \pm 0.268 \\
 \langle k_\perp^2 \rangle_s &= 0.326 \pm 0.144
 \end{aligned}$$

- Central values go down
- Size of minuit errors are similar

In both cases, the χ^2 profile can be reasonably approximated with a quadratic, Hessian approx. works well, MINUIT errors give reliable estimates of the uncertainty on the free parameters.

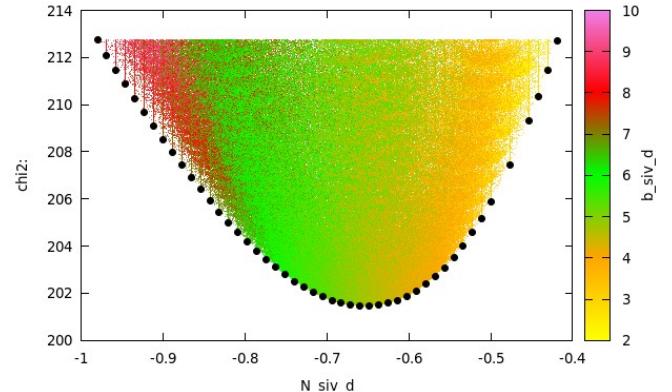
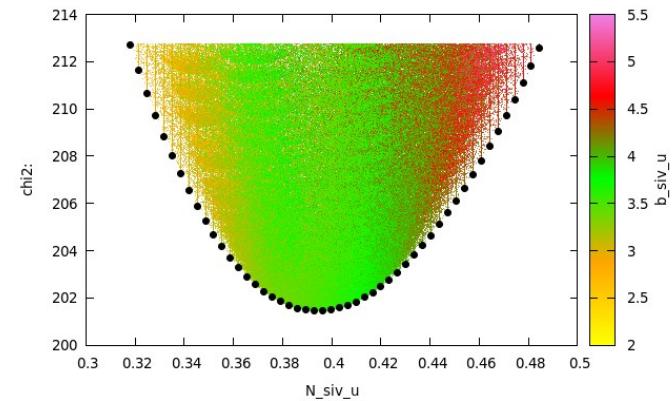
Study of the uncertainties in the extraction of the Sivers function

Parameter correlations



χ^2 scans: some examples

Reference fit
+
Collinear twist-3
evolution



In both cases, the χ^2 profile can be reasonably approximated with a quadratic, Hessian approx. works well, MINUIT errors give reliable estimates of the uncertainty on the free parameters.

Study of the uncertainties in the extraction of the Sivers function

Study of Low-x Uncertainties

(include α_u and α_d in the parametrization of the Sivers function)

Attempt to minimize bias from parametric form

Reference fit – no evolution
5 params.

alpha fit – no evolution
7 params.

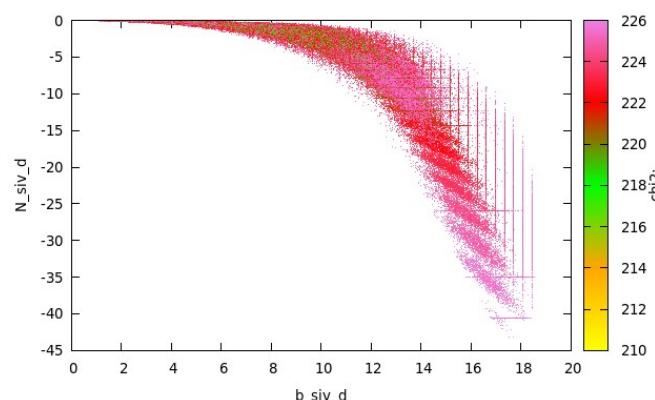
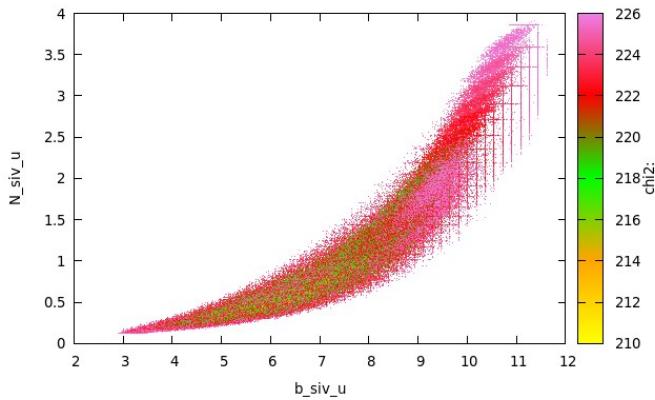
χ^2 does not improve !	
$\chi_{pt}^2 = 0.96$	$\chi_{pt}^2 = 0.96$
$pts = 220$	$pts = 220$
$\langle k_\perp^2 \rangle = 0.6 \pm$	$\langle k_\perp^2 \rangle = 0.6 \pm$
$\langle p_\perp^2 \rangle = 0.2 \pm$	$\langle p_\perp^2 \rangle = 0.2 \pm$
$(hermes) \langle k_\perp^2 \rangle = 0.57 \pm$	$(hermes) \langle k_\perp^2 \rangle = 0.57 \pm$
$(hermes) \langle p_\perp^2 \rangle = 0.12 \pm$	$(hermes) \langle p_\perp^2 \rangle = 0.12 \pm$
$\alpha_{u_v} = 0 \pm$	$\alpha_{u_v} = 0.073 \pm 0.455$
$\alpha_{d_v} = 0 \pm$	$\alpha_{d_v} = -0.075 \pm 0.831$
$\beta_{u_v} = 5.427 \pm 1.588$	$\beta_{u_v} = 5.931 \pm 3.859$
$\beta_{d_v} = 6.448 \pm 3.643$	$\beta_{d_v} = 5.707 \pm 7.431$
$N_{u_v} = 0.396 \pm 0.085$	$N_{u_v} = 0.503 \pm 0.751$
$N_{d_v} = -0.633 \pm 0.234$	$N_{d_v} = -0.482 \pm 0.610$
$\langle k_\perp^2 \rangle_s = 0.296 \pm 0.148$	$\langle k_\perp^2 \rangle_s = 0.303 \pm 0.167$

Alpha and N parameters are totally correlated

For the alpha-fit, the χ^2 profile is NOT quadratic, Hessian approx. does not work, MINUIT errors do NOT give reliable estimates of the uncertainty on the parameters, especially on the N parameters.

Study of the uncertainties in the extraction of the Sivers function

Parameter correlations



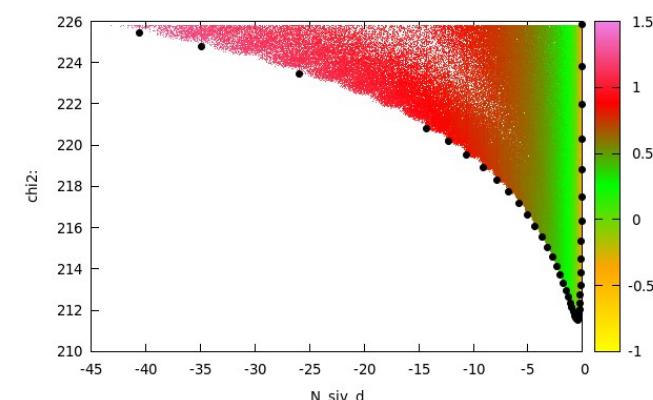
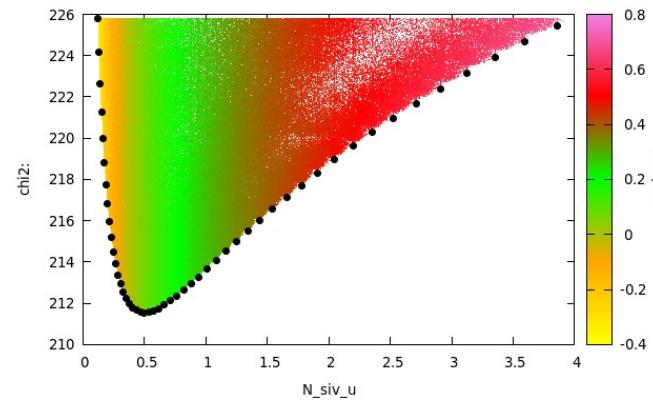
Study of Low-x Uncertainties

(include α_u and α_d
in the parametrization
of the Sivers function)

no-evolution

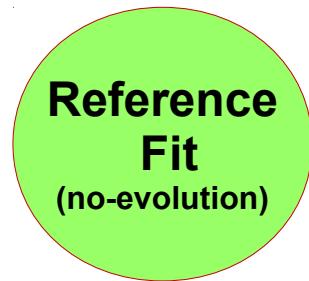
220 data points
7 free-parameters
 $\Delta\chi^2 = 14.34$

χ^2 scans

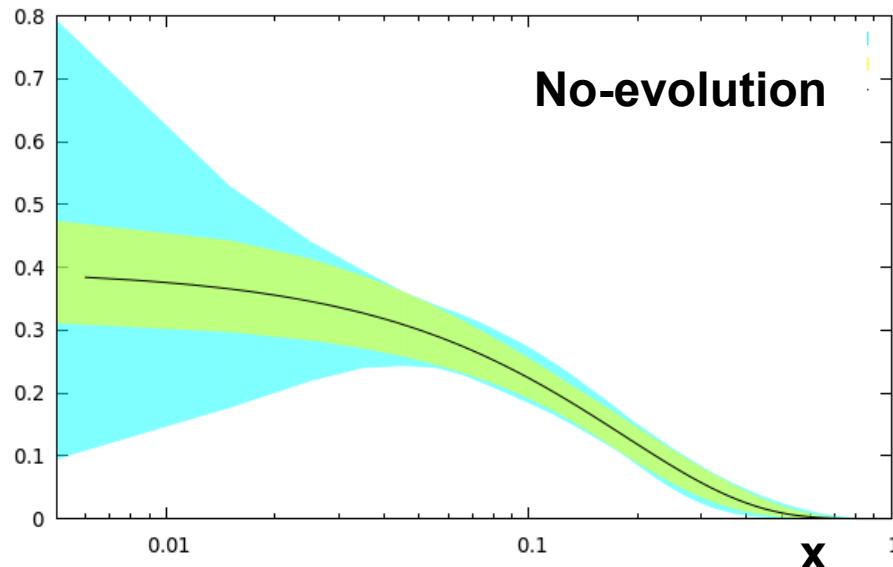


For the alpha-fit, the χ^2 profile is NOT quadratic, Hessian approx. does not work,
MINUIT errors do NOT give reliable estimates of the uncertainty on the parameters,
especially on the N parameters.

Uncertainty bands – Sivers first moment



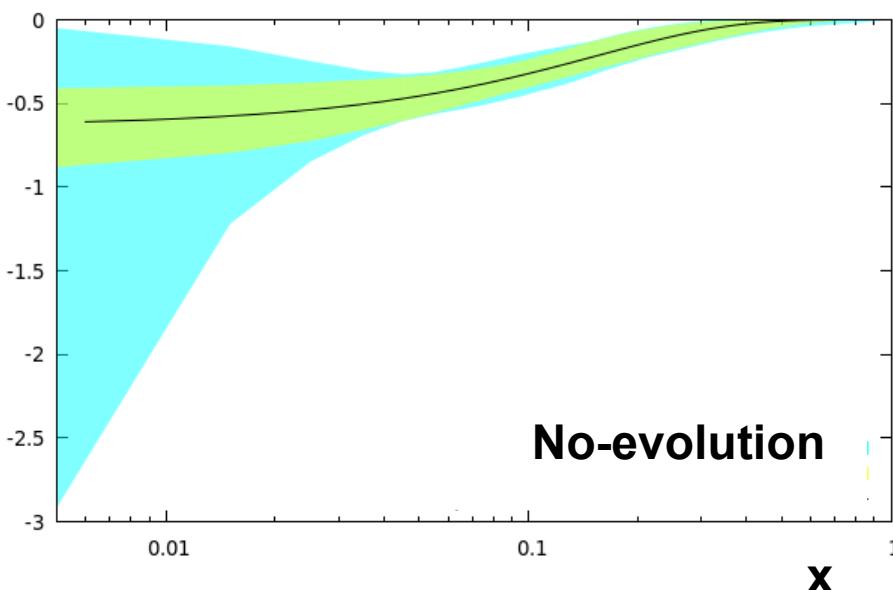
**Sivers
First moment
u-valence**



**Study of Low-x
Uncertainties**

(include α_u and α_d
in the parametrization
of the Sivers function)

**Sivers
First moment
d-valence**



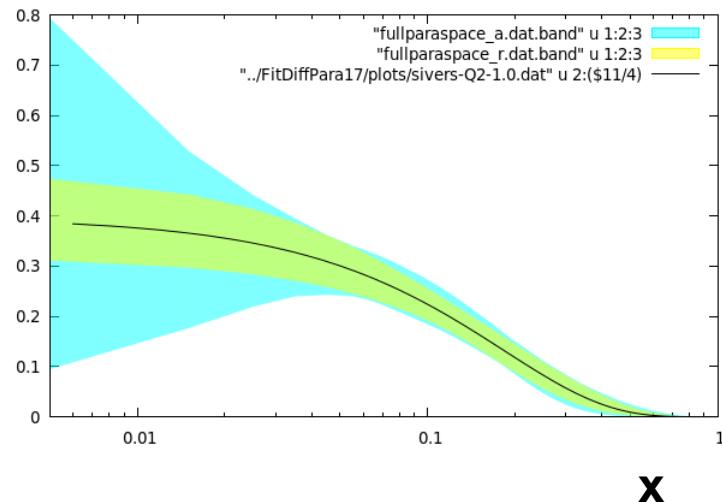
Comparing alpha-fit with and without twist-3 evolution

Sivers
First moment
u-valence

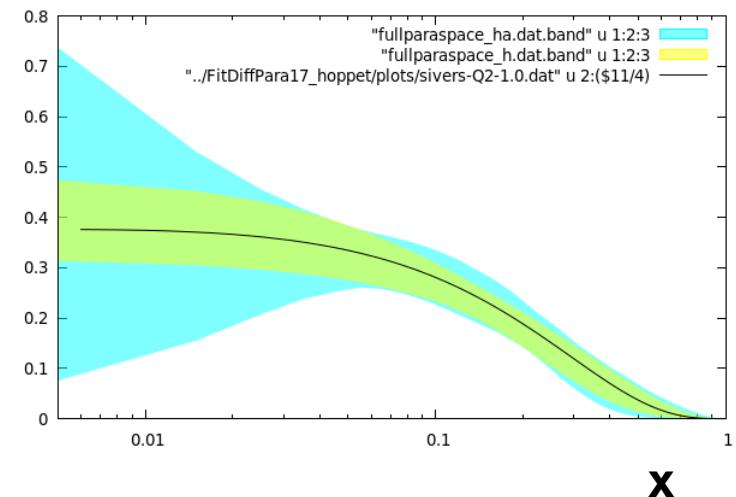
Study of Low-x
Uncertainties

(include α_u and α_d
in the parametrization
of the Sivers function)

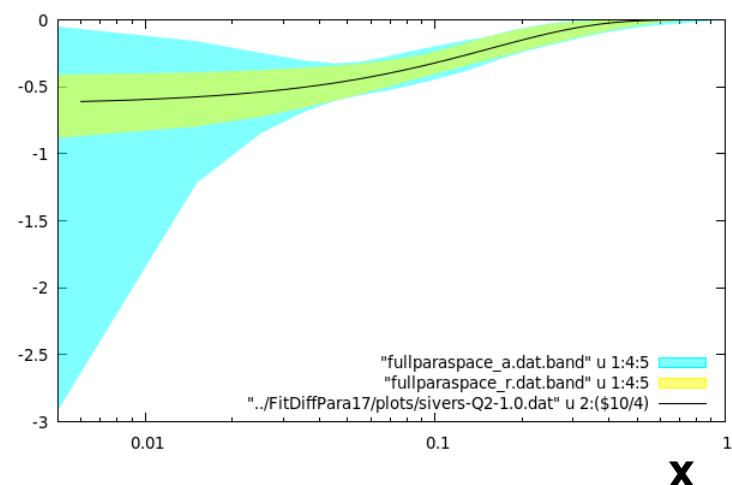
No-evolution



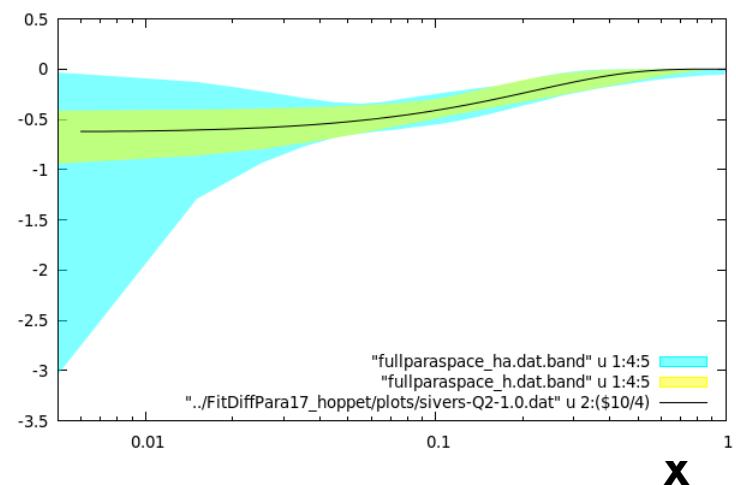
Twist-3 evolution



No-evolution

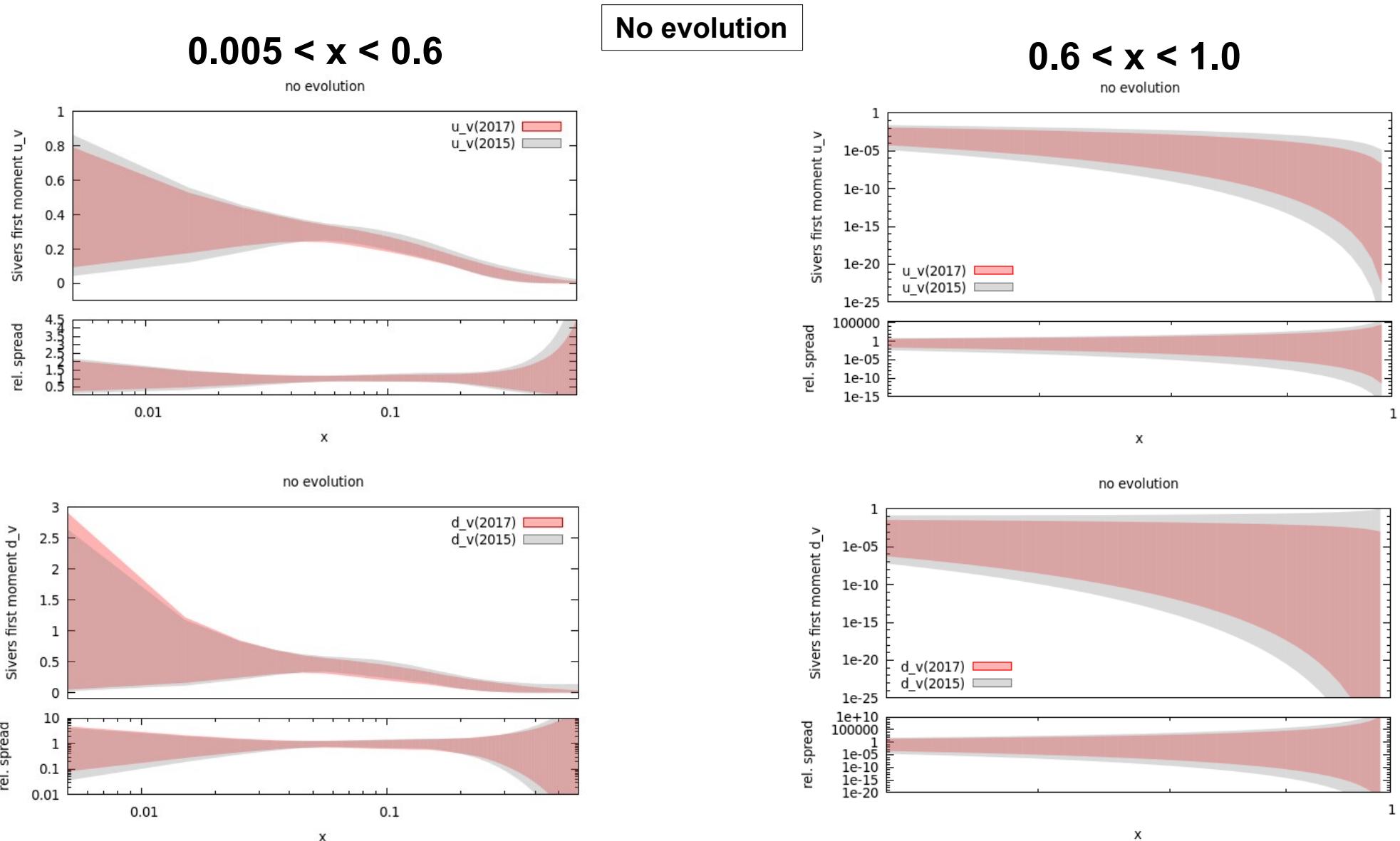


Twist-3 evolution



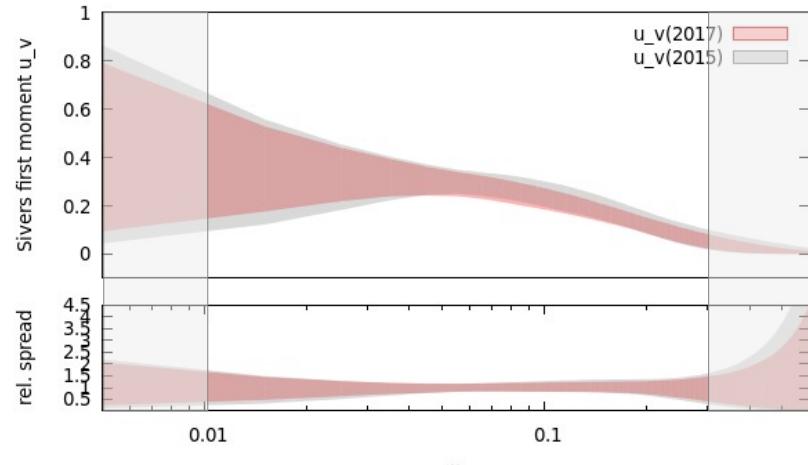
2015 vs 2017 COMPASS data

Uncertainties in the extraction of the Sivers function



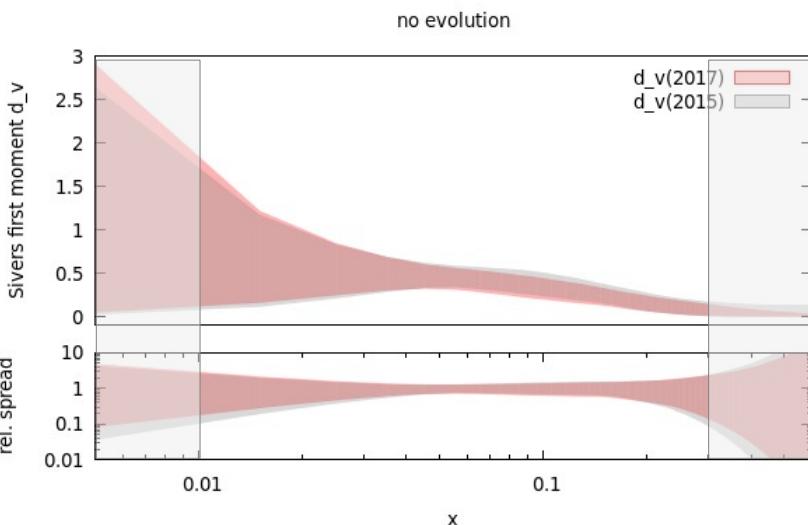
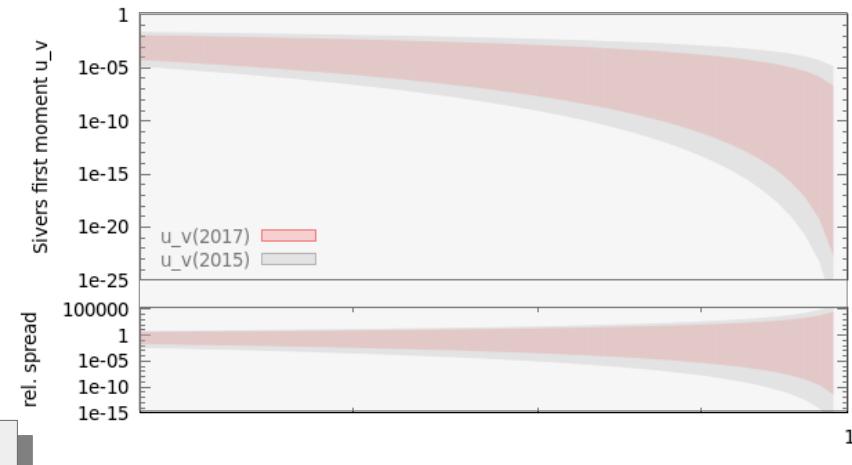
Uncertainties in the extraction of the Sivers function

$0.005 < x < 0.6$
no evolution

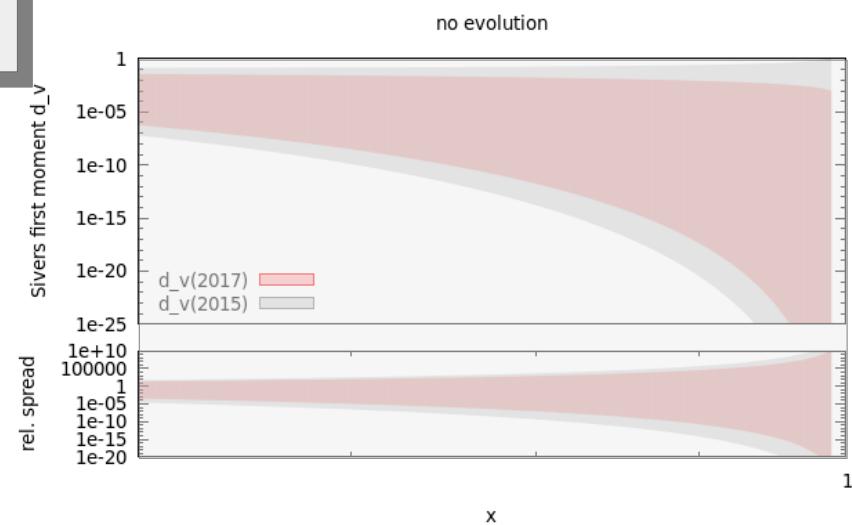


No evolution

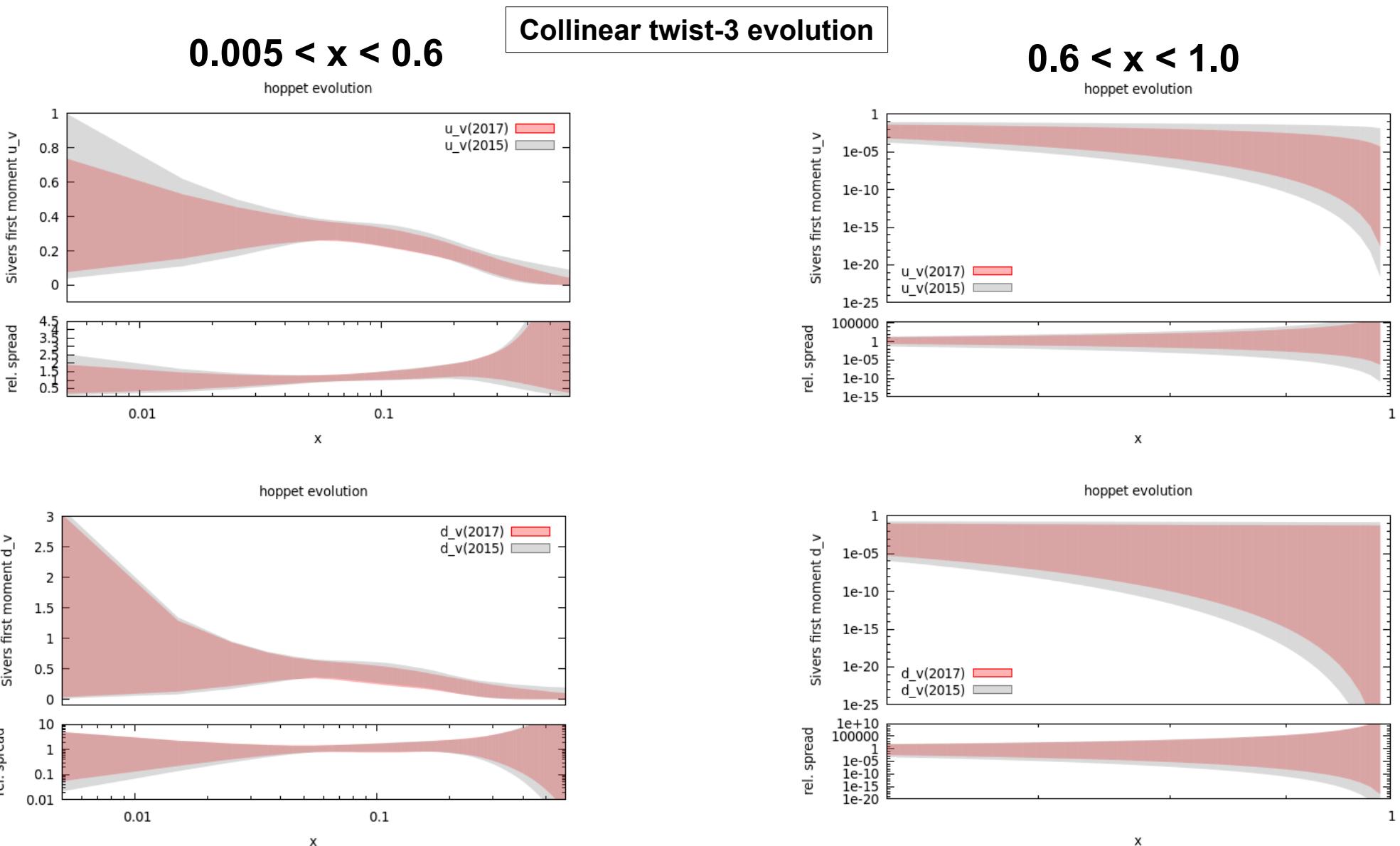
$0.6 < x < 1.0$
no evolution



Shaded areas
indicate regions
where no data
are available

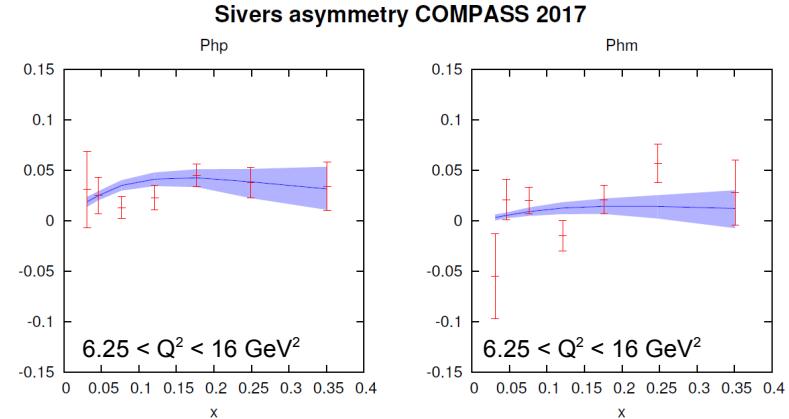
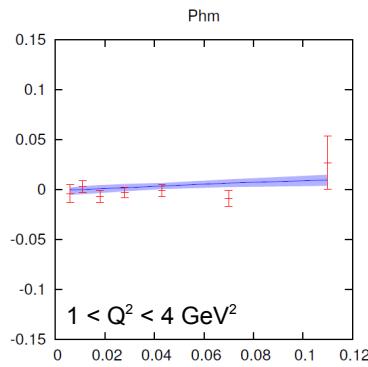
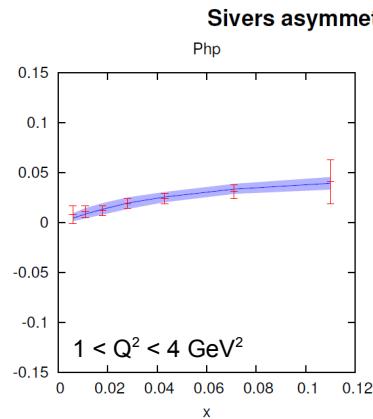


Uncertainties in the extraction of the Sivers function

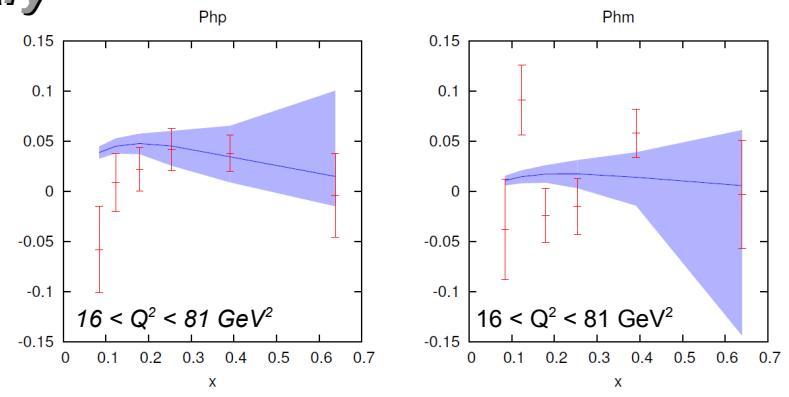
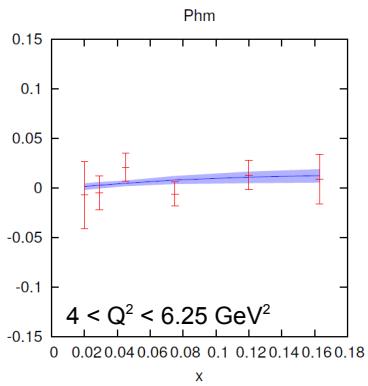
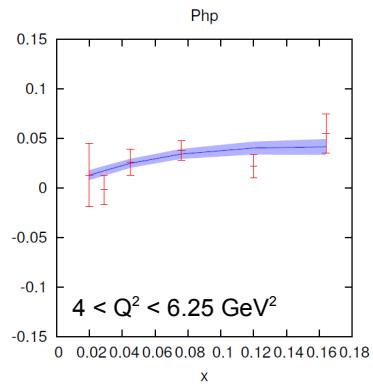


COMPASS 2015 vs. COMPASS 2017

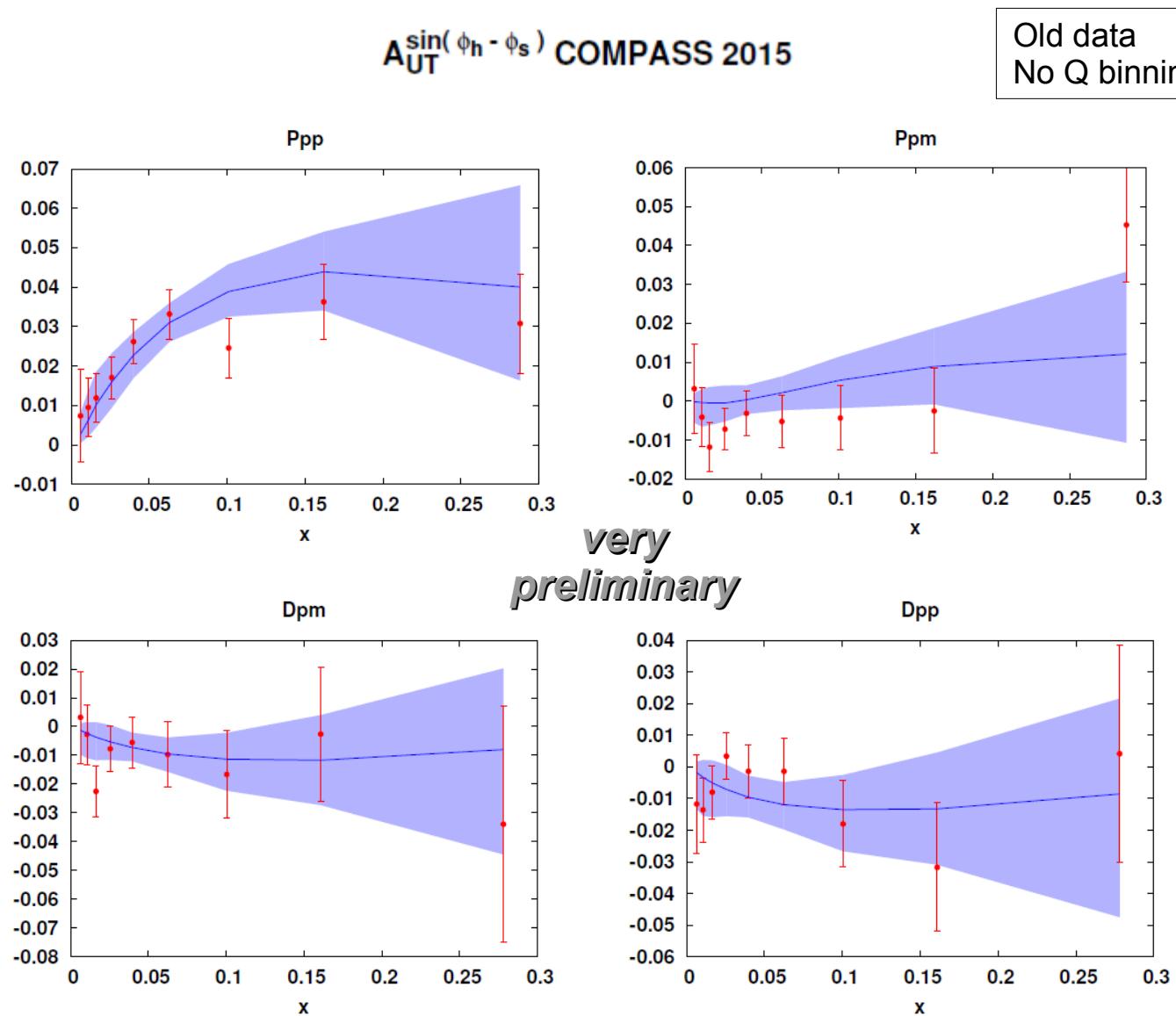
New binning helps reducing uncertainties in the phenomenological extraction of the Sivers function
(low-Q low-x bins)



*very
preliminary*



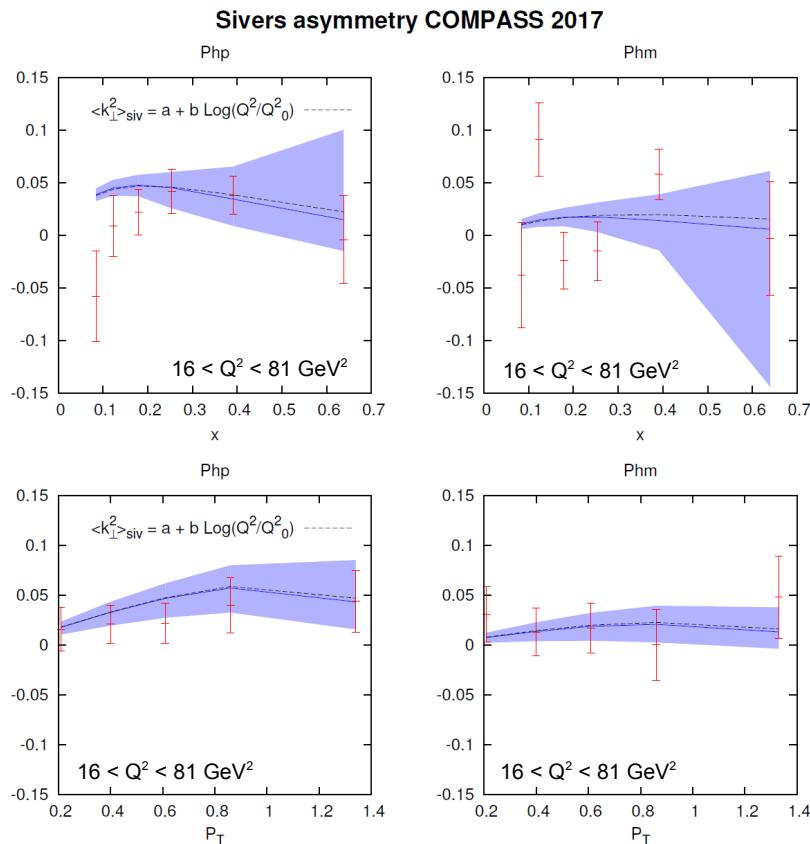
COMPASS 2015 vs. COMPASS 2017



COMPASS 2015 vs. COMPASS 2017

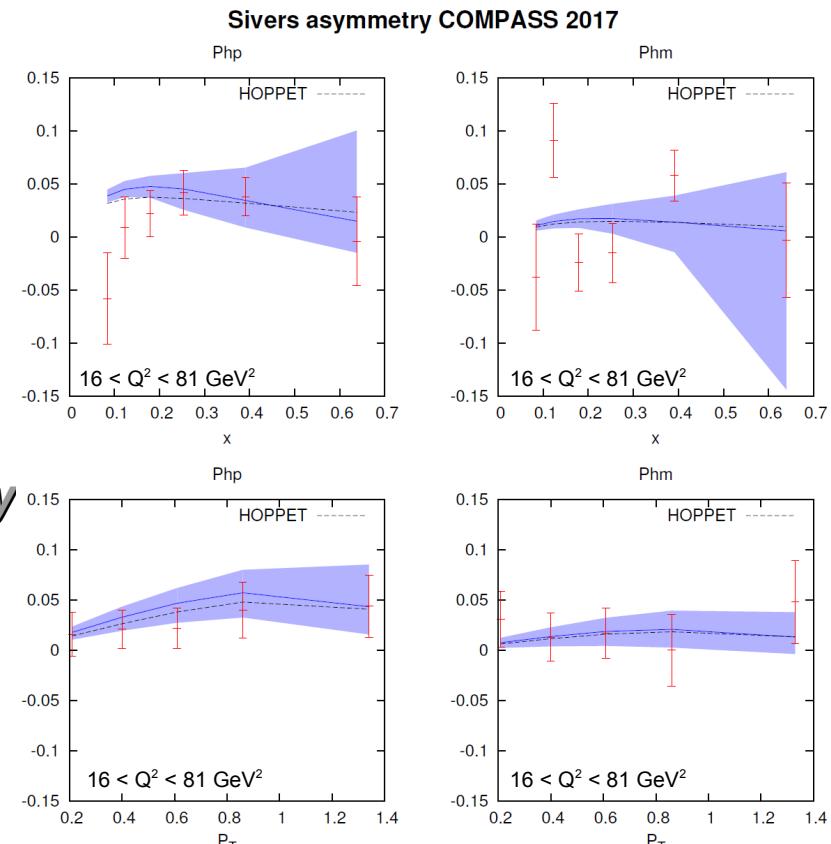
New binning highlights the Q^2 evolution of the Sivers function
(large- Q large- x bins)

TMD evolution



No visible difference between TMD evolution
and no-evolution case in P_T distributions

Collinear Twist-3 evolution



*very
preliminary*

Differences between twist-3 evolution and no-evolution
is evident in P_T distributions as well as in x distributions

Outlooks and perspectives

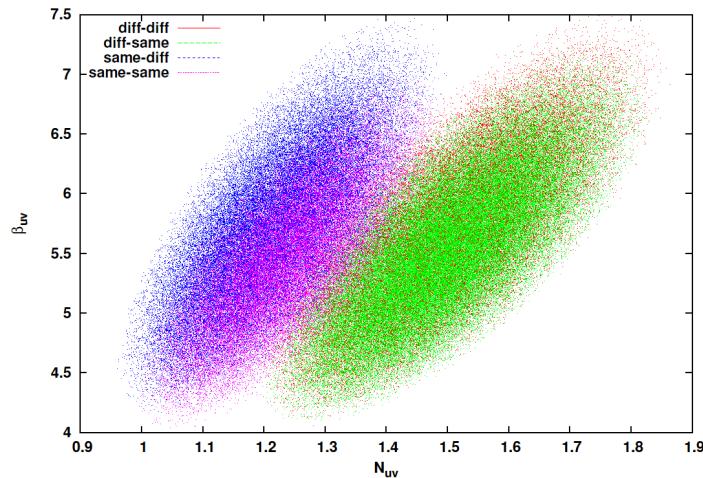
- Phenomenological studies of TMDs, TMD factorization and TMD extraction have come a long way. Some issues remain open and need further investigation
- P_T distributions of unpolarized SIDIS cross sections need to be measured (over the largest possible P_T range) and further investigated on the phenomenological point of view.
- Simultaneous fits of SIDIS, Drell-Yan and e^+e^- annihilation data are highly recommended, but they should be performed within a consistent and solid framework where they can be implemented.
- New data allow for a much more precise extraction of the Sivers function.
 - ✓ Need for a detailed study of the uncertainties affecting the phenomenological extractions
 - ✓ Need for more flexible parametric forms, which do no bias the final results
- Data selection is crucial in global fitting:
 - ✓ not too many
(only data within the ranges where the TMD evolution schemes work should be considered)
 - ✓ not too few
 - ✓ (too strict a selection can bias the fit results and neglect important information from experimental data)

Many thanks for your attention !

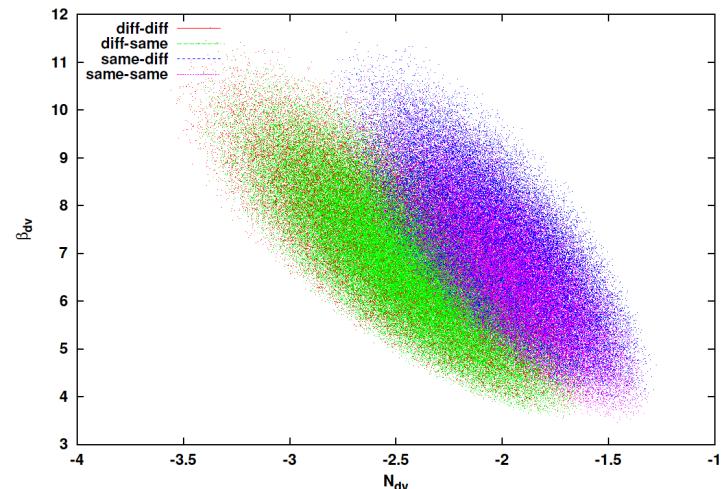
Back up slides

New extraction of the Sivers function

u – valence parameters

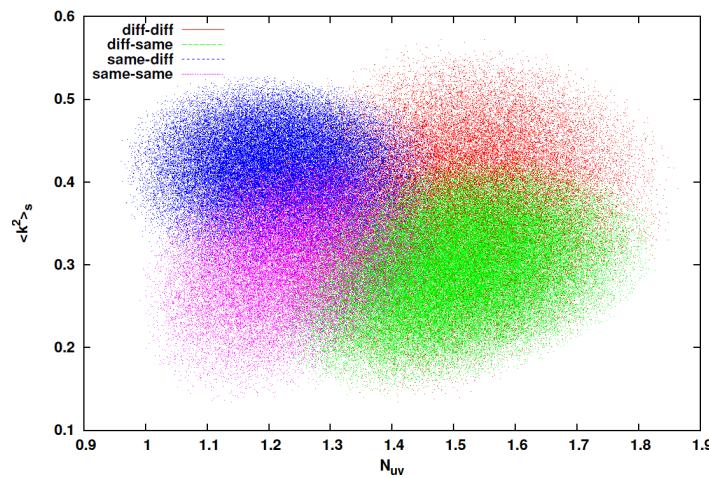


d – valence parameters



No-evolution

Sivers widths (COMPASS) vs. N_{uv}



Sivers widths - HERMES vs. COMPASS

