Theoretical overview on the proton radius puzzle

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and

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International Workshop on Hadron Structure and Spectroscopy
Bonn, 19–21 March 2018

Beginning

The puzzle is clear:

- Measuring the proton radius using electrons gives overall a "big" radius.
- Measuring the proton radius with muons gives a "small" radius.
- The difference is 5.6 σ.
- Numbers,

$$R_E = 0.8751(61) \text{ fm, from CODATA 2014, using electrons only}$$
$$R_E = 0.84037(39) \text{ fm, from muonic hydrogen Lamb shift}$$

Topics

- Introductory material
  - radius from electron scattering
  - radius from atomic physics
  - radius using muons (also atomic physics)
- Selected theory
  - appreciation of theory for atomic energy levels (above)
  - ongoing reanalysis of electron scattering data
  - two photon exchange corrections
  - beyond the standard model possibilities

Radius from elastic electron scattering, $e^- p \to e^- p$

- There are form factors for electric (E) and magnetic (M) charge distributions.
- Cross section is given by

$$\frac{d\sigma}{d\Omega} \propto G_E^2(Q^2) + \frac{\tau}{\varepsilon} G_M^2(Q^2)$$

$$\left[\tau = Q^2/4m_p^2 \; ; \quad 1/\varepsilon = 1 + 2(1+\tau)\tan^2(\theta_e/2)\right]$$

- Low Q^2 is mainly sensitive to G_E.
- DEFINE (for historical reasons) charge radius by,

$$\boxed{R_E^2 = -6 \left(dG_E/dQ^2 \right)_{Q^2=0}}$$

- From real data, need to extrapolate to $Q^2 = 0$.

Scattering data

- Most extensive current data comes from Mainz.

- Data, Bernauer et al., PRL 2010 and later articles.
- Low Q^2 range, 0.004 to 1 GeV^2
- From their eigenanalysis,

$$R_E \text{ or } R_p = 0.879(8)\,\text{fm}$$

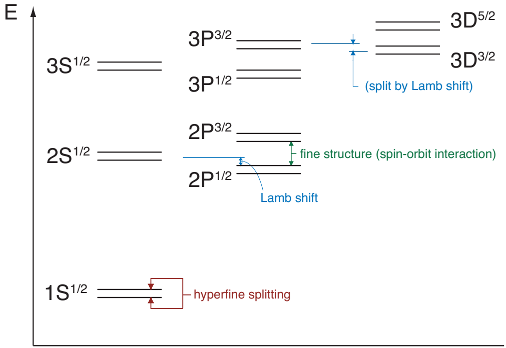
Proton radius from atomic measurements

- Competition for scattering experiments.
- Proton radius effects atomic energy levels.

$$E = E_{\text{QED}} + \delta_{\ell 0} \frac{2 m_r^3 Z^4 \alpha^4}{3 n^3} R_E^2 + E_{\text{TPE}} + \text{very small corrections}$$

- E_{TPE} = two photon exchange corrections (calculated: will discuss)
- Accurate measurements of energy splitting and accurate calculation of QED effects allows determination of proton radius.

Just in case: Hydrogen energy levels



Definitely not to scale:

- Scale for big splittings is Rydberg, Ryd $= \frac{1}{2} m_e \alpha^2 \approx 13.6$ eV.
- Fine structure and Lamb shift are $\mathcal{O}(\alpha^2\,\text{Ryd})$.
- Hyperfine splitting is $\mathcal{O}(m_e/m_p) \times (\alpha^2\,\text{Ryd})$.

History

- Averages from the Committee on Data for Science and Technology (CODATA)
- There have been 8 CODATA reports.

Year	Proton radius (fm)	
2014	0.8751(61)	mostly atomic
2010	0.8775(51)	"
2006	0.8768(69)	"
2002	0.8750(68)	"
1998	0.8545(120)	election scattering
1986	–	no R_E quoted
1973	–	"
1969	0.805(11)	electron scattering

- (Only for 2002 and later is the proton radius among the constants CODATA provided recommended values for.)
- What happened in or about year 2000?

Requirements for calculation

- QED

$$E_{\text{QED}} = \frac{1}{2} m_r \alpha^2 \left[1 + \ldots + \underbrace{\mathcal{O}\left(\frac{\alpha}{2\pi}\right)^3}_{1.6\times 10^{-9}} + \underbrace{\mathcal{O}\left(\frac{\alpha}{2\pi}\right)^4}_{1.8\times 10^{-12}} + \ldots \right]$$

- leading proton size correction

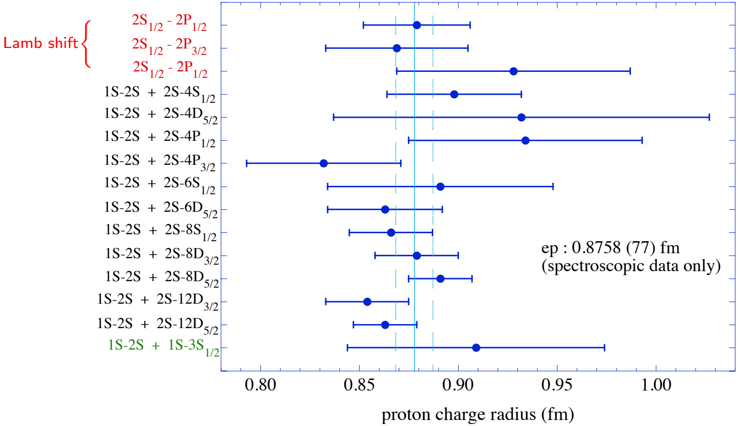
$$\Delta E_{\text{proton size}} = \frac{1}{2} m_r \alpha^2 \cdot \underbrace{\frac{4\alpha^2}{3n^3} \cdot \underbrace{(m_r R_E)^2}_{6.7\times 10^{-6}}}_{6\times 10^{-11}}$$

for $R_E = 1$ fm and $n = 2$.

- Hence need $\mathcal{O}(\alpha/2\pi)^4$ corrections. Became available about year 2000.

Proton radii from hydrogen energy level splittings

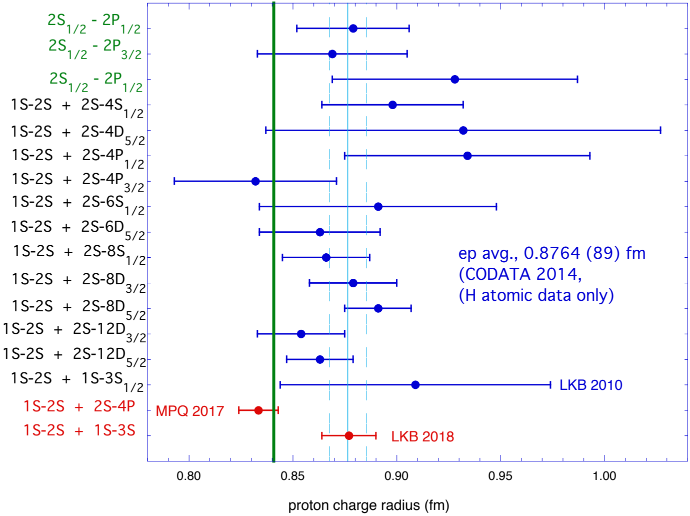
Now can get proton radius from atomic splitting. As of early 2016:



- Crucial observation:
  electron scattering radius and radius from electronic hydrogen agree

Notes

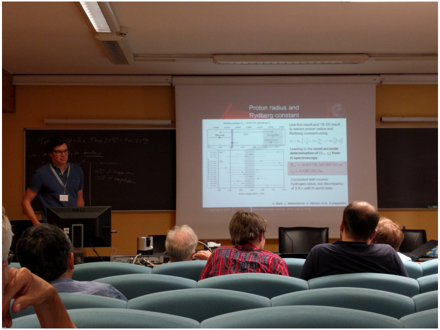
- So from electrons we have result that $R_E \approx 0.88$ fm.
- But,
  - Sub 1% atomic error obtained by dividing by $\sqrt{\text{no. of meas.}} = \sqrt{15}$
  - Should we instead divide by $\sqrt{\text{no. indep. labs}} \approx \sqrt{3}$?
  - Would like new experiments with individual sub 1% error. Two are now available.
  - Have made the radius situation more interesting.



2S1/2 - 2P1/2
2S1/2 - 2P3/2
2S1/2 - 2P1/2
1S-2S + 2S-4S1/2
1S-2S + 2S-4D5/2
1S-2S + 2S-4P1/2
1S-2S + 2S-4P3/2
1S-2S + 2S-6S1/2
1S-2S + 2S-6D5/2
1S-2S + 2S-8S1/2
1S-2S + 2S-8D3/2
1S-2S + 2S-8D5/2
1S-2S + 2S-12D3/2
1S-2S + 2S-12D5/2
1S-2S + 1S-3S1/2
1S-2S + 2S-4P
1S-2S + 1S-3S
MPQ 2017
LKB 2010
LKB 2018
ep avg., 0.8764 (89) fm
(CODATA 2014,
(H atomic data only)
0.80
0.85
0.90
0.95
1.00
proton charge radius (fm)

Re the $2S$-$4P$ splitting measurement

- "MPQ 2017" announced at proton radius workshop June 2016



- Data heard around the world,

$$R_p(2S\text{-}4P) = 0.8297(91)\,\text{fm}$$

- Now have proton radius puzzle for ordinary hydrogen all by itself!

“Crucial” comment regarding R_E and atomic physics

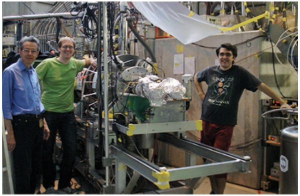
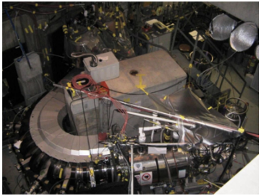
- Also a crucial question: why in atomic physics do we use the derivative of G_E to define the proton radius? Why not, for example, derivative of F_1?
- Answer: do the perturbation theory needed to find proton size effect on atomic energy levels.
- QED calculations use point protons.
- Calculate perturbative term using extra part of proton current,

$$\text{proton current} \rightarrow$$
$$\bar{u}(p') \left(\gamma_\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu} q^\nu}{2m_p} F_2(Q^2) \right) u(p) - \bar{u}(p')\gamma_\mu u(p)$$

- Work through and find result $\propto G'_E(Q^2)\big|_{Q^2=0}$
- Since atomic results measures $G'_E(0)$, quote $R_p = R_E$, to match.

Muon measurement announced in 2010

- Can do analogous measurements with muonic atoms.
- Muons weigh 200× what electron does. Muons orbit 200× closer. Proton looks 200× bigger and proton size effects are magnified.
- Opportunity to obtain more accurate proton radius, despite short muon lifetime.


- Done by CREMA for $2S$-$2P$ splitting (Lamb shift)
- Obtained

$$\boxed{R_p = 0.84087(39)\,\text{fm}}$$

Conflict!

Repeat

$$\boxed{R_p = 0.84087(39)\,\text{fm}}$$

- Delivered on uncertainty limit.

  Current box:

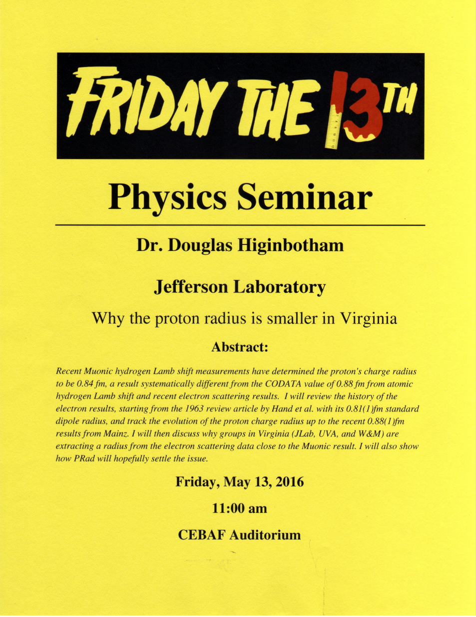
(fm)	atomic	scattering
electron	0.8759 (77)	0.879(8)
muon	0.84087 (39)	insufficient data

New section: Revisionist analyses of scattering data

- Point: Measurements at finite Q^2. Need to extrapolate to $Q^2 = 0$ to obtain charge radius.
- Many authors have taken their own look at how to analyze the data and do the extrapolation when there is the targeted purpose of finding the proton radius.
- Many, but not all, have gotten also low radius results, using ostensibly the same data sets as other who get larger radii.
- There have come to be camps
  - Minimalist camp: Fit data using fit functions that have few parameters, but using only a low-Q^2 selection of the data.
  - Extended fit camp: Fit data with functions having relatively many parameters, but using the full data set.
- A few references (apologies ...)

minimalist	more extended
Meissner et al. (2015)	original Mainz (Bernauer et al.)
Horbatsch & Hessels (2016)	Hill & Paz
Higinbotham et al. (2016)	Graczyk & Juszczak (2014)
Griffioen et al. (2016)	Arrington & Sick (2015)
Yan, Higinbotham, et al (2018)	Lee, Arrington, & Hill (2015)
Hayward & Griffioen (2018)	Ye, Arrington, Hill, and Lee (2018)

Talk announcement



FRIDAY THE 13TH

Physics Seminar

Dr. Douglas Higinbotham

Jefferson Laboratory

Why the proton radius is smaller in Virginia

Abstract:

Recent Muonic hydrogen Lamb shift measurements have determined the proton's charge radius to be 0.84 fm, a result systematically different from the CODATA value of 0.88 fm from atomic hydrogen Lamb shift and recent electron scattering results. I will review the history of the electron results, starting from the 1963 review article by Hand et al. with its 0.81(1)fm standard dipole radius, and track the evolution of the proton charge radius up to the recent 0.88(1)fm results from Mainz. I will then discuss why groups in Virginia (JLab, UVA, and W&M) are extracting a radius from the electron scattering data close to the Muonic result. I will also show how PRad will hopefully settle the issue.

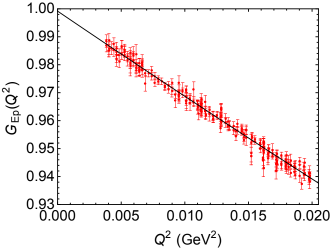
Friday, May 13, 2016

11:00 am

CEBAF Auditorium

One plot

- Minimalist viewpoint: Charge radius requires extrapolation to $Q^2 = 0$. Fits with lots of parameters tend to be less smooth outside data region. Fits to full data set generally require lots of parameters. For charge radius, better to fit to narrower, low Q^2 region of data. Have fewer parameters, less "wiggly" functions, and more faith in extrapolations.



- But consult "Avoiding common pitfalls and misconceptions in extractions of the proton radius," 1606.02159

Recent minimalist discussion

- Analyze robustness of fit functions, with X number of parameters, "robust" meaning that fit function should give correct proton radius—when proton radius is known, as it is for data (or pseudo data) manufactured using a generating function.
- Use lots of different generating functions (see list on next slide)
- Try lots of fit functions

One example of fit function

- from Yan, Higinbotham, et al., with three parameter fit function

$$G(Q^2) = p_0(1 + p_1 Q^2)/(1 + p_2 Q^2)$$

- data manufactured to match pRad anticipated data range and density.

TABLE VIII. The fitting results using the rational-function fitter ($N = 1$, $M = 1$), when different generators are used. Notation as in Table I.

Generator	R(input) (fm)	R(mean) (fm)	δR (fm)	RMS (fm)
Dipole	0.8500	0.8503	0.0003	0.0097
Monopole	0.8500	0.8499	−0.0001	0.0099
Gaussian	0.8500	0.8509	0.0009	0.0094
Kelly-2004	0.8630	0.8631	0.0001	0.0096
Arrington-2004	0.8682	0.8686	0.0004	0.0094
Arrington-2007	0.8965	0.8965	0.0000	0.0094
Venkat-2011	0.8779	0.8777	−0.0002	0.0096
Bernauer-2014	0.8868	0.8844	−0.0024	0.0097
Alarcón-2017	0.8500	0.8499	−0.0001	0.0096
Alarcón-2017 (codata)	0.8750	0.8758	0.0008	0.0093
Alarcón-2017 (μ)	0.8400	0.8407	0.0007	0.0096
Ye-2018	0.8790	0.8750	−0.0040	0.0097
Ye-2018 (re-fix)	0.8500	0.8514	0.0014	0.0096

- Radii reproduced well, uncertainties in fit ("RMS (fm)") not too large.
- No bias: meaning radii missed on high side as often as on low side. (Examples with bias exist.)

Minimalist fits to real data, with bias analysis

- Hayward and Griffioen, using only $Q^2 < 0.15$ GeV^2, and four parameter continued fraction fit, found

$$R_E = 0.851(16) \text{ fm, from 20th century data}$$
$$R_E = 0.859(2) \text{ fm, from Main 21st century data}$$

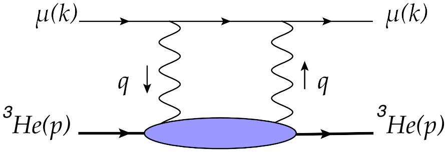
- Hope: The studies are proceeding with serious testing on psuedodata and with analysis of reliability and robustness of fit procedures, and may lead to some criteria for agreement. (May even force use of more parameters!)

Scattering radii future

New scattering experiments coming

- PRad (JLab) does electron scattering down to $Q^2 = 0.0002$ GeV^2. Have data under analysis.
- Initial state radiation experiment at Mainz. First results published, more accurate results to come.
- A2 experiment at Mainz, observing final proton in TPC
- MUSE (PSI) will do both muon and electron scattering, down to 0.002 GeV^2

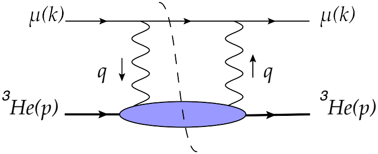
- One of the “other corrections”:
  not the biggest term, but the biggest source of uncertainty.
  E.g.,



- Blob is off shell proton or any higher state. Makes calculation hard.
- How good are we?
- How good do we have to be?

Dispersive calculation

- Some calculate by noting putting the intermediate states on shell
  (a) gives the Imaginary part of the whole diagram, and
  (b) means each half of the diagram is an amplitude for a real scattering process, and hence can be gotten from scattering data.



- What matters is the lower vertex, so can use electron scattering data.
- Mostly need low Q^2, low energy data
- Reconstruct whole diagram using dispersion relations.

Begin with the proton

- Theory for Lamb shift splitting, with numbers for proton,

$$\Delta E_L^{\text{theo}} = \Delta E_{\text{QED}} - \frac{m_r^3 Z^4 \alpha^4}{12} R_p^2 - \Delta E_{\text{TPE}}$$
$$= 206.0336(15) - 5.2275(10) R_p^2 + 0.0332(20)$$

(units are meV and fm)

- TPE number from Birse and McGovern, following CEC and Vdh; ongoing consideration using other techniques
- Faith,

$$\Delta E_L^{\text{theo}} = \Delta E_L^{\text{expt}} = 202.3706(23)\,\text{meV}$$

- Solve,

$$R_p = 0.84087(39)\,\text{fm} \qquad [0.038\%]$$

- If the TPE were perfect,

$$R_p = 0.84087(32)\,\text{fm}$$

- Conclude: for the proton theorists have done their job. Uncertainty in TPE not dominant.

Deuteron

- Trouble: the deuteron is loosely bound, a little energy turns it into other states. Proton remains just a proton until there is enough energy to make a pion.
- Statement "Mostly need low Q^2, low energy data" turns into "need really low Q^2, really low energy data". The data in this region is sparse (and old).
- Poor situation for dispersion theory people, who turn electron scattering data into energy splittings.
- A use of different data is
  - proton-proton and neutron-proton scattering $\rightarrow$ nuclear potentials
  - use these potentials to calculate nuclear bound states,
  - and nuclear hadronic intermediate states contribution to TPE.
- Nuclear potential method succeeds well for deuteron

Deuteron

- Theory with numbers for deuteron is now,

$$\Delta E_L^{\text{theo}} = 228.7766(10) - 6.1103(3) R_d^2 + \Delta E_{\text{TPE}}$$

- and there are now two ways to obtain the TPE,

how	*who*	ΔE_{TPE} (meV)
Nuclear potentials	Hernandez *et al.*	1.6900(200)
Nuclear potentials	Pachucki-Wienczek	1.7170(200)
Dispersion theory	Carlson *et al.*	2.0100(7400)
Summary	Krauth *et al.*	1.7096(200)

- Work out, with $\Delta E_L^{\text{expt}} = 202.8785(34)\,\text{meV}$

$$R_d = 2.12562(78)\,\text{fm}$$

- If TPE be perfect,

$$R_d = 2.12562(15)\,\text{fm}$$

^{3}He

- For dispersion theorists, better case than the deuteron because the binding is stronger, the thresholds are higher, and there is data near the thresholds, which is the important region for this calculation.
- With ^{3}He numbers,

$$\Delta E_L^{\rm theo} = 1644.4643(150) - 103.5184(98) R_T^2 + \Delta E_{\rm TPE}$$

- and for the TPE,

how	*who*	$\Delta E_{\rm TPE}$ (meV)
Nuclear potentials	Hernandez *et al.* (2016)	15.46(39)
Dispersion theory	CEC, Gorchtein, Vanderhaeghen	15.14(49)
Summary	Franke *et al.*	15.30(52)

^{3}He — How good do we have to be?

- comparison will be to current electron scattering data for R_T
- direct electron scattering on ^{3}He: $R_T = 1.973(14)\,\text{fm}$
- can do somewhat better using ^{4}He data, $R_\alpha = 1.681(4)$ and isotope shift (accurate measurement of $R_T^2 - R_\alpha^2$):

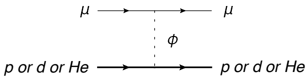
$$R_T = 1.968(11)\ \text{fm} \qquad \text{[**and maybe now improved**]}$$

- How well will the μ-^{3}He Lamb shift do? Use the result given for ΔE_{TPE} and work out the anticipated uncertainty:

$$R_T = 1.96xxx(13)\,\text{fm}$$

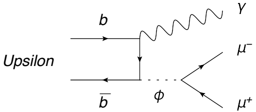
- Uncertainty from corrections still about $8\times$ smaller than that from e^- scattering.
  (Although, (13) $\rightarrow$ (2) if TPE were perfect.)
- Will easily separate results from different isotope shift measurements (barring BSM possibilities).

Energy deficit due to exchange of not-otherwise-discovered boson?



If so, there are constraints.

1. Must couple to $\mu^\pm$
   - But not (or only weakly) to $e^\pm$. (To $\tau^\pm$ not clear.)
2. Must couple to u, d quarks.
   - But not (or only weakly) to b, or it would be seen in $\Upsilon \to \mu^+\mu^-\gamma$.



   - Nor (or only weakly) to c, or it would be seen in $J/\psi \to \mu^+\mu^-\gamma$.
   - And coupling to neutron $\ll$ coupling to proton. (Precision n-nucleus scattering has no Coulomb term, allowing tight limits on exotic terms.)

Break in summary: BSM for ^{3}He

- BSM here means (Tucker-Smith & Yavin; Battel, McKeen & Pospelov; CC & Rislow)
  - proton radius is fixed number
  - observed energy discrepancy is real
  - and due to BSM μ-philic interaction
- Model somehow:
  - vector interaction, new exchange boson ϕ of some mass
  - coupling to $\mu \gg$ coupling to e
  - coupling to hadron like dark photon, *i.e.*, $\propto Z$
- Get result from energy deficit in hydrogen upon scaling to T,

$$\Delta E^{T}_{\mathrm{L,BSM}} = Z^4 \left(\frac{m_r^T}{m_r^p}\right)^3 \frac{f(x_T)}{f(x_p)} \Delta E^{p}_{\mathrm{L,BSM}} \stackrel{m_\phi \gg \text{few MeV}}{=} 6.59\,\mathrm{meV}$$

$$\text{for } f(x) = x^4/(1+x)^4 = m_\phi^4/(Z m_r \alpha + m_\phi)^4$$

- The TPE in this case has a 0.52 meV uncertainty: good enough to kill/confirm BSM idea (for many m_ϕ).

BSM for ^{3}He

- Reports that CREMA finds ^{3}He radius compatible with electron scattering number, with small error limit.
- Incompatible with 6.59 meV shift expected from BSM explanation of original puzzle, for m_ϕ (mass of BSM force carrier) not small.
- Does this kill BSM idea?
  - maybe
  - maybe not
- One difference between ^{3}He and hydrogen is size of atomic state. ^{3}He is factor 2 smaller.
- Recall zero mass exchange particle (photon) gives no $2P$-$2S$ splitting. Something long range to ^{3}He can look like short range to hydrogen. Light boson exchange can give $\approx$ no splitting in ^{3}He but notable splitting for H.
- Works numerically—for present uncertainty limits—for $m_\phi \approx 1$ MeV.

BSM Summary continued

Continued constraint list,

3. Cannot be heavy (meaning here, $>$ few MeV), or else would also affect muonic measurements of ^{3}He and ^{4}He radii.
   - But cannot be massless, or it would not give $2S$-$2P$ splitting at all.
   - Circa 1 MeV for m_ϕ o.k.

Hence have BSM boson with targeted couplings and specific mass requirements.
Likelihood?
Couplings to muons cannot be avoided, and so if it exists it has—sooner or later—to turn up in other muonic situations, *e.g.*,

- $(g-2)_\mu$. But maybe o.k. here.
- Radiative correction to $W \to \mu\nu$. But maybe o.k. if we insist that coupling of new particle has only renormalizable interactions.
- Contribution to radiative corrections to K decay, $K \to \mu\nu e^+ e^-$ (if any coupling at all to electrons).

Ending

- Remarkable: After 8 years, the problem persists.
- Interesting: little discussion of the correctness of the μ-H Lamb shift data.
- Good new data coming, in spectroscopy and scattering.
- Topics here were
  - precision atomic theory
  - revised scattering data analyses
  - TPE correction calculations
  - BSM possibilites
- Opinion (or short list of possibilities): Either
  - The puzzle isnt a puzzle: The electron based radius measurements will reduce to the muonic value.
    - The scattering analysis is under discussion, and more data coming
    - The spectroscopy measurements by themselves have a puzzle.
  - But: BSM ideas have to be entertained if one insists on a large radius from electrons and a smaller one from muons.

Extras

Beyond the end

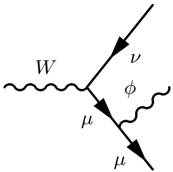
H-spectrosopy future

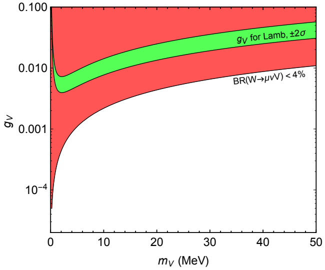
May also expect:

- York University (Canada): Ordinary hydrogen 2S-2P Lamb shift. (Maybe this year??)
- Laboratoire Kastler Brossel (Paris): 1S-3S transition
- More from Garching
- NIST (USA): Measure Rydberg using Rydberg states, very high n states, uncontaminated by proton size. (Very relevant: recall previous discussion.)
- + National Physical Lab (U.K.), several $2S$-nS, nD transitions

Possible W decay constraints

- Remark of Karshenboim, McKeen, and Pospelov: there is fast growth with energy of amplitudes involving massive vector particles
- If light new particle ϕ or V coupling to muon, it gives large radiative correction to W decay via $W \to \mu\nu V$, larger than measured error in W decay rate.

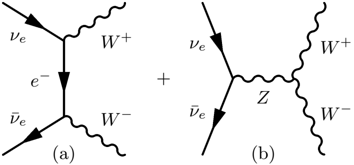




Red: forbidden
Fig. based on Karshenboim et al. (2014)

Relevant to this

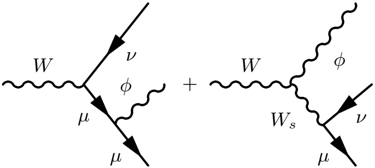
- Reminiscent of (from early days of W.S. model),



- Left diagram grew unpleasantly at high energy, right diagram cancelled it at high energy, was small at lower energy

Here

- Should have interaction also with W to make theory renormalizable.



- Problem ameliorated (see Freid and me, PRD (2015))

Bias in fits?

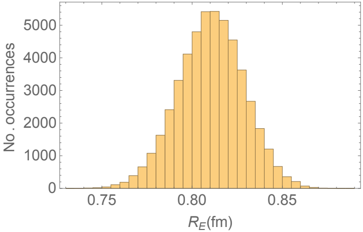
- Meaning of bias in this context:
  - Have set of data, and have function and a procedure to fit data, and obtain measurable fundamental quantities
  - Bias: function and/or procedure systematically gives results too high or too low.
- Test selected procedure using generated data
  - Choose analytic function to underlie test data
  - Present example: pick known function for GE . Part of point: exact RE for this function known.
  - Generate data set with gaussian fluctuations about chosen function.
  - Use selected procedure to fit data, and extract desired fundamental constants
  - Compare to known correct result.

Bias test case I

- With one trial, won't reproduce known parameter of starting point because of statistical fluctuations.
- Run many trials.
  - Do we reproduce known parameter of starting point, on average?
  - What is standard deviation of statistical fluctuations about most likely result?
- Try: Generate data using dipole form for $G_E(Q^2)$.
- Known outcome, $R_E = 0.81125$ fm.
- 219 data points with $0.004 < Q^2 < 0.02\ \mathrm{GeV}^2$
- Fit to $G_E = a\,(1 - R_E^2 Q^2/6 + c\,Q^4)$

Outcome of test case I

- 50,000 runs (50,000 "experiments")



- Extracted values: $R_E = 0.810$ fm, $\sigma_E = 0.018$ fm.
- See no bias, decent accuracy.

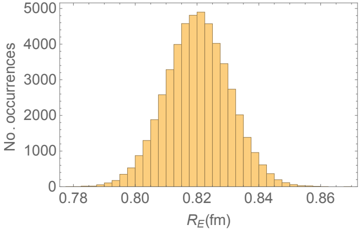
Bias test case II

- Example: generate data using Kelly form factor, over a wider range, $0.004 < Q^2 < 1$ GeV^2, 200 points, 5% uncertainties.
- Kelly gives $R_E = 0.863$ fm
- Fit to dipole form,

$$G_E(Q^2) = \frac{a}{\left(1 + {\frac{1/12}{R}}^{2}_{E} Q^2\right)^2}$$

Outcome of test case II

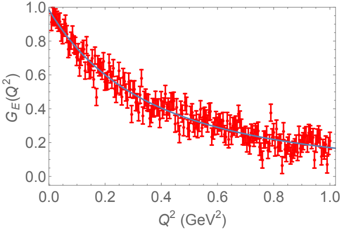
- another 50,000 runs



- Result off: $R_E = 0.820$ fm, $\sigma_E = 0.010$ fm, $\chi^2/\text{dof} = 0.99$
- Bias! 0.04 fm low. Even though χ^2 o.k., and eyeball test o.k. $\rightarrow$

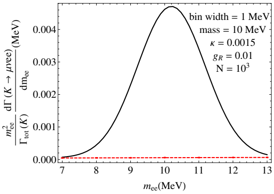
More on the outcome of test case II

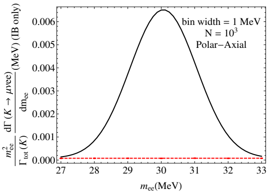
- Plot for one run



- Bias exists: there are fit functions and/or fit procedures (*e.g.*, selection of data range) that lead to systematically high or low results when fitting arguably good representations of real data.
- But not for the low-Q^2 fits to the electron scattering data.

Re: BSM and $K \to \mu\nu e^+e^-$, 2





Note: TREK experiment (E36) at JPARC (Japan) will observe 10^10 kaon decays, or about 200,000 $K \to \mu\nu e^+e^-$ events, about 1000 per MeV bin in the mass range we are considering. (Thanks to M. Kohl)