

# **Open questions and issues in light meson spectroscopy**

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# (Open) issues - an incomplete list ...

- What are the properties of the light scalar nonet?  
( $f_0(500)$  ( $\sigma$ ),  $f_0(980)$ ,  $a_0(980)$ ,  $K_0^*(700)$  ( $\kappa$ ))
- How many  $I^G(J^{PC}) = 0^+(0^{++})$  states are there between 1 and 2 GeV?  
(candidates:  $f_0(1370)$ ,  $f_0(1500)$ ,  $f_0(1710)$ )
- Which  $\eta$ -states ( $I^G(J^{PC}) = 0^+(0^{-+})$ ) are there between 1400 and 1480 MeV?  
(candidates:  $\eta(1405)$ ,  $f_0(1475)$ )
- Which  $a_1$  ( $I^G(J^{PC}) = 1^-(1^{++})$ ) states are there below 1500 MeV?  
(candidates:  $a_1(1260)$  and  $a_1(1420)$ )

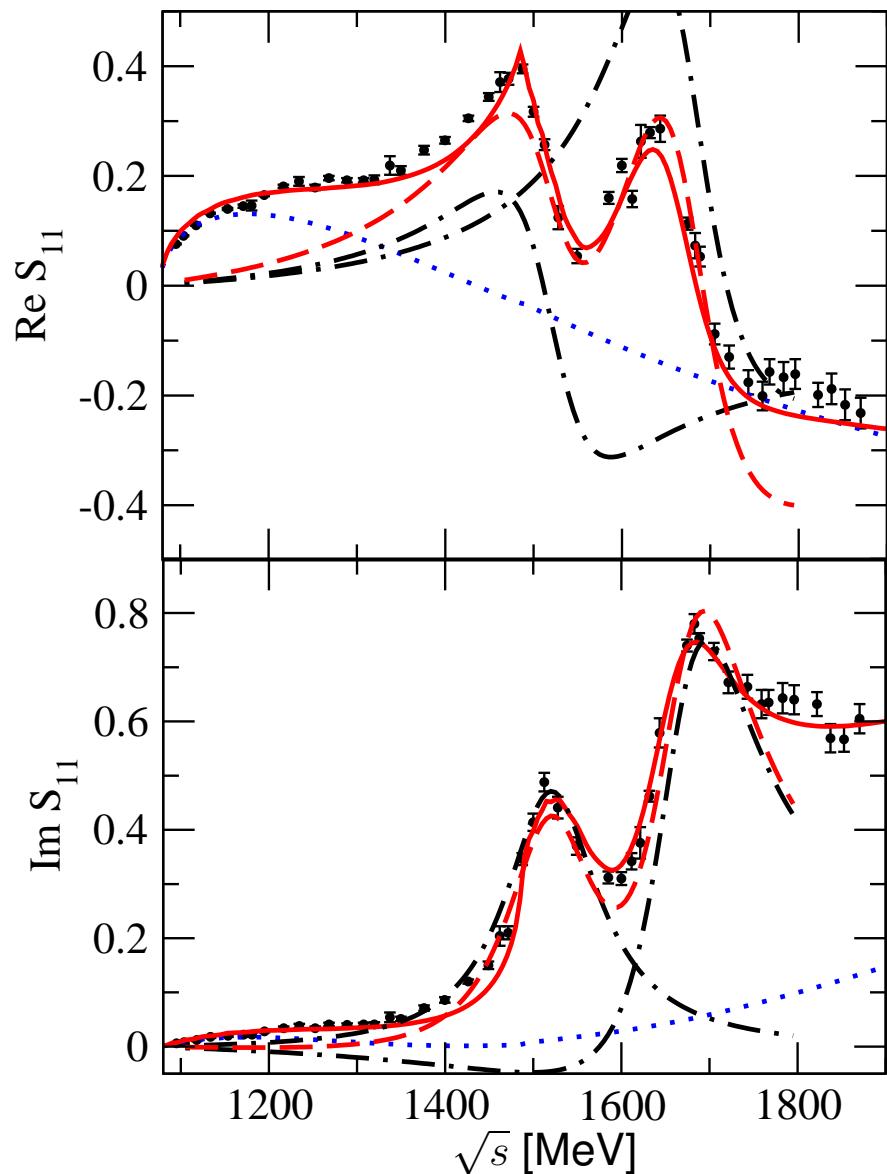
I will briefly discuss these issues, focussing on the first ...

# What is a state?

The  **$S$ -matrix** is characterized by its **analytic structure** (in  $s$ ).

- branch points (and the corresponding cuts)
  - at each channel opening for  $s > s^{\text{thres}}$ : **right hand cut**
  - in the crossed channels for  $s < s^{\text{thres}}$ : **left hand cut**
  - inside the un-physical sheet
- poles on the physical sheet: **bound states**
  - only for real  $s < s_{\min}^{\text{thres}}$  (**no other singul. allowed here**)
- poles on the un-physical sheet (closest to the physical one)
  - for real  $s < s_{\min}^{\text{thres}}$ : **virtual state**
  - for complex  $s$ : **resonance**

# More on Resonances



M. Döring et al. NPA829(2009)170

A resonance is **uniquely** and **unambiguously** characterized by its **pole position** and **residues**

$$BR_{\text{pole}}(i) = \frac{|\text{res}_i|^2 \sigma_i}{2 |m_{\text{pole}}| (\Gamma_{\text{pole}}/2)}$$

with  $\sigma_i$  = phase space channel i

used e.g. for  $f_0(500) \rightarrow \gamma\gamma$

Morgan & Pennington ZPhys48(1990)623

Thus, naively one may write

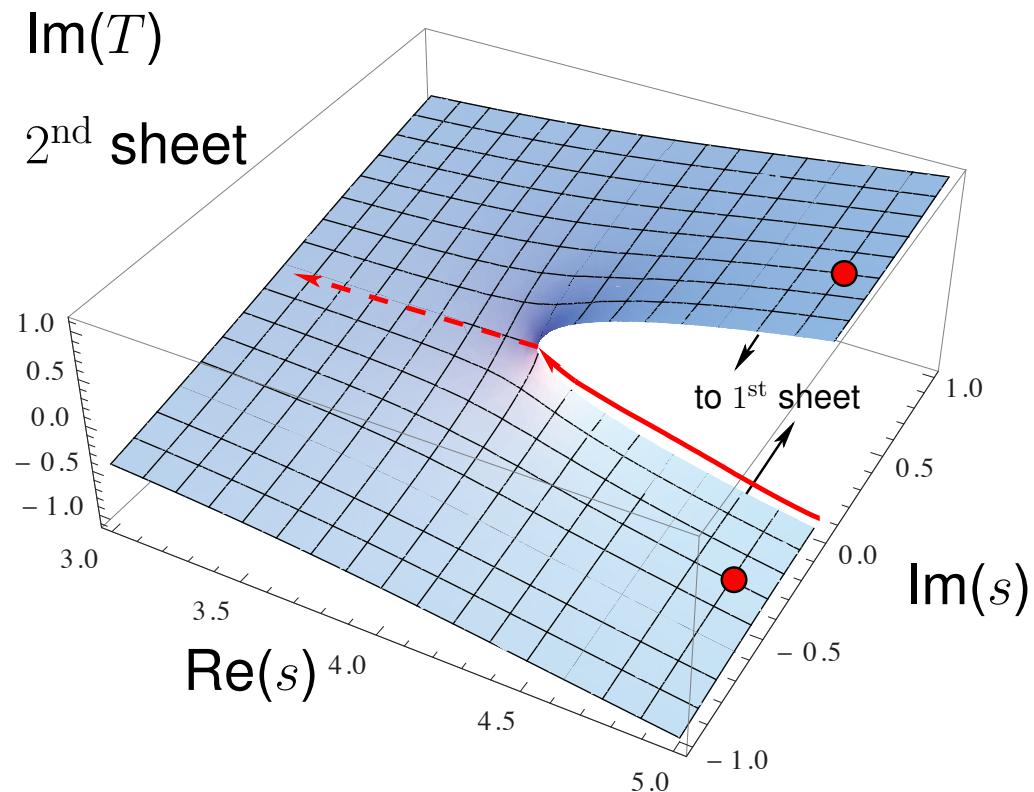
$$T_{ij} = - \sum_r \frac{\text{res}_i^r \text{res}_j^r}{s - s_r}$$

which is a sum of **Breit-Wigners**

But, in general this is **wrong!**

# Reason I: Analyticity

- For real  $s < s_{\min}^{\text{thres}}$ ,  $S$  is real → Branchpoint at  $s = s^{\text{thres}}$
- $S(s^*) = S^*(s)$  → pole at  $s$  implies pole at  $s^*$



For narrow resonances:

In resonance region:  
only lower pole matters

At threshold:  
both poles important!

For broad resonances:  
always both important

**Keep track of the cuts!**

## Reason II: Unitarity

For one channel only one has:  $\text{Im}(T) = \sigma|T|^2$

where  $\sigma = \sqrt{1 - 4m^2/s}$  is the phase space. Then for

$$T = -\frac{\text{res}_{(1)}^2}{s - M_1^2 + iM_1\Gamma_1} - \frac{\text{res}_{(2)}^2}{s - M_2^2 + iM_1\Gamma_2}$$

we get using  $\sigma |\text{res}_{(i)}|^2 = M_i\Gamma_i(s)$  (for  $\text{res}_{(i)}$  real)

$$\text{Im}(T) = \frac{\text{res}_{(1)}^2\Gamma_1 M_1}{(s - M_1^2)^2 + M_1^2\Gamma_1^2} + \frac{\text{res}_{(2)}^2\Gamma_2 M_2}{(s - M_2^2)^2 + M_2^2\Gamma_2^2}$$

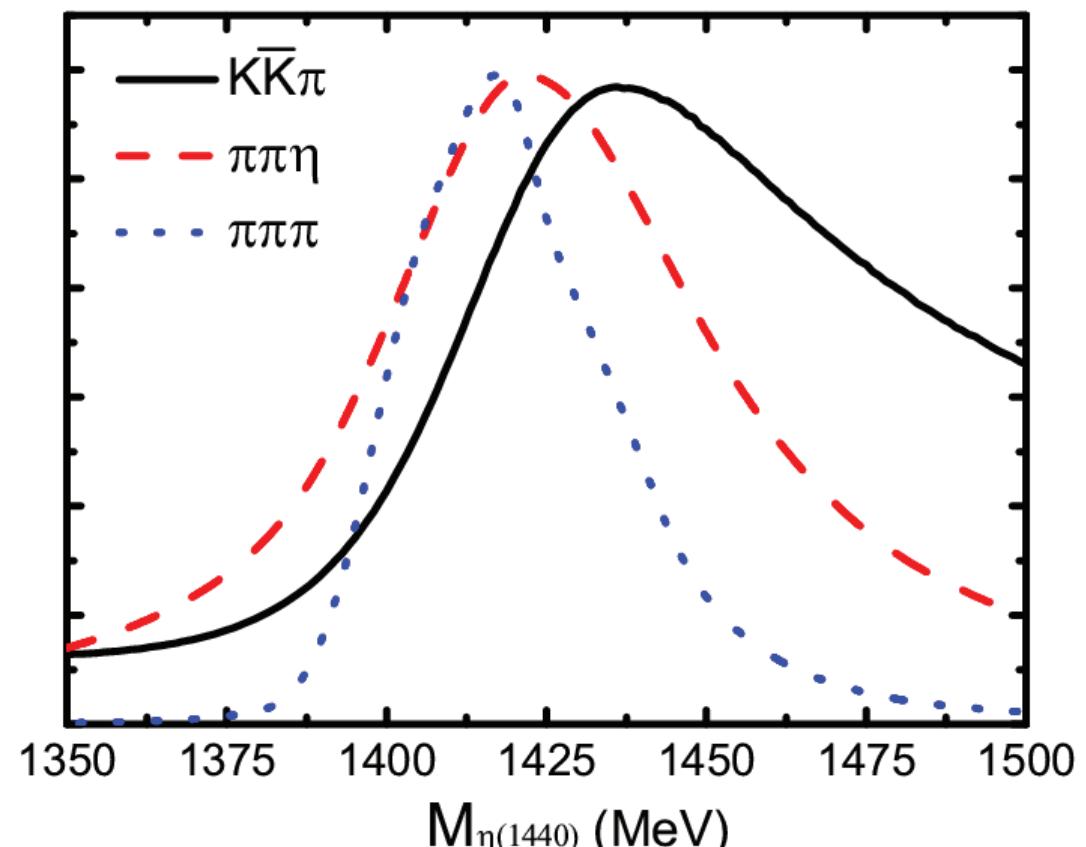
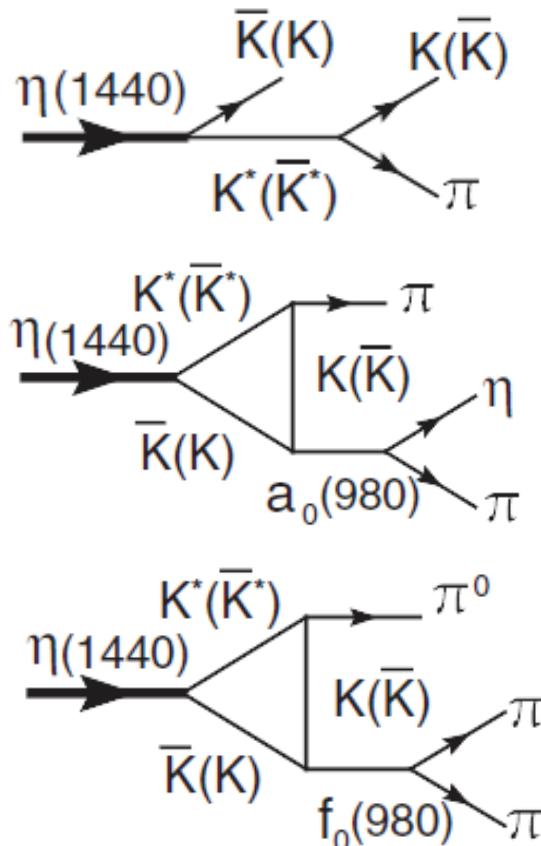
$$\sigma|T|^2 = \frac{\text{res}_{(1)}^2\Gamma_1 M_1}{(s - M_1^2)^2 + M_1^2\Gamma_1^2} + \frac{\text{res}_{(2)}^2\Gamma_2 M_2}{(s - M_2^2)^2 + M_2^2\Gamma_2^2}$$

$$+ 2\sigma \text{Re} \left( \frac{\text{res}_{(1)}^2}{s - M_1^2 + iM_1\Gamma_1} \frac{\text{res}_{(2)}^2}{s - M_2^2 - iM_1\Gamma_2} \right)$$

Interference term violates unitarity!

# Reason III: Only poles are physical

Line shapes and peak positions and thus BW-parameters are channel dependent — only pole–locations are physical!



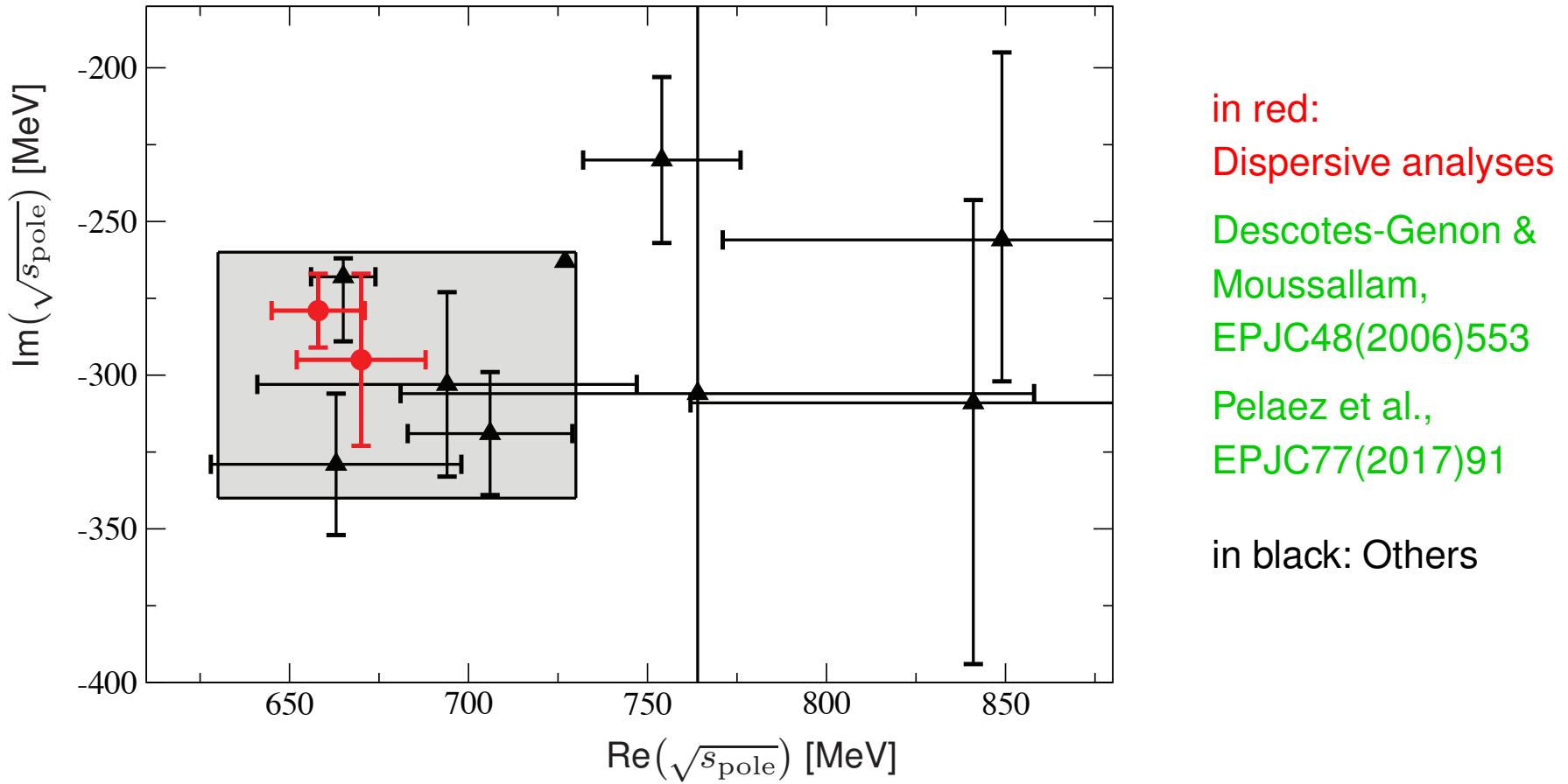
Wu et al., PRL108, 081803 (2012)

Analogue:  $a_1(1420)$  as triangle singul. in tail of  $a_1(1260)$

Mikhasenko et al., PRD91(2015)094015

# Status of the $\kappa$ pole position

In the next issue of the RPP of the PDG based on



the name will be changed to  $K_0^*(700)$  (or  $\kappa$ ) and we will quote

$$\sqrt{s_{\text{Pole}}^\kappa} = (630 - 730) - i(260 - 340) \text{ MeV}$$

as OUR ESTIMATE also in the summary tables

# Imposing Unitarity

- Properly constructed models fit to large sets of data  
a la Bonn-Gatchina, Jülich-Bonn, SAID
- or recent parametrization of near threshold cross sections  
C.H. et al., PRL 115(2015)202001 and Guo et al. PRD93(2016)074031
- or Dispersion Theory: Starting point: Im-part of form factor  $F_i$

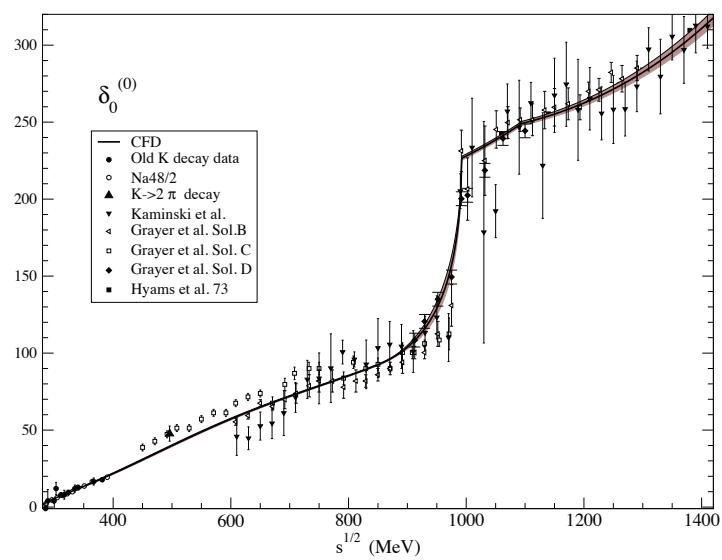
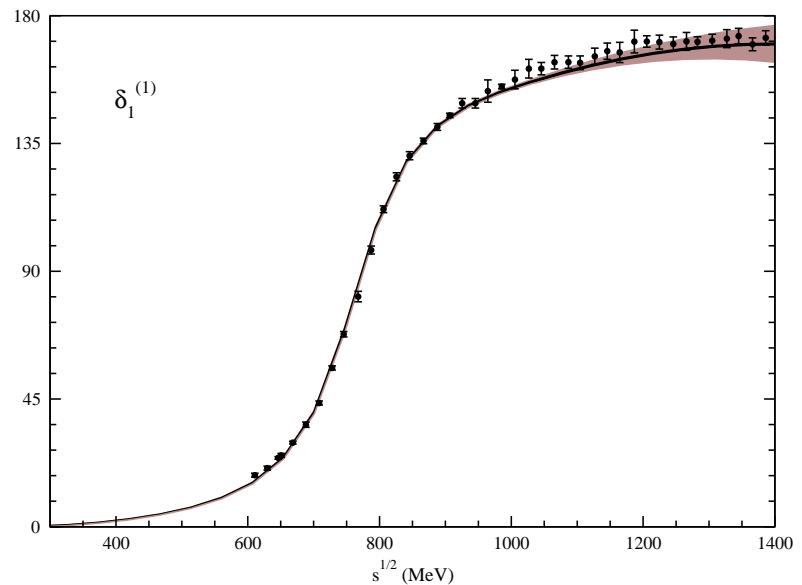
$$\text{Im}(F_i) = \sum_k T_{ik}^* \sigma_k F_k \quad \rightarrow \text{Dispersion Integral(s)}$$

for single channel  $\rightarrow$  Watson theorem and Omnès function

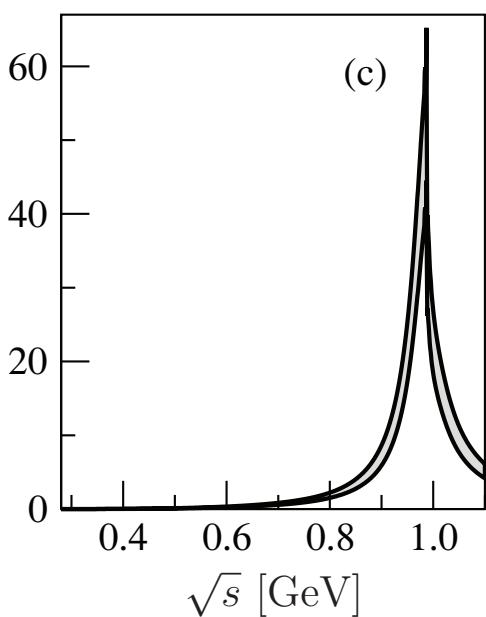
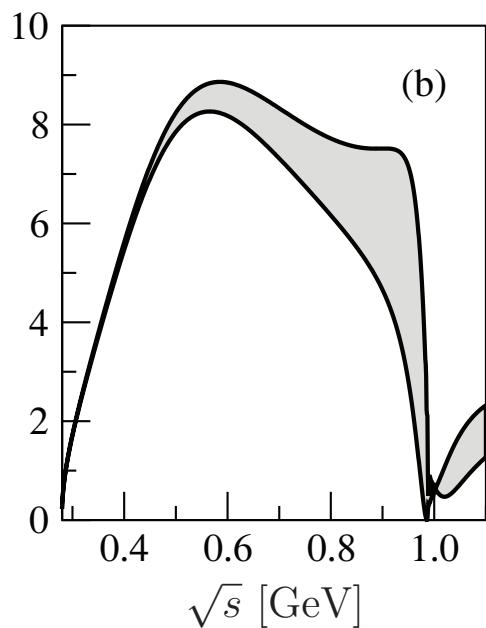
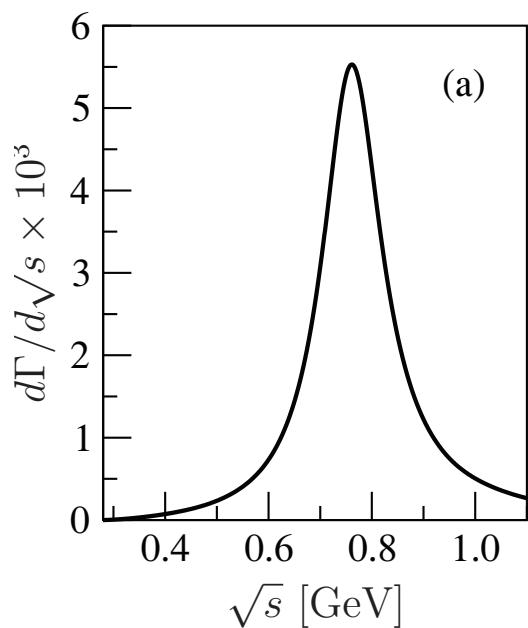
$$\Omega(s) = \exp \left( \frac{s}{\pi} \int_{4m_\pi^2}^\infty ds' \frac{\delta(s')}{s'(s' - s - i\epsilon)} \right) \quad \text{and} \quad F(s) = P(s)\Omega(s)$$

- $\rightarrow \Omega(s)$  is universal and fixed in elastic regime
- $\rightarrow P(s)$  reaction specific and contains e.g.  
higher thresholds, inelastic resonances, left-hand cuts

# Status for $\Omega(s)$ : $\pi\pi$ *S*- and *P*-waves

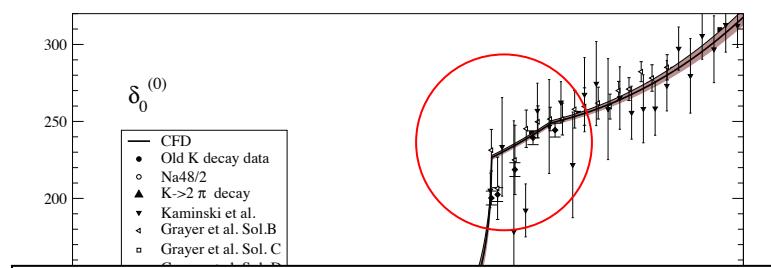
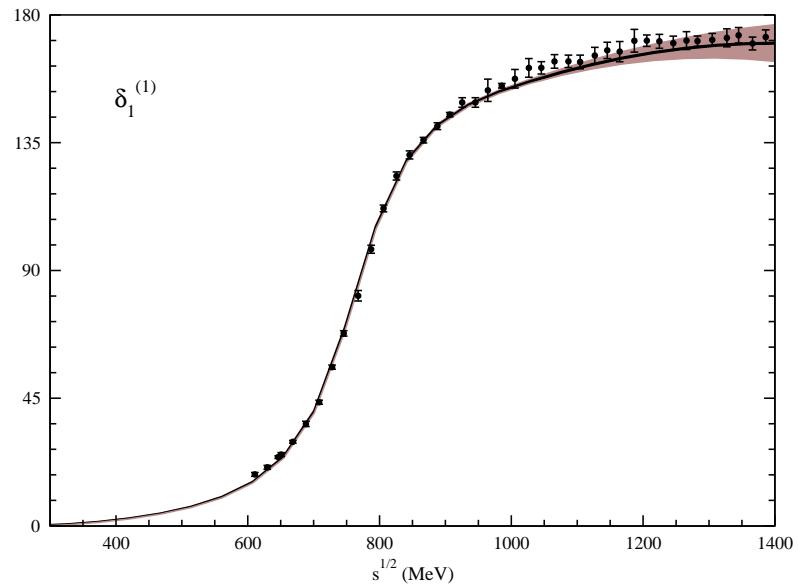


give for  $\Omega_1^1(s)(p+p')^\mu = \langle \pi\pi | \bar{q}\gamma^\mu q | 0 \rangle$   $\Gamma_\pi^n(s) = \langle \pi\pi | (\bar{u}u + \bar{d}d)/2 | 0 \rangle$   $\Gamma_\pi^s(s) = \langle \pi\pi | \bar{s}s | 0 \rangle$



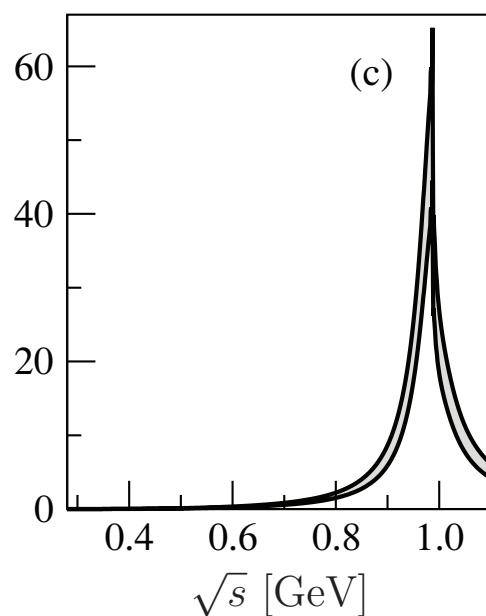
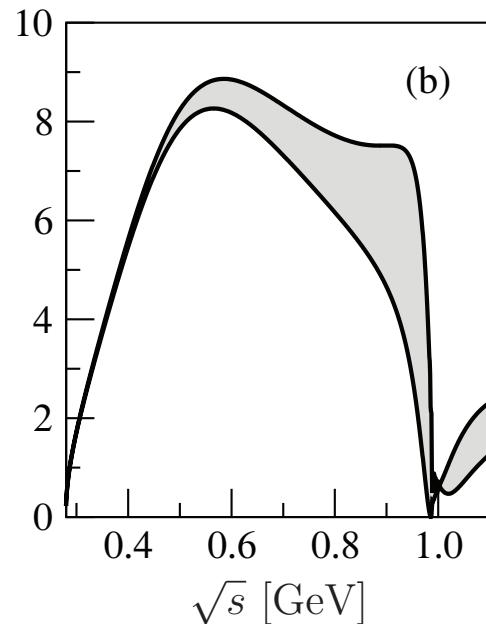
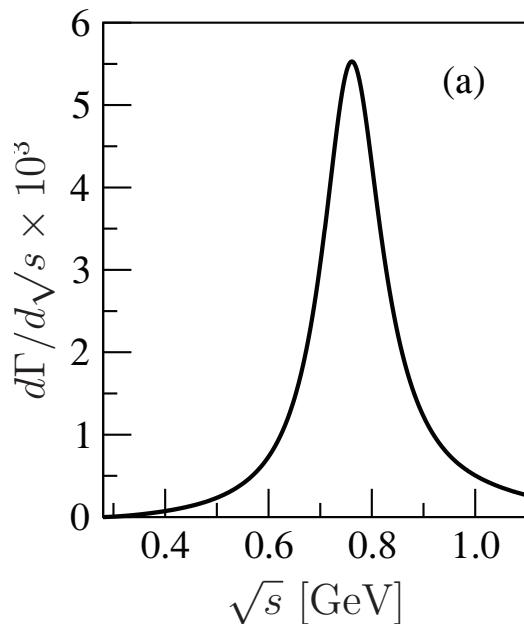
$\delta$ : Garcia-Martin et al., PRD83(2011)074004; FF's: Daub et al., JHEP01(2013)179

# Status for $\Omega(s)$ : $\pi\pi$ *S*- and *P*-waves



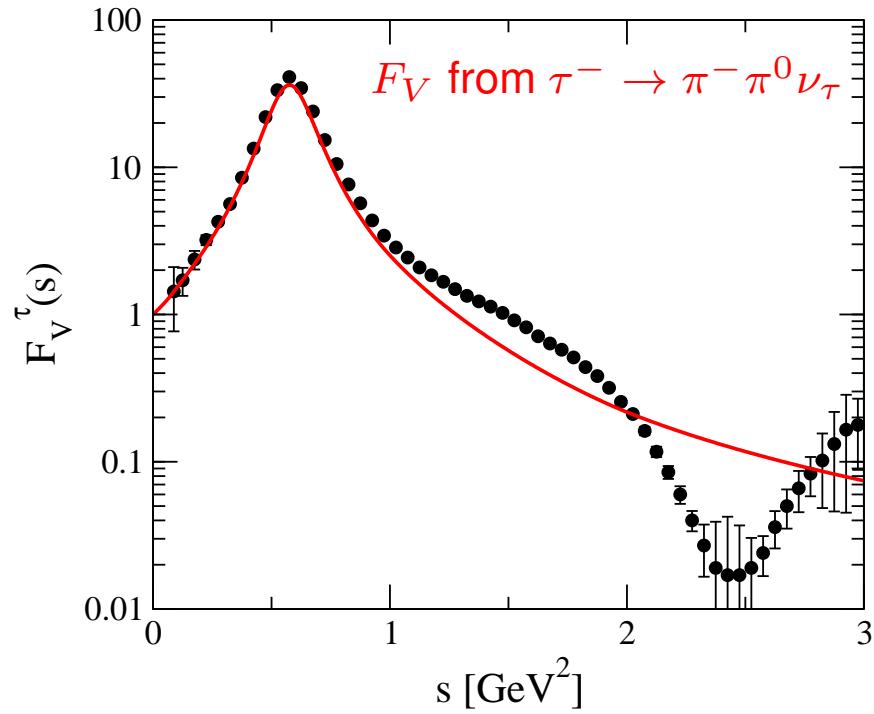
Region of uncertainty - see also  
 Caprini et al., EPJC72(2012)1860  
 Büttiker et al., EPJC33(2004)409  
 Dai, Pennington, PRD90(2014)036004

give for  $\Omega_1^1(s)(p+p')^\mu = \langle \pi\pi | \bar{q}\gamma^\mu q | 0 \rangle$   $\Gamma_\pi^n(s) = \langle \pi\pi | (\bar{u}u + \bar{d}d)/2 | 0 \rangle$   $\Gamma_\pi^s(s) = \langle \pi\pi | \bar{s}s | 0 \rangle$



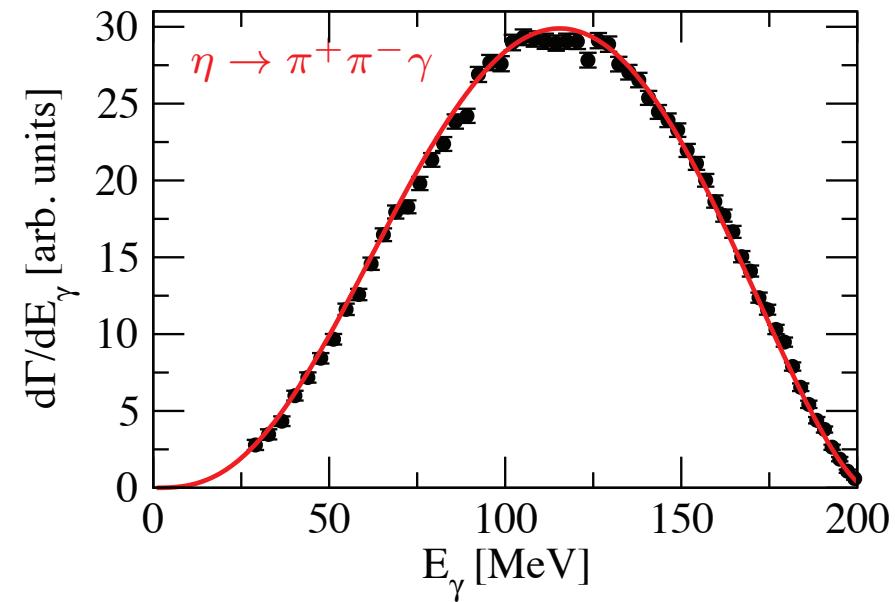
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# Universality of FSI

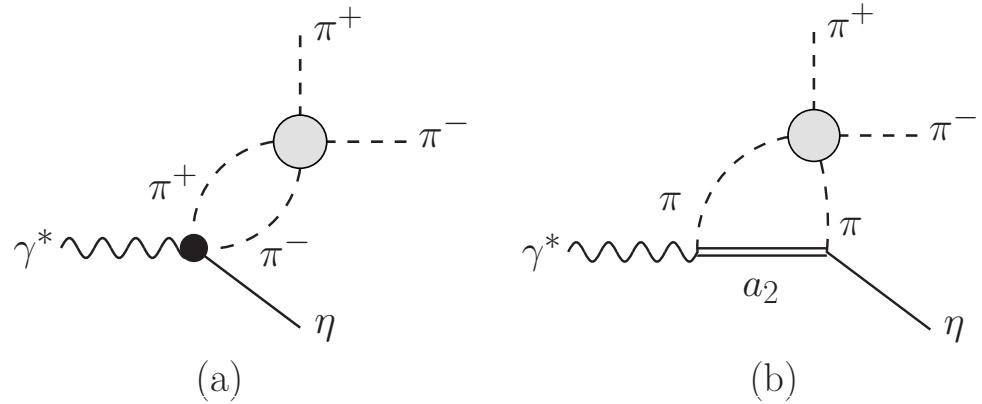


red lines: p-wave Omnes  
 $\times$  kinematic factors

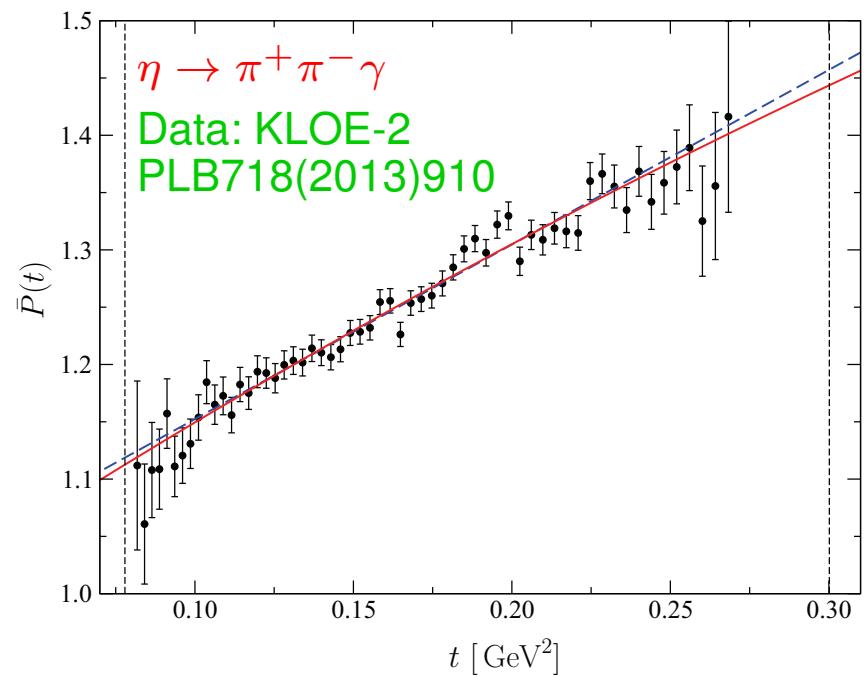
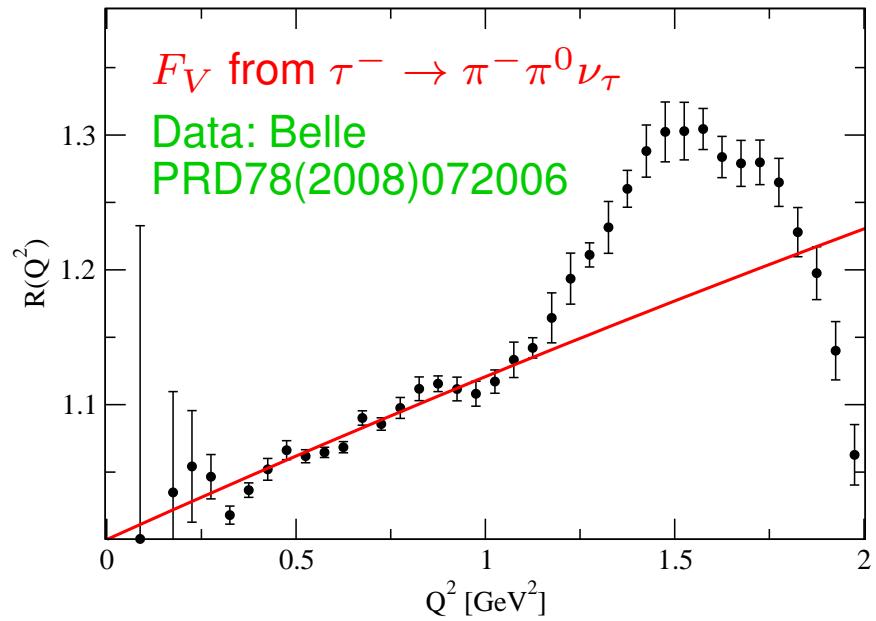
- bulk described properly
- there are deviations  
 $\rightarrow P(s)$  not constant!



We need to add for  $\eta$ -decay:



# Universality of FSI



We write  $F_V(Q^2) = R(Q^2)\Omega(Q^2)$

We find

- $R(Q^2)$  linear for  $Q^2 < 1 \text{ GeV}^2$
- deviations by  $\rho'$  &  $\rho''$

C.H. et al., EPJC73(2013)2668

Inclusion of the left-hand cut  
from  $a_2(1320)$  for  $\eta \rightarrow \pi\pi\gamma$

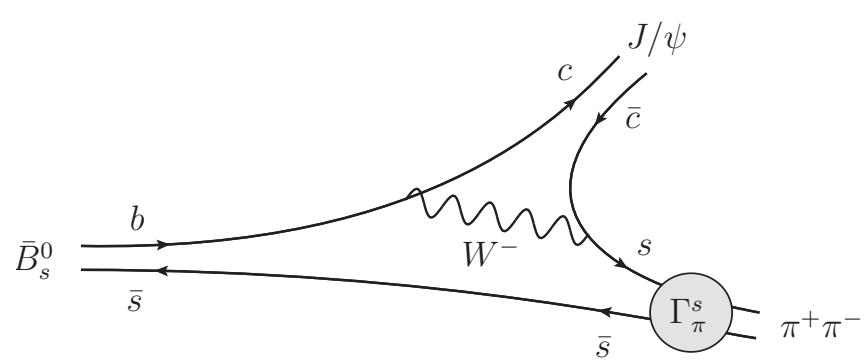
$$F_{a_2}(s_{\pi\pi}) = \Omega(s_{\pi\pi}) \left\{ A(1 + \alpha_\Omega[a_2] s_{\pi\pi}) + \right.$$

$$+ \frac{\cos \delta_1(s') \hat{F}_{a_2}(s')}{|\Omega(s')|}$$

$$\left. + \frac{s_{\pi\pi}^2}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^2} \frac{\sin \delta_1(s') \hat{F}_{a_2}(s')}{|\Omega(s')|(s' - s_{\pi\pi})} \right\},$$

Kubis and Plenter, EPJC75(2015)283

# Application: $\bar{B}_{s/d}^0 \rightarrow J/\psi \pi\pi$



- $\bar{B}_s^0$ : clean  $\bar{s}s$  source
- $\bar{B}_d^0$ : clean  $\bar{d}d$  source
- $\pi J/\psi$  interactions negligible  
→ no left-hand cuts

Ideal testing ground for formalism

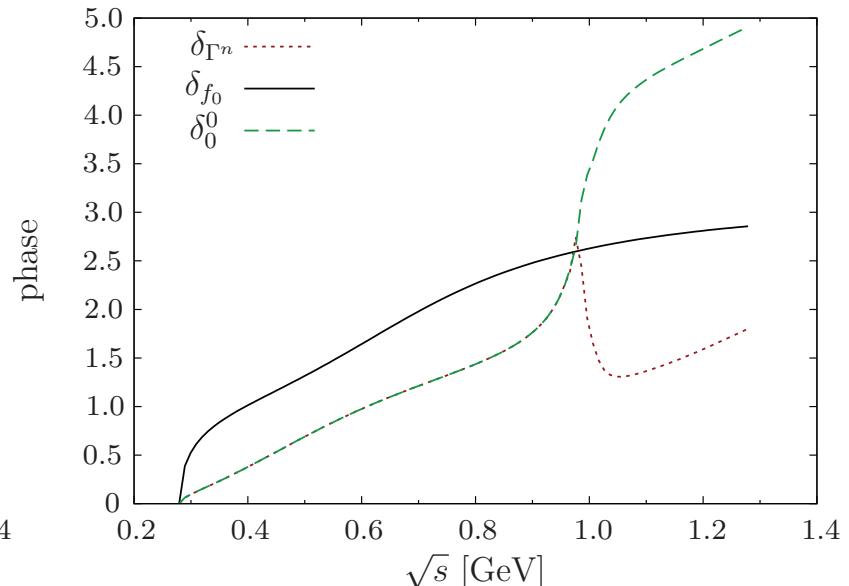
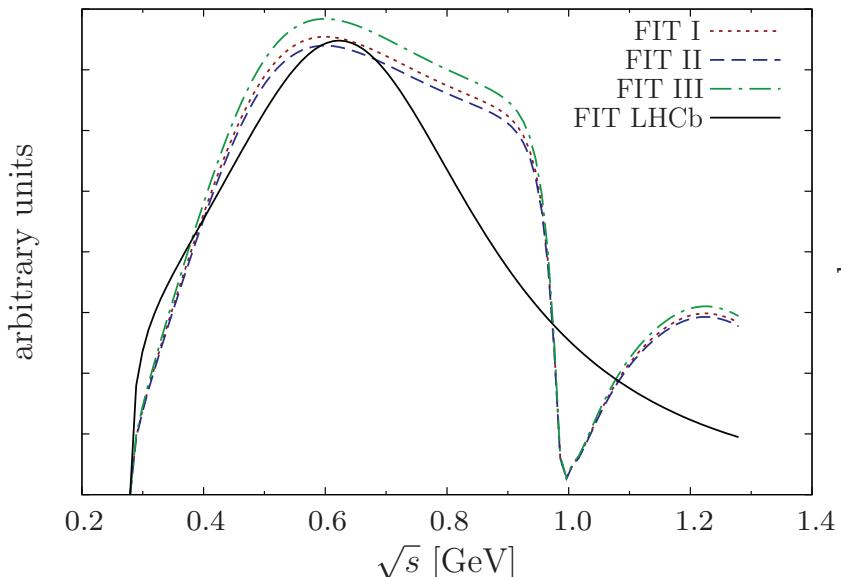
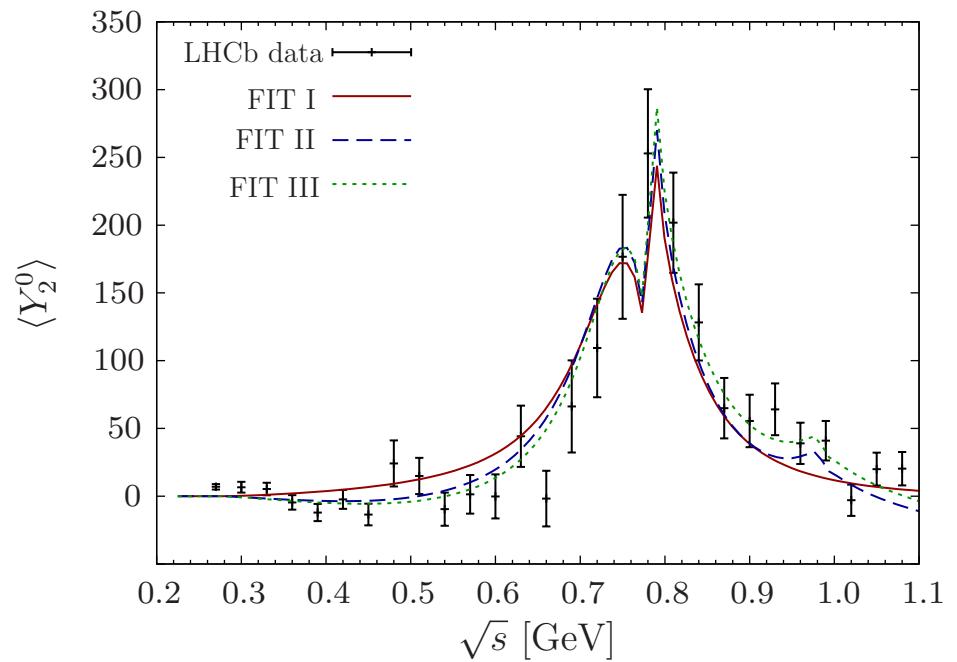
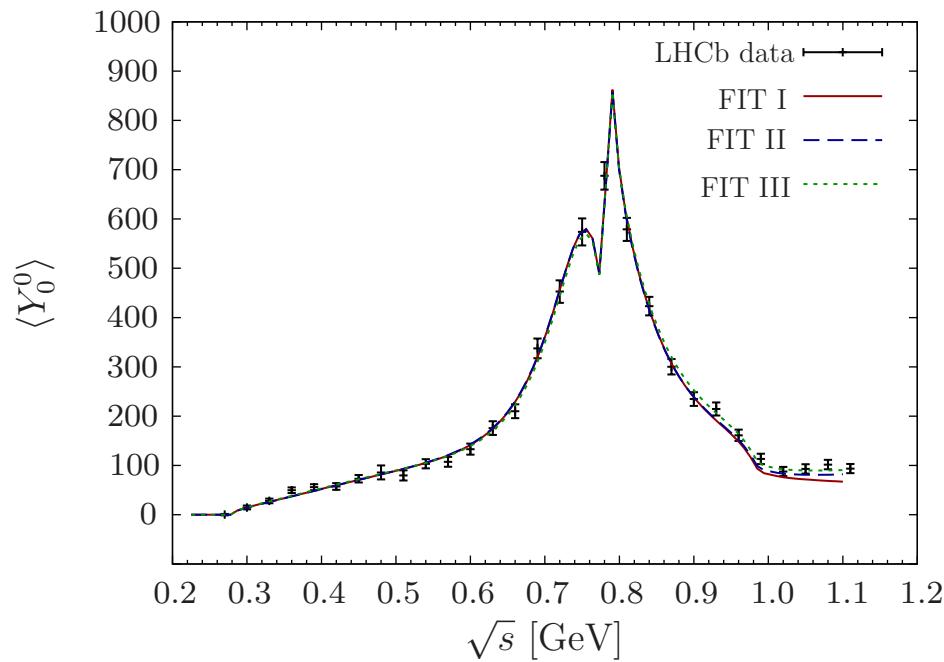
e.g. for  $\bar{B}^0$ -decay:

- phenomenological analysis: inclusion of BW-functions for  $f_0(500)$ ,  $\rho(770)$ ,  $\omega(782)$  → **14 parameters** LHCb, PRD90(2014)012003
- dispersive approach for  $S$ - and  $P$ -waves → **3-4 parameters**  
3 normalizations; eventually allowing for additional slope

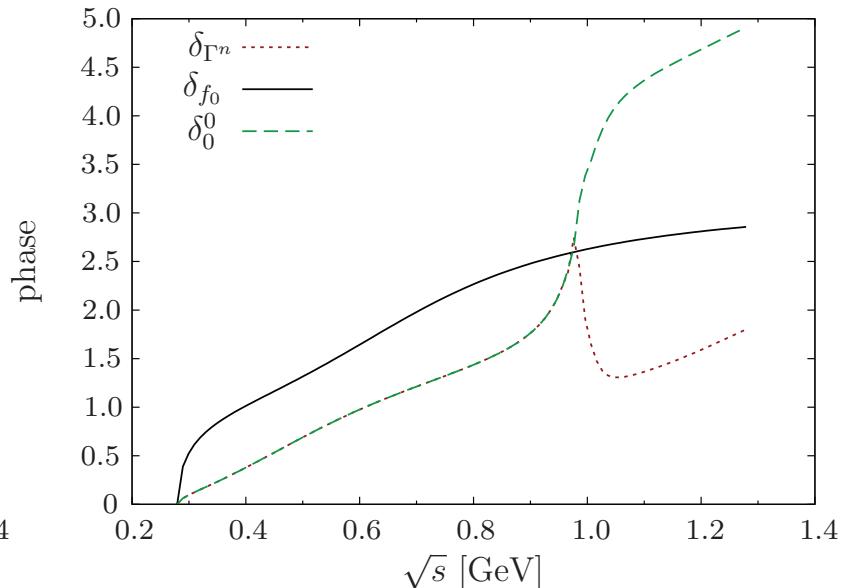
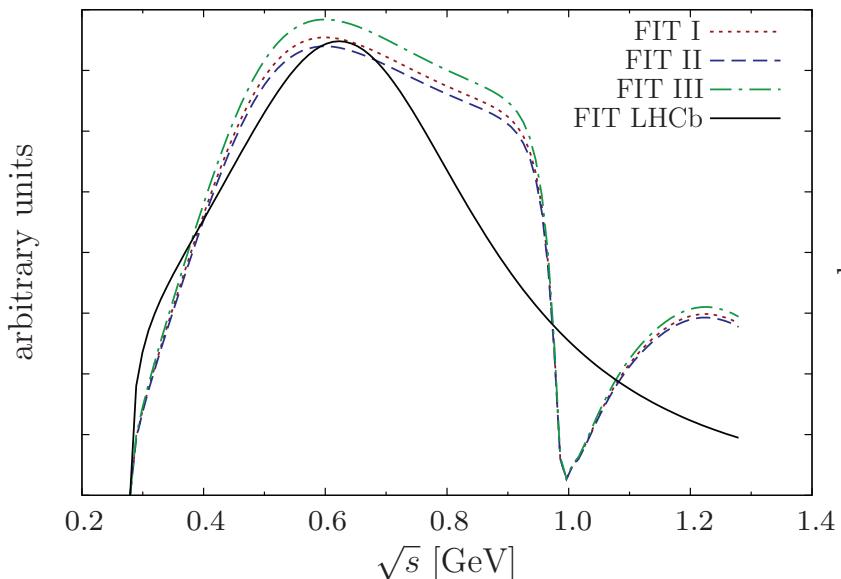
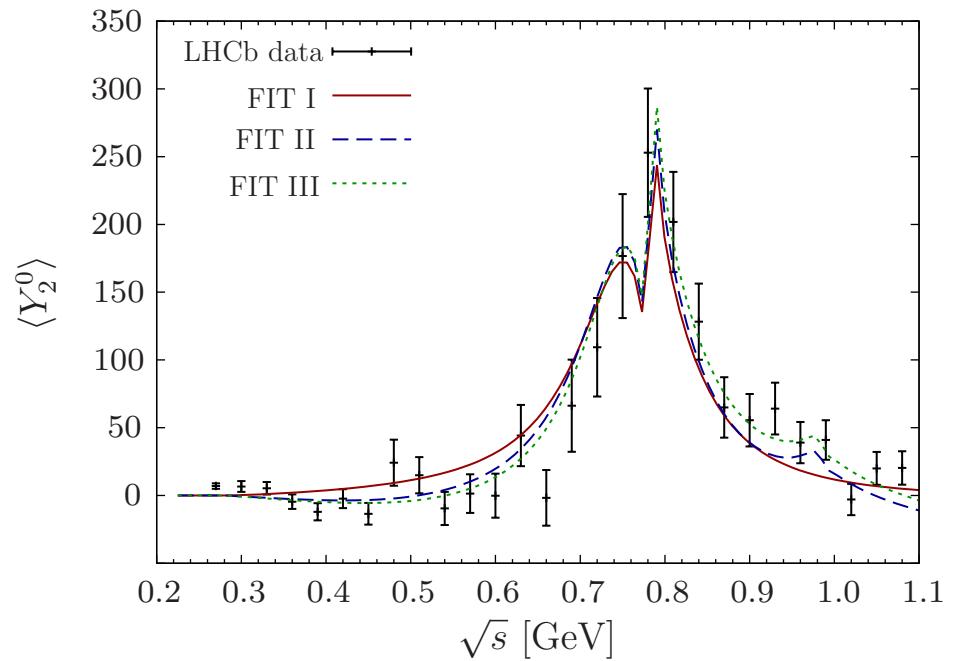
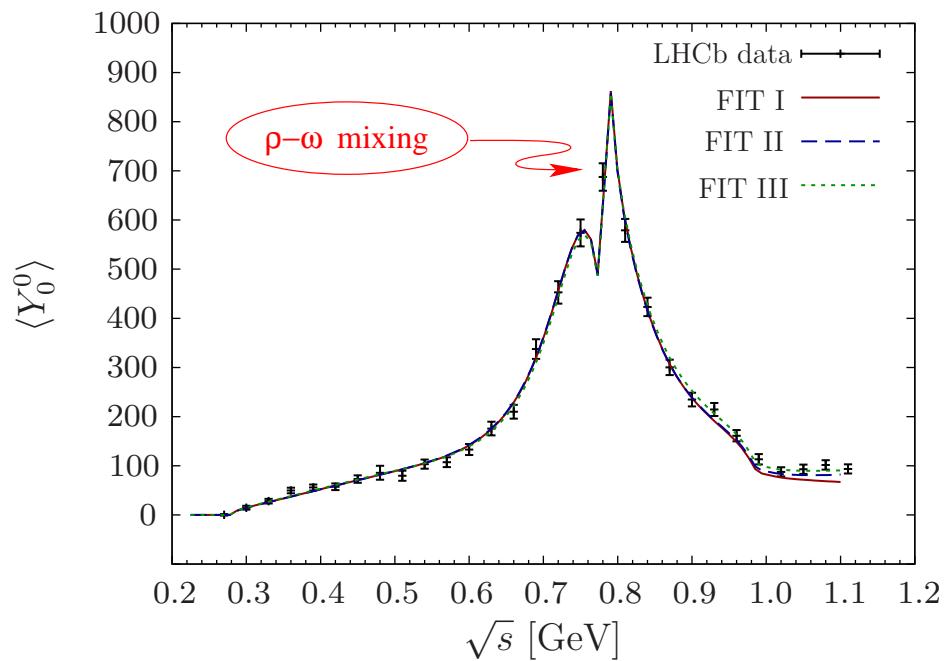
Daub, C.H., Kubis, JHEP1602(2016)009

Fits are of similar quality!

# Results for $\bar{B}^0 \rightarrow J/\psi \pi\pi$



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- The absence of the  $f_0(980)$  in the LHCb fit was interpreted as incompatibility with tetraquark structure (at  $8\sigma$ )

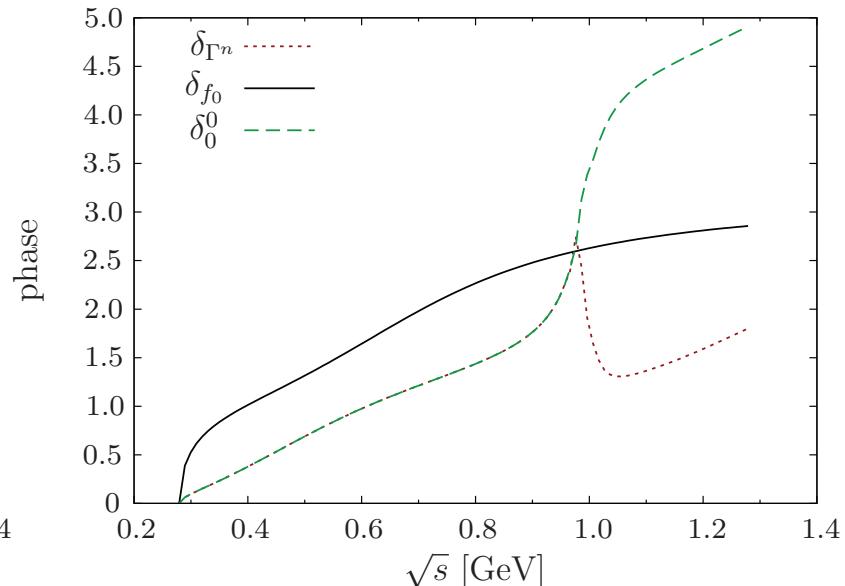
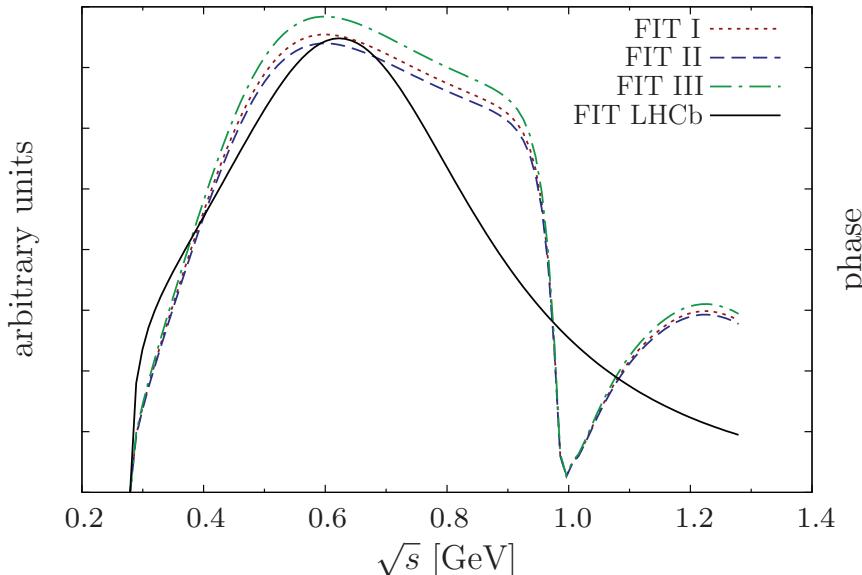
R. Aaij et al. (LHCb Collaboration) PRD90(2014)012003

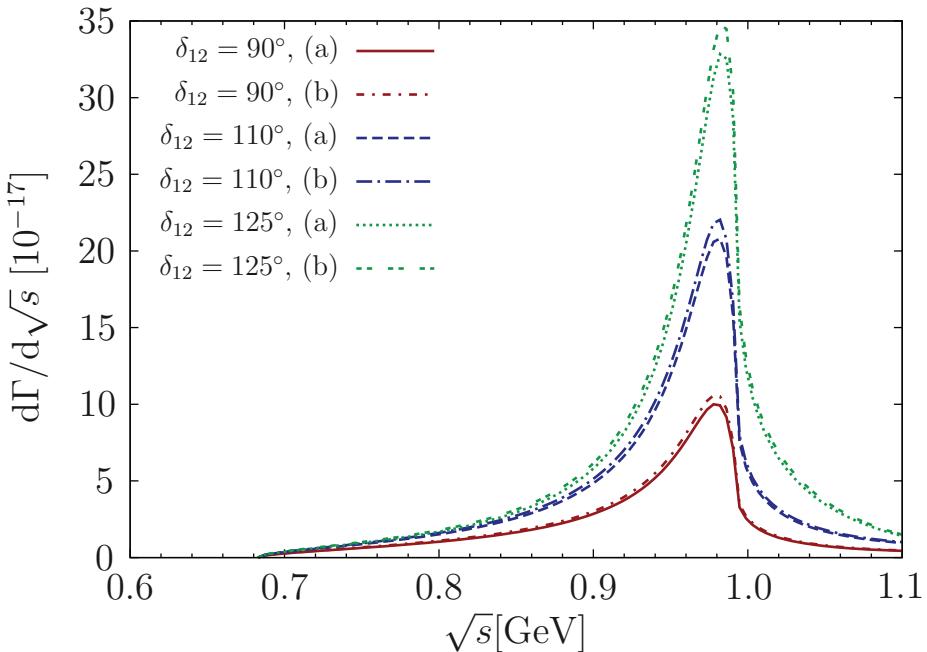
- Re-analysis shows that this conclusion was premature

Daub, C.H., Kubis, JHEP1602(2016)009

- Light scalars consistent with two-meson states

For a review on had.-molecules: F. K. Guo et al., Rev. Mod. Phys. 90(2018)015004

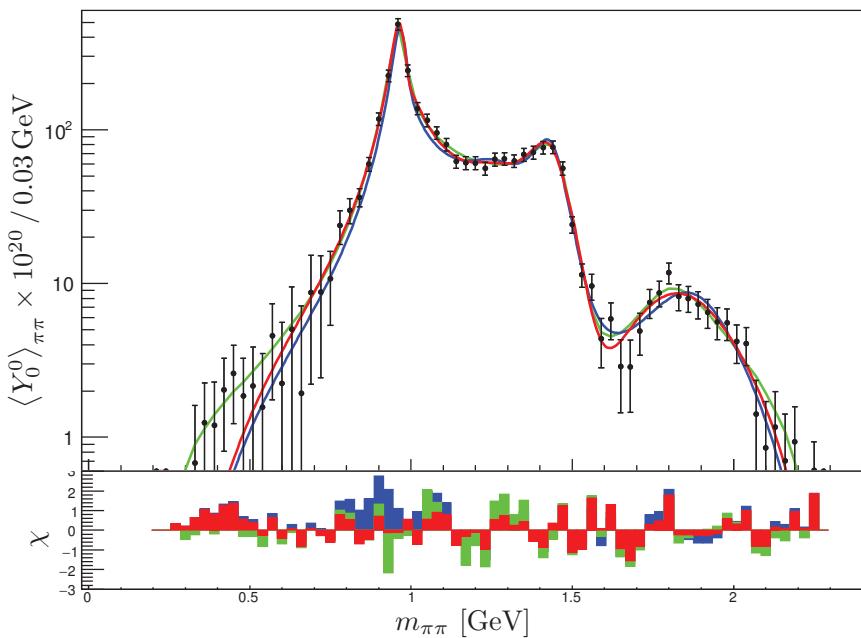




Data for  $\bar{B}_d^0 \rightarrow J/\psi \pi \eta$  will allow one to fix final uncertainty in  $\eta \pi$  scattering phase shifts

Albaladejo et al., JHEP1704(2017)010  
employing

Albaladejo & Moussallam EPJC75(2015)488



Analysis of  $B_s \rightarrow J/\psi \pi \pi$  extended to higher energies: shown fits by LHCb, 2 res., 3 res.

Ropertz et al., in preparation  
Data: LHCb, PRD89(2014)092006  
adapting formalism of

C.H., PLB715(2012)170

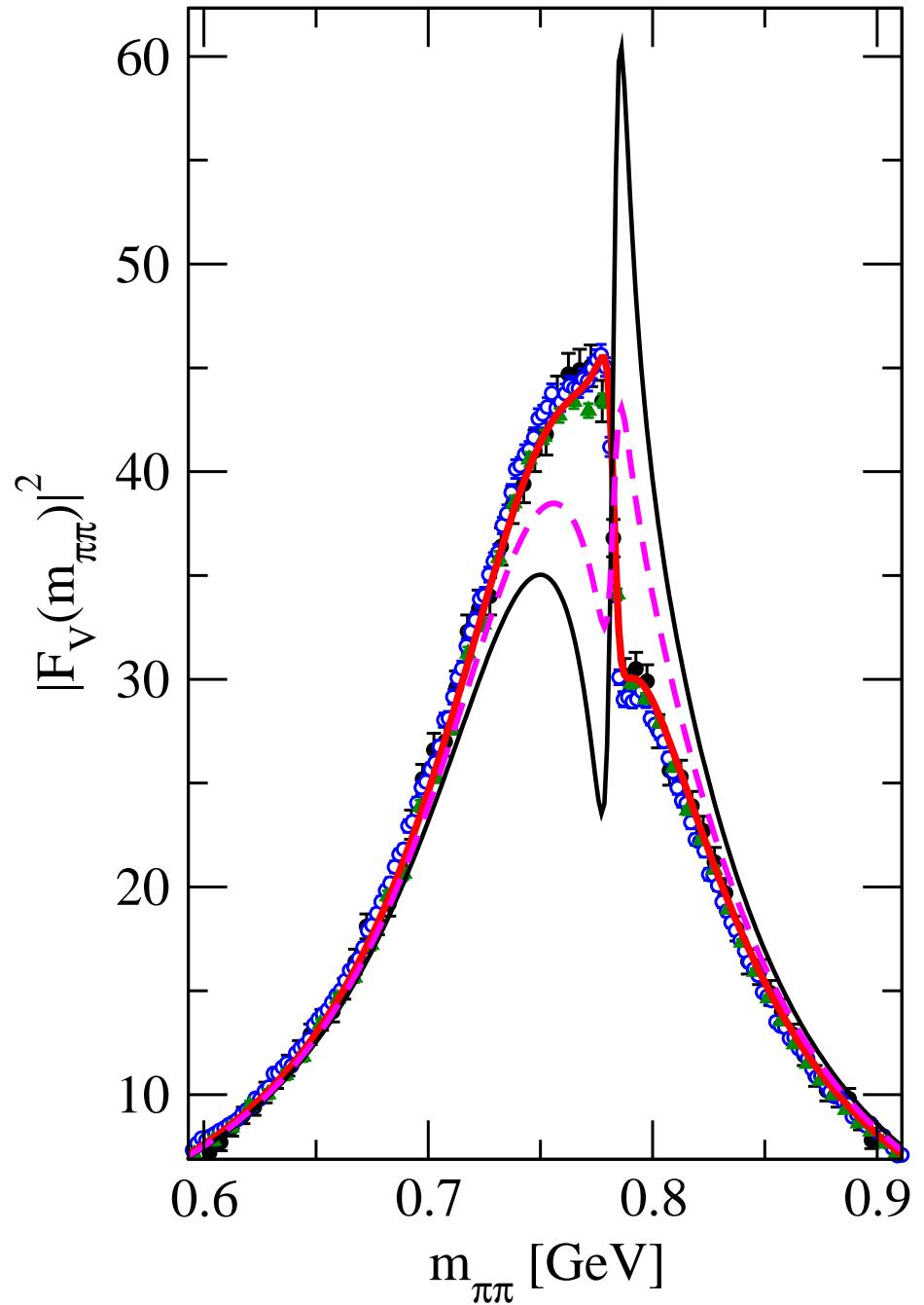
# Summary

- States are characterized by their **pole positions and residues**
- Breit-Wigner fits should in general be **avoided**
- Where ever possible **phase information** should be employed
  - ▷ either via **dispersion theory**  
for other examples see, e.g.,  
 $\gamma\gamma \rightarrow \pi\pi$ : Hoferichter et al. 2011, Moussallam 2011, Mao et al. 2009, Pennington et al. 2008  
 $\omega/\phi \rightarrow \pi\pi\pi$ : Niecknig, Kubis, Schneider 2012, Danilkin et al. (2015)
  - ▷ or via **models consistent with analyticity and unitarity**  
coupled channel analyses for  $\pi N$ ,  $\pi\pi N$ ,  $\gamma N$  a la Bonn-Gatchina, Jülich-Bonn, SAID

The theoretical tools are becoming available to

- extract the hadron spectrum from data in a controlled way
- treat hadronic final state interactions rigorously
- hunt for physics beyond the Standard Model

# On the effect of $\rho$ - $\omega$ mixing



$\gamma$  is coupling to the charges

$$\begin{aligned} j_{\text{em}}^\mu &= \frac{2}{3}\bar{u}\gamma^\mu u - \frac{1}{3}\bar{d}\gamma^\mu d \\ &= \frac{1}{2}(\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d) \\ &\quad + \frac{1}{6}(\bar{u}\gamma^\mu u + \bar{d}\gamma^\mu d). \end{aligned}$$

isovector/isoscalar: +1/3

$\bar{B}_d^0$ : clean  $\bar{d}d$  source

$$\begin{aligned} \bar{d}\gamma^\mu d &= -\frac{1}{2}(\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d) \\ &\quad + \frac{1}{2}(\bar{u}\gamma^\mu u + \bar{d}\gamma^\mu d). \end{aligned}$$

isovector/isoscalar: -1