Soft gluon emission in top pair production

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Top-pairs at the LHC

$$p + p \longrightarrow t + \bar{t} + X$$

Additional final state radiation

Top pair production QCD corrections at hadron colliders are known to NNLO

M. Czakon, D. Heymes, A. Mitov

$$p + p \longrightarrow t + \bar{t} + H(\text{or}\ \ W/Z) + X$$

Top pair + boson production QCD corrections at hadron colliders are known to NLO

ex. MadGraph5_aMC@NLO, OpenLoops, and other NLO providers

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Additional final state radiation

Top pai

are kno

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For both processes, the theoretical predictions can be improved by resumming soft gluon emission effect to NNLL accuracy

For top pair production, in the tail of differential distributions terms which become large in the limit $\hat{s} \gg m_t^2$ are also relevant. We resum them to NNLL' accuracy

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ex. MadGraph5_aMC@NLO, OpenLoops, and other NLO providers

Outline & Goals of the Talk

i. Explain in brief what we are resumming

ii. Top pair + H/W/Z: show some results for the total cross section and several differential distributions (obtained with an in-house Monte Carlo) at NLO+NNLL accuracy

iii. Top pair production: show results for the pair invariant mass distribution and the top quark transverse momentum distribution at NNLO+NNLL' accuracy

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Warning: emphasis on resummation effects rather than on resummation techniques: I will skip a lot of very important technical details (can discuss after the talk)

“Pair” Invariant Mass kinematics

- For top pair production, we have two tree-level partonic processes

$$q(p_1) + \bar{q}(p_2) \to t(p_3) + \bar{t}(p_4)$$
$$g(p_1) + g(p_2) \to t(p_3) + \bar{t}(p_4)$$

- Define the invariants

$$\hat{s} = (p_1 + p_2)^2 \qquad M^2 = (p_3 + p_4)^2$$

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Partonic center of mass energy (squared)

Invariant mass of the heavy particles in the final state

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If real radiation in the final state is present, $\hat{s} \neq M^2$

$$z = \frac{M^2}{\hat{s}}$$

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If real radiation in the final state is

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Partonic threshold limit

$$z \longrightarrow 1$$

Factorization in a nutshell

Differential cross section:

$$\frac{d^2\sigma}{dM^2} = f\!f(z) \otimes C(z)$$

Factorization in a nutshell

Differential cross section:

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Partonic luminosity

$$ff = \int_y^1 \frac{dx}{x} f_{i/N_1}(x) f_{j/N_2}\left(\frac{y}{x}\right)$$

Factorization in a nutshell

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Partonic luminosity

Hard scattering kernel
(partonic cross section)

Factorization in a nutshell

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In the soft emission limit a clear scale hierarchy emerges:

$$\hat{s}, M^2, m_t^2 \gg \hat{s}(1-z)^2 \gg \Lambda_{\mathrm{QCD}}^2$$

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$$\underbrace{\hat{s}, M^2, m_t^2}_{\text{Hard scales}} \gg \underbrace{\hat{s}(1-z)^2}_{\text{Soft scale}} \gg \Lambda^2_{\mathrm{QCD}}$$

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In this limit, the partonic cross section factors into two parts:

$$C_{ij} = \mathrm{Tr}\left[\underbrace{\mathbf{H}_{ij}\left(M, \{p_i\}, \mu\right)}_{\text{Hard function (virtual corrections)}} \underbrace{\mathbf{S}_{ij}\left(\sqrt{\hat{s}}(1-z), \{p_i\}, \mu\right)}_{\text{Soft function (real soft emission)}}\right]$$

Hard function
(virtual corrections)

Soft function
(real soft emission)

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both are matrices
(in color space)

Factorization in a nutshell

Differential cross section:

$$\frac{d^2\sigma}{\ldots} = ff(z) \otimes C(z)$$

In

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Soft gluon emission resummation:
use RG equations satisfied by H and S to resum terms which are singular in the $z \to 1$ limit

$$P_k(z) = \left[\frac{\ln^k(1-z)}{1-z}\right]_+$$

(In reality we work in Mellin space where the plus distributions turn into ln N)

Hard
(virtual corrections)

(in color space)

n
(real soft emission)

Factorization in a nutshell

Differential cross section:

$$\frac{d^2\sigma}{dM^2} = ff(z) \otimes C(z)$$

In the soft emission limit a clear scale hierarchy emerges:

In

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This resummation program can be carried out with relatively few changes also for the production of a

Top-pair + H (or W/Z)

Hard function
(virtual corrections)

both are matrices
(in color space)

Soft function
(real soft emission)

Mass logs in top pair production

- The LHC has the ability to probe top quarks with an energy of a few TeV → "boosted" tops, $\hat{s} \gg m_t^2$
- Within the soft gluon corrections, there are terms which are further enhanced by $\ln(m_t^2/\hat{s})$
- Essentially, in the small mass limit the hard function and the soft function factor further:

$$H_{ij}^m(M, m_t, \cos\theta, \mu_f) = H_{ij}(M, \cos\theta, \mu_f)\, C_D^2(m_t, \mu_f)$$

$$\widetilde{s}_{ij}^m\left(\ln\frac{M^2}{\bar{N}^2\mu_f^2}, M, m_t, \cos\theta, \mu_f\right) = \widetilde{s}_{ij}\left(\ln\frac{M^2}{\bar{N}^2\mu_f^2}, M, \cos\theta, \mu_f\right)\, \widetilde{s}_D^2\left(\ln\frac{m_t}{\bar{N}\mu_f}, \mu_f\right)$$

- A framework for the simultaneous resummation of threshold and small mass logarithms in top-pair differential cross sections at the LHC is available

AF, B. Pecjak, and L.L. Yang ('12)

D. Forde, B. Pecjak, X. Wang and L.L. Yang ('16)

Types and accuracy of the resummation

- We have all of the elements needed to carry out soft gluon (partonic threshold) resummation to NNLL accuracy (for both top pair and top pair + H/W/Z)
- Resummation in the soft-boosted limit can be carried out to NNLL' accuracy (i.e. massless hard and soft functions + fragmentation to NNLO, technically $\mathrm{NNLL}_{s+b'}$)
- The matching of NNLL soft gluon resummation to NLO/NNLO calculations well understood
- The soft - boosted resummation formula in alpha_s is the small mass limit of the threshold resummation formula at any fixed order in alpha_s. This can be used to match the two resummation procedures and obtain NNLO+NNLL' predictions

Top-quark pair + H (or W/Z): NLO+NNLL (soft-limit) results

Total cross section

+

Four differential distributions

A. Broggio, AF, G. Ossola, B.D. Pecjak, R. Sameshima, L.L. Yang

JHEP 1609 (2016) 089 [arXiv:1607.05303]
JHEP 1702 (2017) 127 [arXiv:1611.00049]
JHEP 1704 (2017) 105 [arXiv:1702.00800]

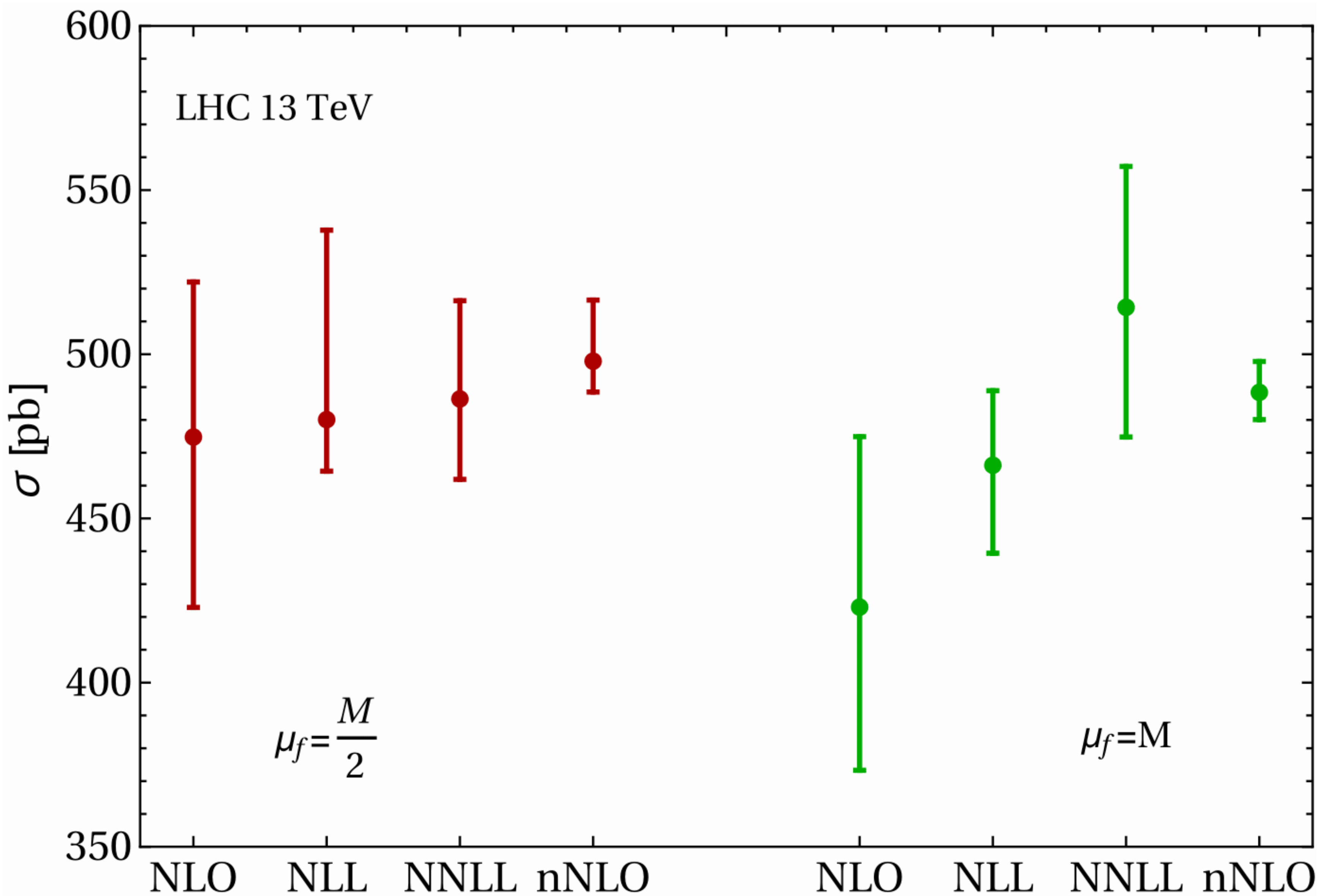
Total cross section @ 13 TeV

order	PDF order	code	σ [fb]
LO	LO	MG5_aMC	$378.7^{+120.5}_{-85.2}$
NLO	NLO	MG5_aMC	$474.8^{+47.2}_{-51.9}$
NLO+NLL	NLO	MC +MG5_aMC	$480.1^{+57.7}_{-15.7}$
NLO+NNLL	NNLO	MC +MG5_aMC	$486.4^{+29.9}_{-24.5}$

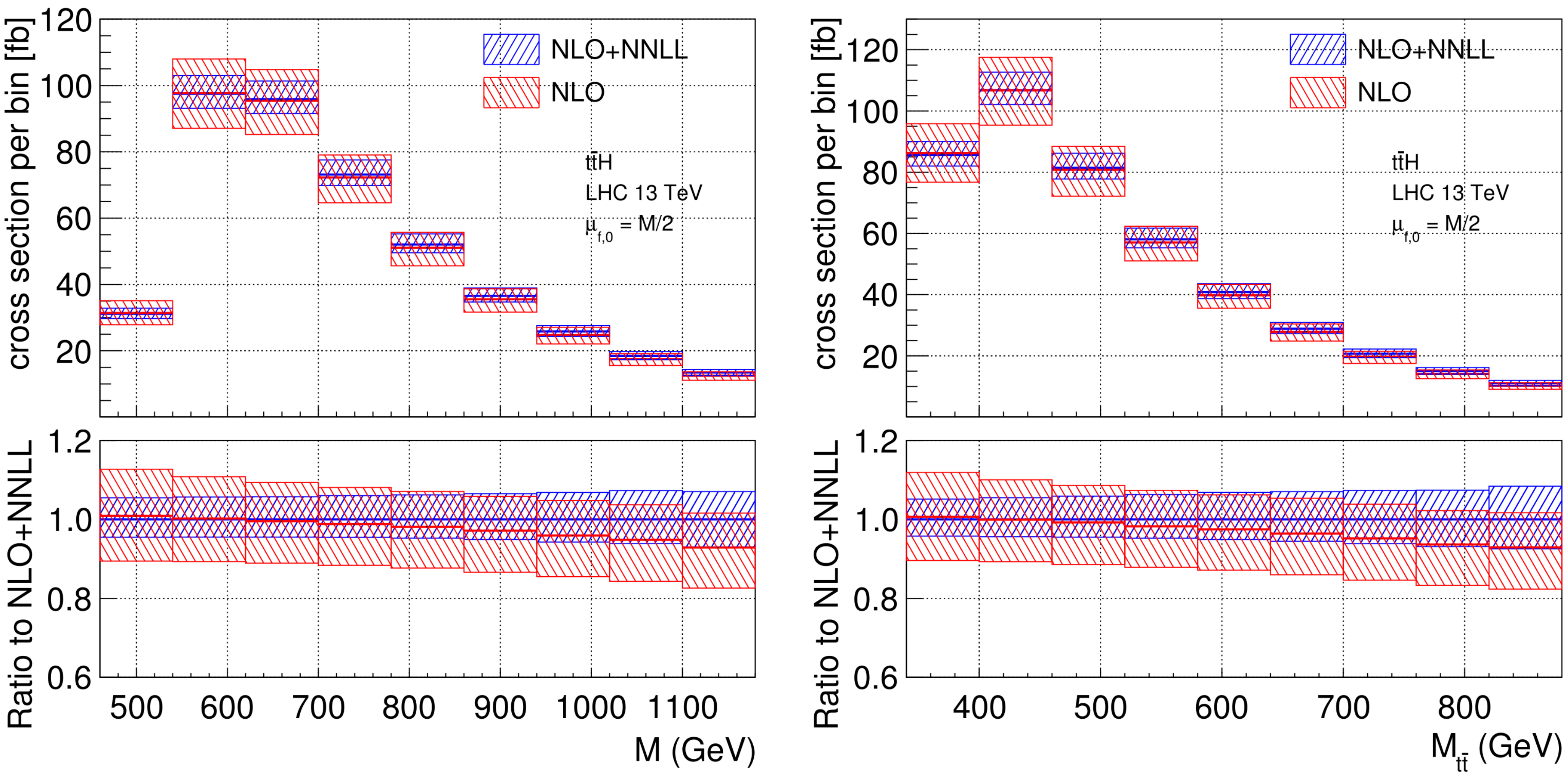
$t\bar{t}H$ boson production $\mu_f = M/2\,, \mu_h = M\,, \mu_s = M/N$

The scale uncertainties get progressively smaller when moving from NLO to NLO+NLL to NLO+NNLL, and the higher-order results are roughly within the range predicted by the uncertainty bands of the lower-order ones.

Comparison among predictions at different factorization scales

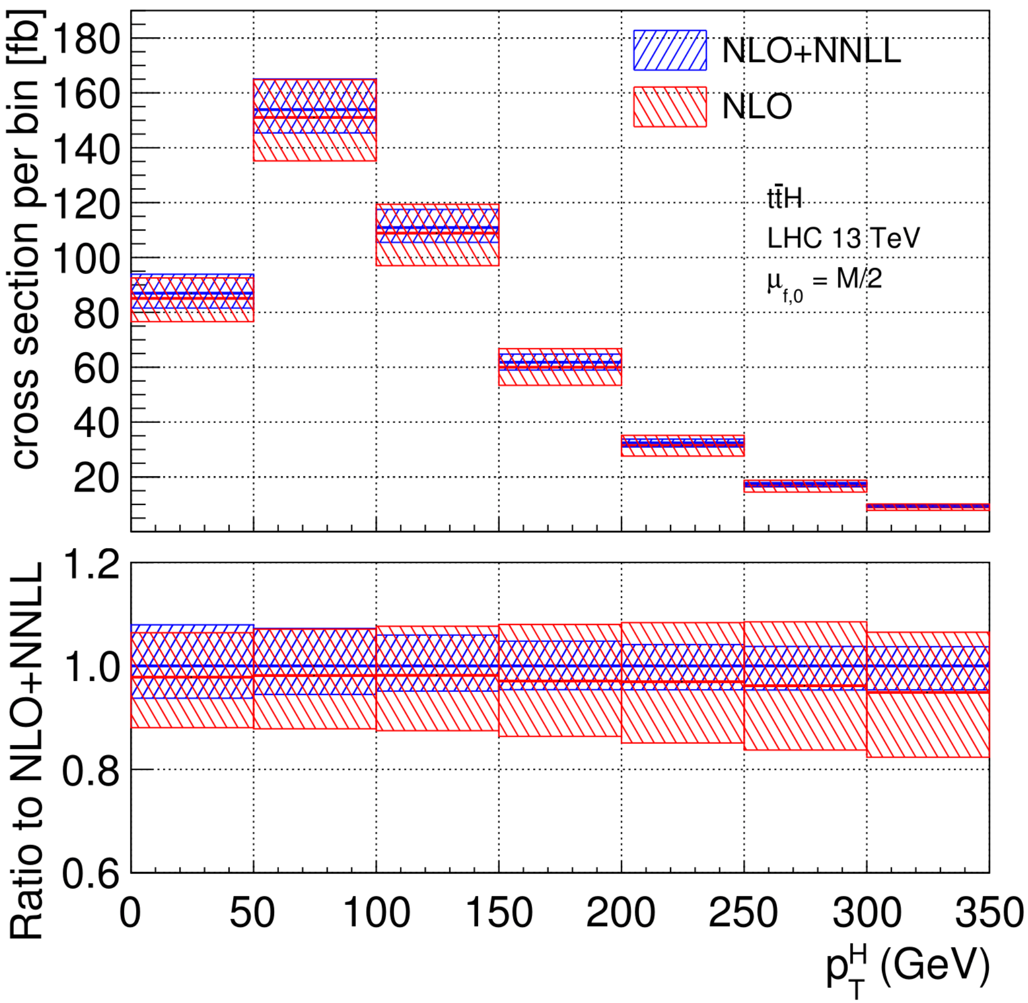


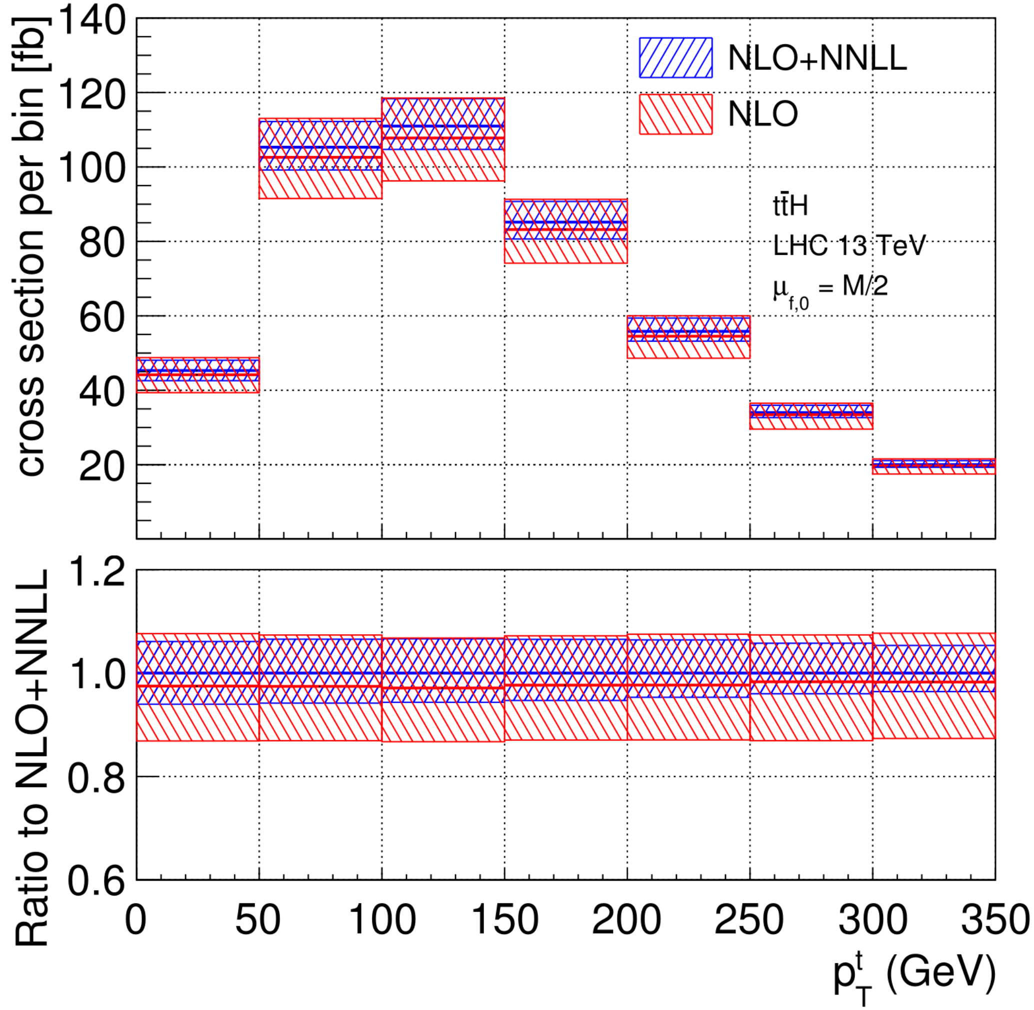
tTH distributions at NLO+NNLL



NLO+NNLL distributions overlap with the upper part of the NLO bands.
The NLO+NNLL bands are narrower than the NLO bands

tTH distributions at NLO+NNLL





NLO+NNLL distributions overlap with the upper part of the NLO bands.
The NLO+NNLL bands are narrower than the NLO bands

Top-quark pair production: NNLO+NNLL' (soft-boosted) results

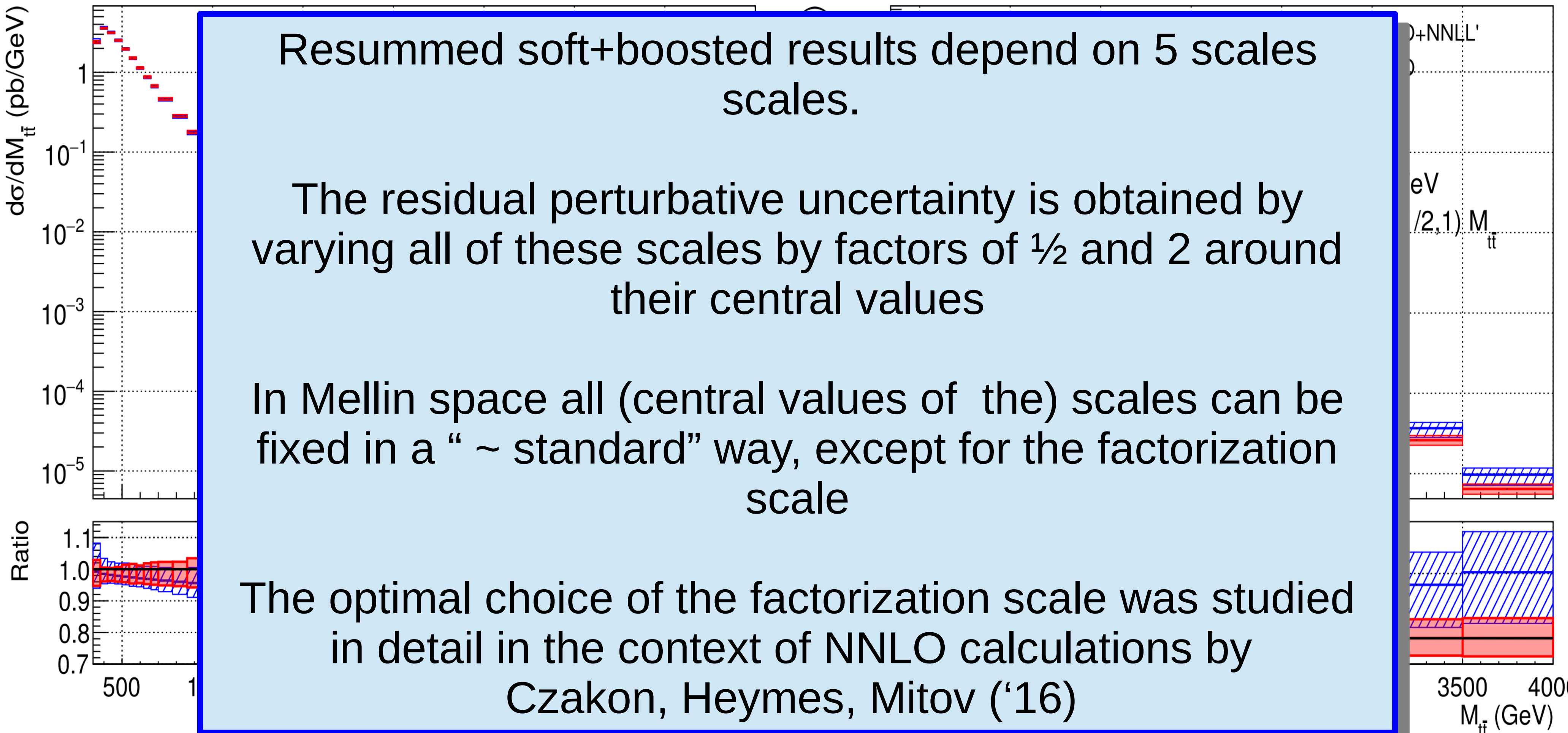
Pair invariant mass

Top-quark transverse momentum

M. Czakon, AF, D. Heymes, A. Mitov, B. Pecjak, D. Scott, X. Wang and L.L. Yang

(In progress, preliminary results)

tT M distribution at NNLO+NNLL’



Resummed soft+boosted results depend on 5 scales scales.

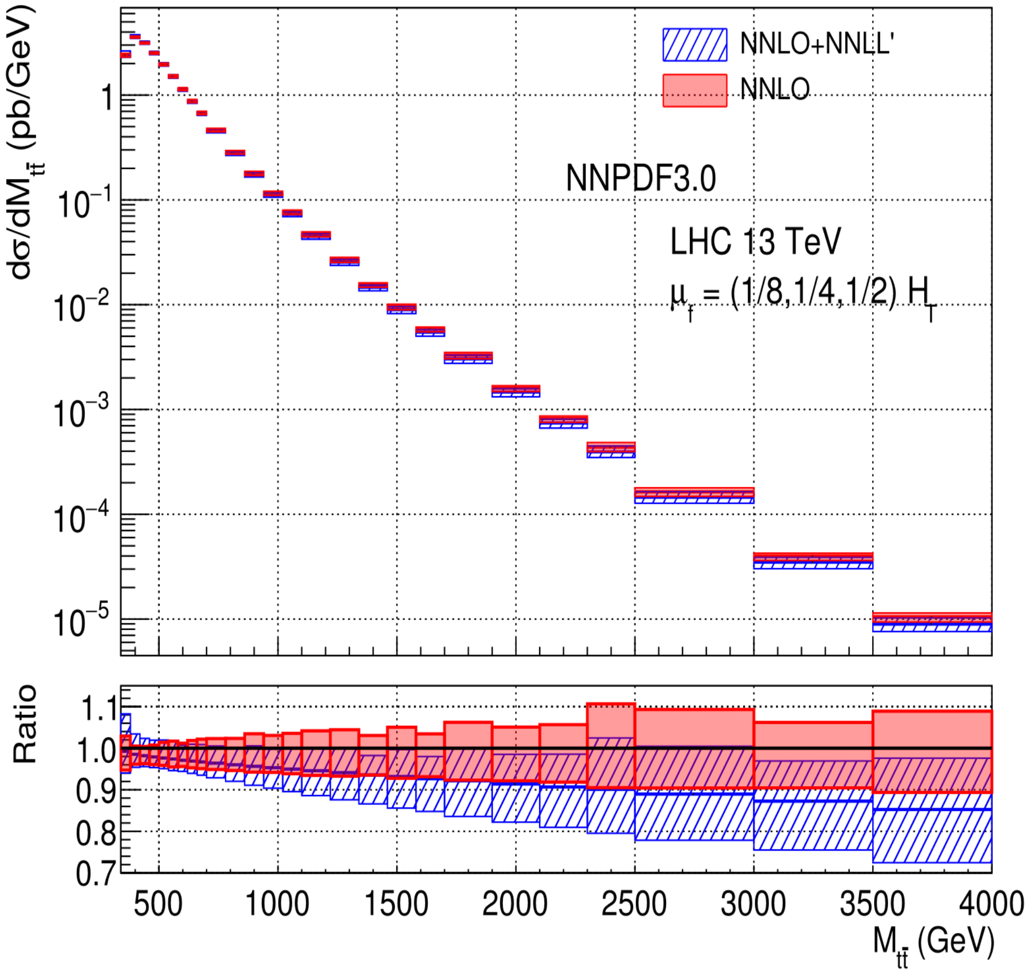
The residual perturbative uncertainty is obtained by varying all of these scales by factors of ½ and 2 around their central values

In Mellin space all (central values of the) scales can be fixed in a “ ~ standard” way, except for the factorization scale

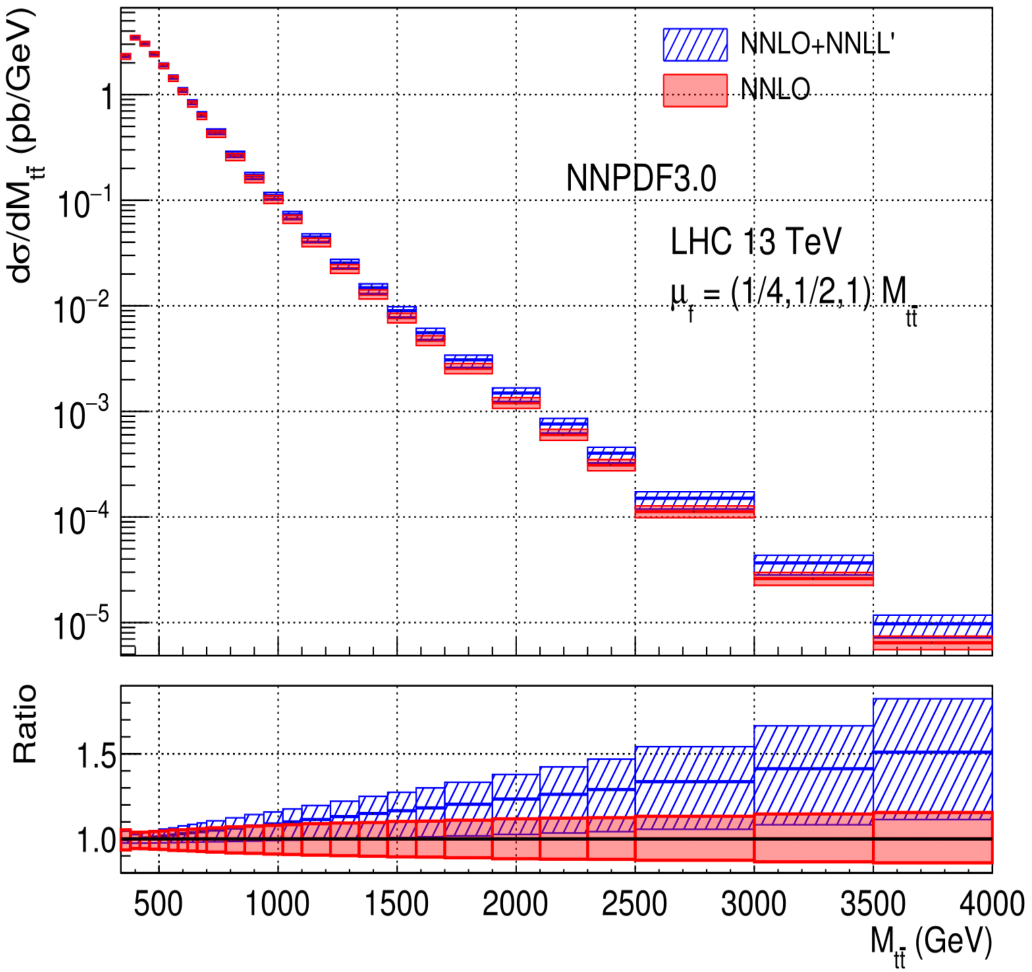
The optimal choice of the factorization scale was studied in detail in the context of NNLO calculations by Czakon, Heymes, Mitov (‘16)

tT M distribution at NNLO+NNLL’

$$H_T = \sqrt{m_t^2 + p_{T,t}^2} + \sqrt{m_t^2 + p_{T,\bar{t}}^2}$$

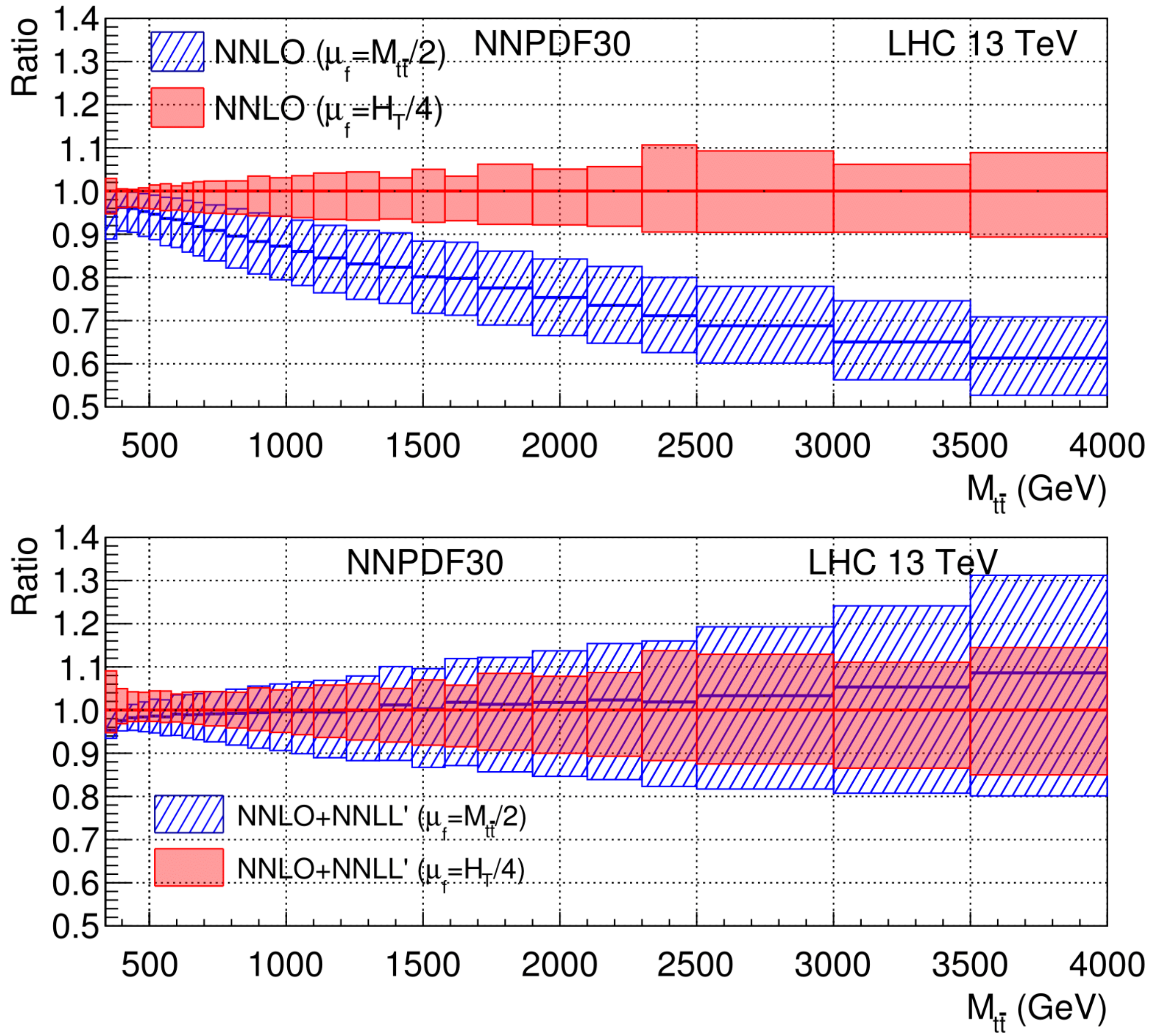


$$M_{t\bar{t}} = \sqrt{(p_t + p_{\bar{t}})^2}$$



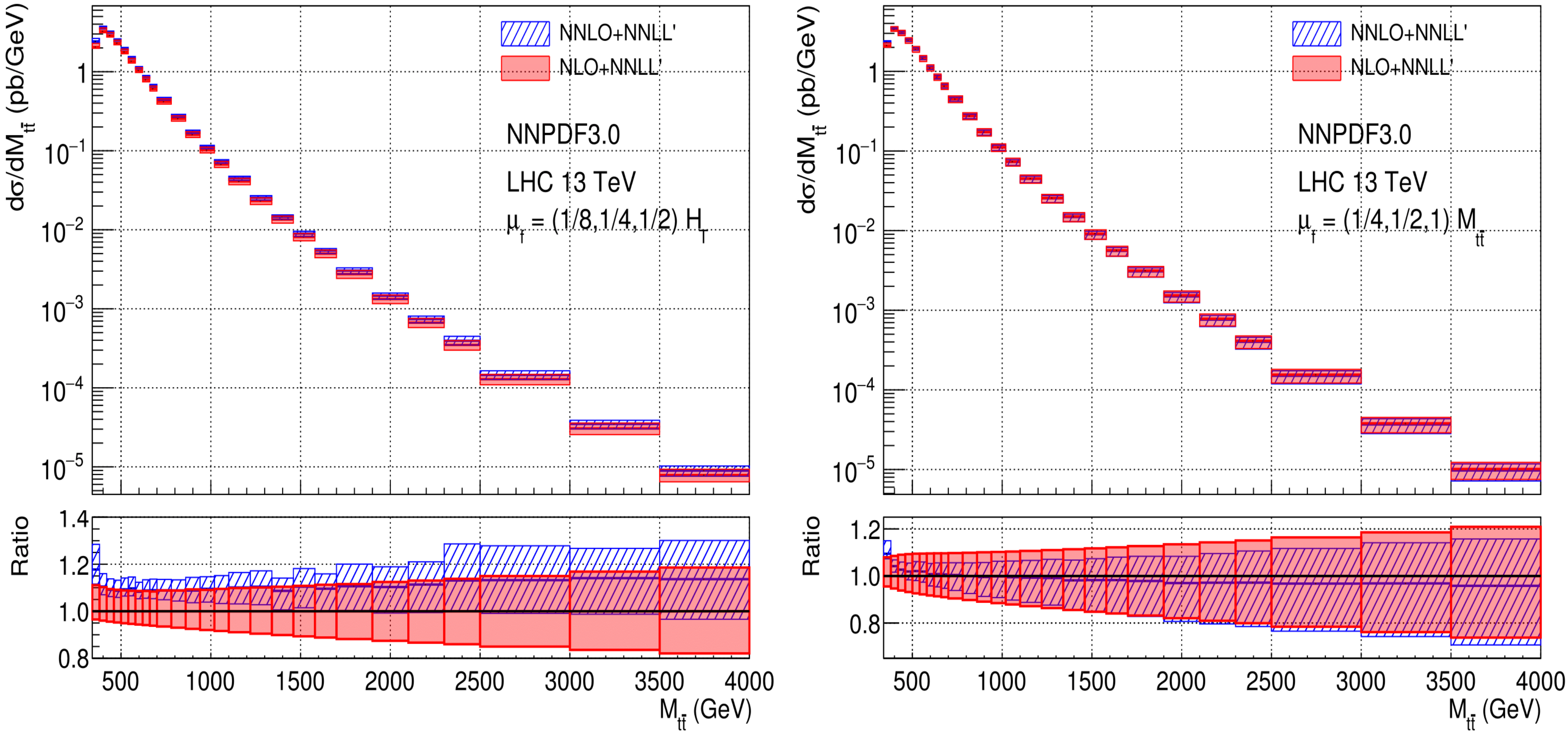
NNLL’ corrections soften the tail in the left plot (H_T/4 scale), but they produce the opposite effect in the right plot (M/2 scale)

tT M distribution convergence



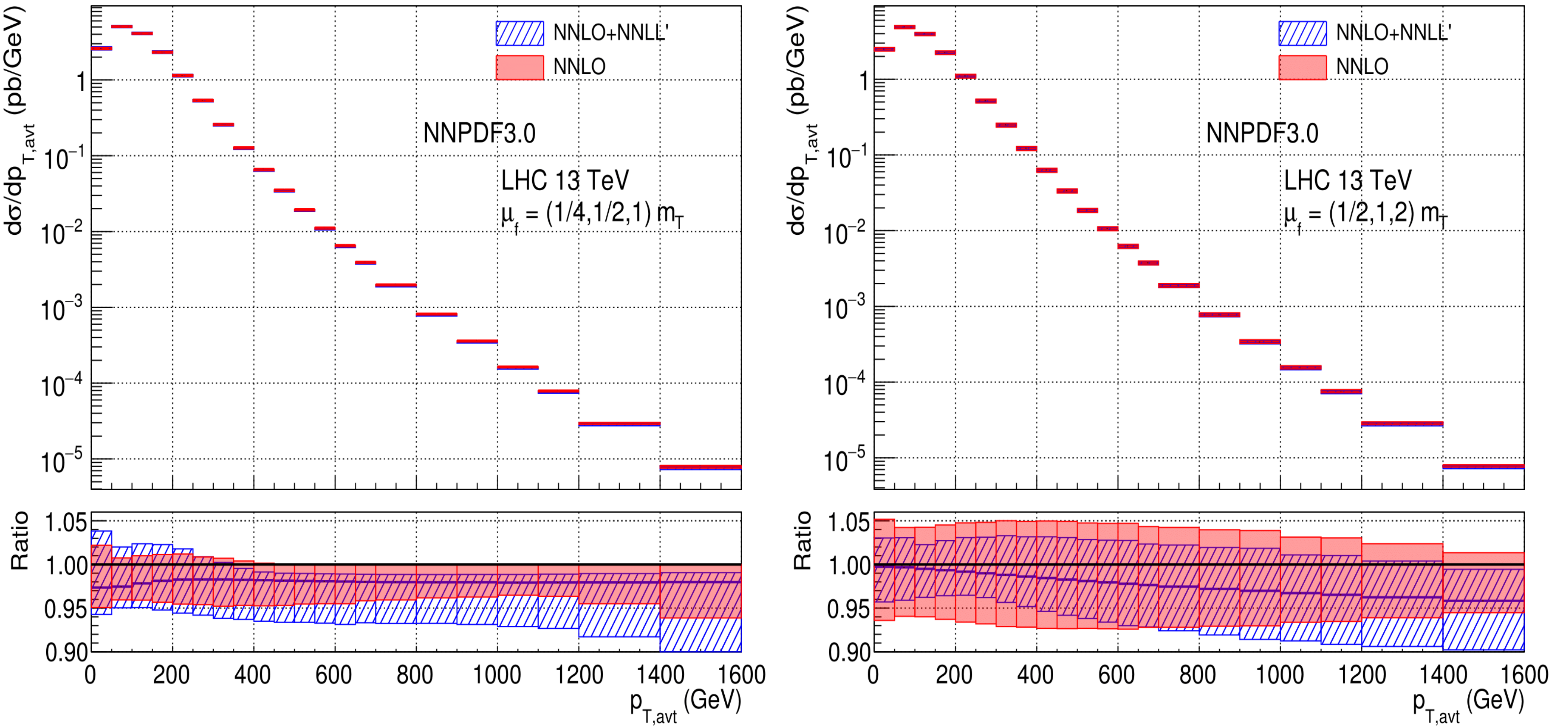
The NNLO results demonstrate a strong dependence on the factorization scale at high M. The NNLO+NNLL' results are more consistent with each other

tT M distribution convergence



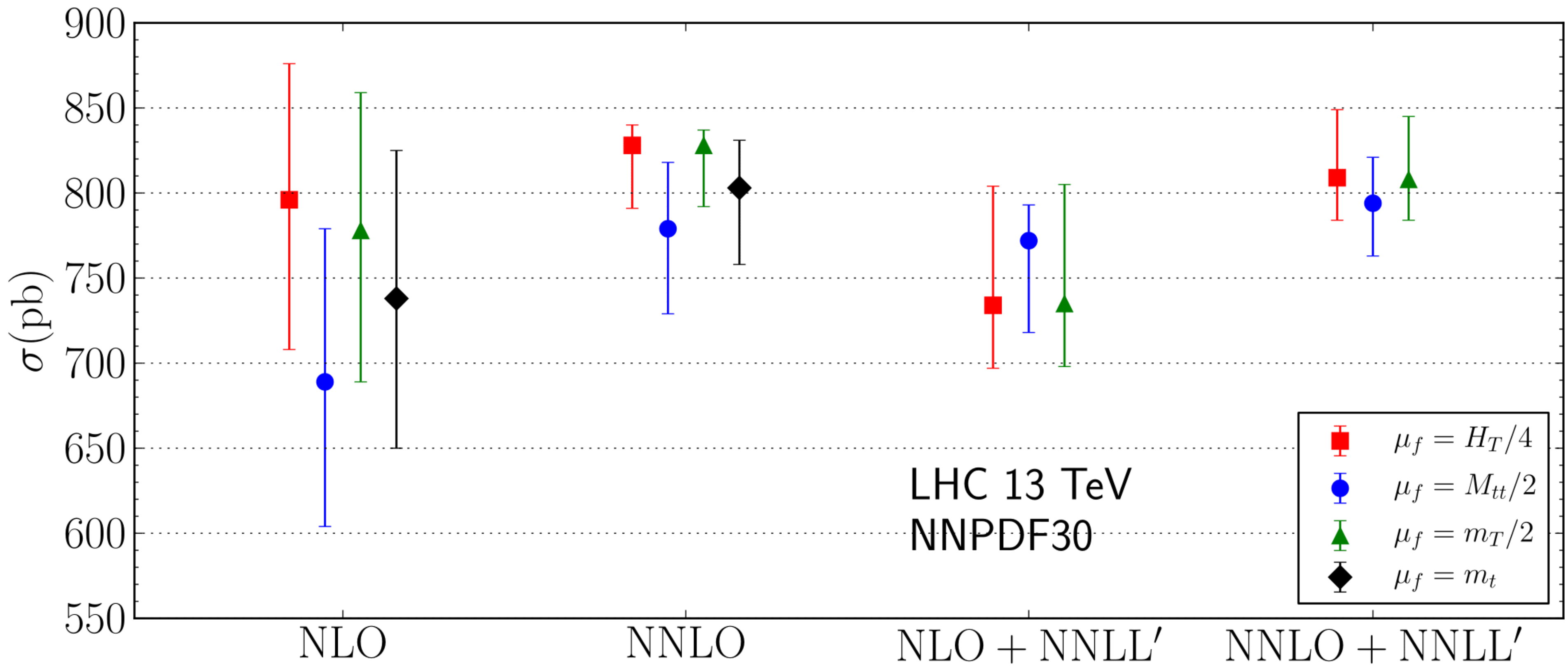
NNLO power corrections are more important at H_T/4 than at M/2 and in both cases they are more important in the lower bins of M

Top quark pT distribution at NNLO+NNLL’



The resummed calculation has less of an effect on the pT distribution that on the M distribution, producing results which lie close to the NNLO calculations

Top pair production total cross section



Results for dynamical scale choices are obtained by integrating the differential cross sections obtained in the previous sections.

The impact of the resummation on the total cross section compared to NNLO is minimal, but NNLO+NNLL' results for the three scale choices are very similar

Conclusions

- We implemented a method to study partonic threshold corrections to top pair (+ H/W/Z) boson production
- In top pair production, we also resum “small” mass logs, important in the tail of the distributions
- NLO+NNLL results in the soft emission limit are available for top pair + H/Z/W production (total cross section + diff. distributions)
- [preliminary] NNLO+NNLL’ (soft+ “boost.”) results are available for the invariant mass and top transverse momentum distributions in top pair production

Backup Slides

How to resum soft logs (very short version)

- The hard and soft functions are free from large logarithms and can be evaluated in fixed order perturbation theory
- The hard and soft functions satisfy RGEs regulated by anomalous dimensions which can also be calculated up to a given order in the strong coupling constant α_s
- By solving the RGEs one can resum large corrections depending on the ratio of hard and soft scales
- In practice, it is more convenient to solve the RGEs in Laplace space or Mellin space, where convolutions become regular products

$$\frac{d\sigma}{dM^2} \propto \int_\tau^1 \frac{dz}{z} ff\left(\frac{\tau}{z}\right) \mathrm{Tr}\left[\mathbf{H}\mathbf{S}(z)\right] \to \frac{d\tilde{\sigma}}{dM^2} \propto \tilde{ff}\,\mathrm{Tr}\left[\mathbf{H}\tilde{\mathbf{s}}\right]$$

$$\left[\tau = M^2/s\,, \quad s = (\text{collider energy})^2\right]$$

Mellin space

- The resummation can also be carried out in Mellin space (by taking the Mellin transform of the factorized cross section), similar to “direct QCD” resummation

$$\tilde{c}(N,\mu) = \int_0^1 dz\, z^{N-1} \int d\mathrm{PS}_{t\bar{t}H} \mathrm{Tr}\left[\mathbf{H}\left(\{p\},\mu\right)\mathbf{S}\left(\sqrt{\hat{s}}(1-z),\{p\},\mu\right)\right]$$

- The total cross section can be then recovered with an inverse Mellin transform

$$\sigma = \frac{1}{2s}\int_{\tau_{\min}}^1 \frac{d\tau}{\tau}\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dN\,\tau^{-N}\widetilde{f\!f}(N,\mu)\int d\mathrm{PS}_{t\bar{t}H}\,\widetilde{c}\,(N,\mu)$$

Resummed kernels in Mellin space

- RG evolution is used to obtain $\widetilde{c}$ at the scale μ_f

$$\widetilde{c}(\mu_f) = \mathrm{Tr}\left[\widetilde{\mathbf{U}}(\mu_f,\mu_h,\mu_s)\,\mathbf{H}(\mu_h)\,\widetilde{\mathbf{U}}^{\dagger}(\mu_f,\mu_h,\mu_s)\widetilde{\mathbf{s}}\,(\mu_s)\right]$$

- By rewriting $\alpha_s(\mu_f)$ and $\alpha_s(\mu_s)$ as a function of $\alpha_s(\mu_h)$

$$\widetilde{\mathbf{U}} = \exp\left\{\frac{4\pi}{\alpha_s(\mu_h)} g_1\left(\lambda,\lambda_f\right) + g_2\left(\lambda,\lambda_f\right) + \frac{\alpha_s(\mu_h)}{4\pi} g_3\left(\lambda,\lambda_f\right) + \cdots\right\}$$

$$\times\, \mathbf{u}(\{p\},\mu_h,\mu_s)$$

$$\lambda = \frac{\alpha_s(\mu_h)}{2\pi}\beta_0 \ln\frac{\mu_h}{\mu_s}\,, \qquad \lambda_f = \frac{\alpha_s(\mu_h)}{2\pi}\beta_0 \ln\frac{\mu_h}{\mu_f}$$

NNLL resummation requires:

1) g_1, g_2, g_3, u 2) Soft function to NLO 3) Hard function to NLO

Matching: technicalities

$$d\sigma^{\mathrm{(N)NLO+NNLL}_m} = d\sigma^{\mathrm{NNLL}_m} + \overbrace{\left(d\sigma^{\mathrm{(N)NLO}} - d\sigma^{\mathrm{NNLL}_m}\Big|_{\substack{\mathrm{(N)NLO} \\ \mathrm{expansion}}} \right)}^{\text{NNLO terms subleading in the soft limit}}$$

$$d\sigma^{\mathrm{(N)NLO+NNLL}'} = d\sigma^{\mathrm{NNLL}'_b} + \overbrace{\left(d\sigma^{\mathrm{NNLL}_m} - d\sigma^{\mathrm{NNLL}_m}\Big|_{m_t \to 0} \right)}^{\text{soft terms subleading in } m_t \to 0}$$
$$+ \underbrace{\left(d\sigma^{\mathrm{(N)NLO}} - d\sigma^{\mathrm{NNLL}'_{\mathrm{b+m}}}\Big|_{\substack{\mathrm{(N)NLO} \\ \mathrm{Expansion}}} \right)}_{\text{NNLO terms subleading in the soft limit}}$$

Scales

- In fixed order calculations, one needs to choose a default value for the factorization scale $\mu_{f,0}$
- In resummed calculations we also have to pick a value for the default hard and soft scales $\mu_{h,0}$, $\mu_{s,0}$
- In order to keep the hard and soft functions free from large logarithms in Mellin space, one chooses

$$\mu_{h,0} = M \qquad \mu_{s,0} = \frac{M}{\bar{N}}$$

- The choice of the default value for the factorization scale is more delicate (see following slides)

Scales in the boosted-soft limit

- In partonic threshold resummation one needs to choose a default value of 3 scales $\mu_{f,0}$, $\mu_{h,0}$, $\mu_{s,0}$

- In the soft boosted limit the hard function and the soft function further factor in the product of two functions. This fact gives rise to two addictional scales $\mu_{dh,0}$, $\mu_{ds,0}$

- In Mellin space, one chooses

$$\mu_{dh,0} = m_t \qquad \mu_{ds,0} = \frac{m_t}{\bar{N}}$$

- In order to determine the residual scale uncertainty, all scales are varied separately and the variations are combined in quadrature

Scale uncertainty

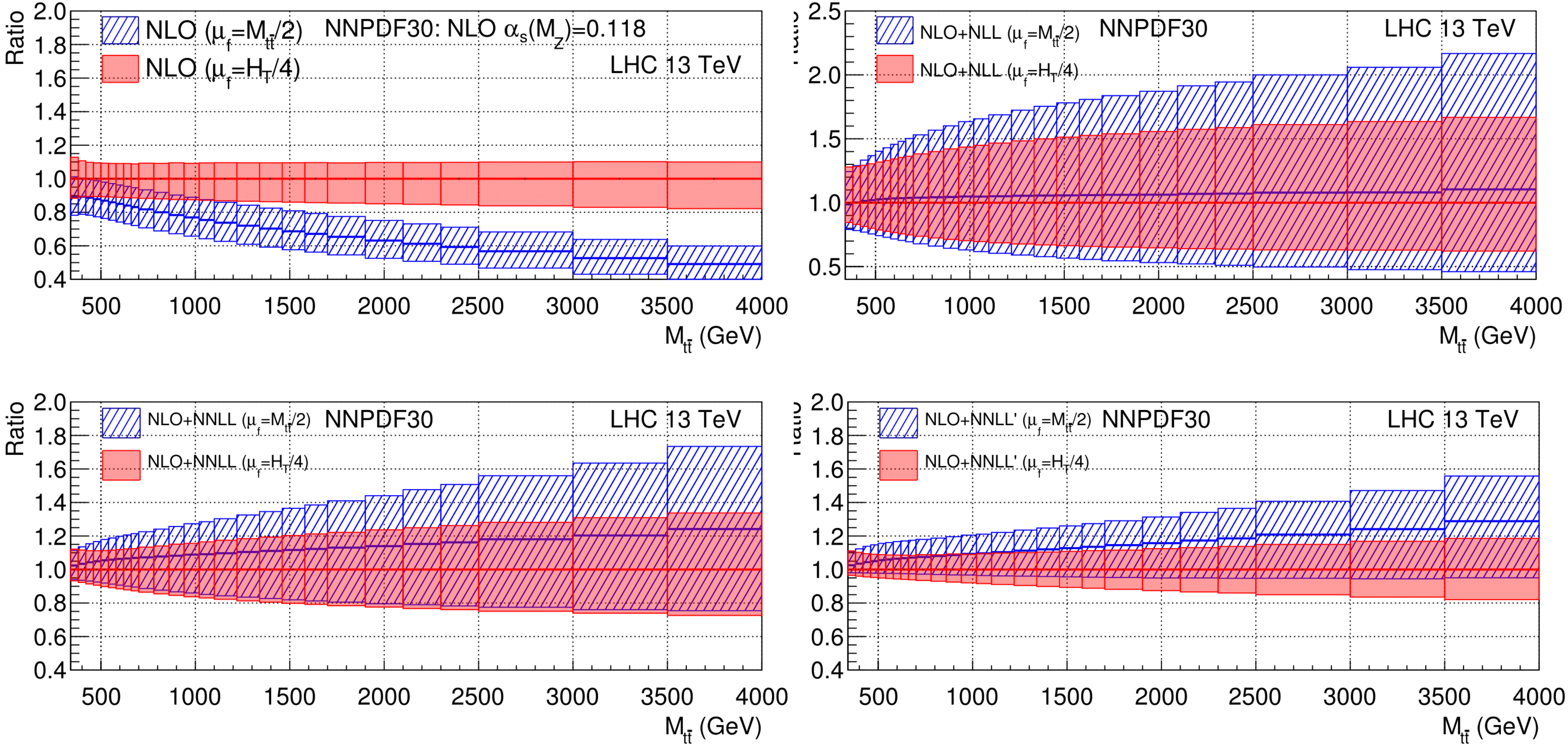
- In fixed order results, the scale uncertainty is evaluated by varying $\mu_f \in [\mu_{f,0}/2, 2\mu_{f,0}]$
- For resummed results, we vary all scales (hard, soft and factorization) independently in the range $\mu_i \in [\mu_{i,0}/2, 2\mu_{i,0}]$
- For an observable O (the total cross section, or the value of a differential cross section in a given bin) one evaluates (for $i = s, f, h \text{ and } k_i = \mu_{i,0}/M$)

$$\Delta O_i^+ = \max\{O(\kappa_i = 1/2), O(\kappa_i = 1), O(\kappa_i = 1)\} - O(\kappa_i = 1)$$

$$\Delta O_i^- = \min\{O(\kappa_i = 1/2), O(\kappa_i = 1), O(\kappa_i = 1)\} - O(\kappa_i = 1)$$

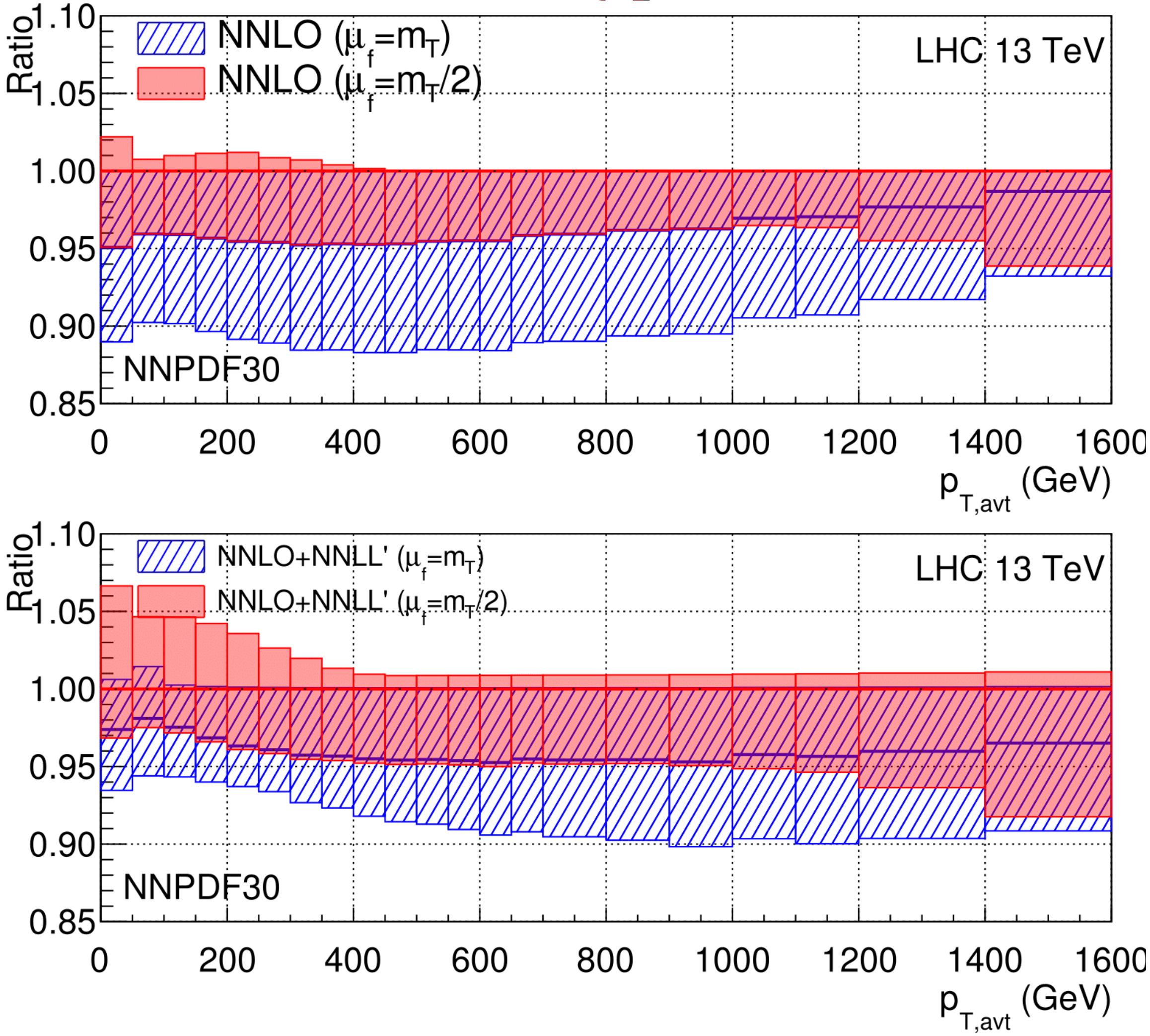
- The quantities $\Delta O_i^+ (\Delta O_i^-)$ are then combined in quadrature in order to obtain the scale uncertainty above (below) the central value

tT M distribution convergence



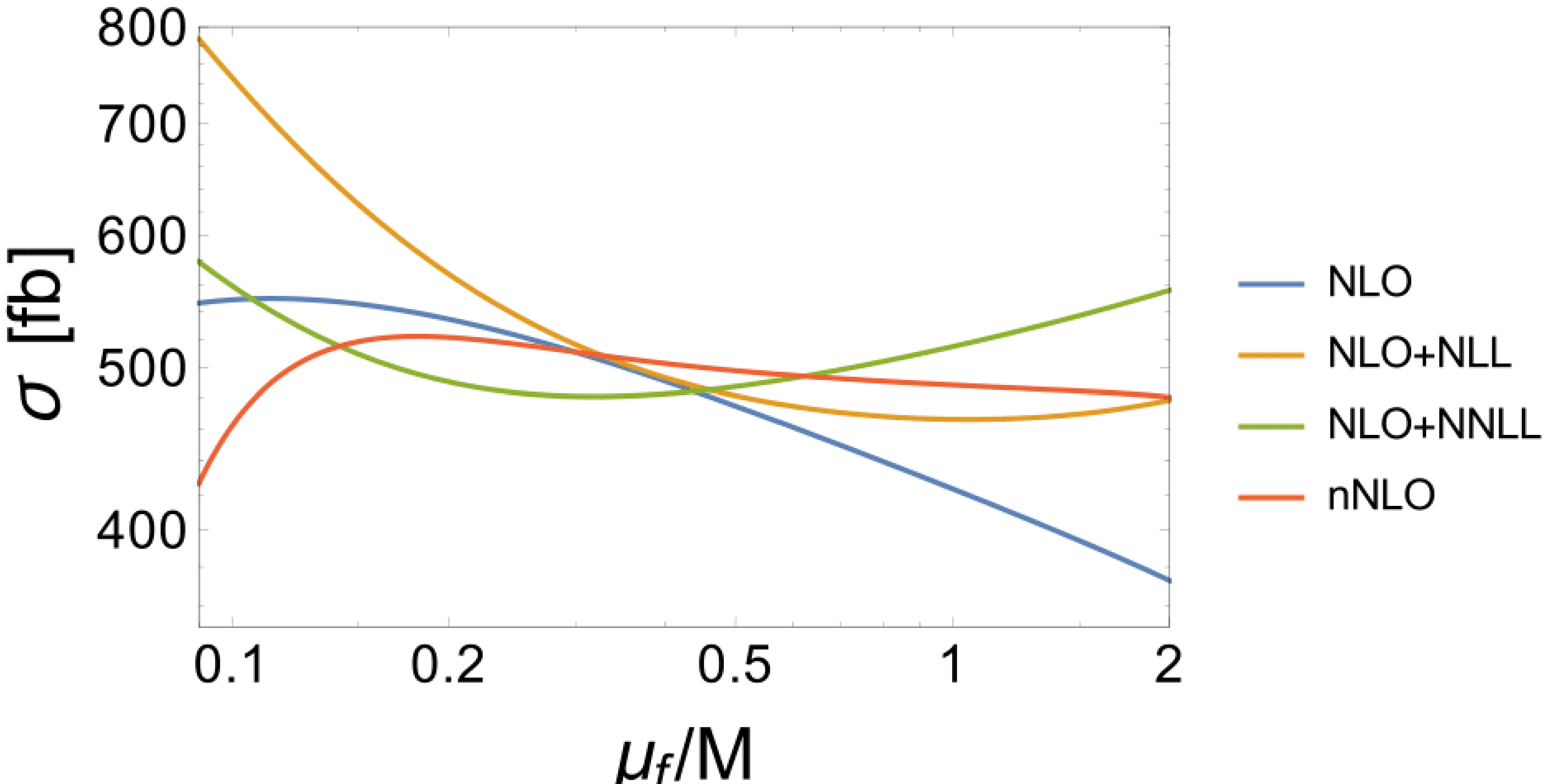
NLO, NLO+NLL, NLO+NNLL and NLO+NNLL' results evaluated for the two different choices of the factorization scale

Top quark pT distribution convergence



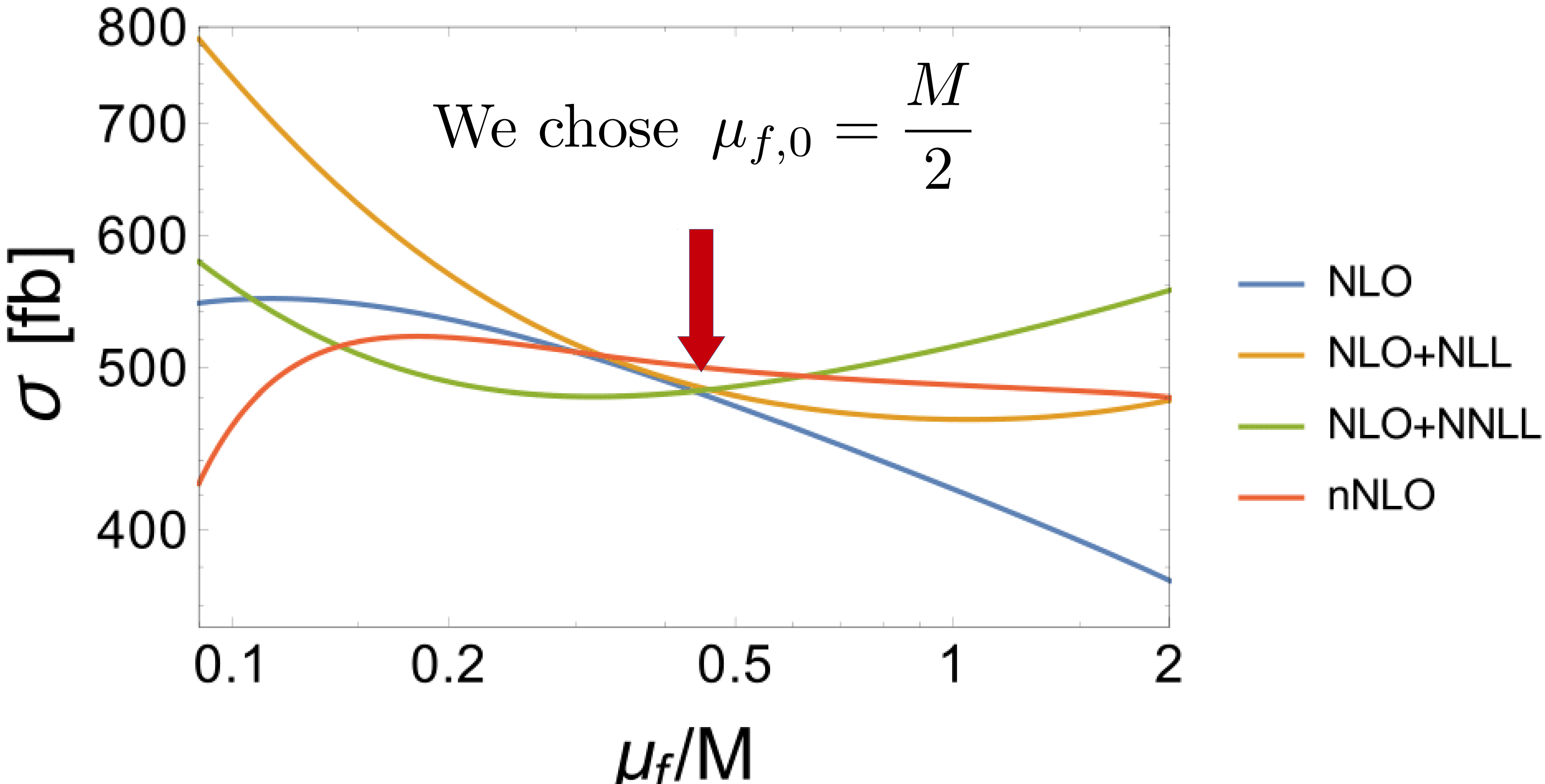
The resummed calculation has less of an effect on the pT distribution that on the M distribution, producing results which lie close to the NNLO calculations

tTH production scale dependence



The factorization scale should be chosen such in such a way that logarithms of the ratio µ_f /M are not large. Since we are working in the partonic threshold limit it is natural to choose a dynamical value for the factorization scale which is correlated with M

tTH production scale dependence



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Total cross section @ 13 TeV

order	PDF order	code	σ [fb]
LO	LO	MG5_aMC	$378.7^{+120.5}_{-85.2}$
app. NLO	NLO	MC	$473.3^{+0.0}_{-28.6}$
NLO no qg	NLO	MG5_aMC	$482.1^{+10.9}_{-35.1}$
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NLO+NLL	NLO	MC +MG5_aMC	$480.1^{+57.7}_{-15.7}$
NLO+NNLL	NNLO	MC +MG5_aMC	$486.4^{+29.9}_{-24.5}$
nNLO (Mellin)	NNLO	MC +MG5_aMC	$497.9^{+18.5}_{-9.4}$
$(\mathrm{NLO+NNLL})_{\mathrm{exp.}}$	NNLO	MC +MG5_aMC	$482.7^{+10.7}_{-21.1}$

$t\bar{t}H$ boson production $\mu_f = M/2$

Total cross section @ 13 TeV

order	PDF order	code	σ [fb]
LO	LO	`MG5_aMC`	$293.5^{+85.2}_{-61.7}$
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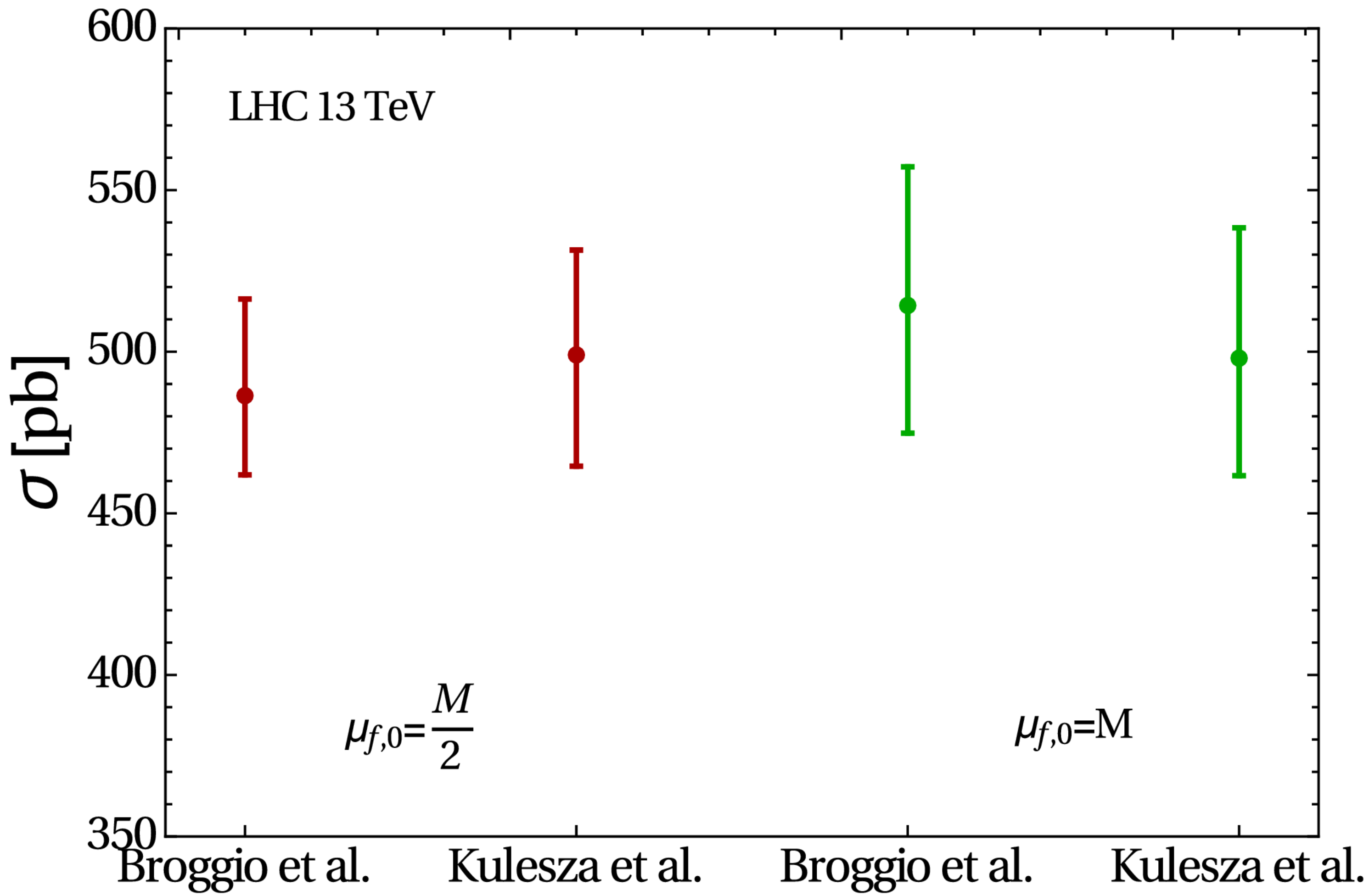
NLO no qg	NLO	`MG5_aMC`	$447.0^{+35.1}_{-40.4}$
NLO	NLO	`MG5_aMC`	$423.0^{+51.9}_{-49.7}$
NLO+NLL	NLO	MC +`MG5_aMC`	$466.2^{+22.9}_{-26.8}$
NLO+NNLL	NNLO	MC +`MG5_aMC`	$514.3^{+42.9}_{-39.5}$
nNLO (Mellin)	NNLO	MC +`MG5_aMC`	$488.4^{+9.4}_{-8.3}$
(NLO+NNLL)$_{\text{exp.}}$	NNLO	MC +`MG5_aMC`	$485.7^{+6.8}_{-15.0}$

$t\bar{t}H$ boson production $\mu_f = M$

Comparison with 1704.03363



Broggio et al MMHT 14 PDFs $m_t = 173.2\ \mathrm{GeV}$
Kulesza et al PDF4LHC15_100 $m_t = 173.0\ \mathrm{GeV}$

Approximate, color averaged hard & soft functions in Kulesza et al.