On Gravitational Corrections to the Electroweak Vacuum Decay

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Mainly based on
- Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia (JHEP) arXiv:1307.3536
- Giudice, Isidori, Salvio, Strumia (JHEP) arXiv:1412.2769
- Salvio, Strumia, Tetradis, Urbano (JHEP) arXiv:1608.02555
Testing the Standard Model (SM) at ultrahigh energies

Two important goals of the LHC:
- testing the SM
- discovering new physics (NP)

But one can also complementary test the SM at even higher energies: e.g. the stability bound
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Precise “running” of $\lambda$ and its $\beta$-function

\[ \beta_\lambda \equiv \frac{d\lambda}{d \ln \mu} \]

[ Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia (2014) ]
Result for the stability of the electroweak (EW) vacuum

Phase diagram of the SM:

[Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia (2014)]

The stability of the electroweak vacuum is violated at the $\sim 3\sigma$ level

We see the main uncertainty is due to the top mass, $M_t$
Result for the stability bound

\[ M_h > 129.6 \text{ GeV} + 2.0(M_t - 173.34 \text{ GeV}) - 0.5 \text{ GeV} \frac{\alpha_3(M_Z) - 0.1184}{0.0007} \pm 0.3_{\text{th}} \text{ GeV} \]

The stability bound is violated at the \( \sim 3\sigma \) level

Since the experimental error on the Higgs mass is small it is better to express the bound in terms of the pole top mass:

\[ M_t < (171.53 \pm 0.15 \pm 0.23\alpha_3 \pm 0.15 M_h) \text{ GeV} = (171.53 \pm 0.42) \text{ GeV}. \]

[Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia (2014)]
Is the meta-stability worrisome?
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Flat space analysis:

Left: Probability of EW vacuum decay.

Right: The life-time of the EW vacuum, with 2 different assumptions for future cosmology: universes dominated by the cosmological constant ($\Lambda$CDM) or by the dark matter (CDM).

[Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia (2014)]
Details on the calculation of the probability of vacuum decay

The probability $d\mathcal{P}/dV \, dt$ per unit time and volume of creating a bubble of true vacuum within a space volume $dV$ and time interval $dt$ is

$$d\mathcal{P} = dt \, dV \, \Lambda_B^4 \, e^{-S_B}$$

$S_B$ is the action of the bounce of size $R \equiv \Lambda_B^{-1}$: the bounce $h$ is an SO(4) symmetric Euclidean solution

$$h'' + \frac{3}{r} h' = \frac{dV}{dh}, \quad \text{with boundary conditions} \quad h'(0) = 0, \quad h(\infty) = h_{\text{EW}}$$

[Coleman (1977); Coleman, Callan (1977)]
Gravitational corrections

We are looking at extremely high energies, sometimes reaching the Planck mass, $\bar{M}_{\text{Pl}}$. Does gravity play a role?

One can address this question in Einstein gravity (compatible with all experiments)

$$\mathcal{L} = \mathcal{L}_{\text{Einstein}} + \mathcal{L}_{\text{SM}} - \xi |H|^2 R$$

However, Einstein gravity is an effective theory with a cutoff $\sim \bar{M}_{\text{Pl}}$

→ consider an expansion in $E/\bar{M}_{\text{Pl}}$ (it is not restrictive: for $E > \bar{M}_{\text{Pl}}$ the theory breaks down)
Some details on the inclusion of gravity

(First down in [Coleman, de Luccia (1980)])

The bounce equation becomes a Higgs-gravity system of equations

\[ h'' + 3 \frac{\rho'}{\rho} h' = \frac{dV}{dh} - \xi h \mathcal{R}, \quad \rho'^2 = 1 + \frac{\rho^2/\bar{M}_{Pl}^2}{3(1 + \xi h^2/\bar{M}_{Pl}^2)} \left( \frac{h'^2}{2} - V - 6 \frac{\rho'}{\rho} \xi hh' \right) \]

where \( \mathcal{R} \) is the Ricci scalar for the metric

\[ ds^2 = dr^2 + \rho(r)^2 d\Omega^2 \]

\( d\Omega \) is the volume element of the unit 3-sphere

[Salvio, Strumia, Tetradis, Urbano (2016)]
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\[ h'' + 3 \frac{\rho'}{\rho} h' = \frac{dV}{dh} - \xi h \mathcal{R}, \quad \rho'^2 = 1 + \frac{\rho^2 / \tilde{M}_{P1}^2}{3(1 + \xi h^2 / \tilde{M}_{P1}^2)} \left( \frac{h'^2}{2} - V - 6 \frac{\rho'}{\rho} \xi h h' \right) \]

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which is adequate to describe the gravitational corrections within Einstein gravity.

[Salvio, Strumia, Tetradis, Urbano (2016)]
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\[ h'' + 3 \frac{\rho'}{\rho} h' = \frac{dV}{dh} - \xi h \mathcal{R}, \quad \rho'^2 = 1 + \frac{\rho^2 / \bar{M}^2_{P_1}}{3(1 + \xi h^2 / \bar{M}^2_{P_1})} \left( \frac{h'^2}{2} - V - 6 \frac{\rho'}{\rho} \xi h h' \right) \]

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We developed a perturbation theory in \( 1/R\bar{M}_{P_1} \) (weak gravity expansion) which is adequate to describe the gravitational corrections within Einstein gravity.

[Salvio, Strumia, Tetradis, Urbano (2016)]
Impact of Einstein gravity on the phase diagram of the SM

This updates the plot in [Salvio, Strumia, Tetradis, Urbano (2016)] by using more recent measurements of $M_t$. 

![Phase Diagram](image-url)
Recall naturalness:

\[ \delta M_h^2 \sim \frac{g^2 M_{NP}^2}{(4\pi)^2} \]
Unnaturalness of the SM + Einstein gravity

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A problem:

gravity introduces a large scale \( M_{Pl} \gg \text{TeV} \)
Naturalness and gravity

We do not know if $G_N$ simply describes a coupling constant or signals the presence of new degrees of freedom with mass $\tilde{M}_{P1} = 1/\sqrt{8\pi G_N}$

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But (Einstein) gravitational interactions increase as energy increases.

Idea (softened gravity): consider theories where the power-law increase of the gravitational coupling stops at $\Lambda_G \ll \bar{M}_{P1}$.

The gravitational contribution to the Higgs mass is then

$$\delta M_h^2 \approx \frac{G_N \Lambda_G^4}{(4\pi)^2}$$

Requiring naturalness $\rightarrow \Lambda_G \lesssim 10^{11}$ GeV

[Giudice, Isidori, Salvio, Strumia (2014)]
Softened gravity and the stability of the EW vacuum

Given that gravity becomes soft at high energies it negligibly affect the stability issue

(checked in a concrete realization of softened gravity)

However, other UV completion of Einstein gravity, such as string theory can affect these results

But, Planck-scale physics cannot suppress sub-Planckian contributions to SM vacuum decay, which can only be affected by new physics at lower energies.
Conclusions

- In the pure SM the vacuum stability is excluded at roughly $3\sigma$ level

- These calculations involve the extrapolation of the SM potential up to Planckian energies so one may wonder if gravity changes the result

- We included Einstein gravity within its regime of validity and found that the corrections are small, even including $\xi$

- We assumed a desert between the EW and the Planck scale. What about naturalness? We discussed that a modification of gravity which softened the strength of gravity at high energy leads to negligible modifications
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Disclaimer: New physics below $\bar{M}_{\text{Pl}}$ may or may not change completely the results. So these calculations are useful as tests of the SM hypothesis and are possible means to find further evidences for new physics.
Thank you very much for your attention