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Power Corrections for $\bar{B} \to X_u\, l\, \bar{\nu}$ and $\bar{B} \to X_s \gamma$

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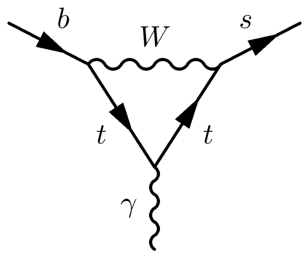
S.J. Lee, M. Neubert, GP	PRD **75** 114005 (2007)
M. Benzke, S.J. Lee, M. Neubert, GP	*in preparation*

Introduction

Reminder

- Underlying process of $\bar{B} \to X_u\, l\, \bar{\nu}$: $b \to u$ arises at tree level in the SM
- Underlying process of $\bar{B} \to X_s \gamma$: $b \to s\gamma$ arises as a loop effect in the SM



- Gives rise to the operator:

$$Q_{7\gamma} = \frac{-e}{8\pi^2} m_b \bar{s} \sigma_{\mu\nu} F^{\mu\nu} (1+\gamma_5) b$$

- Part of the effective Hamiltonian

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left(C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3,\ldots,10} C_i Q_i + C_{7\gamma} Q_{7\gamma} + C_{8g} Q_{8g} \right) + \text{h.c.}$$

- For now consider only $Q_{7\gamma}$

Factorization at Leading Power

- Experimental cuts force charmless inclusive B decay spectra

  to the end point region:

  - $\bar{B} \to X_u \, l \, \bar{\nu}$: $P_X^2 < M_D^2$
  - $\bar{B} \to X_s \gamma$: $E_\gamma > E_{\rm cut}$

  where $P_X^2 \sim m_b \, \Lambda_{\rm QCD}$

- At the end point region, spectra of

  - $\bar{B} \to X_u \, l \, \bar{\nu}$
  - $Q_{7\gamma} - Q_{7\gamma}$ contribution to $\bar{B} \to X_s \gamma$

  obey a **leading power** factorization formula

  (Korchemsky, Sterman '94; Bauer, Pirjol, Stewart '01)

$$\bar{B} \to X_s \gamma : \qquad d\Gamma_s^{77} \sim H_s \cdot J \otimes S + \ldots$$

$$\bar{B} \to X_u \, l \, \bar{\nu} : \qquad d\Gamma_u \sim H_u \cdot J \otimes S + \ldots$$

- S (leading) shape function, non-perturbative

- H_i and J calculable using PT in α_s

Factorization at Leading Power

- Progress in the perturbative calculation:

$$\bar{B} \to X_s \gamma : \qquad d\Gamma_s^{77} \sim H_s \cdot J \otimes S + \ldots$$

$$\bar{B} \to X_u \, l \, \bar{\nu} : \qquad d\Gamma_u \sim H_u \cdot J \otimes S + \ldots$$

- 2005: H_s calculated at $\mathcal{O}(\alpha_s^2)$

  (Melnikov, Mitov '05, ...)

- 2006: J calculated at $\mathcal{O}(\alpha_s^2)$

  (Becher, Neubert '06)

- 2008: H_u calculated at $\mathcal{O}(\alpha_s^2)$

  (Bonciani, Ferroglia '08; Asatrian, Greub, Pecjak '08; Beneke, Huber, Li '08; Bell '08)

  See also Ben Pecjak's talk

- Current status: leading power $H \cdot J \otimes S$ at $\mathcal{O}(\alpha_s^2)$

Subleading Shape Functions

- Study of power corrections simplified by using SCET
- Using an effective field theory is
  - Systematic: not missing any terms
  - Improvable: can be carried to higher orders
- 2004: Using SCET, study of **one** type of power corrections

  subleading shape functions (subleading "twist")

  for $X_u\, l\, \bar{\nu}$ and $Q_{7\gamma} - Q_{7\gamma}$ contribution to $\bar{B} \to X_s \gamma$

  $$d\Gamma \sim H \cdot J \otimes S + \frac{1}{m_b} \sum_i H \cdot J \otimes s_i + \ldots$$

  (K.S.M. Lee, Stewart '04; Bosch, Neubert, GP '04; Beneke, Campanario, Mannel, Pecjak '04)

  See GP talk at Vub/SF Workshop SLAC 2004

  Supersedes earlier studies
- The subleading shape function s_i are non perturbative

  appear in the factorization formula at tree level

Beyond Leading Power

- Status at the end of 2008

$$\bar{B} \to X_s \gamma: \qquad d\Gamma_s^{77} \sim H_s \cdot J \otimes S + \frac{1}{m_b} \sum_i H \cdot J \otimes s_i + \ldots$$

$$\bar{B} \to X_u \, l \, \bar{\nu}: \qquad d\Gamma_u \sim H_u \cdot J \otimes S + \frac{1}{m_b} \sum_i H \cdot J \otimes s_i + \ldots$$

Leading power $H \cdot J \otimes S$ at $\mathcal{O}(\alpha_s^2)$

- How to improve?
  - Consider α_s/m_b suppressed terms

    Part II: Subleading Jet Function for $\bar{B} \to X_u \, l \, \bar{\nu}$ and $\bar{B} \to X_s \gamma$
  - Complete SSF analysis for $\bar{B} \to X_s \gamma$

    Part III: Factorization at Subleading Power

    and Irreducible Uncertainties in $\bar{B} \to X_s \gamma$

Subleading Jet Functions in Inclusive B Decays

GP; JHEP **0906** 083 (2009)

What are the power corrections at $1/m_b$?

- At leading power

$$d\Gamma \sim H \cdot J \otimes S + \ldots$$

- Beyond leading power have:

$H \cdot j_i \otimes S$	$H \cdot J \otimes s_i$
Subleading jet functions	Subleading shape functions

- Hard function **always** $\mathcal{O}(1)$ quantities (depend on m_b)

$\Rightarrow$ Only subleading jet and shape functions

- Confirmed at $\mathcal{O}(\alpha_s)$ by analysis of momentum regions
- Subleading shape functions

  $H \cdot J \otimes s_i$ with H, J at $\mathcal{O}(\alpha_s^0)$

  known since 2004
- What about subleading jet functions (SJF)?

How can we calculate the SJF?

- Partial rate calculated via

$$d\Gamma \sim \text{Disc} \int d^4x\, e^{iqx} \,\langle \bar{B}|T\{J^\dagger(0),\, J(x)\}|\bar{B}\rangle$$

- Match QCD current $J_{\rm QCD}$ onto SCET
- Enough to match at tree level, but to second order in $\sqrt{\Lambda_{\rm QCD}/m_b}$

  Conveniently already done by

  Beneke, Chapovsky, Diehl, Feldmann '02
- Power suppression in currents arises from
  - Collinear fields $\mathcal{A}_\perp,\ i\mathcal{D}_\perp,\, n\cdot\mathcal{A}$
  - Soft fields and their covariant derivatives $S^\dagger D_\mu h$
  - or both

How can we calculate the SJF?

- $H \cdot J \otimes s_i$ were calculated using soft suppressed currents
- $H \cdot j_i \otimes S$ are calculated using collinear suppressed currents
- "All" we need to do is to combine the various currents

$$
\begin{aligned}
J^{(0)} &= \bar{\mathfrak{X}}\,\Gamma\left(S^\dagger h\right)_{x_-}, \\
J^{(1)} &= -\,\bar{\mathfrak{X}}\,\frac{\not{\bar n}}{2}\,\not{\mathcal{A}}_{\perp hc}\,\frac{1}{i\bar n\cdot\overleftarrow{\partial}}\,\Gamma\left(S^\dagger h\right)_{x_-} - \bar{\mathfrak{X}}\,\Gamma\,\frac{\not{n}}{2m_b}\,\not{\mathcal{A}}_{\perp hc}\left(S^\dagger h\right)_{x_-}, \\
J^{(2)} &= -\,\bar{\mathfrak{X}}\,\Gamma\frac{\not{n}}{2m_b}\,n\cdot\mathcal{A}_{hc}\left(S^\dagger h\right)_{x_-} - \bar{\mathfrak{X}}\,\Gamma\,\frac{1}{i\bar n\cdot\partial}\,n\cdot\mathcal{A}_{hc}\left(S^\dagger h\right)_{x_-} \\
&\quad -\,\bar{\mathfrak{X}}\,\Gamma\,\frac{1}{i\bar n\cdot\partial}\,\frac{(i\not{\mathcal{D}}_{\perp hc}\not{\mathcal{A}}_{\perp hc})}{m_b}\left(S^\dagger h\right)_{x_-} + \bar{\mathfrak{X}}\,\frac{i\overleftarrow{\not{\mathcal{D}}}_{\perp hc}}{m_b}\,\frac{1}{i\bar n\cdot\overleftarrow{\partial}}\,\frac{\not{\bar n}}{2}\,\Gamma\,\frac{\not{n}}{2}\,\not{\mathcal{A}}_{\perp hc}\left(S^\dagger h\right)_{x_-}
\end{aligned}
$$

and the collinear Lagrangian insertion

$$
\mathcal{L} = \bar\xi^{(0)}\,\frac{\not{\bar n}}{2}\left(in\cdot D_{hc}^{(0)} + i\not{D}_{\perp hc}^{(0)}\,W\,\frac{1}{i\bar n\cdot\partial}\,W^\dagger\,i\not{D}_{\perp hc}^{(0)}\right)\xi^{(0)}
$$

in

$$
d\Gamma \sim \text{Disc}\int d^4x\,e^{iqx}\,\langle\bar B|T\{J^\dagger(0),\,J(x)\}|\bar B\rangle
$$

What are the Subleading Jet Functions?

- Leading jet function is the disc. of TOP of two quarks

$$\frac{\not{n}}{2}\delta_{kl}J(p^2) = \frac{1}{\pi}\,\mathrm{Disc}\, i\int d^4x\, e^{-ip\cdot x}\langle\Omega|\,T\left\{\mathcal{X}_k(0),\,\bar{\mathcal{X}}_l(x)\right\}|\Omega\rangle$$

- SJF are the disc. of TOP of two quarks and one or two gluons, e.g.

$$\frac{\not{n}}{2}\delta_{kl}\,j_n(p^2) = \frac{1}{\pi}\,\mathrm{Disc}\, i\int d^4x\, e^{-ip\cdot x}\langle\Omega|\,T\left\{\mathcal{X}_k(0),\,\left[\bar{\mathcal{X}}\not{n}\cdot\mathcal{A}_{hc}\right]_l(x)\right\}|\Omega\rangle$$

- Explicit definitions can be found in GP JHEP **0906** 083 (2009)
- In general there are 8 subleading jet functions,

  but only 6 contribute to semileptonic and radiative B decays
- Subleading jet functions are perturbative, what do they look like?
- For end point region $p^2 \sim m_b\Lambda_{\mathrm{QCD}}$

  Leading jet function $\quad J(p^2) = \delta(p^2) + \mathcal{O}(\alpha_s) \quad \Rightarrow J \sim \dfrac{1}{\Lambda_{\mathrm{QCD}}}$
- Subleading jet functions $j_i \sim 1$

  subleading jet function $\quad j_i(p^2) = \alpha_s\left[\mathrm{const.} + \ln\left(\dfrac{p^2}{\mu^2}\right)\right] + \mathcal{O}(\alpha_s^2)$

What are the 1-Loop Expressions of the SJF?

$$
\begin{aligned}
j_{11}^S(p^2,\mu) &= \theta(p^2)\frac{C_F\alpha_s(\mu)}{4\pi}\big(-1\big)+\mathcal{O}(\alpha_s^2)\\
j_{11}^A(p^2,\mu) &= 0+\mathcal{O}(\alpha_s^2)\\
j_n(p^2,\mu) &= \theta(p^2)\frac{C_F\alpha_s(\mu)}{4\pi}\left(5+4\ln\frac{\mu^2}{p^2}\right)+\mathcal{O}(\alpha_s^2)\\
j_{n'}(p^2,\mu) &= \theta(p^2)\frac{C_F\alpha_s(\mu)}{4\pi}\left(-6+6\ln\frac{\mu^2}{p^2}\right)+\mathcal{O}(\alpha_s^2)\\
j_K(p^2,\mu) &= \theta(p^2)\frac{C_F\alpha_s(\mu)}{4\pi}\left(-\frac{5}{2}-2\ln\frac{\mu^2}{p^2}\right)+\mathcal{O}(\alpha_s^2)\\
j_G(p^2,\mu) &= \theta(p^2)\frac{C_F\alpha_s(\mu)}{4\pi}\big(-1\big)+\mathcal{O}(\alpha_s^2)\\
j_S(p^2,\mu) &= \theta(p^2)\frac{C_F\alpha_s(\mu)}{4\pi}\left(-\frac{3}{2}\right)+\mathcal{O}(\alpha_s^2)\\
j_A(p^2,\mu) &= \theta(p^2)\frac{C_F\alpha_s(\mu)}{4\pi}\left(\frac{1}{2}\right)+\mathcal{O}(\alpha_s^2).
\end{aligned}
$$

What is the SJF Contribution to the Rate?

- $Q_{7\gamma} - Q_{7\gamma}$ contribution to $\bar{B} \to X_s\,\gamma$

$$\frac{d\Gamma}{dE_\gamma} = -\frac{G_F^2\alpha}{4\pi^4} E_\gamma^3 |V_{tb}V_{ts}^*|^2\, \overline{m}_b^2\, |C_{7\gamma}^{\rm eff}|^2\, W(P_+)$$

$$\begin{aligned} W^{\,\rm SJF} = \int d\omega \quad \Bigg[& \quad \frac{1}{m_b}\Big(4 j_K(p_\omega^2,\mu) + 4 j_n(p_\omega^2,\mu) + 2 j_G(p_\omega^2,\mu)\Big) \\ + & \quad \frac{4}{\bar{n}\cdot p}\, j_{n'}(p_\omega^2,\mu) \\ + & \quad \frac{2\bar{n}\cdot p}{m_b^2}\Big(j_{11}^S(p_\omega^2,\mu) - j_{11}^A(p_\omega^2,\mu)\Big)\Bigg]\, S(\omega) \end{aligned}$$

- At the lowest order in α_s

$$W^{\,\rm SJF} = \int d\omega\, \frac{1}{m_b}\,\frac{C_F\alpha_s(\mu)}{4\pi}\,\theta(p_\omega^2)\left[32\,\ln\frac{\mu^2}{p_\omega^2} - 18\right] S(\omega) + \mathcal{O}(\alpha_s^2)$$

where

$$p_\omega^2 = \bar{n}\cdot p\,(n\cdot p + \omega)$$

What is the SJF Contribution to the Rate?

- $\bar{B} \to X_u\, l\, \bar{\nu}$

$$\frac{d^3\Gamma}{dP_+\, dP_-\, dP_l} = \frac{G_F^2 |V_{ub}|^2}{16\pi^3} (M_B - P_+) \Big[(P_- - P_l)(M_B - P_- + P_l - P_+)\, \tilde{W}_1$$

$$+ \quad (M_B - P_-)(P_- - P_+) \frac{\tilde{W}_2}{2} + (P_- - P_l)(P_l - P_+) \tilde{W}_{\rm comb} \Big]$$

$$\begin{aligned}
\tilde{W}_1^{\rm SJF} &= -\int d\omega \bigg[\frac{1}{\bar{n}\cdot p} \Big(2 j_{n'}(p_\omega^2,\mu) + j_{11}^S(p_\omega^2,\mu) + j_{11}^A(p_\omega^2,\mu) \Big) \\
&\qquad + \frac{1}{m_b} \Big(2 j_K(p_\omega^2,\mu) + j_G(p_\omega^2,\mu) \Big) \bigg] S(\omega) \\
\tilde{W}_2^{\rm SJF} &= -\int d\omega\, \frac{2}{\bar{n}\cdot p} \Big[j_{11}^S(p_\omega^2,\mu) + j_{11}^A(p_\omega^2,\mu) \Big] S(\omega) \\
\tilde{W}_{\rm comb}^{\rm SJF} &= -\int d\omega \bigg[\Big(\frac{4}{m_b} - \frac{2}{\bar{n}\cdot p} \Big) \Big(j_{11}^S(p_\omega^2,\mu) + j_{11}^A(p_\omega^2,\mu) \Big) \\
&\qquad + \frac{2}{\bar{n}\cdot p} \Big(j_n(p_\omega^2,\mu) - j_G(p_\omega^2,\mu) \Big) \bigg] S(\omega)
\end{aligned}$$

What is the SJF Contribution to the Rate?

- At the lowest order in α_s

$$\tilde{W}_1^{\rm SJF} = -\int d\omega \frac{C_F\alpha_s(\mu)}{4\pi}\,\theta(p_\omega^2)\left[\frac{1}{\bar{n}\cdot p}\left(12\ln\frac{\mu^2}{p_\omega^2}-11\right)-\frac{1}{m_b}\left(4\ln\frac{\mu^2}{p_\omega^2}+6\right)\right]S(\omega)$$

$$\tilde{W}_2^{\rm SJF} = \int d\omega \frac{C_F\alpha_s(\mu)}{4\pi}\,\theta(p_\omega^2)\,\frac{2}{\bar{n}\cdot p}\,S(\omega)$$

$$\tilde{W}_{\rm comb}^{\rm SJF} = -\int d\omega \frac{C_F\alpha_s(\mu)}{4\pi}\,\theta(p_\omega^2)\left[\frac{1}{\bar{n}\cdot p}\left(8\ln\frac{\mu^2}{p_\omega^2}+14\right)-\frac{4}{m_b}\right]S(\omega)$$

- If using "BLNP" approach where

$$y=\frac{P_- - P_+}{M_B - P_+}$$

Some of the subleading terms are absorbed into the leading order formula.

- Only $\tilde{W}_1$ changes

$$\tilde{W}_{1,\,\rm BLNP}^{\rm SJF} = -\int d\omega \frac{C_F\alpha_s(\mu)}{4\pi}\,\theta(p_\omega^2)\left[\frac{1}{\bar{n}\cdot p}\left(12\ln\frac{\mu^2}{p_\omega^2}-15\right)-\frac{1}{m_b}\left(4\ln\frac{\mu^2}{p_\omega^2}+2\right)\right]S(\omega)$$

How is the new analysis related to the old one?

- Demonstrate using $Q_{7\gamma} - Q_{7\gamma}$ contribution to $\bar{B} \to X_s\, \gamma$

  Similar results for $\bar{B} \to X_u\, l\, \bar{\nu}$

- Two terms at subleading power:
  - "Hadronic"

$$W^{\,\rm SSF} = \frac{2}{m_b} \int d\omega\, \delta(n\cdot p + \omega) \Big[\omega\, S(\omega) - s(\omega) + t(\omega) - u(\omega) + v(\omega)$$

$$+ \quad \pi\alpha_s\, f_u(\omega) + \pi\alpha_s\, f_v(\omega)\Big] + \mathcal{O}(\alpha_s)$$

  - "Kinematic"

$$W^{\rm Kin.} = \frac{1}{m_b}\frac{C_F\,\alpha_s(\mu)}{4\pi} \int d\omega\, \theta(\omega{+}n{\cdot}p) \left[32\, \ln \frac{\omega + n\cdot p}{m_b} + 30\right] S(\omega){+}\mathcal{O}(\alpha_s^2)$$

- Subleading shape function contribution

$$W^{\,\rm SJF} = \frac{1}{m_b}\frac{C_F\alpha_s(\mu)}{4\pi} \int d\omega\;\, \theta(\omega{+}n{\cdot}p) \left[32\, \ln \frac{\mu^2}{m_b(\omega + n\cdot p)} - 18\right] S(\omega){+}\mathcal{O}(\alpha_s^2)$$

- SJF contribution looks similar to "kinematic"

  but argument of log and constant are different...

How is the puzzle resolved?

- "Kinematic" correction is derived from incorrect matching

$$W^{\text{Kin.}} = \frac{1}{m_b}\frac{C_F\,\alpha_s(\mu)}{4\pi}\int d\omega\,\theta(\omega+n\cdot p)\left[32\,\ln\frac{\omega+n\cdot p}{m_b}+30\right]S(\omega)+\mathcal{O}(\alpha_s^2)$$

- The correct splitting of log and constant

$$32\ln\frac{\omega+n\cdot p}{m_b}+30 \quad = \quad 32\,\ln\frac{\mu^2}{m_b(\omega+n\cdot p)}-18 \quad + \quad 32\,\ln\frac{(\omega+n\cdot p)^2}{\mu^2}+48$$

$\downarrow$ Subleading Jet Functions

$\downarrow$ Subleading Shape Functions

- The second part is **already** included in the SSF contribution
- Confirmed twice
  - Analysis of regions
  - 1-loop parton expressions for SSF
- Including both "Kinematic" and "Hadronic" leads to **double counting**

What is the Bottom Line?

- The correct subleading power term is the sum of

$$W^{\,\mathrm{SSF}} = \frac{2}{m_b}\int d\omega\,\delta(n\cdot p+\omega)\Big[\omega\,S(\omega,\mu) - s(\omega,\mu) + t(\omega,\mu) - u(\omega,\mu) + v(\omega,\mu)$$

$$+ \quad \pi\alpha_s\,f_u(\omega,\mu) + \pi\alpha_s\,f_v(\omega,\mu)\Big] + \mathcal{O}(\alpha_s)$$

and

$$W^{\,\mathrm{SJF}} = \frac{1}{m_b}\,\frac{C_F\alpha_s(\mu)}{4\pi}\int d\omega\;\theta(\omega{+}n{\cdot}p)\left[32\;\ln\frac{\mu^2}{m_b(\omega+n\cdot p)} - 18\right]S(\omega){+}\mathcal{O}(\alpha_s^2)$$

and **not** as is currently done

- Need to modify treatment and modeling of SSF

  to account for their non zero one loop contribution

  Only ωS and u need to be modified

- New analysis not implemented yet

- Although α_s and $1/m_b$ suppressed, effect can be non-negligible

  e.g. constant change from $+30$ to -18

Factorization at Subleading Power and Irreducible Uncertainties in $\bar{B} \to X_s \gamma$

S.J. Lee, M. Neubert, GP; PRD **75** 114005 (2007)

M. Benzke, S.J. Lee, M. Neubert, GP; *in preparation*

How To Make a Photon?

- Produce it directly...

$$Q_{7\gamma} = \frac{-e}{8\pi^2} m_b \bar{s} \sigma_{\mu\nu} F^{\mu\nu} (1 + \gamma_5) b$$

How To Make a Photon?

- Produce it directly...

$$Q_{7\gamma} = \frac{-e}{8\pi^2} m_b \bar{s}\sigma_{\mu\nu} F^{\mu\nu}(1+\gamma_5)b$$

- Or make a gluon or a quark pair

$$\begin{aligned} Q_{8g} &= \frac{-e}{8\pi^2} m_b \bar{s}\sigma_{\mu\nu} G^{\mu\nu}(1+\gamma_5)b \\ Q_1^q &= (\bar{q}b)_{V-A}(\bar{s}q)_{V-A} \quad (p = u, c) \end{aligned}$$

and convert them to a photon

How To Make a Photon?

- Produce it directly...

$$Q_{7\gamma} = \frac{-e}{8\pi^2} m_b \bar{s}\sigma_{\mu\nu}F^{\mu\nu}(1+\gamma_5)b$$

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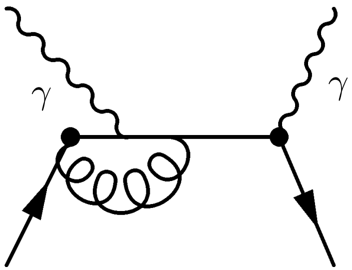
$$Q_{8g} = \frac{-e}{8\pi^2} m_b \bar{s}\sigma_{\mu\nu}G^{\mu\nu}(1+\gamma_5)b$$

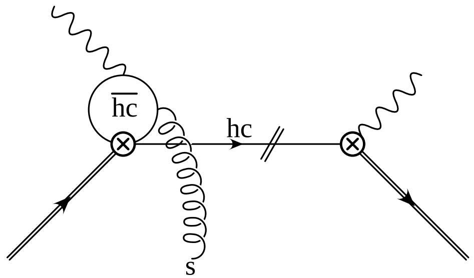
$$Q_1^q = (\bar{q}b)_{V-A}(\bar{s}q)_{V-A} \quad (p=u,c)$$

and convert them to a photon

- But it will cost you..

α_s or $1/m_b$





Effective Hamiltonian

- For $\bar{B} \to X_u \, l \, \bar{\nu}$ only need one operator
- for $\bar{B} \to X_s \, \gamma$ need Effective Hamiltonian

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left(C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3,\ldots,10} C_i Q_i + C_{7\gamma} Q_{7\gamma} + C_{8g} Q_{8g} \right) + \text{h.c.}$$

  - At leading power only $Q_{7\gamma} - Q_{7\gamma}$ contribute
  - At higher orders need other $Q_i - Q_j$ contributions
  - Most important: $Q_{7\gamma}, Q_{8g}$, and Q_1

$$\begin{aligned} Q_{7\gamma} &= \frac{-e}{8\pi^2} m_b \bar{s} \sigma_{\mu\nu} (1+\gamma_5) F^{\mu\nu} b \\ Q_{8g} &= \frac{-g_s}{8\pi^2} m_b \bar{s} \sigma_{\mu\nu} (1+\gamma_5) G^{\mu\nu} b \\ Q_1^q &= (\bar{q} b)_{V-A} (\bar{s} q)_{V-A} \quad (q = u, c) \end{aligned}$$

Total Rate

- Previous studies of $Q_i - Q_j$ contributions focus on $\Gamma(\bar{B} \to X_s\,\gamma)$

  and mostly on α_s suppressed effects

- Common lore:

  like $\Gamma(\bar{B} \to X_u\,l\,\bar{\nu})$ non perturbative effects arise at $1/m_b^2$

- Hints that not all is well
  - $Q_{8g} - Q_{8g}$ (Ali, Greub '95; Kapustin, Ligeti, Politzer '95)
  - $Q_1 - Q_{7\gamma}$ (Voloshin '96; Ligeti, Randall, Wise '97; Grant, Morgan, Nussinov, Peccei '97; Buchalla, Isidori, Rey '97)
  - No local OPE for $\Gamma(\bar{B} \to X_s\gamma)$ (Ligeti, Randall, Wise '97)

- **Never** a systematic study!

  In fact largest uncertainty from $Q_{7\gamma} - Q_{8g}$ was **missed**!

  (Lee, Neubert, GP '06)

  Non perturbative effects in $\Gamma(\bar{B} \to X_s\,\gamma)$ arise at $1/m_b$

- What do we find from a systematic analysis?

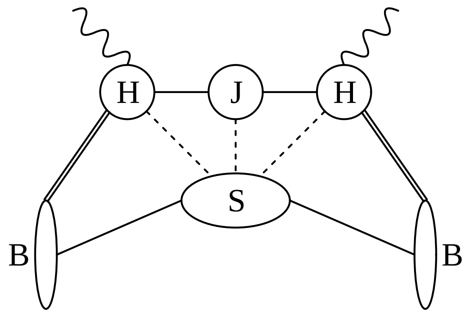
New Factorization Formula: Schematically

At the endpoint region

- Considering only $Q_{7\gamma} - Q_{7\gamma}$... factorization formula for $d\Gamma/dE_\gamma$

  (Korchemsky, Sterman '94; Bauer, Pirjol, Stewart '01)

$$H \cdot J \otimes S$$



New Factorization Formula:: Schematically

At the endpoint region

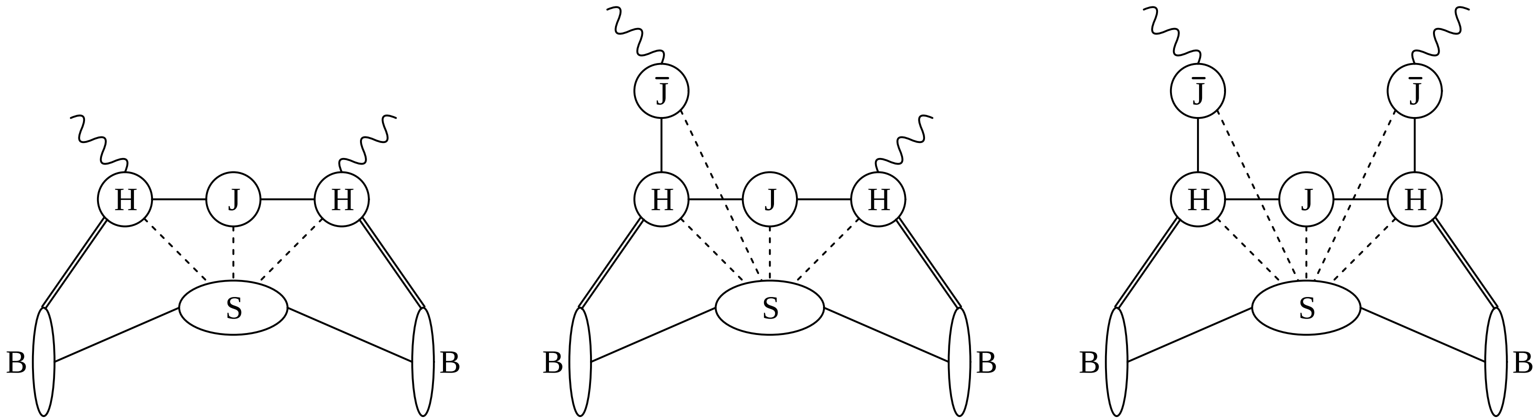
- Considering only $Q_{7\gamma} - Q_{7\gamma}$ factorization formula for $d\Gamma/dE_\gamma$

  (Korchemsky, Sterman '94; Bauer, Pirjol, Stewart '01)

- Considering also other operators **new** factorization formula

  for $d\Gamma/dE_\gamma$ (Benzke, Lee, Neubert, GP)

$H \cdot J \otimes S$ $\quad$ $H \cdot J \otimes S \otimes \bar{J}$ $\quad$ $H \cdot J \otimes S \otimes \bar{J} \otimes \bar{J}$



- No analog for semileptonic decays

Resolved Photon Contribution

- For $Q_i \neq Q_{7\gamma}$, γ does not couple directly to weak vertex

  γ coupling to light partons $\Rightarrow$ new "resolved photon contribution"

  Probe hadronic structure at the scale $\sqrt{E_\gamma \Lambda_{\rm QCD}}$

  Need new jet function $\bar{J}$!

- Resolved photon contribution for $d\Gamma/dE_\gamma$
  - arise at order $1/m_b$ (and higher)
  - "single" resolved contribution (one $\bar{J}$):

    $Q_{7\gamma} - Q_{8g}$, $Q_{7\gamma} - Q_1$
  - "double" resolved contribution (two $\bar{J}$'s):

    $Q_{8g} - Q_{8g}$, $Q_{8g} - Q_1$, $Q_1 - Q_1$

- New soft functions (S)
  - Contain non localities in two light-cone directions
  - Have non zero normalization $\Rightarrow$ No local OPE for total rate

Resolved Photon Contribution

- New factorization formula :

  Lowest order terms arise at $1/m_b$ and at tree level

- We need
  - $Q_{7\gamma} - Q_{7\gamma}$ $\surd$
  - $Q_{8g} - Q_{8g}$
  - $Q_{7\gamma} - Q_{8g}$ ($\overline{\text{hc}}$ gluon, hc gluon)
  - $Q_{7\gamma} - Q_1$
  - $Q_{8g} - Q_1$ $1/m_b^2$
  - $Q_1 - Q_1$ $1/m_b^2$

New factorization formula: Explicitly

- New factorization formula up to $1/m_b^2$

$$\frac{d\Gamma}{dE_\gamma} = \frac{G_F^2\alpha|V_{tb}V_{ts}^*|^2}{2\pi^4}\,\overline{m}_b^2(\mu)\,E_\gamma^3\left[|H_\gamma(\mu)|^2\int_{-p_+}^{\bar\Lambda} d\omega\, m_b\, J\big(m_b(\omega+p_+),\mu\big)\,S(\omega,\mu)\right.$$
$$\left. + \frac{1}{m_b}\sum_{i\le j}\mathrm{Re}\big[C_i^*(\mu)\,C_j(\mu)\big]\,F_{ij}(E_\gamma,\mu) + \mathcal{O}\left(\frac{1}{m_b^2}\right)\right]$$

$$F_{77}(E_\gamma,\mu) = \frac{C_F\alpha_s(\mu)}{4\pi}\int_{-p_+}^{\bar\Lambda} d\omega\left(16\ln\frac{m_b(\omega+p_+)}{\mu^2}+9\right)S(\omega,\mu) + F_{77}^{\rm SSF}(E_\gamma,\mu)$$

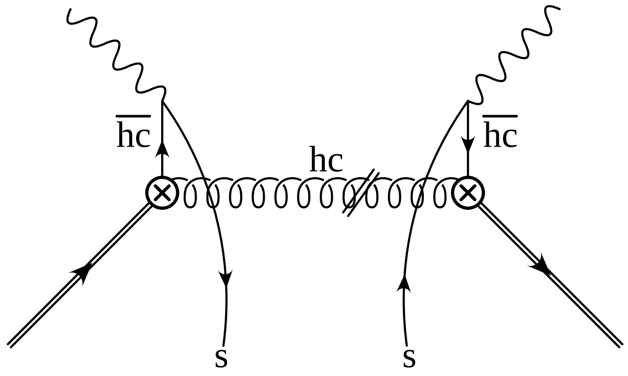
$$F_{88}(E_\gamma,\mu) = \frac{C_F\alpha_s(\mu)}{4\pi}\int_{-p_+}^{\bar\Lambda} d\omega\left(\frac{2}{9}\ln\frac{m_b(\omega+p_+)}{\mu^2}-\frac{1}{3}\right)S(\omega,\mu) + 4\pi\alpha_s(\mu)\,f_{88}(-p_+,\mu)$$

$$F_{78}(E_\gamma,\mu) = \frac{C_F\alpha_s(\mu)}{4\pi}\,\frac{10}{3}\int_{-p_+}^{\bar\Lambda} d\omega\,S(\omega,\mu) + 4\pi\alpha_s(\mu)\,\mathrm{Re}\left[f_{78}^{(\rm I)}(-p_+,\mu)+f_{78}^{(\rm II)}(-p_+,\mu)\right]$$

$$F_{17}(E_\gamma,\mu) = \frac{C_F\alpha_s(\mu)}{4\pi}\left(-\frac{2}{3}\right)\int_{-p_+}^{\bar\Lambda} d\omega\,S(\omega,\mu) + \sum_{q=c,u}\left(\frac{\lambda_q}{-\lambda_t}\right)\mathrm{Re}\,f_{17,c}(-p_+,\mu)$$

$$F_{11}(E_\gamma,\mu) = F_{18}(E_\gamma,\mu) = \frac{C_F\alpha_s(\mu)}{4\pi}\,\frac{2}{9}\int_{-p_+}^{\bar\Lambda} d\omega\,S(\omega,\mu)$$

$Q_{8g} - Q_{8g}$



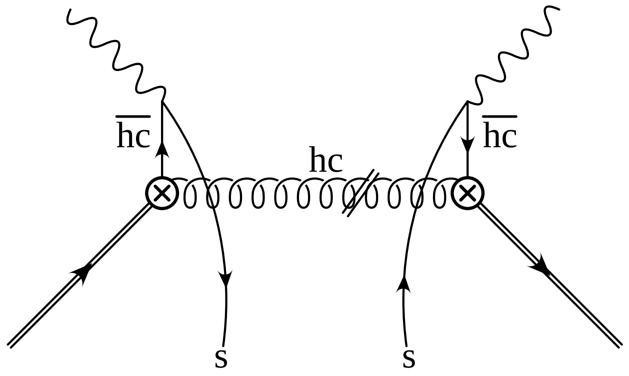
$$f_{88}(-p_+,\mu) = \frac{2}{9}\int d\omega\,\delta(p_+ + \omega)\int \frac{d\omega_1}{\omega_1 + i\epsilon}\int \frac{d\omega_2}{\omega_2 - i\epsilon}\, g_{88}^{\text{cut}}(\omega,\omega_1,\omega_2,\mu)$$

- Where $g_{88}(\omega,\omega_1,\omega_2,\mu)$ is F.T. of

$$\frac{1}{2M_B}\sum_{\mathcal{X}_s}\delta(\omega + n\cdot p_{\mathcal{X}_s} - \bar{\Lambda})\ \langle \bar{B}|\bar{h}(0)\cdots s(u\bar{n})|\mathcal{X}_s\rangle\langle\mathcal{X}_s|\bar{s}(r\bar{n})\cdots h(0)|\bar{B}\rangle$$

where $\cdots$ is a known color and Dirac structure

$Q_{8g} - Q_{8g}$



- New factorization formula

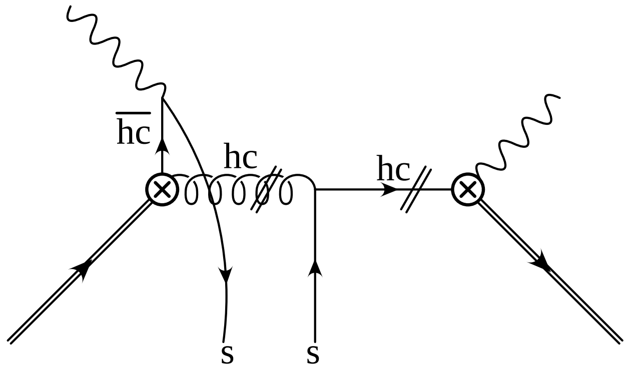
$$f_{88}(-p_+,\mu) = \frac{2}{9}\int d\omega\,\delta(p_+ + \omega)\int\frac{d\omega_1}{\omega_1 + i\epsilon}\int\frac{d\omega_2}{\omega_2 - i\epsilon}\,g_{88}^{\rm cut}(\omega,\omega_1,\omega_2,\mu)$$

$$= H\int m_b\,d\omega\,J\big(m_b(n\cdot p + \omega)\big)$$

$$\times \int 2E_\gamma\,d\omega_1\,\bar{J}(2E_\gamma\omega_1)\int 2E_\gamma\,d\omega_2\,[\bar{J}(2E_\gamma\omega_2)]^*\,g_{88}(\omega,\omega_1,\omega_2,\mu)$$

where $H = \frac{2}{9} + \mathcal{O}(\alpha_s)$, $J = \delta(p^2) + \mathcal{O}(\alpha_s)$, $\bar{J} = 1/(p^2 + i\epsilon) + \mathcal{O}(\alpha_s)$

$Q_{7\gamma} - Q_{8g}$: hc gluon



$$f_{78}^{(\mathrm{I})}(-p_+,\mu) = \frac{4}{3}\int d\omega\,\delta(n\cdot p+\omega)\int\frac{d\omega_1}{\omega_1+i\epsilon}\int\frac{d\omega_2}{\omega_1-i\epsilon}\,[\,\bar{g}_{78}(\omega,\omega_1,\omega_2,\mu)-\bar{g}_{78}^{\mathrm{cut}}(\omega,\omega_1,\omega_2,\mu)]$$

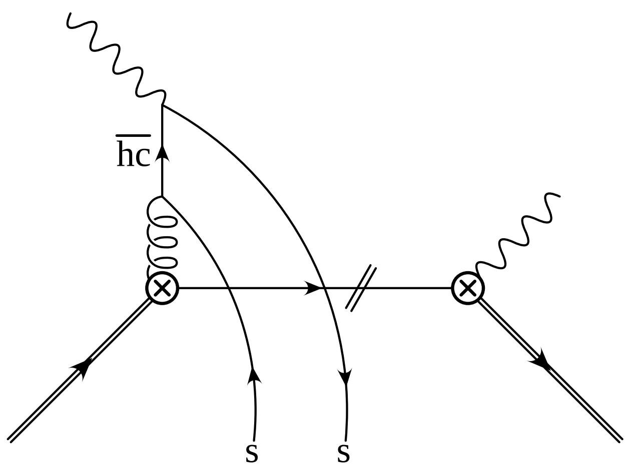
$\bar{g}_{78}(\omega,\omega_1,\omega_2,mu)$ F.T. of

$$\frac{1}{2M_B}\langle\bar{B}|\bar{h}(tn)\cdots s(un)s(r\bar{n})\cdots h(0)|\bar{B}\rangle$$

$\bar{g}_{78}^{\mathrm{cut}}(\omega,\omega_1,\omega_2,mu)$ F.T. of

$$\frac{1}{2M_B}\sum_{\mathcal{X}_s}\delta(\omega+n\cdot p_{\mathcal{X}_s}-\bar{\Lambda})\;\langle\bar{B}|\bar{h}(0)\cdots s(un)|\mathcal{X}_s\rangle\langle\mathcal{X}_s|\bar{s}(r\bar{n})\cdots h\big(0)|\bar{B}\rangle$$

$Q_{7\gamma} - Q_{8g}$: $\overline{\text{hc}}$ gluon



$$f_{78}^{(\text{II})}(-p_+,\mu) = \int d\omega\, \delta(n\cdot p+\omega) \int d\omega_1 \int d\omega_2\, \frac{1}{\omega_1-\omega_2+i\epsilon}$$
$$\times \left[\left(\frac{1}{\omega_1+i\epsilon} + \frac{1}{\omega_2-i\epsilon} \right) g_{78}^{(1)}(\omega,\omega_1,\omega_2,\mu) - \left(\frac{1}{\omega_1+i\epsilon} - \frac{1}{\omega_2-i\epsilon} \right) g_{78}^{(5)}(\omega,\omega_1,\omega_2,\mu) \right]$$

- Where $g_{78}^{(i)}(\omega,\omega_1,\omega_2,mu)$ F.T. of $\left(\Gamma^{(1)} = \bar{n}\!\!\!/,\ \Gamma^{(5)} = \bar{n}\!\!\!/\gamma_5\right)$

$$\frac{1}{2M_B}\langle \bar{B}|\bar{h}(tn)\cdots h(0)\sum_q e_q\, \bar{q}(r\bar{n})\cdots\Gamma^i\cdots q(s\bar{n})|\bar{B}\rangle$$

- New source of CP asymmetry in $\bar{B}\to X_s\gamma$

$Q_{7\gamma} - Q_{8g}$ $\overline{\text{hc}}$ gluon: Total rate

- Using VIA (Lee, Neubert, GP '06)

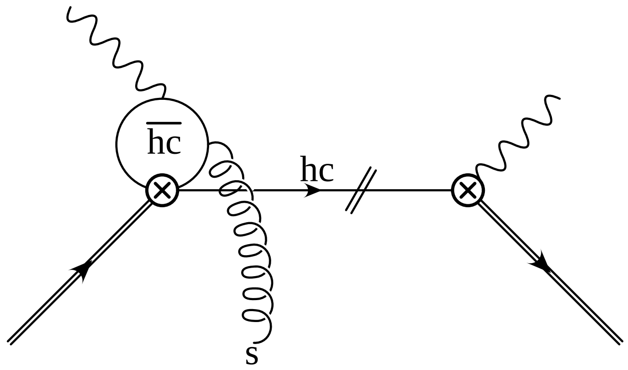
$$\frac{\Delta\Gamma_{\text{VIA}}}{\Gamma_{77}} \approx (-0.3\%, -3\%)$$

- Same terms also give **leading contribution** to isospin asymmetry

$$\Delta_{0-} \equiv \frac{\Gamma(\bar{B}^0 \to X_s\gamma) - \Gamma(B^- \to X_s\gamma)}{\Gamma(\bar{B}^0 \to X_s\gamma) - \Gamma(B^- \to X_s\gamma)} \approx (1\%, 9.5\%) \textbf{ in VIA}$$

- Asymmetry measured by BaBar
  - [PRD **72** 052004 (2005)]: $\Delta_{0-} = -0.6\% \pm 6.3\%$, $E_\gamma > 1.9$ GeV
  - [PRD **77** 051103(R) (2008)]: $\Delta_{0-} = -6\% \pm 16.3\%$, $E_\gamma > 2.2$ GeV

$Q_{7\gamma} - Q_1$



$$f_{17,q}(-p_+,\mu) = \frac{2}{3}\int d\omega\, \delta(n\cdot p+\omega) \int \frac{d\omega_1}{\omega_1+i\varepsilon}\left[1-F\left(\frac{m_q^2-i\varepsilon}{2E_\gamma\,\omega_1}\right)\right] g_{17}(\omega,\omega_1,\mu)$$

$g_{17}(\omega,\omega_1)$ F.T. of

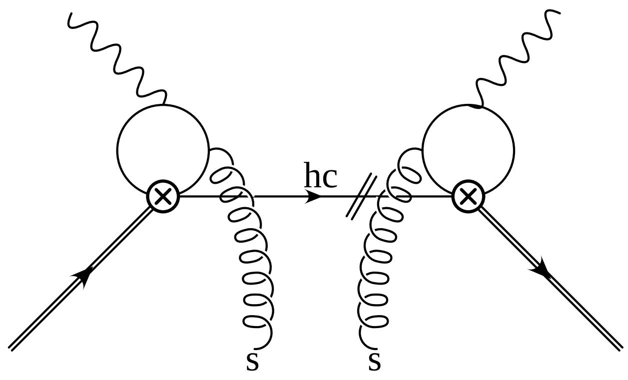
$$\frac{\langle \bar{B}|\big(\bar{h}S_n\big)(tn)\,\not{\bar{n}}(1+\gamma_5)\big(S_n^\dagger S_{\bar{n}}\big)(0)\,i\gamma_\alpha^\perp \bar{n}_\beta\,\big(S_{\bar{n}}^\dagger\, gG_s^{\alpha\beta} S_{\bar{n}}\big)(s\bar{n})\,\big(S_{\bar{n}}^\dagger h\big)(0)|\bar{B}\rangle}{2M_B}$$

- Non trivial $\bar{J}$ from loop: $F(r) = 4r\arctan^2\left[(4r-1)^{-1/2}\right]$
- Taking $m_c \sim m_b$, can expand $\bar{J}$

  Reproduces Voloshin's $\lambda_2/9m_c^2$ in the total rate, still a SSF for spectrum
- Taking $m_c^2 \sim m_b\Lambda_{\text{QCD}}$, $\bar{J}$ should not be expanded

$Q_{8g} - Q_1$ and $Q_1 - Q_1$

- $Q_{8g} - Q_1$ and $Q_1 - Q_1$ give $1/m_b^2$ suppressed contribution, e.g.



New Contributions - Summary

- At order $1/m_b$ and at tree level find 4 contributions:

Pair	Factorization	S
$Q_{8g} - Q_{8g}$	$H \cdot J \otimes S \otimes \bar{J} \otimes \bar{J}$	$\langle \bar{h} ... q\, \bar{q} ... h \rangle$
$Q_{7\gamma} - Q_{8g}$ ($\overline{\text{hc}}$ gluon)	$H \cdot J \otimes S \otimes \bar{J}$	$\langle \bar{h} ... h\, \bar{q} ... q \rangle$
$Q_{7\gamma} - Q_{8g}$ (hc gluon)	$H \cdot J \otimes S \otimes \bar{J}$	$\langle \bar{h} ... q\, \bar{q} ... h \rangle$
$Q_{7\gamma} - Q_1$	$H \cdot J \otimes S \otimes \bar{J}$	$\langle \bar{h} ... G ... h \rangle$

4-q soft functions differ by position and Dirac structure

Modeling

- Very hard to estimate magnitude of new contributions:
  - Spectrum: dependence on $p_+ = m_b - 2E_\gamma$ unknown
  - Total Rate: Integrating over E_γ, removes non locality in n direction

    Does not remove non-locality in $\bar{n}$ direction, contribution still non local
  - Moments of the new soft functions are usually unknown
- Most problematic:
  - g_{88}^{cut} F.T.

$$\frac{1}{2M_B}\sum_{\mathcal{X}_s}\delta(\omega + n\cdot p_{\mathcal{X}_s} - \bar{\Lambda})\ \langle\bar{B}|\bar{h}(0)\cdots s(u\bar{n})|\mathcal{X}_s\rangle\langle\mathcal{X}_s|\bar{s}(r\bar{n})\cdots h\big(0)|\bar{B}\rangle$$

  - $\bar{g}_{78}$ F.T.

$$\frac{1}{2M_B}\langle\bar{B}|\bar{h}(tn)\cdots s(un)s(r\bar{n})\cdots h(0)|\bar{B}\rangle$$

  - $\bar{g}_{78}^{\text{cut}}$ F.T.

$$\frac{1}{2M_B}\sum_{\mathcal{X}_s}\delta(\omega + n\cdot p_{\mathcal{X}_s} - \bar{\Lambda})\ \langle\bar{B}|\bar{h}(0)\cdots s(un)|\mathcal{X}_s\rangle\langle\mathcal{X}_s|\bar{s}(r\bar{n})\cdots h\big(0)|\bar{B}\rangle$$

- Model according to scaling in Λ_{QCD}?

Modeling

- $g_{78}^{(i)}$ F.T. $\quad \left(\Gamma^{(1)} = \bar{n}\!\!\!/,\, \Gamma^{(5)} = \bar{n}\!\!\!/\gamma_5\right)$

$$\frac{1}{2M_B}\,\langle \bar{B}|\bar{h}(tn)\cdots h(0)\sum_q e_q\,\bar{q}(r\bar{n})\cdots\Gamma^i\cdots q(s\bar{n})|\bar{B}\rangle$$

- **Integral** of $g_{78}^{(i)}$ over ω can be modeled in VIA as product of LCDAs

$$\int_{-\infty}^{\bar{\Lambda}} d\omega\, g_{78}^{(1,5)}(\omega,\omega_1,\omega_2,\mu) = -e_q\,\frac{f_B^2 m_B}{8}\left(1-\frac{1}{N_c^2}\right)\phi_+^B(\omega_1,\mu)\phi_+^B(\omega_2,\mu)$$

- g_{17} F.T.

$$\frac{\langle \bar{B}|\bar{h}(tn)\cdots gG_s(r\bar{n})\cdots h(0)|\bar{B}\rangle}{2M_B}$$

- **Integral** of $g_{17}(\omega,\omega_1)$ over ω: even function of ω_1

$$\int_{-\infty}^{\bar{\Lambda}} d\omega\, g_{17}(\omega,\omega_1,\mu) = \int_{-\infty}^{\bar{\Lambda}} d\omega\, g_{17}(\omega,-\omega_1,\mu)$$

- Moments in (ω,ω_1) of g_{17} are related to HQET parameters:

$$M_{0,0} = 2\lambda_2 \quad M_{1,0} = -\rho_2 = -\rho_{LS}^3/3 \quad M_{0,1} = 0$$

- Still can find g_{17} that satisfy all these constraints

  but Voloshin's term underestimates its real effect

$\Gamma(\bar{B} \to X_s\gamma)$ in SM

- Experiment
  - Experimental value of $\mathrm{Br}(\bar{B} \to X_s\gamma)$:
    Extrapolated from measured $E_\gamma \sim 1.9$ GeV to $E_\gamma > 1.6$ GeV
    (HFAG Average '08)
    $\mathrm{Br}(\bar{B} \to X_s\gamma, E_\gamma > 1.6 \text{ GeV }) = (3.52 \pm 0.25) \cdot 10^{-4}$ (error 7%)
- Theory NNLO:
  - OPE: Assume 1.6 GeV is in the OPE region
    (Misiak et. al. '06)
    $\mathrm{Br}(\bar{B} \to X_s\gamma, E_\gamma > 1.6 \text{ GeV }) = (3.15 \pm 0.23) \cdot 10^{-4}$ (error 7%)
  - MSOPE: 1.6 GeV is still in MSOPE region
    (Becher, Neubert '06)
    $\mathrm{Br}(\bar{B} \to X_s\gamma, E_\gamma > 1.6 \text{ GeV }) = (2.98 \pm 0.26) \cdot 10^{-4}$ (error 9%)
- Largest error "non perturbative": estimated **5%**

  based on $Q_{7\gamma} - Q_{8g}$ (Lee, Neubert, GP '06)
- Improved numerical estimate based on all $1/m_b$ contributions

  (Benzke, Lee, Neubert, GP, *in preparation*)

  Effect on extraction of $|V_{ub}|$?

Conclusions

- Use of Soft Collinear Effective Theory (SCET)

  allows us to systematically address power corrections

- Factorization in the end-point region for

  $B \to X_u\, l\, \bar{\nu}$ and $Q_{7\gamma} - Q_{7\gamma}$ contribution to $B \to X_s\, \gamma$

$$d\Gamma \sim \overbrace{H \cdot J \otimes S + \frac{1}{m_b} \sum_i h \cdot J_0 \otimes s_i}^{\text{known}} + \overbrace{\frac{1}{m_b} \sum_i h \cdot j_i \otimes S}^{\text{new}} + \mathcal{O}\left(\frac{1}{m_b^2}\right)$$

Conclusions

- Use of Soft Collinear Effective Theory (SCET)

  allows us to systematically address power corrections

- Factorization in the end-point region for

  $B \to X_u\, l\, \bar{\nu}$ and $Q_{7\gamma} - Q_{7\gamma}$ contribution to $B \to X_s\, \gamma$

$$d\Gamma \sim \overbrace{H \cdot J \otimes S + \frac{1}{m_b}\sum_i h \cdot J_0 \otimes s_i}^{\text{known}} + \overbrace{\frac{1}{m_b}\sum_i h \cdot j_i \otimes S}^{\text{new}} + \mathcal{O}\left(\frac{1}{m_b^2}\right)$$

- Including other operators in $B \to X_s\, \gamma$

$$\begin{aligned} d\Gamma_s \quad &\sim \quad H \cdot J \otimes S + \frac{1}{m_b}\sum_i h \cdot J_0 \otimes s_i + \frac{1}{m_b}\sum_i h \cdot j_i \otimes S + \mathcal{O}\left(\frac{1}{m_b^2}\right) \\ &+ \quad \frac{1}{m_b}\sum_i h \cdot J \otimes \bar{s}_i \otimes \bar{J} + \frac{1}{m_b}\sum_i h \cdot J \otimes \bar{s}_i \otimes \bar{J} \otimes \bar{J} + \mathcal{O}\left(\frac{1}{m_b^2}\right) \end{aligned}$$

- In the near future:
  - Can use new SJF analysis to improve precision on $|V_{ub}|$
  - Estimate theoretical uncertainty for $B \to X_s\, \gamma$ rate and spectrum

Backup Slides

What about the other α_s/m_b Corrections?

$$d\Gamma \sim H \cdot J \otimes S + \frac{1}{m_b} \sum_i H \cdot J \otimes s_i + \frac{1}{m_b} \sum_i H \cdot j_i \otimes S + \mathcal{O}\left(\frac{1}{m_b^2}\right)$$

- Naively two types of α_s/m_b corrections
  - s_i with 1-loop coefficients
  - j_i with tree level coefficients
- But s_i and j_i with tree level coefficients

  already account for all "kinematical" corrections
- Experimental improvement lower cuts, e.g. E_γ

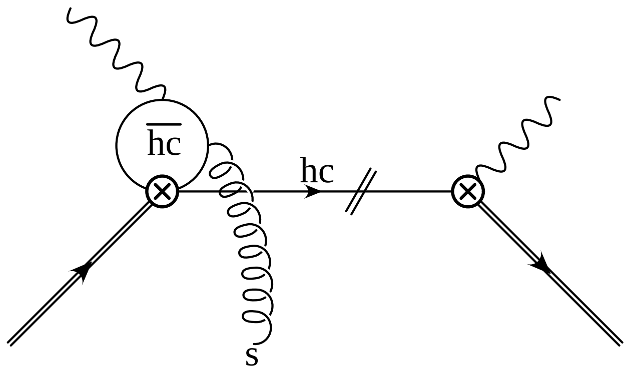
  $\Rightarrow$ increase $p^2 = m_b(m_b - 2E_\gamma)$
  - s_i remain power suppressed = hadronically suppressed
  - $j_i(p^2)$ become less power suppressed = kinematically suppressed

$$j_i(p^2) = \alpha_s \left[\text{const.} + \ln\left(\frac{\mu^2}{p^2}\right)\right] + \mathcal{O}(\alpha_s^2)$$

- Subleading jet functions do **not** introduce new hadronic uncertainties
- Conclusion: Using SJF and SSF with tree level coefficients

  is a consistent approximation

$Q_{7\gamma} - Q_1^u$



- Previous work: up-quark loop contribution strongly CKM suppressed

  but described by an uncalculable long-distance contribution

  (Voloshin '96; Grant, Morgan, Nussinov, Peccei '97; Buchalla, Isidori, Rey '97)

- Effect described by non-local soft matrix element

$$f_{17,u}(-p_+, \mu) = \frac{2}{3}\int d\omega\, \delta(n\cdot p + \omega) \int \frac{d\omega_1}{\omega_1 + i\varepsilon}\, g_{17}(\omega, \omega_1, \mu)$$

Convolution well defined as long as g_{17} non-singular for $\omega_1 = 0$

Estimating $Q_7 - Q_8$: $\overline{\text{hc}}$ gluon for total rate

- Relevant operator

$$\frac{1}{2M_B}\langle \bar{B}|\bar{h}(0)\cdots h(0)\sum_q e_q\,\bar{q}(r\bar{n})\cdots\Gamma^i\cdots q(s\bar{n})|\bar{B}\rangle$$

- Estimated using VIA in (Lee, Neubert, GP '06)

  also affect isospin asymmetry (Lee, Neubert, GP '06)

- Recently, Misiak suggested to use $SU(3)$ flavor to relate effect to isospin asymmetry, based on

$$\sum_q e_q\,\bar{q}\,q = e_u\bar{u}u + e_d\bar{d}d + e_s\bar{s}s$$

  being a pure octet

  $\Rightarrow$ **isospin asymmetry** can be related to effect on **isospin averaged** rate

- Nice argument, but:
  - Strongly assumes $SU(3)$ flavor
  - If we assume only isospin symmetry no relation between isospin asymmetry and isospin averaged rate
  - Only works for (part of) $Q_7 - Q_8$, all other operators don't lead to isospin asymmetry