Overview of Inclusive Determination of $|V_{ub}|$

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Outline

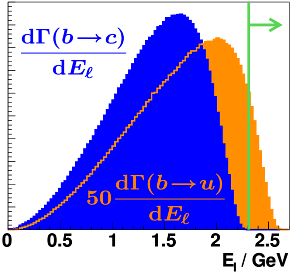
1. (Brief) Overview
2. New Developments
3. Conclusions

$|V_{ub}|$ from Inclusive $B \to X_u \ell \nu$

Removing huge charm background requires stringent phase space cuts

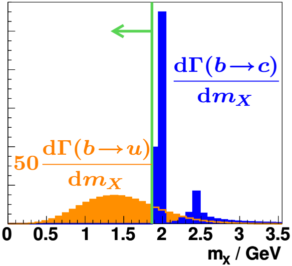
$$\mathcal{B}(B \to X_c \ell \nu)/\mathcal{B}(B \to X_u \ell \nu) \simeq 50$$

- Cuts can drastically enhance perturbative and nonperturbative corrections



Rates become sensitive to b-quark PDFs in B meson

- Determine shape of spectra
- Leading order: Universal shape function (SF) [Neubert (1993); Bigi et al. (1993)]
- $\mathcal{O}(\Lambda_{QCD}/m_b)$: Several more subleading shape functions [Bauer, Luke, Mannel (2001)]
- Need to be extracted from data (like any PDF)



Regions of Phase Space

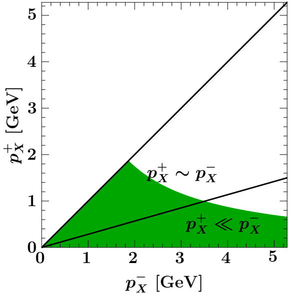
Kinematic variables: $p_X^\pm = E_X \mp |\vec{p}_X|$

Shape function region (SCET region): $p_X^+ \ll p_X^-$

- Leading order in $1/m_b$ requires nonperturbative shape function $S(\omega)$
  [Korchemsky, Sterman (1994); Bauer et al. (2001)]

$$\mathrm{d}\Gamma = H(E_\ell, p_X^\pm) \int \mathrm{d}\omega\, J[p_X^- (p_X^+ - \omega)] S(\omega)$$

- $\mathcal{O}(\alpha_s^2)$ corrections recently completed
  [Becher, Neubert (2005, 2006); Bonciani, Ferroglia; Asatrian et al.; Beneke et al.; Bell (2008)]



Local OPE region: $p_X^+ \sim p_X^-$ (q^2 spectrum, small E_ℓ)

- Leading order in $1/m_b$ given by quark decay (as in $B \to X_c \ell \nu$) known to $\mathcal{O}(\alpha_s, \alpha_s^2 \beta_0)$ [De Fazio, Neubert (1999); Gardi, Ridolfi, Gambino (2006)]

Cut on $m_X < m_D$ does not imply $p_X^+ \ll p_X^- \Rightarrow$ depends on both regions

Non-Experimental Uncertainties

Theoretical uncertainties

- Unknown higher orders in α_s, $1/m_b$ expansions
- Weak annihilation (open question $\Rightarrow$ separate data into B^+ and B^0)

Dominant uncertainties on $|V_{ub}|$ come from input parameters

- m_b: Total rate $\sim |V_{ub}|^2 m_b^5$, partial rates with cuts $\sim |V_{ub}|^2 m_b^{\mathcal{O}(10)}$
  - Need as precise as possible m_b to get precise $|V_{ub}|$
- Shape function(s): Sensitivity depends on phase space region
  - (a) SCET region: small p_X^+, very large E_ℓ
    $\Rightarrow$ Need the full shape (i.e. all moments)
  - (b) Local OPE region: total rate, q^2 spectrum, small E_ℓ
    $\Rightarrow$ Only need 1st moments (i.e. m_b, μ_π^2)
  - (c) something in between: m_X, moderately large E_ℓ
- m_b and SF uncertainties are separate but correlated

How to Best Deal With the Shape Function?

Inclusive $|V_{ub}|$ with small and reliable uncertainty requires

- Consistent treatment of m_b and SF uncertainties and their correlation
- Ability to consistently combine all measurements with different kinematic cuts: E_ℓ, m_X, q^2, p_X^+

Reliable SF uncertainty should reflect the actual information we have

- Perturbative constraints (RGE running and perturbative tail)
- Constraints on moments from m_b, μ_π^2 (λ_1)
- Nonperturbative shape information from $B \to X_s\gamma$ and $B \to X_u\ell\nu$ spectra, including uncertainties and correlations

Current approaches relying on SF models

- Use precise m_b, μ_π^2 from global fits but otherwise fixed model functions
- Current uncertainties are unreliable and likely underestimated

$\Rightarrow$ I do not believe we know inclusive $|V_{ub}|$ to better than 10% at present!

Current Approaches

Current approaches are essentially based on theory for one region and are extrapolated/modeled into other region

	BLNP [Bosch et al. (2004, 2005)]	GGOU [Gambino et al. (2007)]
based on	SCET region	local OPE region
SCET region	correct scale separation $\mathcal{O}(\alpha_s)$ + NLL resummation	$\mathcal{O}(\alpha_s, \alpha_s^2\beta_0)$ + model no resummation
local OPE region	partly $\mathcal{O}(\alpha_s)$, partly model	correct $\mathcal{O}(\alpha_s, \alpha_s^2\beta_0)$ model at large q^2
m_b scheme	tied to SF scheme m_b^{SF}	uses kinetic scheme m_b^{kin}
nonpert. input	universal SF $S(\omega)$ at LO 3 subleading SFs at $1/m_b$	3 nonuniversal functions $F_i(k_+, q^2)$

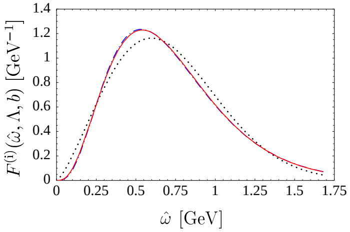
DGE [Andersen, Gardi (2006, 2008)]

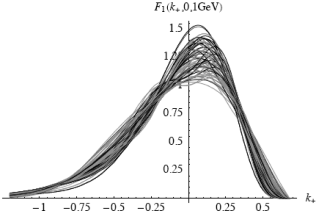
- Effectively uses a fixed model for SF from renormalon resummation
- No subleading power corrections or estimate of model dependence

Parameter Uncertainties

BLNP | **GGOU**

LO SF models for fixed m_b, μ_π^2





	BLNP		GGOU
$\delta\vert V_{ub}\vert(\text{models})$	$\left({}^{+0.4}_{-0.5}{}_{[\text{SF}]} \pm 0.8_{[\text{sub SF}]}\right)\%$	?	${}^{+1.3}_{-0.5}\,\%$
$\delta\vert V_{ub}\vert(m_b, \mu_\pi^2)$	${}^{+4.9}_{-5.4}\,\%$	$\longleftrightarrow$	$3.7\,\%$

Between HFAG averages for ICHEP 08 and Winter 09 m_b increased by 0.5σ

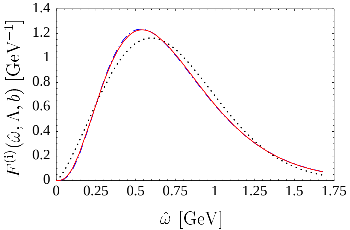
- would expect change in $|V_{ub}|$ by $\simeq -2\%$
- actual change: -6% (BLNP), $+2\%$ (GGOU)

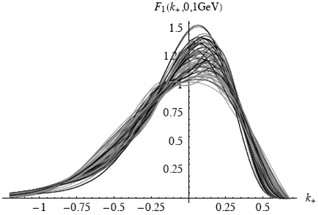
Parameter Uncertainties

BLNP

GGOU

LO SF models for fixed m_b, μ_π^2





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$\delta|V_{ub}|(m_b, \mu_\pi^2)$ ${}^{+4.9}_{-5.4}\,\%$ $\longleftrightarrow$ $3.7\,\%$

Theory uncertainties for BLNP

2008: $\left({}^{+2.2}_{-2.1}{}_{[\text{SF}]} \pm 0.9_{[\text{sub SF}]}\ {}^{+3.7}_{-3.4}{}_{[\text{matching}]} \pm 1.6_{[\text{WA}]}\right)\%$

2009: $\left({}^{+0.4}_{-0.5}{}_{[\text{SF}]} \pm 0.8_{[\text{sub SF}]} \pm 1.5_{[\text{matching}]} \pm 3.4_{[\text{WA}]}\right)\%$

Different Attempts to Avoid the Shape Function

BLL: local OPE at large q^2 [Bauer, Ligeti, Luke (2000, 2001)]

- Formally shape-function independent, important cross check
- Pure q^2 cut: Few events and large nonpert. corrections in local OPE
- m_X-q^2 cut: Reintroduces residual SF sensitivity

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"Shape-function independent" relations

[Leibovich, Low, Rothstein (1999, 2000); Lange, Neubert, Paz (2005)]

- Only avoids parameterizing the SF (same underlying theory)
- Still dependence on subleading shape functions
- Hard to combine different $B \to X_s\gamma$ measurements, no way to include additional knowledge of m_b

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Push experimental cuts deep into $B \to X_c\ell\nu$ background

- Trade off between theory/parameter and systematic uncertainty
- Theory/SF uncertainty now hides in MC signal model and is much harder to deal with consistently or improve in the future
- More useful to measure spectra (or different cuts including correlations)

Comments on Measurements

Monte Carlo signal model depends on the shape function

- Corresponding systematic uncertainty is correlated with m_b and SF uncertainty in the theory
- Can become dominant systematic uncertainty if signal shape is needed for background subtraction, e.g. Babar lepton endpoint [PRD 73, 012006 (2006)]

$\mathcal{B}$ with $E_\ell^{\rm cut}$ [GeV]	2.0	2.1	2.2	2.3
other sys unc. [%]	8.8	8.6	7.9	6.6
SF sys unc. [%]	6.0 – 13.3	3.5 – 8.6	1.6 – 4.0	0.3 – 0.8

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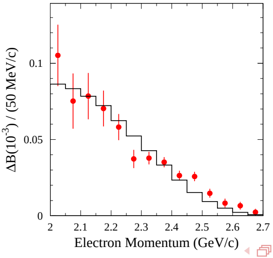
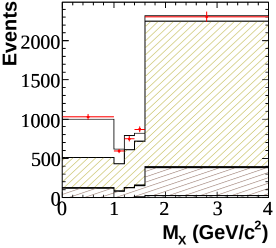
Lepton-endpoint measurements define signal region with an explicit upper cut $E_\ell^\Upsilon < 2.6$ GeV

- Kinematic endpoint in $\Upsilon(4S)$ frame is $E_\ell^\Upsilon < 2.81$ GeV
- Rate is clearly nonzero for $E_\ell^\Upsilon > 2.6$ GeV
- *Impossible* to calculate on theory side



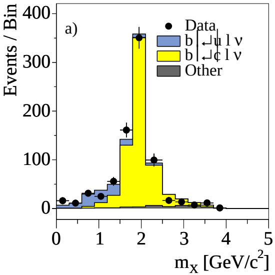
Total Rate Measurements

Belle [arXiv:0907.0379]



$\sigma_{\lvert V_{ub}\rvert}$ for $m_X < m_X^{\rm cut}$			
4.0		**1.67**	**2.5**
4.0	$\sigma_{\rm stat}$ [%]	**7.7**	**18.2**
0.85	$\sigma_{b\to c}$ [%]	**1.0**	**3.8**
3.1	$\sigma_{b\to u}$ [%]	**3.9**	**5.6**

BABAR [PRL 96 (2006) 221801]



- Belle measurement has 8 times statistics with comparable S/B as in previous analyses above and below $m_X \simeq m_D$
- Dominant $B \to X_u \ell \nu$ sensitivity still from $m_X \lesssim m_D$
- Still depends on theory in SF region, but now via Monte Carlo cocktail
- Measurement of total rate important cross check, but eventually would be more useful to have precise measurement of spectrum below $m_X \lesssim m_D$

Strategy Towards Precision $|V_{ub}|$

Minimize SF uncertainties by incorporating all available information on it

- Perturbative constraints (RGE running and perturbative tail)
- Moment constraints from m_b, μ_π^2 (λ_1)
- Shape information from $B \to X_s\gamma$ and $B \to X_u\ell\nu$ spectra

Perform global fit to all available data (similar to inclusive $|V_{cb}|$)

[Bernlochner, Lacker, Ligeti, Stewart, FT, K. Tackmann (work in progress)]

$\longrightarrow$ see Kerstin's talk

- Simultaneously determines $|V_{ub}|$ and inputs (m_b, SF)
- Consistent treatment of m_b and SF uncertainties
- Consistent combination of different $B \to X_u\ell\nu$ measurements

SIMBA - Spectra - Analysis of Inclusive B-Meson

New and Improved Approach to Shape Function

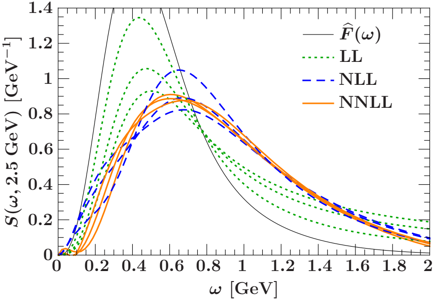
[Ligeti, Stewart, FT (2008)]

Start with perturbative constraints on shape function. Derive factorized form

$$S(\omega, \mu_\Lambda) = \int \mathrm{d}k\, \widehat{C}_0(\omega - k, \mu_\Lambda)\, \widehat{F}(k)$$

$\widehat{C}_0(\omega, \mu_\Lambda)$ perturbative (partonic SF)

- Determines tail consistent with RGE
- Known to $\mathcal{O}(\alpha_s, \alpha_s^2)$
  [Bauer, Manohar (2003); Becher, Neubert (2005)]
- For given $\widehat{F}(k)$ can calculate $S(\omega)$ order by order in α_s
- Vary μ_Λ to estimate perturbative uncertainty



$\mu_\Lambda = (1.1, 1.3, 1.8)$ GeV
+ RGE to $\mu = 2.5$ GeV

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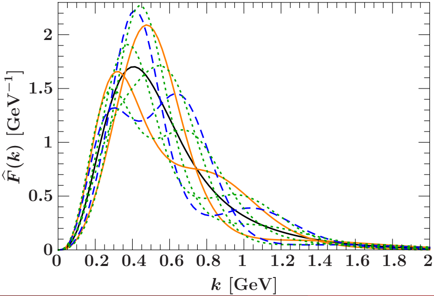
$$S(\omega, \mu_\Lambda) = \int \mathrm{d}k\, \widehat{C}_0(\omega - k, \mu_\Lambda)\, \widehat{F}(k)$$

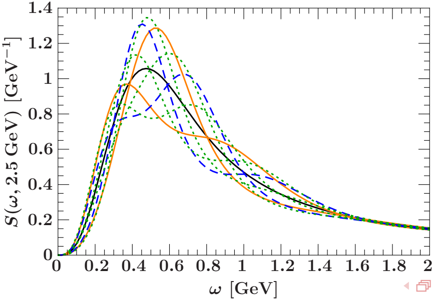
$\widehat{F}(k)$ purely nonperturbative part determines peak region

$\widehat{F}(k)$



$\Rightarrow$

$S(\omega, 2.5\,\mathrm{GeV})$ at LL



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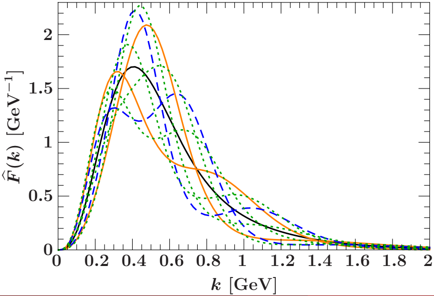
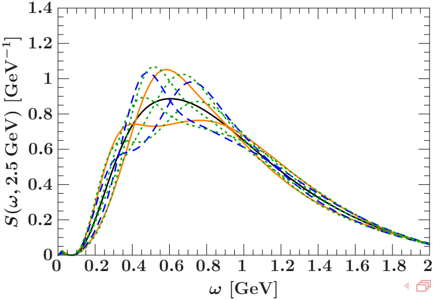


$\Rightarrow$

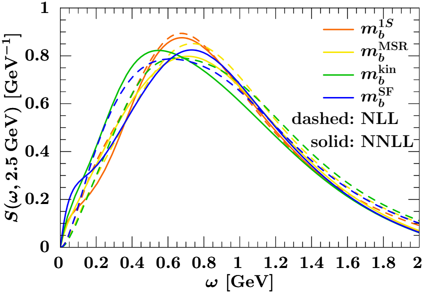
$S(\omega, 2.5\,\mathrm{GeV})$ at NNLL



Different Short Distance Schemes

$\widehat{C}$ and $\widehat{F}$ defined in generic short distance scheme, can use any m_b scheme!

$$
\begin{aligned}
S(\omega) &= \int \mathrm{d}k\, C_0^{\mathrm{pole}}(\omega - k)\, F^{\mathrm{pole}}(k) \\
&= \int \mathrm{d}k\, C_0^{1S}(\omega - k)\, F^{1S}(k) \\
&= \int \mathrm{d}k\, C_0^{\mathrm{kin}}(\omega - k)\, F^{\mathrm{kin}}(k) \\
&= \int \mathrm{d}k\, C_0^{\mathrm{SF}}(\omega - k)\, F^{\mathrm{SF}}(k) = \dots
\end{aligned}
$$



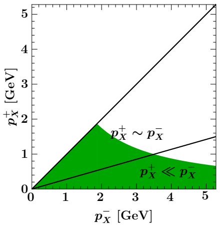
Moments of $\widehat{F}(k)$ given by corresponding SD HQE parameters $\widehat{m}_b$, $\widehat{\lambda}_1$, ... (at any order in α_s), e.g.

$$
\int \mathrm{d}k\, k^n\, F^{1Si}(k) = M_n = \begin{cases} 1 & (n = 0) \\ m_B - m_b^{1S} & (n = 1) \\ -\lambda_1^{\mathrm{i}}/3 + (m_B - m_b^{1S})^2 & (n = 2) \end{cases}
$$

$\Rightarrow$ Can avoid having to switch from different m_b scheme used for $B \to X_c \ell \nu$

Master Formula

[Ligeti, Stewart, FT (to appear)]



Separation $S = \widehat{C} \otimes \widehat{F}$ allows to consistently connect

SCET region: $p_X^+ \ll p_X^-$

Local OPE region: $p_X^+ \sim p_X^-$

for both $B \to X_u \ell \nu$ and $B \to X_s \gamma$

$$\mathrm{d}\Gamma_s = |V_{tb} V_{ts}^*|^2 K_s(E_\gamma) \int \mathrm{d}k \, \widehat{W}_s^{\mathrm{pert}}(E_\gamma, k) \, \widehat{F}(k)$$

$$\mathrm{d}\Gamma_u = |V_{ub}|^2 K_u(E_\ell, p_X^-, p_X^+) \int \mathrm{d}k \, \widehat{W}_u^{\mathrm{pert}}(p_X^-, p_X^+, k) \, \widehat{F}(k)$$

- Combines optimal descriptions for different phase space regions
- Smooth transition between correct fixed-order result in local OPE region and factorized RGE improved result in SCET region
- Not the case in any previous approach!

Setup for Global $|V_{ub}|$ Fit

$$\mathrm{d}\Gamma_s = |V_{tb}V_{ts}^*|^2 K_s(E_\gamma) \int \mathrm{d}k\, \widehat{W}_s^{\mathrm{pert}}(E_\gamma, k)\, \widehat{F}(k)$$

$$\mathrm{d}\Gamma_u = |V_{ub}|^2 K_u(E_\ell, p_X^-, p_X^+) \int \mathrm{d}k\, \widehat{W}_u^{\mathrm{pert}}(p_X^-, p_X^+, k)\, \widehat{F}(k)$$

$$M_n = \int \mathrm{d}k\, k^n \widehat{F}(k)$$

Can now perform a combined fit (similar to $|V_{cb}|$)

- $B \to X_u \ell\nu$ partial rates
  - Normalization determines $|V_{ub}|$
- $B \to X_s\gamma$ and $B \to X_u \ell\nu$ spectra
  - Shapes of distributions constrain $\widehat{F}(k)$
- Known moments of $\widehat{F}(k)$
  - Consistently combines existing constraints on m_b^{1S}, λ_1^{i} (from $B \to X_c\ell\nu$ or anywhere else) with $B \to X_u\ell\nu$ and $B \to X_s\gamma$ data

Conclusions

Towards inclusive $|V_{ub}|$ with reliable uncertainties

- Improved treatment of SF and multiple phase space regions
- Work in progress towards combining all information into global $|V_{ub}|$ fit (see Kerstin's talk)
- Will provide more rigorous uncertainties and test of theory

SIMBA - Analysis of B-Meson Inclusive Spectra

Questions and Comments

- To reduce parametric uncertainties it is better to measure spectra with correlations rather than to trade them for increased and less reliable systematic uncertainties
- Measuring large q^2 spectrum for B^+ and B^0 would be very valuable
- How well do we trust uncertainties due to background subtraction given our poor knowledge of $B \to X_c \ell \nu$ composition?
- Do we trust MC to estimate effect of kaon veto on signal ($s\bar{s}$ popping)?
- Full α_s^2 for triple differential $B \to X_u \ell \nu$ rate would be very valuable