NNLO corrections to $\bar{B} \to X_u l \bar{\nu}$ in the shape-function region

Ben Pecjak

Johannes Gutenberg-Universität Mainz

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Based on work with Christoph Greub and Matthias Neubert

Outline

- partial decay rates in $\bar{B} \to X_u l \bar{\nu}$ in the shape-function region
- numerical results at NNLO in α_s in BLNP
  (NNLO = two loops = α_s^2)

Factorization in the shape-function region

$$d\Gamma \sim H \cdot J \otimes S + \frac{1}{m_b} \sum h \cdot j \otimes S^{\Lambda} + \frac{1}{m_b} \sum h \cdot J^{\Lambda} \otimes S + \ldots$$

- hard-jet-soft factorization
  Korchemsky, Sterman ’94; Akhoury, Rothstein ’95; SCET papers
- $(H \cdot J)$ at NLO in α_s (one loop)
  Bauer, Manohar ’03; Bosch, Lange, Neubert, Paz ’04
- subleading shape-functions (S^{Λ}) at tree level
  Lee, Stewart; Bosch, Neubert, Paz; Beneke, Campanario, Mannel, BP ’05
- subleading jet functions (J^{Λ}) at one loop
  Paz ’09

Today: $H \cdot J \otimes S$ at NNLO in α_s

BLNP and $|V_{ub}|$

Most complete numerical implementation in SCET is “BLNP”

Bosch, Lange, Neubert, Paz ’04; Lange, Neubert, Paz ’05

$$\Gamma_u\Big|_{\rm cut} = |V_{ub}|^2 \left[\Gamma_u^{(0)} + \frac{1}{m_b}\Gamma_u^{(1)} + \frac{1}{m_b^2}\Gamma_u^{(2)}\right]_{\rm BLNP}$$

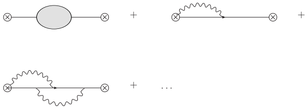
Have included $\Gamma_u^{(0)}$ to NNLO in BLNP “generator”

Greub, BP, Neubert ’09

The two-loop calculations should be useful in all frameworks using factorization in SCET

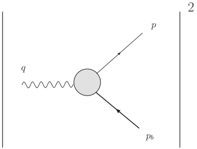
Main ingredients: J and H at NNLO

- J from cut quark propagator in light-cone gauge
  Becher, Neubert '06



- H from matching $b \to u$ current onto SCET
  Bonciani, Ferroglia; Asatrian, Greub, BP; Beneke, Huber, Li; Bell '08

$H(\bar{n} \cdot p, \mu) \propto$



Resummation

$$d\Gamma \sim H(m_b, \mu_f) J(M_X, \mu_f) \otimes \hat{S}(\mu_f)$$

- in S.F. region $m_b \gg M_X$ there are "large" logs $\ln m_b / M_X$
- have model for $\hat{S}$ at some scale μ_0, but needed at arbitrary μ_f

Standard solution in SCET: derive and solve RG-equations

$$\begin{aligned} H(m_b, \mu_f) &= U_H(\mu_f, \mu_h) H(m_b, \mu_h \sim m_b) \\ J(M_X, \mu_f) &= U_J(\mu_f, \mu_i) \otimes J(M_X, \mu_i \sim M_X) \\ \hat{S}(\mu_f) &= U_S(\mu_f, \mu_0) \otimes \hat{S}(\mu_0) \end{aligned}$$

- large logs are "resummed" into the evolution factors $U_{H,J}$
  (missing some three loop anomalous dimensions for complete resummation at NNLO)

Master formula for partial decay rates

$$d\Gamma \sim H(m_b, \mu_h) J(M_X, \mu_i) \otimes U(\mu_h, \mu_i, \mu_0) \otimes \hat{S}(\mu_0)$$

- partial rates formally independent of μ_h, μ_i
- $\mu_h = \mu_i = \mu$ is fixed-order perturbation theory $[H \cdot J](\mu) = C(\mu)$

In numerical analysis, will

- compare partial rates at LO, NLO, NNLO
- let μ_i vary (fixed at $\mu_i = 1.5$ GeV in previous analyses)
- compare with fixed order

Would hope to see

- decreased sensitivity to scale μ_i and μ_h (or μ in fixed order)
- (reasonably) small shifts between LO, NLO and NNLO

Input for partial rates

HQETparameters

HQET parameters

- $m_b \equiv m_b^{\mathrm{SF}} = 4.71$ GeV, $\quad \mu_\pi^2 \equiv \mu_\pi^{2,\mathrm{SF}} = 0.2\,\mathrm{GeV}^2$

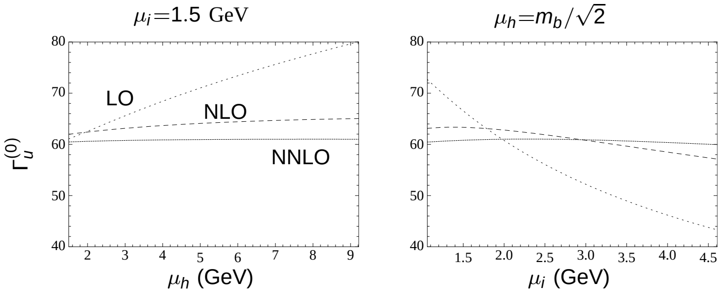
Shape-function

- should be determined from data
- instead, use simple model

$$\hat{S}(\hat{\omega}, \mu_0) = \mathcal{N}(b, \Lambda)\hat{\omega}^{b-1}\exp\left(-\frac{b\hat{\omega}}{\Lambda}\right)$$

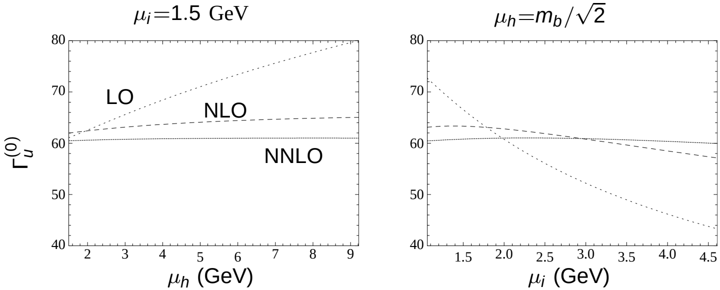
- model parameters b and Λ depend on m_b, μ_π^2 through moment constraints, implemented as in BLNP at $\mu_0 = 1.5$ GeV

Partial rate for $P_+ < 0.66$ GeV



- scale dependence flat
- small shifts between NLO and NNLO

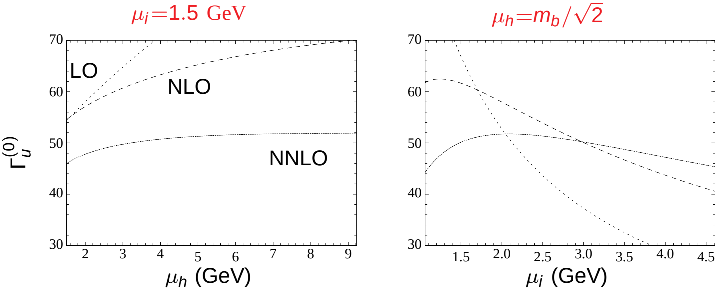
Partial rate for $P_+ < 0.66$ GeV



$$\alpha_s(m_b) = 0.1176!!$$

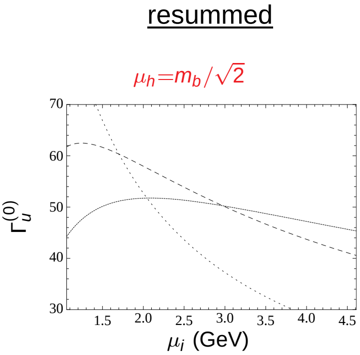
In real life, $\alpha_s(m_b) \sim 0.2$, $\alpha_s(1.5\,\mathrm{GeV}) \sim 0.3$

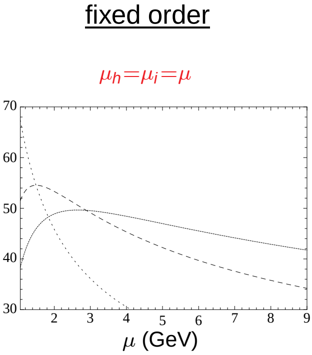
Partial rate for $P_+ < 0.66$ GeV



- reduced dependence on μ_h, μ_i at NNLO
- large negative shift between NLO and NNLO at $\mu_i = 1.5$ GeV
- largest uncertainty associated with μ_i (fixed at $\mu_i = 1.5$ GeV in previous analyses in BLNP)

Comparison with fixed-order perturbation theory

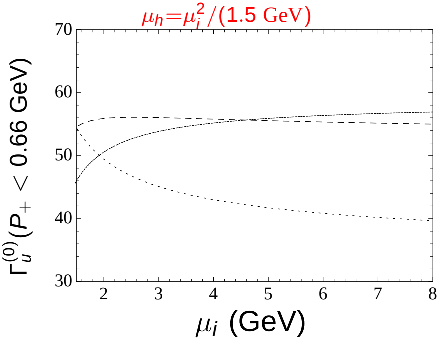




- fixed-order well behaved
- not clear that resummation is necessary
  (although it does stabilize scale dependence at low μ)

Correlated running in SCET

In SCET, one assumes the hierarcy $\mu_h \sim \mu_i^2/\mu_0$.



- NLO so stable as to be misleading
- can play other such games with scales, important fact is that NNLO remains stable under variations

Adding power corrections: partial decay rates in BLNP

- $P_+ < 0.66$ GeV: $$\Gamma_u = |V_{ub}|^2 \left[\Gamma_u^{(0)} + \frac{1}{m_b}\Gamma_u^{(1)} + \frac{1}{m_b^2}\Gamma_u^{(2)}\right]_{\text{BLNP}}$$

	$\Gamma_u/|V_{ub}|^2$	μ_h	μ_i	μ_{pow}	μ_π^{*2}	SSF	tot	m_b^*
NLO	54.28	${}^{+3.49}_{-3.34}$	${}^{+3.68}_{-6.56}$	${}^{+1.91}_{-1.48}$	${}^{+1.27}_{-1.07}$	± 1.41	${}^{+5.87}_{-7.81}$	${}^{+8.20}_{-7.05}$
NNLO	46.77	${}^{+1.44}_{-1.70}$	${}^{+0.11}_{-2.82}$	${}^{+1.91}_{-1.48}$	${}^{+2.08}_{-1.56}$	± 1.41	${}^{+3.69}_{-4.36}$	${}^{+6.34}_{-5.48}$

Input:

- $1/\sqrt{2}\mu_h^{\text{def}} < \mu_h < \sqrt{2}\mu_h^{\text{def}}$; $\mu_h^{\text{def}} = 4.25$ GeV
- $1/\sqrt{2}\mu_i^{\text{def}} < \mu_i < \sqrt{2}\mu_i^{\text{def}}$; $\mu_i^{\text{def}} = 2.0$ GeV
- $m_b^* = \left(4.707^{+0.059}_{-0.053}\right)$ GeV (shape-function scheme)
- $\mu_\pi^{*2} = \left(0.216^{+0.054}_{-0.076}\right)$ GeV^2 (shape-function scheme)
- subleading shape-functions, power corrections as in BLNP

Method	$\lvert V_{ub}\rvert$ [10^{-3}] NLO	$\lvert V_{ub}\rvert$ [10^{-3}] NNLO
$E_l > 2.1\,\mathrm{GeV}$ CLEO	$3.56 \pm 0.40^{+0.48}_{-0.27}{}^{+0.31}_{-0.26}$	$3.81 \pm 0.43^{+0.33}_{-0.21}{}^{+0.31}_{-0.26}$
$E_l > 2.0\,\mathrm{GeV}$ BABAR	$3.97 \pm 0.22^{+0.37}_{-0.23}{}^{+0.26}_{-0.25}$	$4.30 \pm 0.24^{+0.26}_{-0.20}{}^{+0.28}_{-0.27}$
$E_l > 1.9\,\mathrm{GeV}$ BELLE	$4.27 \pm 0.39^{+0.32}_{-0.19}{}^{+0.25}_{-0.22}$	$4.65 \pm 0.43^{+0.27}_{-0.18}{}^{+0.27}_{-0.24}$
$M_X < 1.55\,\mathrm{GeV}$ BABAR	$3.67 \pm 0.18^{+0.29}_{-0.17}{}^{+0.26}_{-0.24}$	$3.96 \pm 0.19^{+0.20}_{-0.13}{}^{+0.26}_{-0.24}$
$P_+ < 0.66\,\mathrm{GeV}$ BELLE	$3.56 \pm 0.31^{+0.30}_{-0.17}{}^{+0.27}_{-0.23}$	$3.84 \pm 0.33^{+0.21}_{-0.13}{}^{+0.26}_{-0.22}$
$P_+ < 0.66\,\mathrm{GeV}$ BABAR	$3.30 \pm 0.23^{+0.27}_{-0.16}{}^{+0.25}_{-0.22}$	$3.55 \pm 0.24^{+0.19}_{-0.13}{}^{+0.24}_{-0.21}$

Table: In the columns labeled $|V_{ub}|$ the first error is experimental, the second is the sum of all theoretical and parametric errors *except* for that from m_b^*, and the third is that from m_b^*

Summary

NNLO calculation for partial rates in $\bar{B} \to X_u l \bar{\nu}$ now complete
(to leading order in $1/m_b$ in the shape-function region)

Numerical analysis shows that the NNLO corrections

- reduce perturbative uncertainty compared to NLO
- raise $|V_{ub}|$ by roughly 5-10% compared to NLO with scale choices and parameter values typically used in BLNP analysis

Backup slides

Comparison with large-β_0 (BLM) limit

Sometimes large–β_0 (BLM) results used in absence of NNLO results ($\beta_0 = 11/3C_A - 2/3n_l$)

$$A_{\mathrm{QCD}} = \cdots + \alpha_s^2 \left[C_F^2 A_a + C_F C_A A_{na} + C_F n_l A_{n_l} \right]$$

$$A_{\mathrm{BLM}} = \cdots + \alpha_s^2 \left[\qquad\qquad\qquad\qquad -\frac{3}{2} C_F \beta_0 A_{n_l} \right]$$

$\Gamma_u(P_+ < 0.66 \text{ GeV})\Big|_{\mu_h=\mu_i=1.5 \text{ GeV}}$

QCD: $54.43 + 0.11\,[\alpha_s] + \big(-3.68\,[h] - 0.26\,[j] - 4.61\,[hj] = -8.55 \big)\,[\alpha_s^2] = 45.99$

BLM: $54.43 + 0.11\,[\alpha_s] + \big(-14.1\,[h] + 14.1\,[j] \qquad = -0.02 \big)\,[\alpha_s^2] = 54.52$

- poor agreement for hard (h) and jet (j) contributions
- better in sum, but still $\sim 20\%$ difference