# SIMBA – A Global Fit Approach to $|V_{ub}|$

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SIMBA
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# Where Can We Gain?

Room for improvement in current $|V_{ub}|$ determinations at several levels

Theoretical Treatment (see Frank's talk)

- Combination of different phase space regions
- Move away from fixed parametrizations of shape function
  - ⋆ Instead, systematically constrain shape from data
- Pertubative tail and RGE running of shape function

Combine all available information in one fit

- Determine $|V_{ub}|$, $m_b$, shape function simultaneously
  - ⋆ Employ strategy that proved successful for $|V_{cb}|$
- Combine different decay modes, measurements and experiments
  - ⋆ Different $B \to X_s\gamma$ spectra
  - ⋆ Different $B \to X_u\ell\nu$ partial BFs (and spectra)
  - ⋆ External constraints on $m_b$, $\lambda_1$ (from $B \to X_c\ell\nu$ or other)

# Factorizing the Shape Function

[Ligeti, Stewart, F Tackmann (2008)]

Factorize nonperturbative and perturbative contributions to shape function:

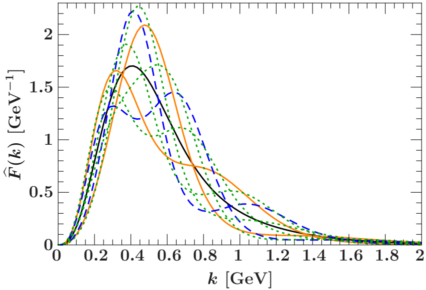
$$S(\omega, \mu_\Lambda) = \int \mathrm{d}k \, \widehat{C}_0(\omega - k, \mu_\Lambda) \, \widehat{F}(k)$$

$\widehat{F}(k)$ nonperturbative part

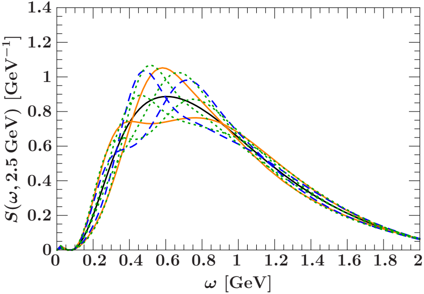
- Determine from data

$\widehat{C}_0(\omega, \mu_\Lambda)$ perturbative part

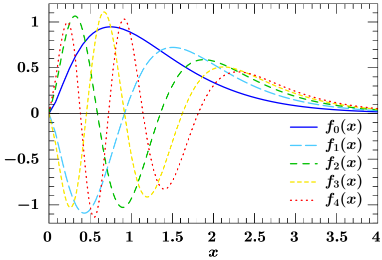
- Compute

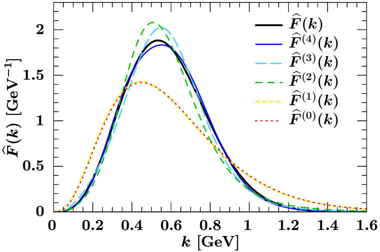



$\Rightarrow$



# Basis Expansion for $\widehat{F}(k)$



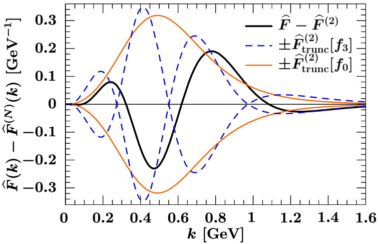

Design suitable orthonormal basis for $\widehat{F}(k)$ (formally model independent)

$$\widehat{F}(\lambda x) = \frac{1}{\lambda}\left[\sum_{n=0}^{\infty} c_n f_n(x)\right]^2 \quad \text{with} \quad \int \mathrm{d}k\, \widehat{F}(k) = \sum_{n=0}^{\infty} c_n^2 = 1$$

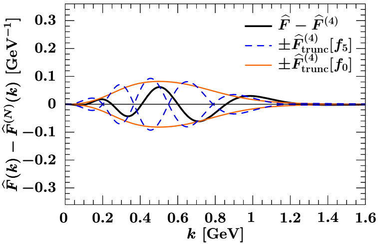
- Builds an orthonormal basis on top of any given model function
- Keep terms up to $n \leq N$ as required by precision of data
- Experimental uncertainties and correlations can be properly captured by uncertainties and correlations in basis coefficients $c_n$

# Estimating Residual Model Dependence

Truncation error at $N = 2$




Truncation error at $N = 4$




Truncating series at $n \leq N$ introduces residual dependence on basis model

- Overall size of truncation error scales with $1 - \sum_{n=0}^{N} c_n^2$
- Can test expansion by varying $N$ and underlying basis model
- Choose final $N$ so that truncation error is small compared to experimental uncertainties in coefficients

$\Rightarrow$ Allows for systematic, fully data driven SF uncertainties

# Getting the Inputs: Stumbling Blocks

## Dealing with experimental cuts

- $B \to X_u \ell \nu$ partial BF for $E_{\text{cut}} < E_\ell^\Upsilon < \mathbf{2.6\,GeV}$
- Impossible for theory to calculate

## Dealing with experimental correlations

- Correlations missing for many sets of partial BFs and spectra
  - ⋆ BABAR leptonic tag $B \to X_s \gamma$ spectrum, $E_\ell^\Upsilon$ partial BFs, Belle hadronic tag partial BFs, ...
  - ⋆ This excludes a lot of valuable inputs

## Dealing with experimental smearing

- Cannot use spectra if significantly affected by efficiency and resolution

## Wishlist for experimental results

- Correlations between partial branching fractions and spectra
- Efficiency and resolution matrices/corrections for spectra

# Disclaimer

What I am showing is work in progress

- Some theory parts still missing from fit
- No theoretical uncertainties or correlations yet

$\Rightarrow$ What follows is a proof-of-concept
(central values and uncertainties subject to change)

# $B \to X_s\gamma$: Fitting for the Shape Function

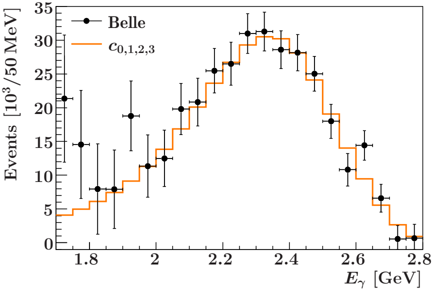
## Experimental inputs

- Belle $605\,\text{fb}^{-1}$
  - ⋆ Thanks to Belle, especially Antonio Limosani, for providing the covariance matrix, experimental efficiency and resolution!
  - ⋆ Efficiency and resolution effects folded into theory predictions
- BABAR sum-over-exclusive-modes ($80\,\text{fb}^{-1}$)
- BABAR hadronic tag ($210\,\text{fb}^{-1}$)

## First test: convergence of expansion ($\lambda = 0.5\,\text{GeV}$)

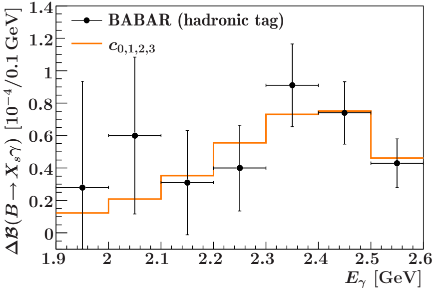
| | 4 coefficients | 5 coefficients |
|---|---|---|
| $c_0$ | $0.987 \pm 0.008$ | $0.986 \pm 0.009$ |
| $c_1$ | $0.150 \pm 0.034$ | $0.152 \pm 0.034$ |
| $c_2$ | $0.052 \pm 0.044$ | $0.057 \pm 0.058$ |
| $c_3$ | $0.025 \pm 0.061$ | $0.026 \pm 0.062$ |
| $c_4$ | | $0.009 \pm 0.068$ |

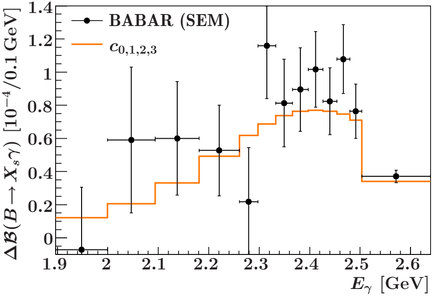
# $B \to X_s\gamma$: Fitting for the Shape Function




## Fit for 4 coefficients

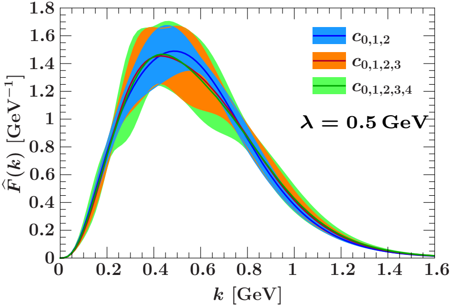
- $\chi^2$ fit including experimental correlations
- Fits validated with pseudo experiments



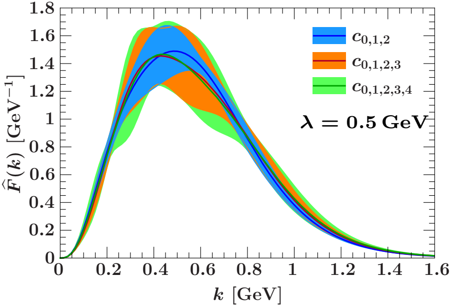
# $B \to X_s\gamma$: Fitting $\widehat{F}(k)$ from Data

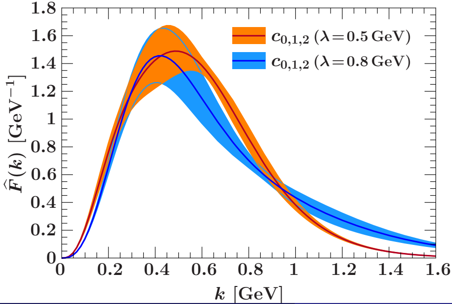
- $\widehat{F}(k)$ with different number of coefficients consistent and converging



# $B \to X_s\gamma$: Fitting $\widehat{F}(k)$ from Data

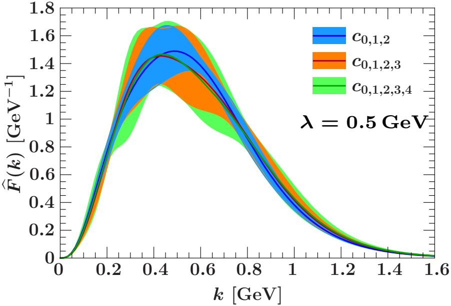
- $\widehat{F}(k)$ with different number of coefficients consistent and converging
- Different bases help to determine truncation error

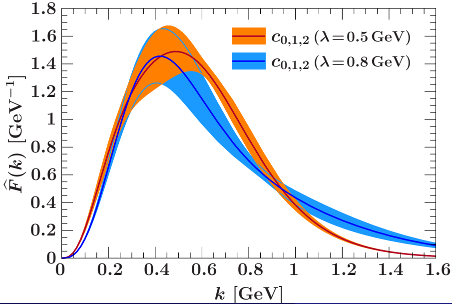


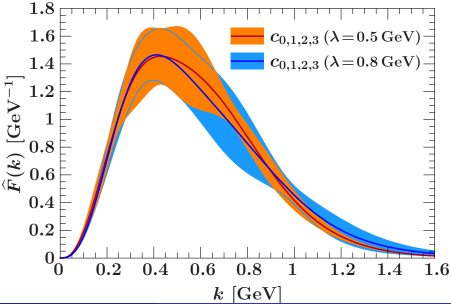



# $B \to X_s\gamma$: Fitting $\widehat{F}(k)$ from Data

- $\widehat{F}(k)$ with different number of coefficients consistent and converging
- Different bases help to determine truncation error
  - ⋆ Here: need 4 coefficients for consistency between bases



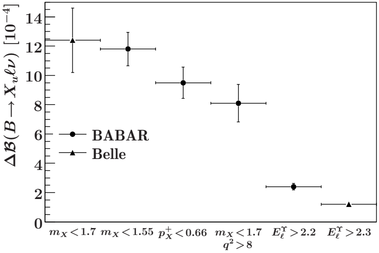



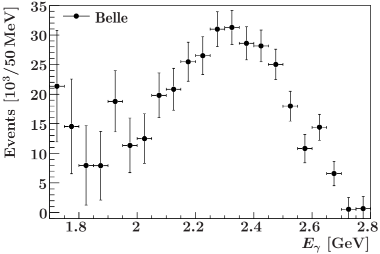
# Combining All Inputs: Fitting for $|V_{ub}|$

As proof-of-concept, fit to

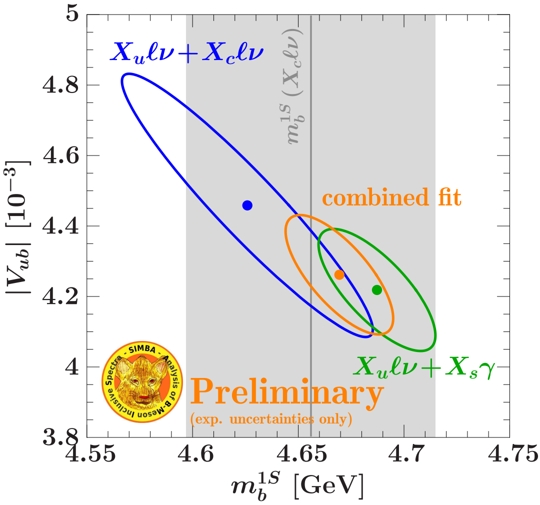
- $B \to X_u \ell\nu$ hadronic tag
  - ⋆ BABAR: $m_X, m_X - q^2, p_X^+$
  - ⋆ Belle: $m_X$
- $B \to X_u \ell\nu$ lepton endpoint
  - ⋆ BABAR: $E_\ell^\Upsilon > 2.2\,\mathrm{GeV}$
  - ⋆ Belle: $E_\ell^\Upsilon > 2.3\,\mathrm{GeV}$
- 3 $B \to X_s\gamma$ spectra
- $m_b^{1S}, \lambda_1$ from $B \to X_c \ell\nu$
  - ⋆ Global fit in 1S scheme (fresh from Christoph's talk)

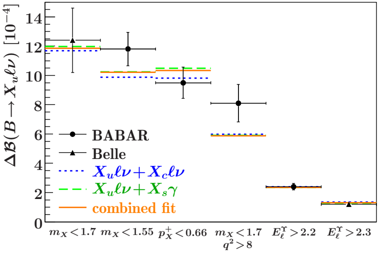
$$m_b^{1S} = (4.66 \pm 0.05)\,\mathrm{GeV}$$
$$\lambda_1 = (-0.34 \pm 0.05)\,\mathrm{GeV}^2$$

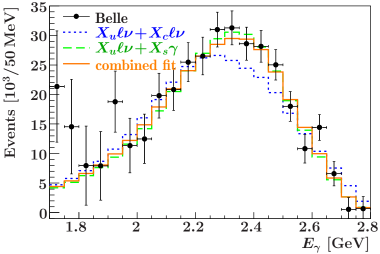


# Combining All Inputs: Fitting for $|V_{ub}|$




- No theory uncertainties yet

Wrong $E_\gamma$ spectrum without $B \to X_s\gamma$

# Conclusions

- Successfully combine partial BFs in $B \to X_u \ell \nu$, photon spectra from $B \to X_s \gamma$ and constraints from $B \to X_c \ell \nu$ to determine $|V_{ub}|$ as well as the shape of the shape function
- What I showed is work in progress, expect details to change
- Competitive experimental uncertainties on both $|V_{ub}|$ and $m_b$
- Theoretical uncertainties forthcoming
  - ⋆ Parametric uncertainties ($m_b$, SF) already included!
- Hardly any gain from $m_b$ and $\lambda_1$ constraints from $B \to X_c \ell \nu$ thanks to new Belle $B \to X_s \gamma$ spectrum
- First attempt to determine SF shape beyond first moments from data
  - ⋆ Allows for rigorous treatment of SF uncertainties in $|V_{ub}|$

# Backup

# Setup for Global $|V_{ub}|$ Fit

Use expansion $\widehat{F}(k) = \left[\sum_n c_n f_n(k)\right]^2$:

$$\mathrm{d}\Gamma(s\gamma) = |V_{td}V_{ts}^*|^2 \sum_{n,m} c_n c_m K(E_\gamma) \int \mathrm{d}k\, \widehat{W}_{\mathrm{pert}}(E_\gamma, k) f_n(k) f_m(k)$$

$$\mathrm{d}\Gamma(u\ell\nu) = |V_{ub}|^2 \sum_{n,m} c_n c_m K(E_\ell, p_X^\pm) \int \mathrm{d}k\, \widehat{W}_{\mathrm{pert}}(p_X^\pm, k) f_n(k) f_m(k)$$

$$M_j(m_b^{1S}, \lambda_1^{\mathrm{i}}) = \sum_{n,m} c_n c_m \int \mathrm{d}k\, k^j\, f_n(k) f_m(k)$$

Perform combined fit (similar to $|V_{cb}|$)

- $B \to X_u \ell\nu$ partial rates
  - ⋆ Normalization determines $|V_{ub}|$
- $B \to X_s\gamma$ and $B \to X_u\ell\nu$ spectra
  - ⋆ Shapes of distributions constrain $\widehat{F}(k)$ through basis coefficients $c_n$
- Known moments of $\widehat{F}(k)$
  - ⋆ Consistently combines existing constraints on $m_b^{1S}$, $\lambda_1^{\mathrm{i}}$ (from $B \to X_c\ell\nu$ or anywhere else) with $B \to X_u\ell\nu$ and $B \to X_s\gamma$ data