

Waiting for Truths and Analyzing Facts in

$$B \rightarrow l \nu X_c$$

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OPE has been applied to $B \rightarrow l \nu X_c$
 - total width and moments -
 with considerable success leading to a high
 accuracy description of high quality data:

$$\Delta |V(bc)| = \pm 2 \%$$

Essential foundations of success story:

- heavy quark symmetry
- $1/m_Q$ expansions

Estimating **theoretical** uncertainties?

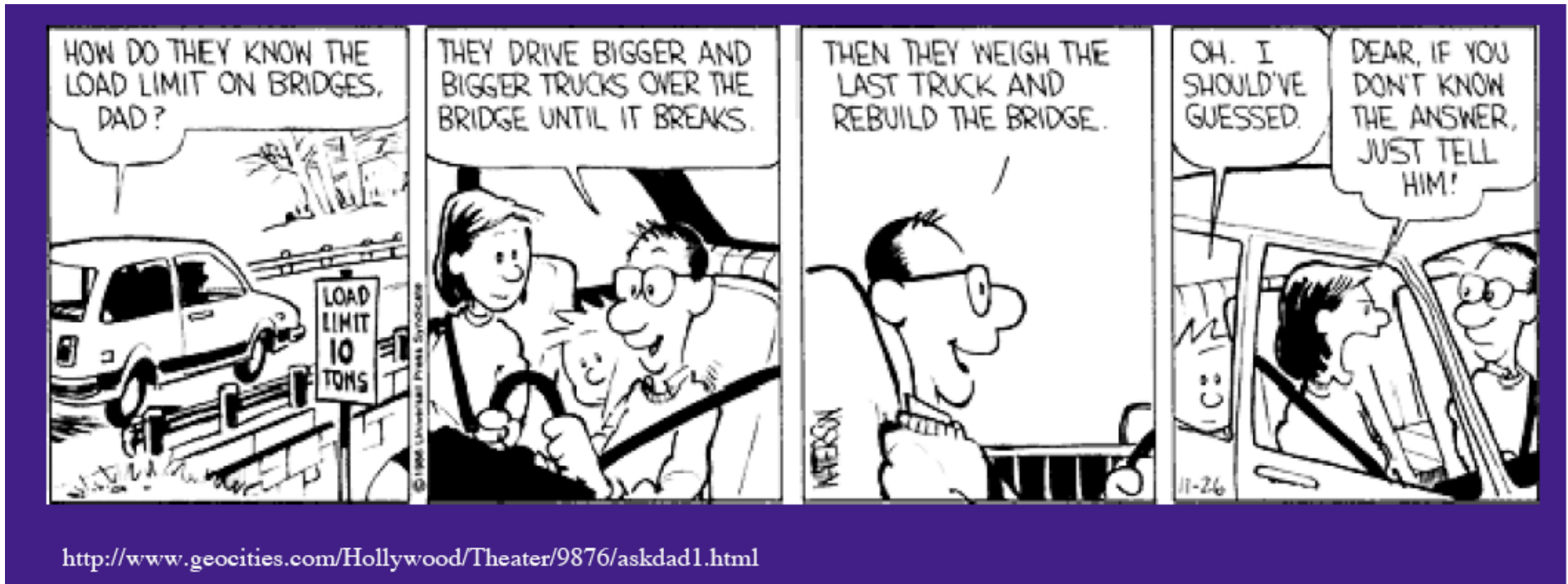
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Estimating theoretical uncertainties?

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- *in practice* need an *a posteriori* analysis!

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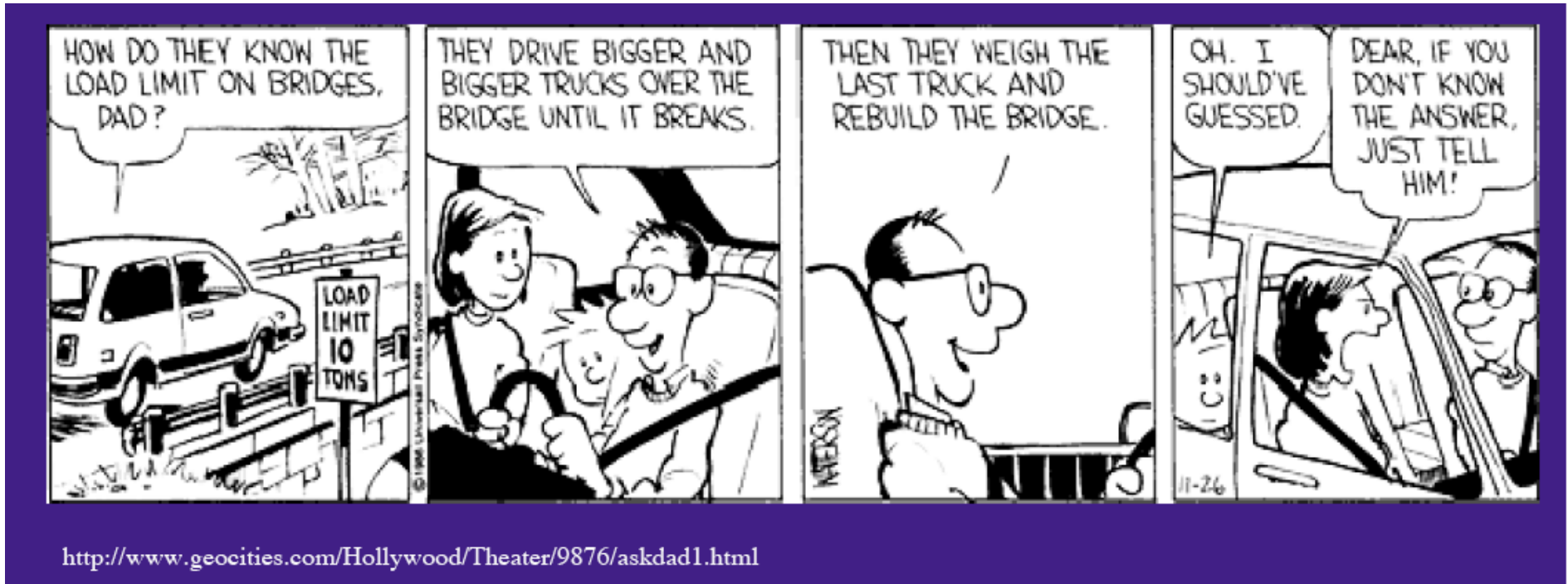
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[with thanks to Marinus Bigi]

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One applies a **theoretical** framework/technology to more and more **processes & observables** - **till it fails!**

Heavy Quark Symmetry & Heavy Quark Expans.

$$\sim H_{\text{Pauli}} = -A_0 + (i\partial - \mathbf{A})^2/2m_Q + \boldsymbol{\sigma} \cdot \mathbf{B}/2m_Q \rightarrow -A_0 \quad \text{as } m_Q \rightarrow \infty$$

i.e.,

infinitely heavy **static** quark, **without spin** dynamics,
only colour Coulomb potential!

→ hadrons H_Q labeled by total spin S and by $j_q = l_q + s_q$:

□ **ground states**: $[S | l_q | j_q] = [0, 1 | 0 | 1/2]$:

PS -- B or D -- & V -- B* or D*

□ **1st excit. states**: $[0, 1 | 1 | 1/2]$ & $[1, 2 | 1 | 3/2]$

4 P wave states: 2 $j_q = 3/2$ **narrow** states

2 $j_q = 1/2$ **broad** states

$m_Q : m_b \text{ or } m_c, \text{ yet } 1/m_b < 1/m_c!$

(2/3 - 3/4) of $B \rightarrow l\nu X_c$ given by D/D^*

→ charm can act as a heavy flavour

• HQS classification used also for charm!

□ more dubious for higher excitations

📖 what is the rest of X_c made up from?

• P wave states will be present and more

$$D^{**} = X_c - (D + D^*) \left\{ \begin{array}{l} = D_{1/2, 3/2}, l > 1 \text{ \& radial excitations} \\ = \text{non-resonant } D/D^* + \pi \text{'s configurations} \end{array} \right.$$

` OPE machinery' leads to various Heavy Quark Sum Rules

$$\mu_\pi^2(\mu)/3 = \sum_n \varepsilon^{<\mu} (\varepsilon_{1/2}^{(n)})^2 |\tau_{1/2}^{(n)}(1)|^2 + 2 \sum_m \varepsilon^{<\mu} (\varepsilon_{1/2}^{(n)})^2 |\tau_{3/2}^{(m)}(1)|^2$$

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Sum rules saturate (approximately) at which n, m and μ ?

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□ no accurate mass predictions

1/2 P waves could be lighter or heavier than 3/2 P waves

□ no reliable predictions on decay patterns

analysis of HQ SR
using observed values of HQP &
assuming expected (approximate) saturation
yields for P wave production

$$\varepsilon_{1/2}^{(0)} \sim (300 - 500) \text{ MeV vs. } \varepsilon_{3/2}^{(0)} \sim 450 \text{ MeV}$$

$$\tau_{1/2}^{(0)}(1) \sim 0.14 - 0.32 \text{ vs. } \tau_{1/2}^{(0)}(1) \sim 0.6$$

narrow `3/2' have to dominate over broad `1/2'

Quark Models - BT model:

satisfies HQS, Bjorken and Spin SRs, yet no $1/m_c$ correction

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LQCD -- yet no $1/m_c$ correction

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unconventional scenarios: radial excitations?
 - 2 structures $0^-, 1^-$ vs. $0^+, 1^+$ for P waves

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✧ not searched for unconventional modes like

$D^{(n)} \rightarrow D/D^*+\eta$?

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- Most valuable analogy will be $B_s \rightarrow l \nu D_s(^*)\pi$'s etc. ;
finding $B \rightarrow l \nu D_s K$ would be a ‘game changer’

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 - ✧ novel lessons on non-perturbative dynamics
 - ✧ even of practical concerns - will affect in particular values of integrated moments!