



# Global fit overview

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Joint workshop on  $|V_{ub}|$  and  $|V_{cb}|$  ( Vxb 2009)  
October 29-31, 2009, SLAC

# The semileptonic width

- $\Gamma(B \rightarrow X_c l \bar{\nu})$  can be systematically calculated with the operator production expansion (OPE)

$$\Gamma_{\text{sl}}(b \rightarrow c) = \frac{G_F^2 m_b^5(\mu)}{192 \pi^3} |V_{cb}|^2 (1 + A_{\text{ew}}) A^{\text{pert}}(r, \mu)$$

$$[ z_0(r) \left( 1 - \frac{\mu_\pi^2(\mu) + \mu_G^2(\mu) + \frac{\rho_D^3(\mu) + \rho_{LS}^3(\mu)}{m_b(\mu)}}{2m_b^2(\mu)} \right) - 2(1-r)^4 \frac{\mu_G^2(\mu) - \frac{\rho_D^3(\mu) + \rho_{LS}^3(\mu)}{m_b(\mu)}}{m_b^2(\mu)} + d(r) \frac{\rho_D^3(\mu)}{m_b^3(\mu)} + \dots ] \quad r = m_c^2(\mu)/m_b^2(\mu)$$

from [Benson et al.,  
Nucl. Phys. B665,  
367 (2003)]

○ ... HQ parameters (non-calculable;  
contain soft QCD physics)

- At each order in  $1/m_b$ , the expectation values of local operator products (heavy quark parameters) are multiplied by perturbatively calculable coefficients

# Other observables in B decays

- Moments of the lepton energy spectrum in  $B \rightarrow X_c l \nu$

$$R_n(E_{\text{cut}}, \mu) = \int_{E_{\text{cut}}} (E_\ell - \mu)^n \frac{d\Gamma}{dE_\ell} dE_\ell, \quad \langle E_\ell^n \rangle_{E_{\text{cut}}} = \frac{R_n(E_{\text{cut}}, 0)}{R_0(E_{\text{cut}}, 0)}$$

- Moments of the hadronic mass spectrum in  $B \rightarrow X_c l \nu$

$$\langle m_X^{2n} \rangle_{E_{\text{cut}}} = \frac{\int_{E_{\text{cut}}} (m_X^2)^n \frac{d\Gamma}{dm_X^2} dm_X^2}{\int_{E_{\text{cut}}} \frac{d\Gamma}{dm_X^2} dm_X^2}$$

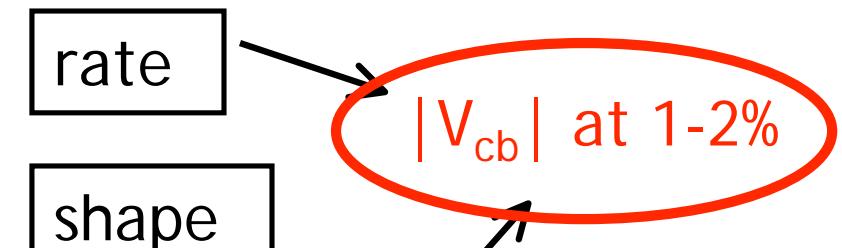
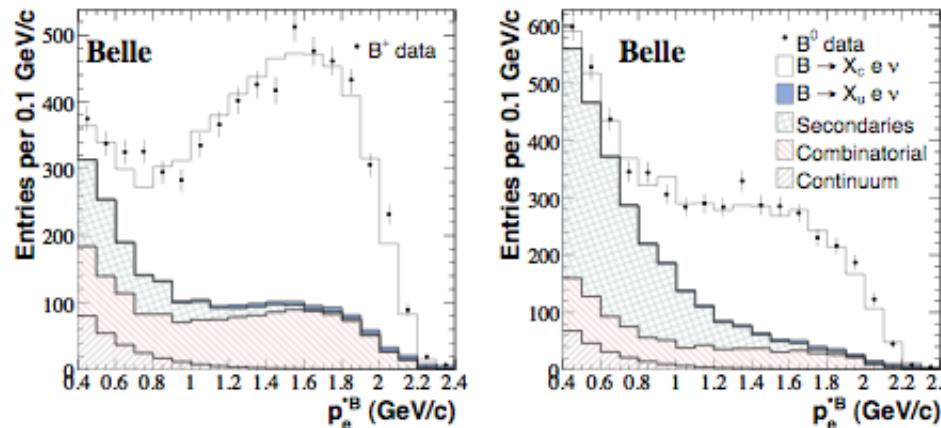
- Moments of the photon energy spectrum in  $B \rightarrow X_s \gamma$

$$\langle E_\gamma^n \rangle_{E_{\text{cut}}} = \frac{\int_{E_{\text{cut}}} E_\gamma^n \frac{d\Gamma}{dE_\gamma} dE_\gamma}{\int_{E_{\text{cut}}} \frac{d\Gamma}{dE_\gamma} dE_\gamma}$$

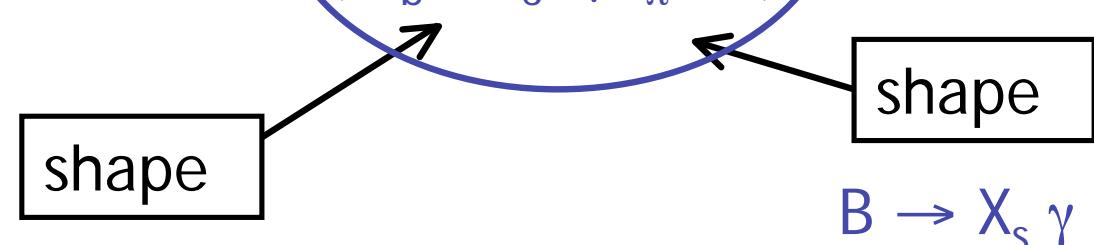
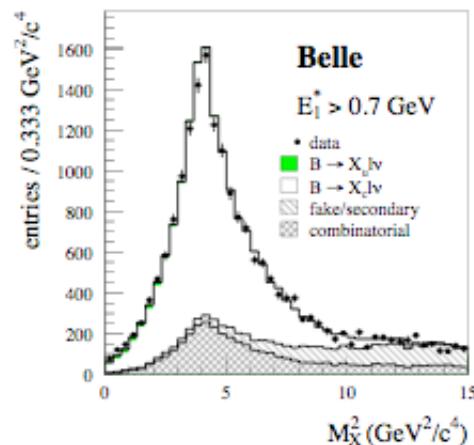
The OPEs of these inclusive observables contain the same HQ parameters

- Non-perturbative parameters can be **measured** from inclusive observables in B decays

## Inclusive $E_1$ spectrum



## Inclusive $M_X^2$ spectrum



# Global analysis of B decays

- Dedicated predictions for each observable
  - $\langle E_I^n \rangle_{E_I > E_{\text{cut}}} = f^{(n)}(E_{\text{cut}}, m_b, \text{HQ param.})$
  - $\langle M_X^{2n} \rangle_{E_I > E_{\text{cut}}} = g^{(n)}(E_{\text{cut}}, m_b, \text{HQ param.})$
  - $\langle E_\gamma^n \rangle_{E_\gamma > E_{\text{cut}}} = h^{(n)}(E_{\text{cut}}, m_b, \text{HQ param.})$
- Determine HQ parameters by performing a minimum  $\chi^2$  fit to all available moment measurements
- Take into account correlated experimental and theoretical errors
- External input: average B lifetime  $\tau_B = (1.585 \pm 0.006) \text{ ps}$

# Available calculations

- Kinetic running mass
  - [P.Gambino, N.Uraltsev, Eur.Phys.J. C34, 181 (2004)]
  - [D.Beson, I.Bigi, N.Uraltsev, Nucl.Phys. B710, 371 (2005)]

both calculations up to  $O(1/m_b^3)$

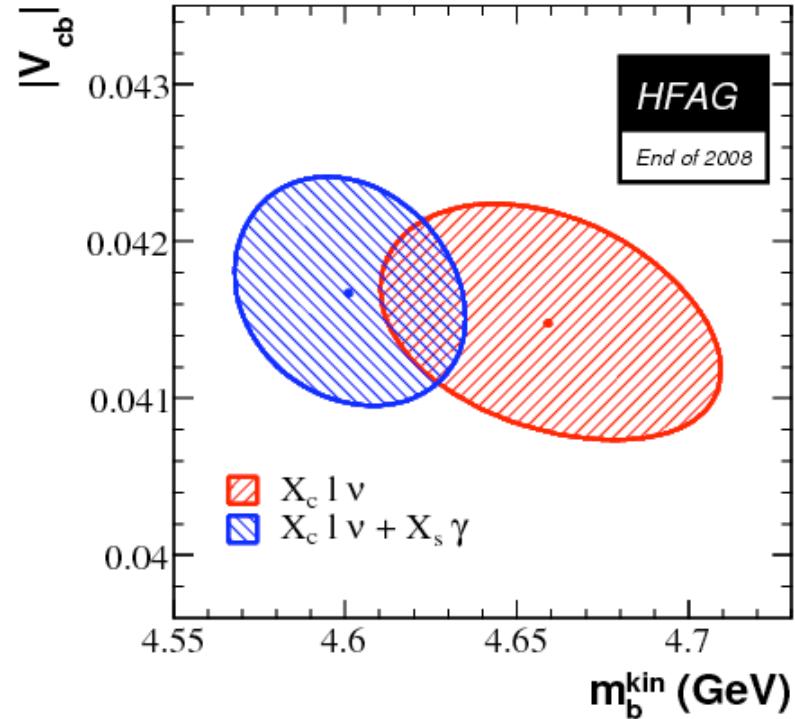
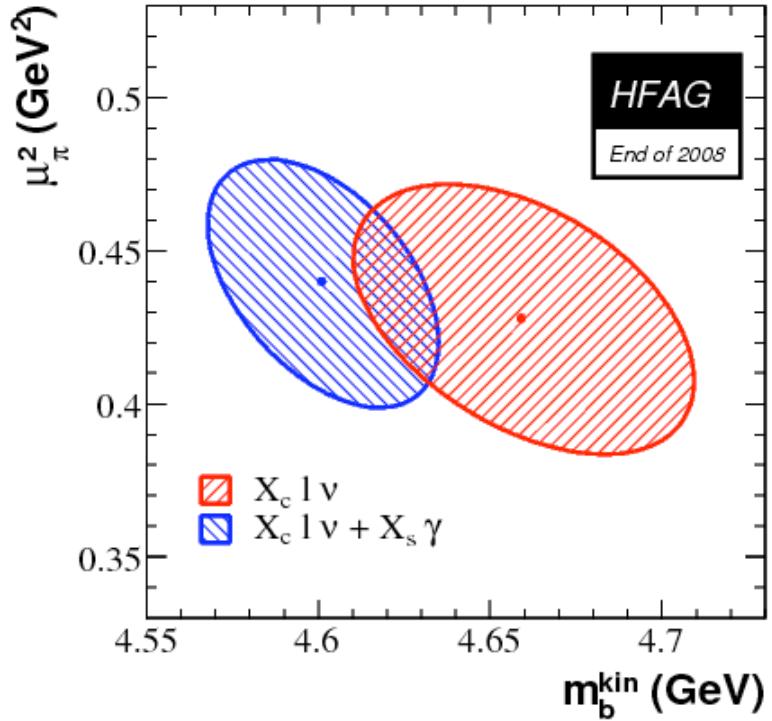
- 1S mass
  - [C.Bauer, Z.Ligeti, M.Luke, A.Manohar, M.Trott, Phys.Rev. D70, 094017 (2004)]
- Non-perturbative parameters in the  $1/m_b$  expansion

	Kinetic scheme	1S scheme
$O(1)$	$m_b, m_c$	$m_b$
$O(1/m_b^2)$	$\mu_\pi^2, \mu_G^2$	$\lambda_1, \lambda_2$
$O(1/m_b^3)$	$\rho_D, \rho_{LS}$	$\rho_1, \tau_{1-3}$

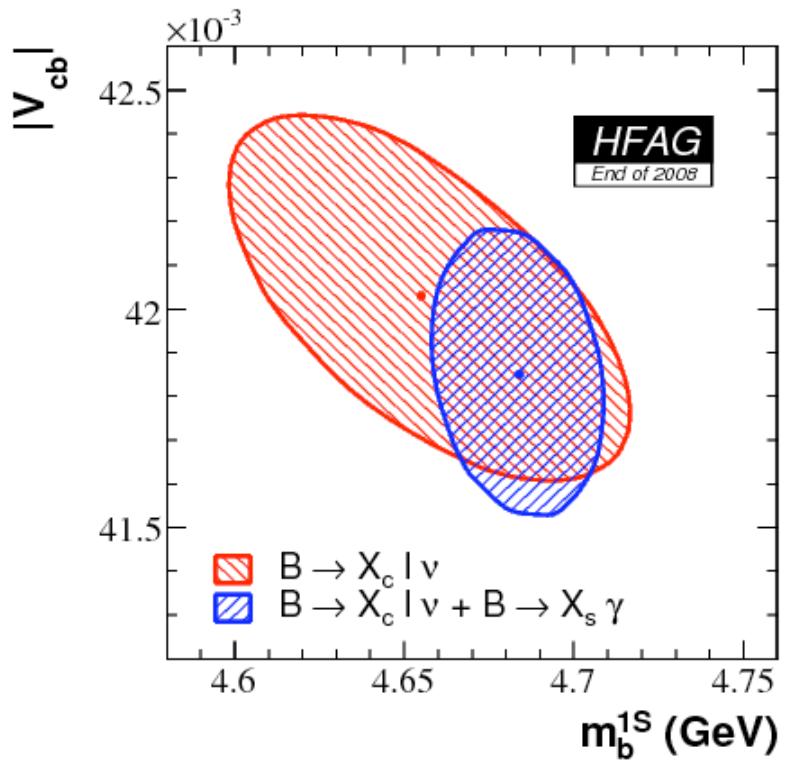
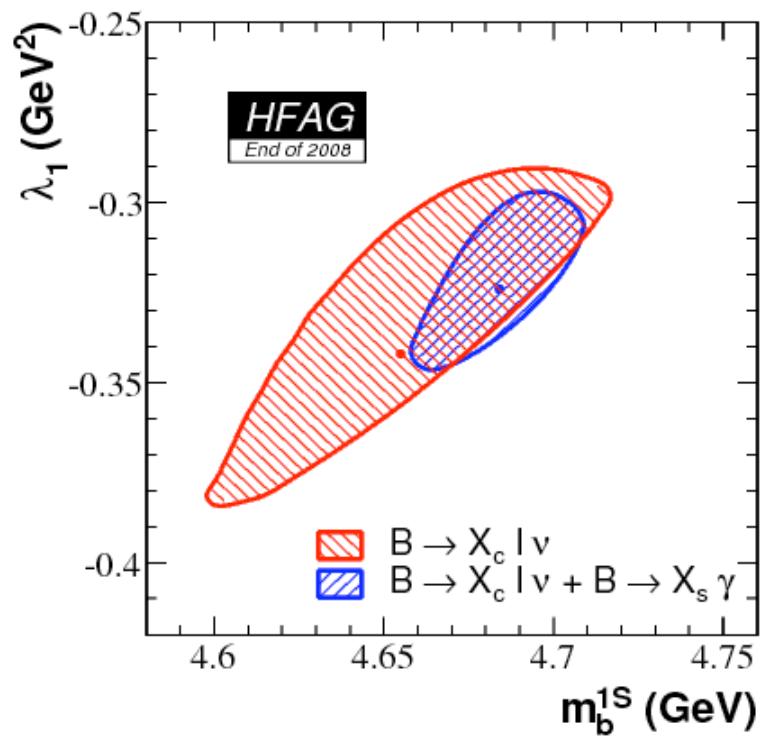
# Measurements used for “end of 08”

BaBar	$\langle E_{\gamma}^n \rangle$ : n=0,1,2,3 [PRD 69, 111104 (2004)] $\langle M_{\chi}^{2n} \rangle$ : n=1,2 [arXiv:0707.2670] preliminary $\langle E_{\gamma}^n \rangle$ : n=1,2 [PRL 97, 171803 (2006)] and [PRD 72, 052004 (2005)]
Belle	$\langle E_{\gamma}^n \rangle$ : n=0,1,2,3 [PRD 75, 032001 (2007)] $\langle M_{\chi}^{2n} \rangle$ : n=1,2 [PRD 75, 032005 (2007)] $\langle E_{\gamma}^n \rangle$ : n=1,2 [arXiv:0804.1580] preliminary
CDF	$\langle M_{\chi}^{2n} \rangle$ : n=1,2 [PRD 71, 051103 (2005)]
CLEO	$\langle M_{\chi}^{2n} \rangle$ : n=1,2 [PRD 70, 032002 (2004)] $\langle E_{\gamma}^n \rangle$ : n=1 [PRL 87, 251807 (2001)]
DELPHI	$\langle E_{\gamma}^n \rangle$ : n=1,2,3 $\langle M_{\chi}^{2n} \rangle$ : n=1,2 [EPJ C45, 35 (2006)]

- 27 moments from BaBar, 25 moments from Belle and 12 moments from other experiments



Input	$ V_{cb}  (10^{-3})$	$m_b^{\text{kin}} (\text{GeV})$	$\mu_\pi^2 (\text{GeV}^2)$	$\chi^2/\text{ndf}$
All moments	$41.67 +/- 0.43(\text{fit}) +/- 0.08(\tau_B) +/- 0.58(\text{th})$	$4.601 +/- 0.034$	$0.440 +/- 0.040$	$29.7/57$
$X_c 1 v$ only	$41.48 +/- 0.47(\text{fit}) +/- 0.08(\tau_B) +/- 0.58(\text{th})$	$4.659 +/- 0.049$	$0.428 +/- 0.044$	$24.1/46$



Input	$ V_{cb}  (10^{-3})$	$m_b^{1S}$ (GeV)	$\lambda_1$ ( $\text{GeV}^2$ )	$\chi^2/\text{ndf}$
All moments	$41.86 +/- 0.33(\text{fit}) +/- 0.08(\tau_B)$	$4.684 +/- 0.024$	$-0.324 +/- 0.023$	$27.9/57$
$X_c l \nu$ only	$42.03 +/- 0.42(\text{fit}) +/- 0.08(\tau_B)$	$4.656 +/- 0.059$	$-0.342 +/- 0.045$	$19.0/46$

# Backup

# Measurements used

Experiment	Hadron moments $\langle M_X^n \rangle$	Lepton moments $\langle E_\ell^n \rangle$	Photons moment $\langle E_\gamma^n \rangle$
BaBar	$n = 2, c = 0.9, 1.1, 1.3, 1.5$	$n = 0, c = 0.6, 1.2, 1.5$	$n = 1, c = 1.9, 2.0$
	$n = 4, c = 0.8, 1.0, 1.2, 1.4$ [10]	$n = 1, c = 0.6, 0.8, 1.0, 1.2, 1.5$	$n = 2, c = 1.9$ [12, 13]
		$n = 2, c = 0.6, 1.0, 1.5$	
		$n = 3, c = 0.8, 1.2$ [11]	
Belle	$n = 2, c = 0.7, 1.1, 1.3, 1.5$	$n = 0, c = 0.6, 1.0, 1.4$	$n = 1, c = 1.8, 1.9$
	$n = 4, c = 0.7, 0.9, 1.3$ [14]	$n = 1, c = 0.6, 0.8, 1.0, 1.2, 1.4$	$n = 2, c = 1.8, 2.0$ [16]
		$n = 2, c = 0.6, 1.0, 1.4$	
		$n = 3, c = 0.8, 1.0, 1.2$ [15]	
CDF	$n = 2, c = 0.7$		
	$n = 4, c = 0.7$ [17]		
CLEO	$n = 2, c = 1.0, 1.5$		$n = 1, c = 2.0$ [19]
	$n = 4, c = 1.0, 1.5$ [18]		
DELPHI	$n = 2, c = 0.0$	$n = 1, c = 0.0$	
	$n = 4, c = 0.0$ [20]	$n = 2, c = 0.0$	
		$n = 3, c = 0.0$ [20]	

# End of 08 -- kinetic scheme

	$ V_{cb}  (10^{-3})$	$m_b^{\text{kin}}$ (GeV)	$m_c^{\text{kin}}$ (GeV)	$\mu_\pi^2$ (GeV $^2$ )	$\rho_D^3$ (GeV $^3$ )	$\mu_G^2$ (GeV $^2$ )	$\rho_{LS}^3$ (GeV $^3$ )
value	41.67	4.601	1.165	0.440	0.192	0.268	-0.193
$\sigma(\text{fit})$	0.43	0.034	0.050	0.040	0.021	0.044	0.086
$\sigma(\tau_B)$	0.08						
$\sigma(\text{th})$	0.58						
$ V_{cb} $	1.000	-0.198	-0.065	0.160	0.312	-0.244	0.110
$m_b^{\text{kin}}$		1.000	0.938	-0.478	-0.188	-0.032	-0.258
$m_c^{\text{kin}}$			1.000	-0.520	-0.120	-0.294	-0.066
$\mu_\pi^2$				1.000	0.455	-0.023	-0.065
$\rho_D^3$					1.000	-0.224	-0.314
$\mu_G^2$						1.000	-0.210
$\rho_{LS}^3$							1.000

Data	$\chi^2/\text{dof}$	$ V_{cb}  (10^{-3})$	$m_b^{\text{kin}}$ (GeV)	$\mu_\pi^2$ (GeV $^2$ )
All moments ( $X_c \ell \nu_\ell$ and $X_s \gamma$ )	29.7/(64 - 7)	$41.67 \pm 0.73$	$4.601 \pm 0.034$	$0.440 \pm 0.040$
$X_c \ell \nu_\ell$ only	24.1/(53 - 7)	$41.48 \pm 0.75$	$4.659 \pm 0.049$	$0.428 \pm 0.044$

# End of 08 -- 1S scheme

	$ V_{cb}  (10^{-3})$	$m_b^{1S}$ (GeV)	$\lambda_1$ (GeV $^2$ )	$\rho_1$ (GeV $^3$ )	$\tau_1$ (GeV $^3$ )	$\tau_2$ (GeV $^3$ )	$\tau_3$ (GeV $^3$ )
value	41.86	4.684	-0.324	0.051	0.125	-0.501	0.125
$\sigma(\text{fit})$	0.33	0.024	0.023	0.010	0.002	0.020	0.002
$\sigma(\tau_B)$	0.08						
$ V_{cb} $	1.000	-0.231	-0.052	-0.052	0.004	-0.177	0.010
$m_b^{1S}$		1.000	0.879	-0.469	-0.035	-0.549	-0.017
$\lambda_1$			1.000	-0.650	-0.025	-0.705	-0.029
$\rho_1$				1.000	-0.160	0.294	-0.046
$\tau_1$					1.000	0.119	-0.004
$\tau_2$						1.000	0.060
$\tau_3$							1.000

Data	$\chi^2/\text{dof}$	$ V_{cb}  (10^{-3})$	$m_b^{1S}$ (GeV)	$\lambda_1$ (GeV $^2$ )
All moments ( $X_c\ell\nu_\ell$ and $X_s\gamma$ )	27.9/(64 - 7)	$41.86 \pm 0.33$	$4.684 \pm 0.024$	$-0.324 \pm 0.023$
$X_c\ell\nu_\ell$ only	19.0/(53 - 7)	$42.03 \pm 0.42$	$4.656 \pm 0.059$	$-0.342 \pm 0.045$