Semi-Leptonic $B \to X_c \ell \bar{\nu}_\ell$ Decays: Overview

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Contents

Set Up: How to compute Inclusive Decays

Heavy Quark Expansion = Operator Product Expansion

(Chay, Georgi, Bigi, Shifman, Uraltsev, Vainstain, Manohar. Wise, Neubert, M,...)

$$\Gamma \propto \sum_X (2\pi)^4 \delta^4(P_B - P_X) |\langle X | \mathcal{H}_{eff} | B(v) \rangle|^2$$

$$= 2\,\mathrm{Im} \int d^4x\, e^{-im_b v\cdot x} \langle B(v) | T\{\widetilde{\mathcal{H}}_{eff}(x) \widetilde{\mathcal{H}}_{eff}^\dagger(0)\} | B(v) \rangle$$

- Perform an OPE: m_b is large

$$\int d^4x e^{-im_b vx} T\{\widetilde{\mathcal{H}}_{eff}(x) \widetilde{\mathcal{H}}_{eff}^\dagger(0)\} = \sum_{n=0}^{\infty} \left(\frac{1}{2m_Q}\right)^n C_{n+3}(\mu) \mathcal{O}_{n+3}$$

- Short distances: $C_{n+3}(\mu)$ perturbatively calculable
- Long Distances: $\langle B(v) | \mathcal{O}_{n+3} | B(v) \rangle$ non-perturbative

- Structure of the expansion:
  Two large scales m_b and m_c

$$\Gamma = \Gamma_0 + \frac{1}{m_b}\Gamma_1 + \frac{1}{m_b^2}\Gamma_2 + \frac{1}{m_b^3}\Gamma_3 + \frac{1}{m_b^4}\Gamma_4 + \frac{1}{m_b^3}\log(m_c)\Gamma_{3,0} + \frac{1}{m_b^3}\frac{\alpha_s(m_b)}{m_c}\Gamma_{3,1} + \frac{1}{m_b^3}\frac{1}{m_c^2}\Gamma_{3,2} + \cdots$$

- The Γ_i and $\Gamma_{i,j}$ are regular as $m_c \to 0$
- The Γ_i and $\Gamma_{i,j}$ have perturbative expansions

- Γ_0 is the decay of a free quark (“Parton Model”)
- Γ_1 vanishes due to Heavy Quark Symmetries
- Γ_2 is expressed in terms of two parameters

$$\begin{aligned} 2M_H\mu_\pi^2 &= \langle H(v)|\bar{Q}_v(i\vec{D})^2 Q_v|H(v)\rangle \\ 2M_H\mu_G^2 &= \langle H(v)|\bar{Q}_v\sigma_{\mu\nu}(iD^\mu)(iD^\nu)Q_v|H(v)\rangle \end{aligned}$$

μ_π: Kinetic energy and μ_G: Chromomagnetic moment

- Γ_3 two more parameters

$$\begin{aligned} 2M_H\rho_D^3 &= -\langle H(v)|\bar{Q}_v(iD_\mu)(ivD)(iD^\mu)Q_v|H(v)\rangle \\ 2M_H\rho_{LS}^3 &= \langle H(v)|\bar{Q}_v\sigma_{\mu\nu}(iD^\mu)(ivD)(iD^\nu)Q_v|H(v)\rangle \end{aligned}$$

ρ_D: Darwin Term and ρ_{LS}: Spin Orbit term

- Higher Orders: Proliferation of parameters

Present state of the $b \to c$ semileptonic Calculations

- Tree level terms up to and including $1/m_b^5$ known
- $\mathcal{O}(\alpha_s)$ and full $\mathcal{O}(\alpha_s^2)$ for the partonic rate known
- $\mathcal{O}(\alpha_s)$ for the μ_π^2/m_b^2 is known
- Status of Quark Masses: Andre Hoangs talk
- In the pipeline:
  - Complete α_s/m_b^2, including the μ_G terms
  - More on “Intrinsic Charm”
- The upshot:

  1.5 - 2 % Relative Uncertainty in V_{cb} (inclusive)

Tree level $\mathcal{O}(1/m_b^4)$ Corrections

- Tree level calculation is straightforward
- Can be generalized to even higher orders
- Problem: Identify the Basic parameters
  (The analogues of μ_π, μ_G etc.)
- Expressions are somewhat lengthy, but are available

  (B. Dassinger, S. Turczyk, TM: JHEP 0703:087 (2007))

Basic Dimension Seven Matrix Elements

- Identify the basic dim-7 Matrix elements

Spin-independent basic parameters of dimension 7

$$2M_B\, m_1^4 = \langle B|\bar{b}_v\, iD_\rho iD_\sigma iD_\lambda iD_\delta\, b_v|B\rangle$$
$$\frac{1}{3}\left(\Pi^{\rho\sigma}\Pi^{\lambda\delta} + \Pi^{\rho\lambda}\Pi^{\sigma\delta} + \Pi^{\rho\delta}\Pi^{\sigma\lambda}\right)$$
$$2M_B\, m_2^4 = \langle B|\bar{b}_v\left[iD_\rho, iD_\sigma\right]\left[iD_\lambda, iD_\delta\right] b_v|B\rangle\ \Pi^{\rho\delta} v^\sigma v^\lambda$$
$$2M_B\, m_3^4 = \langle B|\bar{b}_v\left[iD_\rho, iD_\sigma\right]\left[iD_\lambda, iD_\delta\right] b_v|B\rangle\ \Pi^{\rho\lambda}\Pi^{\sigma\delta}$$
$$2M_B\, m_4^4 = \langle B|\bar{b}_v\left\{iD_\rho, \left[iD_\sigma, \left[iD_\lambda, iD_\delta\right]\right]\right\} b_v|B\rangle\ \Pi^{\sigma\lambda}\Pi^{\rho\delta}$$

$$\Pi_{\mu\nu} = g_{\mu\nu} - v_\mu v_\nu$$

Spin-dependent basic parameters of dimension 7

$$2M_B\, m_5^4 = \langle B|\bar{b}_v \left[iD_\rho, iD_\sigma\right]\left[iD_\lambda, iD_\delta\right]\left(-i\sigma_{\alpha\beta}\right) b_v|B\rangle\ \Pi^{\alpha\rho}\Pi^{\beta\delta} v^\sigma v^\lambda$$

$$2M_B\, m_6^4 = \langle B|\bar{b}_v \left[iD_\rho, iD_\sigma\right]\left[iD_\lambda, iD_\delta\right]\left(-i\sigma_{\alpha\beta}\right) b_v|B\rangle\ \Pi^{\alpha\sigma}\Pi^{\beta\lambda}\Pi^{\rho\delta}$$

$$2M_B\, m_7^4 = \langle B|\bar{b}_v \left\{\left\{iD_\rho, iD_\sigma\right\}, \left[iD_\lambda, iD_\delta\right]\right\}\left(-i\sigma_{\alpha\beta}\right) b_v|B\rangle \\ \Pi^{\sigma\lambda}\Pi^{\alpha\rho}\Pi^{\beta\delta}$$

$$2M_B\, m_8^4 = \langle B|\bar{b}_v \left\{\left\{iD_\rho, iD_\sigma\right\}, \left[iD_\lambda, iD_\delta\right]\right\}\left(-i\sigma_{\alpha\beta}\right) b_v|B\rangle \\ \Pi^{\rho\sigma}\Pi^{\alpha\lambda}\Pi^{\beta\delta}$$

$$2M_B\, m_9^4 = \langle B|\bar{b}_v \left[iD_\rho, \left[iD_\sigma, \left[iD_\lambda, iD_\delta\right]\right]\right]\left(-i\sigma_{\alpha\beta}\right) b_v|B\rangle \\ \Pi^{\rho\beta}\Pi^{\lambda\alpha}\Pi^{\sigma\delta}\,.$$

Physical Interpretation of the m_i

Spin-independent

$$2M_B m_1 = \langle \left((\vec{p})^2 \right)^2 \rangle$$

$$2M_B m_2 = g^2 \langle \vec{E}^2 \rangle$$

$$2M_B m_3 = g^2 \langle \vec{B}^2 \rangle$$

$$2M_B m_4 = g \langle \vec{p} \cdot \mathrm{rot}\, \vec{B} \rangle$$

Spin-dependent

$$2M_B m_5 = g^2 \langle \vec{S} \cdot (\vec{E} \times \vec{E}) \rangle$$
$$2M_B m_6 = g^2 \langle \vec{S} \cdot (\vec{B} \times \vec{B}) \rangle$$
$$2M_B m_7 = g \langle (\vec{S} \cdot \vec{p})(\vec{p} \cdot \vec{B}) \rangle$$
$$2M_B m_8 = g \langle (\vec{S} \cdot \vec{B})(\vec{p})^2 \rangle$$
$$2M_B m_9 = g \langle \Delta(\vec{\sigma} \cdot \vec{B}) \rangle$$

- In the published paper three of the matrix elements were ommitted!
- An erratum and a more sophisticated estimate of the matrix elements will be published soon

Quantitative Results

- Estimate by "Ground State Saturation": (Spatial Components only)

$$\langle B(v)|\bar{b}(iD_{\mu_1})(iD_{\mu_2})(iD_{\mu_3})(iD_{\mu_4})b|B(v)\rangle \approx$$
$$\frac{1}{2M_B}\langle B(v)|\bar{b}(iD_{\mu_1})(iD_{\mu_2})b|B(v)\rangle \langle B(v)|\bar{b}(iD_{\mu_3})(iD_{\mu_4})b|B(v)\rangle$$
$$+\frac{1}{2M_B}\sum_{\text{Pol}}\langle B(v)|\bar{b}(iD_{\mu_1})(iD_{\mu_2})b|B^*(v)\rangle \langle B^*(v)|\bar{b}(iD_{\mu_3})(iD_{\mu_4})b|B(v)\rangle$$

- Calculate via "Trace Formulae"
  $\longrightarrow$ reduce them to μ_π and μ_G (Bigi, Zwicky, Uraltsev)

- With two time derivatives:

$$\langle B(v)|\bar{b}(iD_{\mu_1})(iD_0)(iD_0)(iD_{\mu_4})b|B(v)\rangle$$
$$\approx \bar{\epsilon}^2 \langle B(v)|\bar{b}(iD_{\mu_1})(iD_{\mu_4})b|B(v)\rangle$$

- $\bar{\epsilon}$: Excitation energy to the first excited state
- Numerical values
  ($\mu_\pi^2 = 0.45\,\mathrm{GeV}^2, \mu_G^2 = 0.35\,\mathrm{GeV}^2, \bar{\epsilon} = 400\,\mathrm{MeV}$)

m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	m_9
0.11	-0.07	-0.08	0.39	-0.06	-0.16	0.42	1.26	0.40

(all values in GeV^4)

- Effect has been studied in detail on the moments
- $\rightarrow$ small effects of expected size!
- Effect on the total rate:
  $(\delta\Gamma|_{1/m_b^i} = (\Gamma|_{1/m_i} - \Gamma|_{1/m_{i-1}})/\Gamma_{\text{parton}})$

$$\delta\Gamma\Big|_{1/m_b^4} \approx +0.29\% \quad \delta\Gamma\Big|_{1/m_b^3} \approx -2.84\% \quad \delta\Gamma\Big|_{1/m_b^2} \approx -4.29\%$$

- Impact on V_{cb}: Slight improvement of the uncertainly related to the application of the HQE
  Total improvement small, $\mathcal{O}(0.25\%)$

$\mathcal{O}(\alpha_s \mu_\pi^2/m_b^2)$ corrections

- One-Loop α_s corrections known since a long time
- Corrections to the leading (partonic) rate
- Make use of Reparametrization invariance:

$$v \to v' = v + \frac{k}{m_b}$$

- Relates different orders of the $1/m_b$ expansion
- Valid to all orders in α_s
- $\to$ Compute $\mathcal{O}(\alpha_s)$-Correction with $p_b = m_b v + k$ and expand in k

$$k_\mu k_\nu \to (g_{\mu\nu} - v_\mu v_\nu)\frac{\mu_\pi^2}{3}$$

- For the complete α_s/m_b^2 also the $\mathcal{O}(\alpha_s \mu_G^2/m_b^2)$ Corrections need to be computed
- Significantly more complicated
- $\rightarrow$ Needs the one gluon matrix elements at one loop
- Doable, is in the pipeline E. Lunghi, T. Becher; R. Feger, B. Dassinger, TM
- The knowledge of the partonic α_s^2 corrections also give us the $\alpha_s^2 \mu_\pi^2/m_b^2$ by RPI
- .... and the $\alpha_s m_1/m_b^4$, and the $\alpha_s^2 m_1/m_b^4$

$\mathcal{O}(\alpha_s^2)$ corrections Czarnecki, Pak; Melnikov

- Technically challenging
- Partially numerical calculation
- Analytic Results for limiting cases
- $\rightarrow$ allows for an interpolation
- Recenty: Complete differential distributions available

- Contributions to the Moments ($d\Gamma_0$: Partonic rate)

$$L_n(E_{\rm cut}) = \frac{\langle (E_l/m_b)^n \, \theta(E_l - E_{\rm cut}) d\Gamma \rangle}{\langle d\Gamma_0 \rangle}$$

$$H_n(E_{\rm cut}) = \frac{\langle (E_h/m_b)^n \, \theta(E_l - E_{\rm cut}) d\Gamma \rangle}{\langle d\Gamma_0 \rangle}$$

- Expansion:

$$L_n(E_{\rm cut}) = L_n^{(0)} + \frac{\alpha_s}{\pi} L_n^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \left[\beta_0 L_n^{2,\rm BLM} + L_n^{(2)}\right]$$

$$H_n(E_{\rm cut}) = H_n^{(0)} + \frac{\alpha_s}{\pi} H_n^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \left[\beta_0 H_n^{2,\rm BLM} + H_n^{(2)}\right]$$

- $\beta_0 = 11 - 2N_f/3$ and $\alpha_s = \alpha_s^{\overline{\rm MS}, \rm N_f=5}(m_b)$

n	E_{cut}, GeV	$L_n^{(0)}$	$L_n^{(1)}$	$L_n^{(2,\text{BLM})}$	$L_n^{(2)}$
0	0	1	-1.77759	−1.9170	3.40
1	0	0.307202	−0.55126	−0.6179	1.11
2	0	0.10299	−0.1877	−0.2175	0.394
0	1	0.81483	-1.4394	−1.5999	2.63
1	1	0.27763	-0.49755	−0.5667	1.00
2	1	0.09793	-0.17846	−0.20875	0.382

TABLE I: Lepton energy moments.

Tables from Melnikov

n	E_{cut}, GeV	$H_n^{(0)}$	$H_n^{(1)}$	$H_n^{(2,\text{BLM})}$	$H_n^{(2)}$
1	1	0.334	-0.57728	−0.6118	1.02
2	1	0.14111	-0.23456	−0.2343	0.362

TABLE II: Hadronic energy moments.

Even higher orders: $\mathcal{O}(1/m_b^n)$, $n > 4$ Corrections

- $1/m_b^5$ has been studied in the context of “intrinsic charm”
  Numerical estimates of the $1/m_b^5$ are available
- General Structure of the higher order terms have been studied
- Proliferation of new parameters

Estimates of $1/m_b^5$

- In total 18 parameters (Singlet and Triplet)
- Estimates of the parametes by “Ground State Saturation”
- Full expressions for doubly differential rates are available
- Numerical estimates

$$\frac{\Gamma|^{1/m_b^5}_{\rm complete}}{\Gamma_0} \approx 0.36\% \qquad \frac{\Gamma|^{1/m_b^5}_{1/m_c^2}}{\Gamma_0} \approx 0.46\%$$

Beyond $1/m_b^5$

- Structure of the expansion (@ tree): (see talk by Turczyk)

$$d\Gamma = d\Gamma_0 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^2 d\Gamma_2 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 d\Gamma_3 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^4 d\Gamma_4$$
$$+ d\Gamma_5 \left(a_0 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^5 + a_2 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^2 \right)$$
$$+ \ldots + d\Gamma_7 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^4$$

- Power counting $m_c^2 \sim \Lambda_{\rm QCD} m_b$

- Proliferation of parameters in high orders $1/m_b$:

	Dim 5	Dim 6	Dim 7	Dim 8	Dim 9	Dim 10	Dim 11
1	1	1	4	7	24	60	216
σ	1	1	5	11	48	150	624
tot	2	2	9	18	72	210	840

- At high orders: (n = Dim - 3)

$$N_1(n) \approx \frac{1}{2} \sum_{n_g=1}^{[\frac{n}{2}]} (2n_g - 1)!! \binom{n-2}{n-2n_g}$$

$$N_\sigma(n) \approx \frac{1}{2} \sum_{n_g=1}^{[\frac{n}{2}]-1} (2n_g - 1)!! \binom{n-2}{n-2n_g-2} \binom{2+2n_g}{2}$$

Conclusion

- Heavy Quark Expansion for $b \to c\ell\bar{\nu}_\ell$ seems in good shape
- Theoretical uncertainty in inclusive V_{cb} will be 1%
- After we have the $\alpha_s\mu_G^2$ terms further improvement will be difficult
- Point for Discussion:
  Theory Correlations between the HQE Parameters