Joint Workshop on $|V_{ub}|$ and $|V_{cb}|$, SLAC 29 Oct – 31 Oct 2009

$B \to X_c \ell \nu$ moment measurements (at B-Factories)

Marcello Rotondo
INFN Padova

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Inclusive $|V_{cb}|$: Heavy Quark Expansion

- *HQE connect the inclusive $b \to c \ell \nu$ decay width to $|V_{cb}|$*

$r = m_c/m_b$

$$\Gamma_{SL} \sim |V_{cb}|^2 m_b^5 \; [z_0(r) + 0/m_b + z_2(r, \mu_\pi^2/m_b^2, \mu_G^2/m_b^2) + z_3(r, \rho_D^3/m_b^3, \rho_{LS}^3/m_b^3) + \ldots]$$

- *Similar expressions for moments of various inclusive distributions:*
  - *Hadron mass moments* $\langle m_X^n \rangle_{E>Ecut}$

$$\langle m_X^{2n} \rangle_{E_{cut}} = \frac{\int_{E_{cut}} (m_X^2)^n \frac{d\Gamma}{dm_X^2} dm_X^2}{\int_{E_{cut}} \frac{d\Gamma}{dm_X^2} dm_X^2}$$

  - *Lepton energy moments* $\langle E_{lep}^n \rangle_{E>Ecut}$

$$R_n(E_{cut}, \mu) = \int_{E_{cut}} (E_\ell - \mu)^n \frac{d\Gamma}{dE_\ell} dE_\ell$$

  - *Photon energy moments in $b \to s\gamma$:* $\langle E_\gamma^n \rangle_{E>Ecut}$

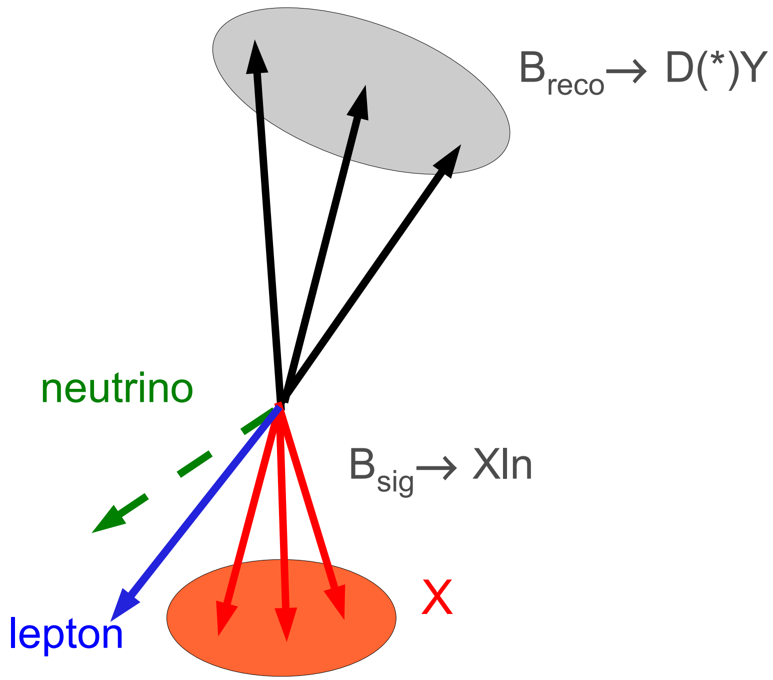
$$\langle E_\gamma^n \rangle_{E_{cut}} = \frac{\int_{E_{cut}} E_\gamma^n \frac{d\Gamma}{dE_\gamma} dE_\gamma}{\int_{E_{cut}} \frac{d\Gamma}{dE_\gamma} dE_\gamma}$$

$$\langle M_X^n \rangle_{E>E_0} = \tau_B \int M_X^n d\Gamma = f\,(E_0,\ m_b,\ m_c,\ \mu_\pi^2,\ \mu_G^2,\ \rho_D^3,\ \rho_{LS}^3)$$

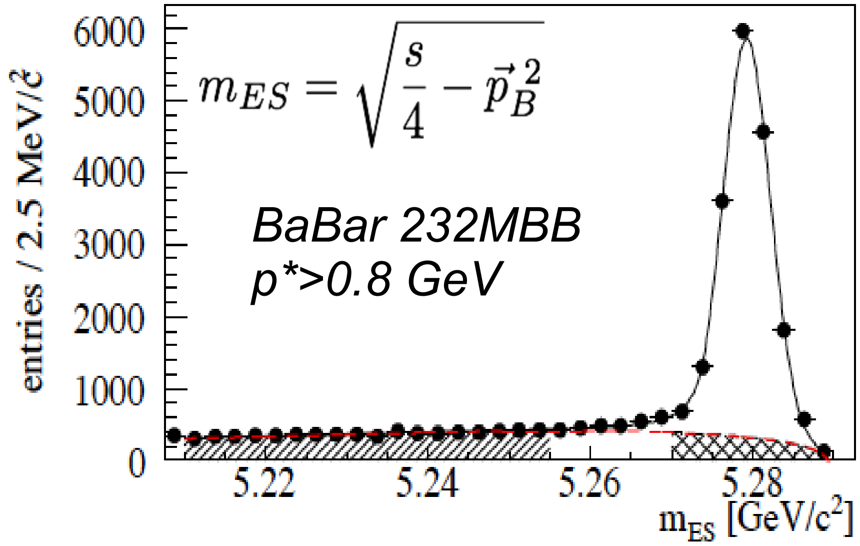
Recent experimental results

- ***BaBar:***
  - *New hadronic moments (232M BB): arXiv:0908.0415 (submitted to PRD)*
  - *Update of moments of E_ℓ spectra (52M BB): arXiv:0908.0415*
- ***Belle***
  - *Hadronic moments (152M BB): PRD75, 032001 (2007)*
  - *E_ℓ moments (152M BB): PRD75, 032005 (2007)*
  - *Moments of the γ energy spectrum and Global Fit (152M BB): PRD78, 032016 (2008)*
- ***Old measurements***
  - *DELPHI E_ℓ and m_X: EPJC45,35 (2006)*
  - *CLEO m_X: PRD70,032002(2005)*
  - *CDF m_X: PRD71,051103(2005)*

Hadronic mass moments at BFactory



- *Fully reconstruct one B: low efficiency ~0.4%*
  - *B momentum known: good resolution*
  - *Flavor determined*
  - *Kinematic quantity in B rest frame*
- *Require 1 recoiling lepton with $p>p_{min}$ in B rest frame*
- *Remaining tracks associated with the X*
  - *Missing mass & energy consistent with a ν*



Combinatorial background and continuum subtracted by fitting m_{ES}

Background modelled with a threshold function
Signal modeled with a Gaussian with exp. tails

Hadronic mass moments

- *Measure hadronic moments as function of minimum lepton momentum:*

- ***BaBar***
  - $\langle m_X^1 \rangle$ to $\langle m_X^6 \rangle$ moments for E_ℓ>0.8,... ,1.9 GeV
  - m_X: all remaining particles, mass hypothesis from particle identification

$$m_X^2 = \left| \sum (p_\pi + p_K + p_p + \gamma) \right|^2$$

$$p_{miss} = p_{\Upsilon(4S)} - p_{reco} - p_X - p_\ell$$

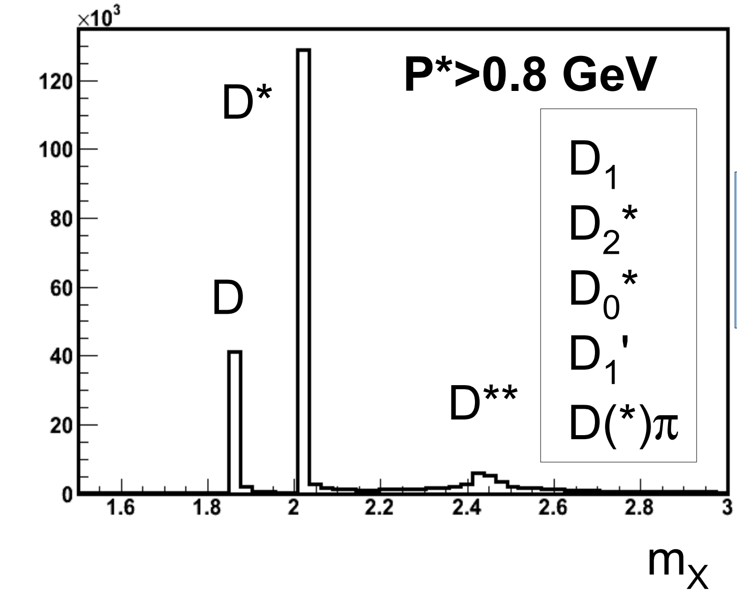
- m_X resolution improved with a kinematic fit:
  - p_{miss} consistent with a ν
  - $\sigma(m_X)$= 0.355 GeV

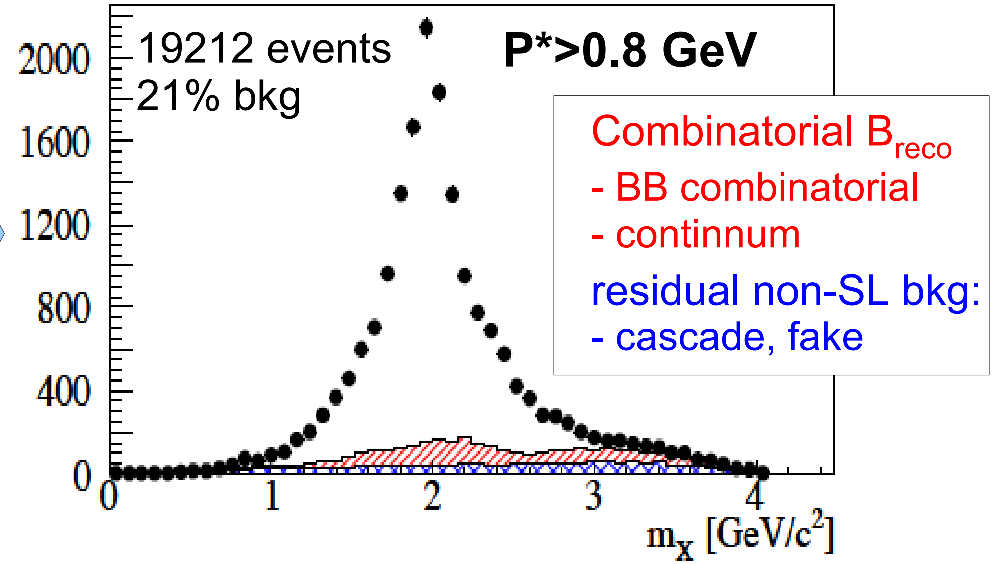
- ***Belle***
  - $\langle m_X^2 \rangle$ and $\langle m_X^4 \rangle$ moments for E_ℓ>0.7, 0.9,... ,1.9 GeV

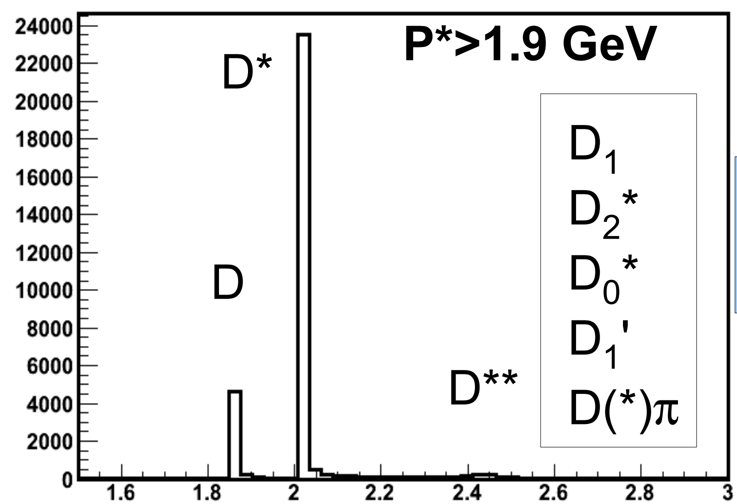
$$m_X^2 = \left| p_{\Upsilon(4S)} - p_{reco} - p_\ell - p_\nu \right|, \quad p_\nu = \left(|p_{miss}|, |\vec{p}_{miss}| \right)$$

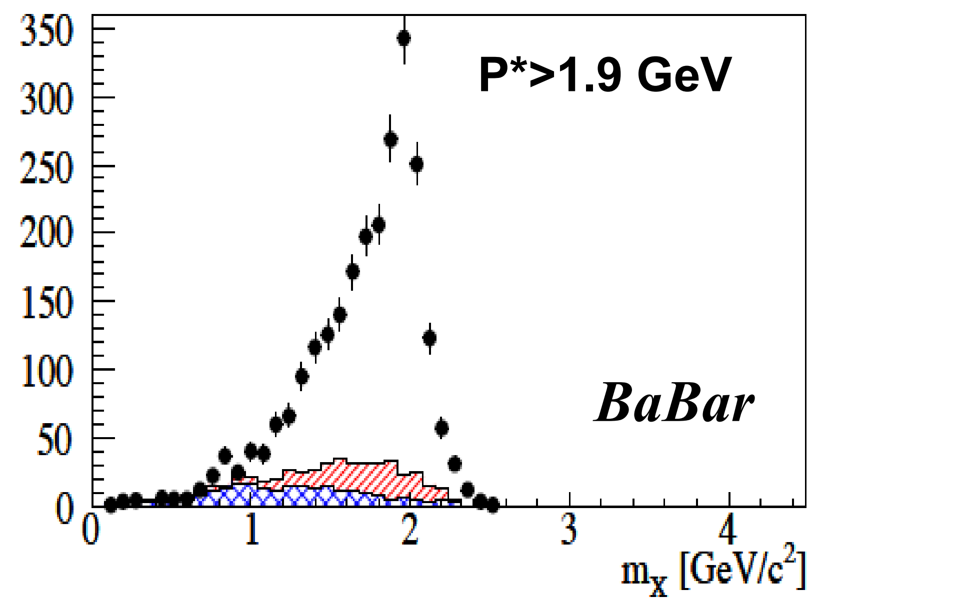
  - $\sigma(m_X^2)$= 0.8 GeV^2

Hadronic mass moments



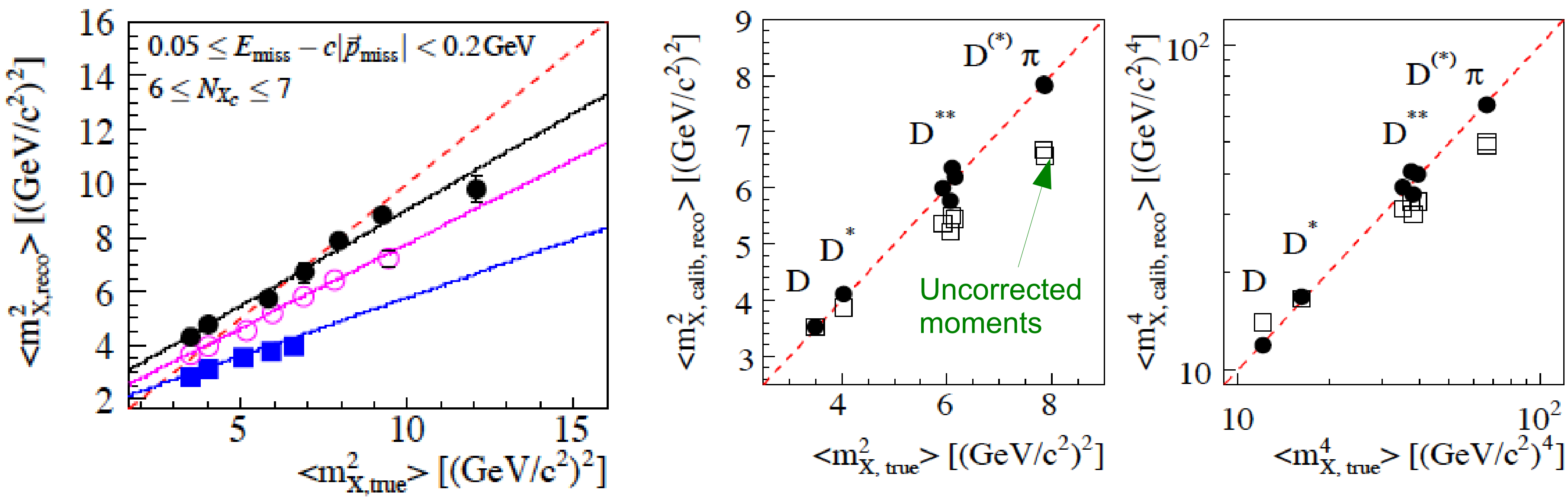






Hadronic mass moments

- *Missing particles: bias the hadronic system: 5-16% effect on <m_X>*
- ***BaBar**: Event by event m_X calibration (linear correction) functions to relate reconstructed m_X to true m_X*
  - *X charged tracks multiplicity, E_{miss}-|$\boldsymbol{P}_{miss}$| and P_ℓ* (3 x 3 x 12)

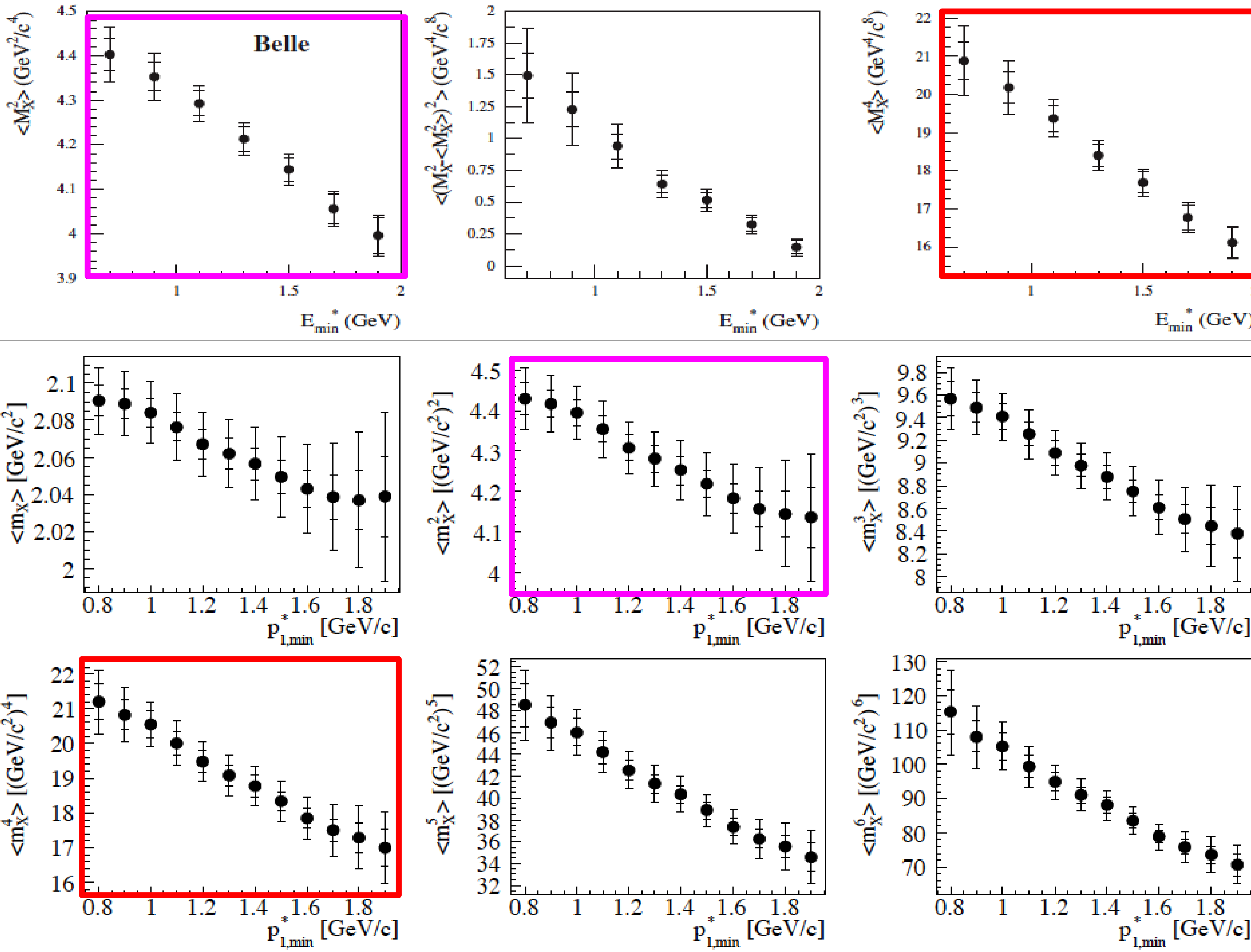


- ***Belle:** unfolding with SVD algorithm, separately for (B^0, B^+), (e, m)*

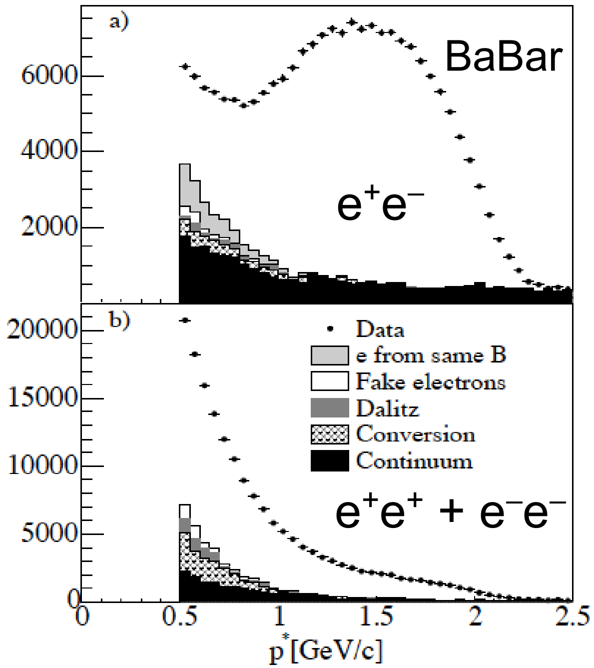
Hadronic mass moments

Belle

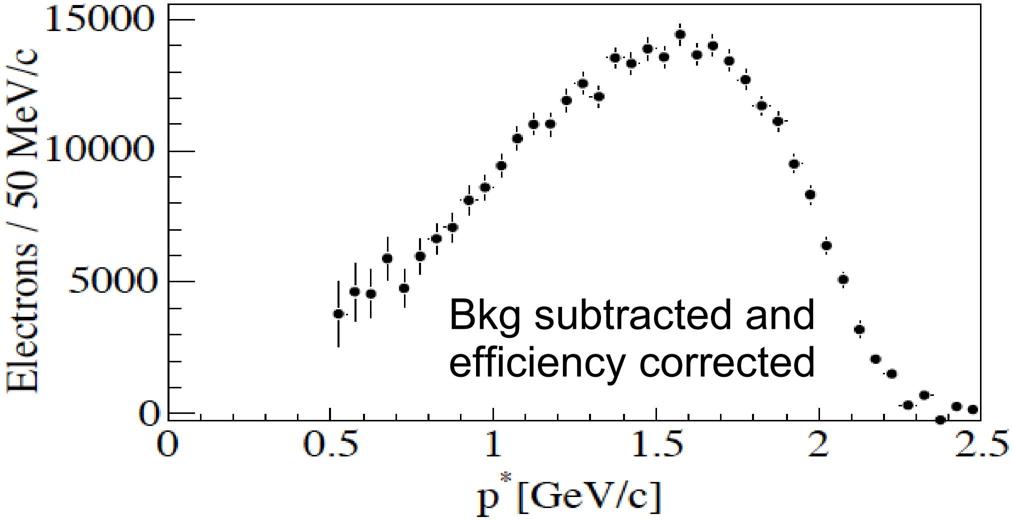
BaBar



Lepton Energy

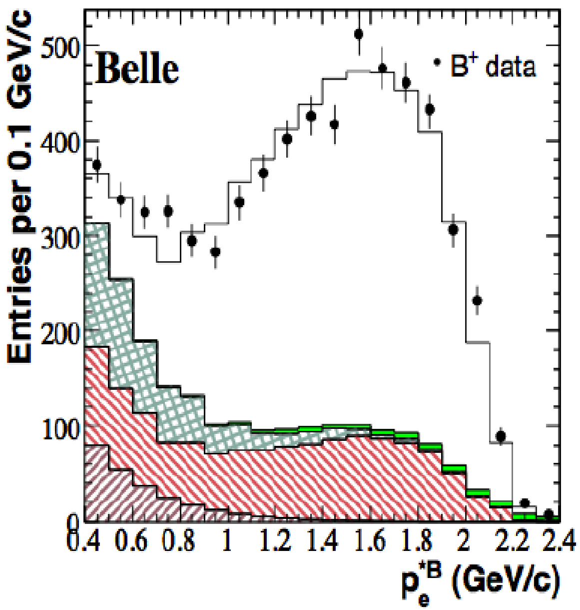


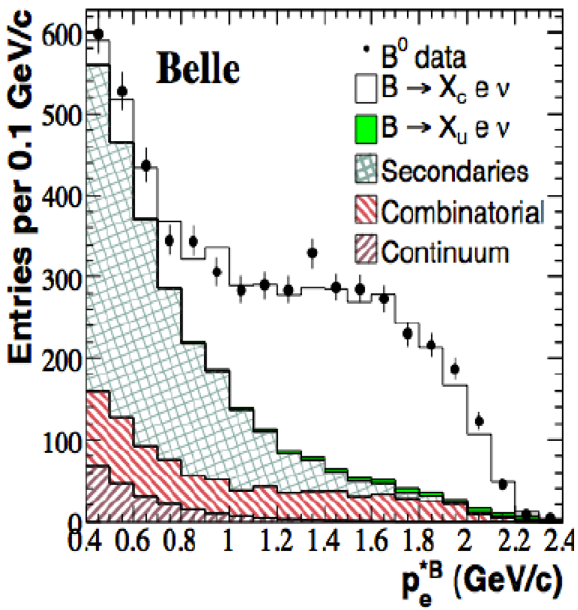
BaBar 52MBB, lepton tag (syst dominated)



Belle with 140MBB:
Use B fully reconstructed events

$\langle E_e^n \rangle$ with E_{cut}=0.4-2.0 GeV
Stat. error > syst. error

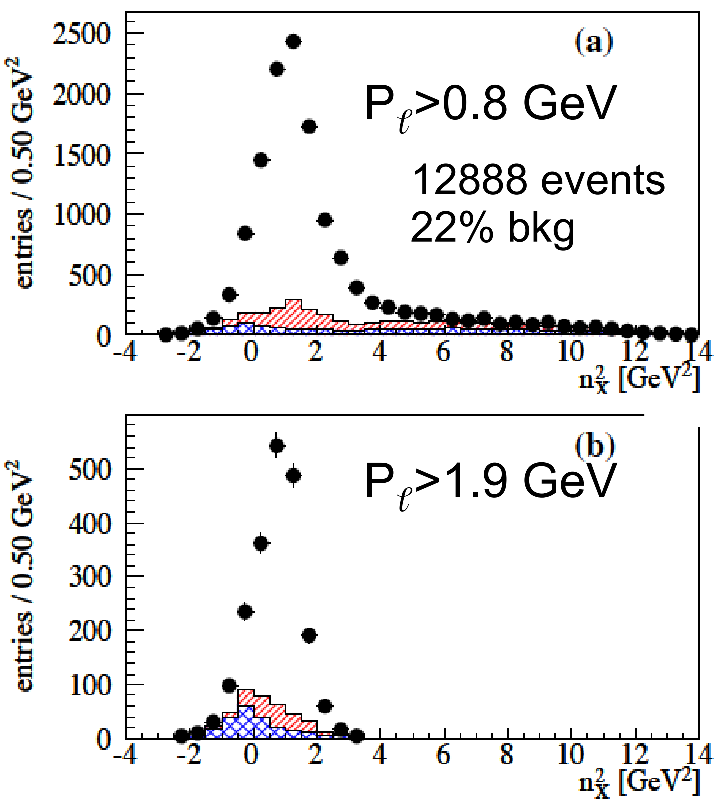


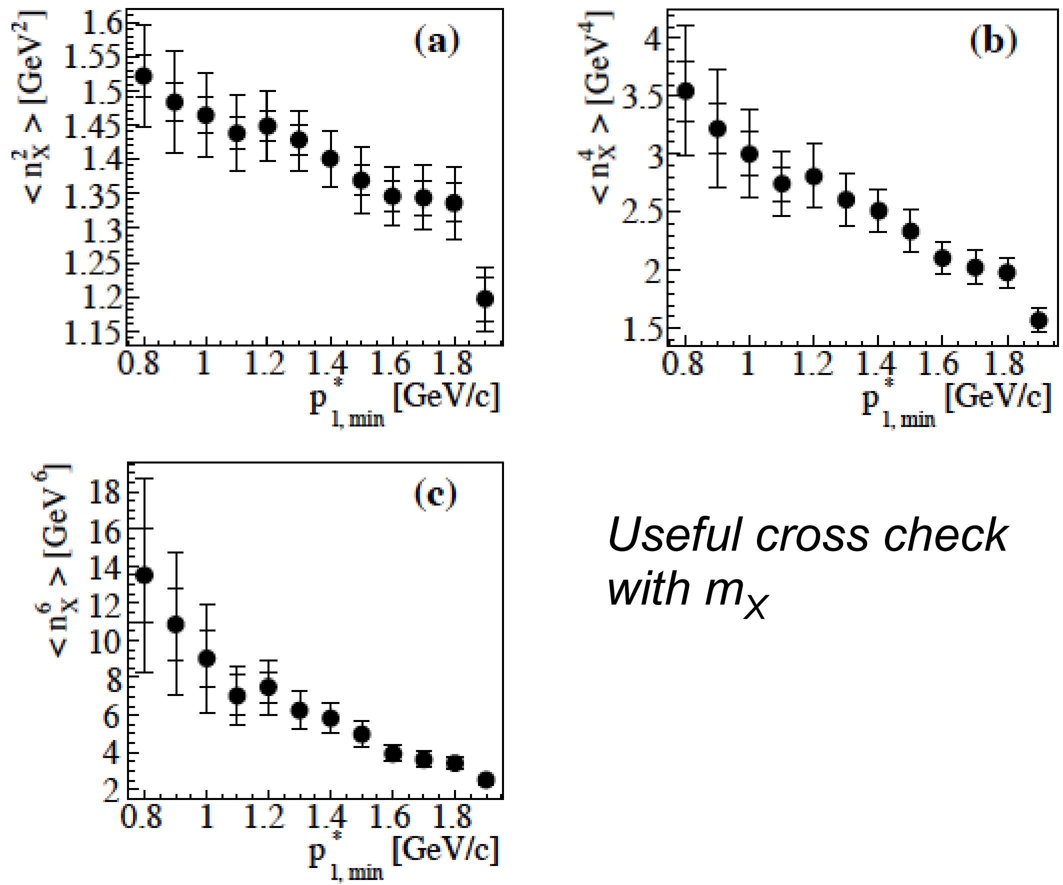


Hadronic Mixed moments

Gambino, Uraltsev
JHEP34,181(2004)

- *Modified moments: $n_X^2 = m_X^2 - 2\Lambda E_X + \Lambda^2$ (with Λ=0.65 GeV)*
  - *Combination of Hadronic Mass and Energy Moments*
  - *Expect higher sensitivity to higher order parameters*





Useful cross check with m_X

HQE fit

- *Moments fitted with a χ^2*
  - *Experimental and theoretical uncertainty have to be properly accounted*

$$\chi^2 = (\vec{Exp} - H\vec{Q}E)^T \frac{1}{C_{exp} + C_{theory}} (\vec{Exp} - H\vec{Q}E)$$

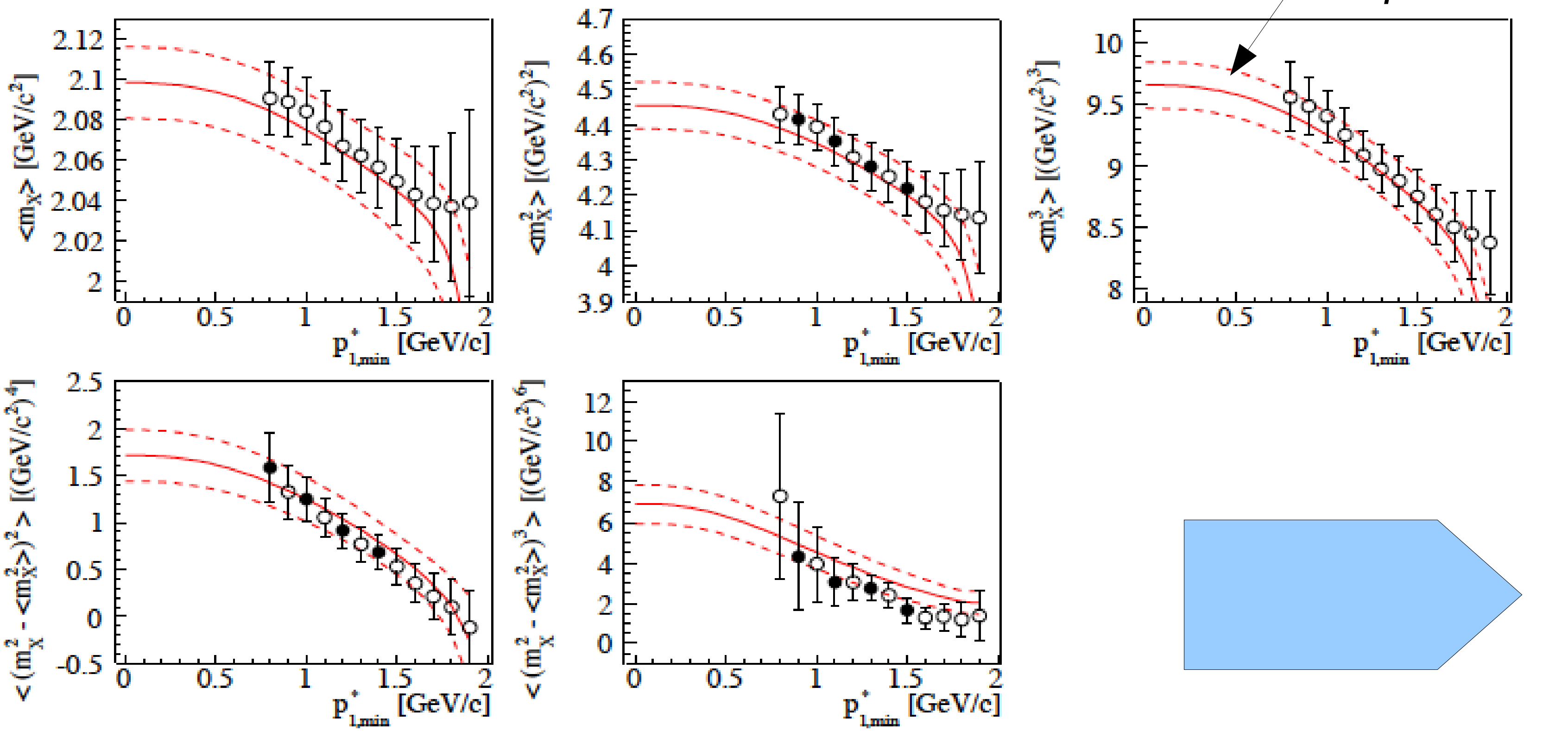
- *Available calculations (both till $1/m_b^3$ order)*
  - *Kinetic running mass (BaBar and Belle)*
  - *1S (Belle)*

Kinetic Scheme
Gambino, Uraltsev, Eur.Phys.J.C34,181(2004)
1S Scheme
Bauer, Ligeti, Luke, PRD64, 113004 ((2004)

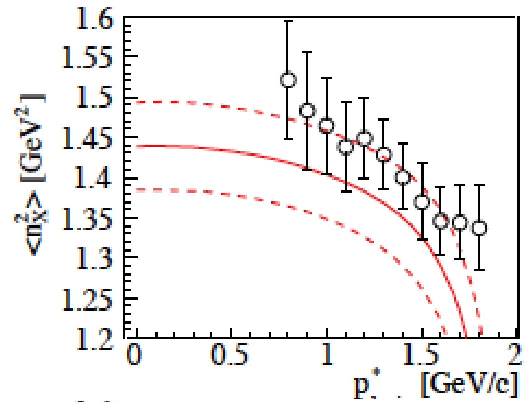
- *Reduce the number of fitted points (reduce experimental correlations)*
- *External constraint: μ_G^2=0.35 $\pm$ 0.07 GeV^2, ρ_{LS}^3=-0.15 $\pm$ 0.10 GeV^3,*
  - *Added as further term in the χ^2*
- *Theoretical uncertainty assumptions*
  - *100% correlation between different E_{lep} cut*
  - *Uncorrelated for moments of different orders and types*

BaBar OPE fit

- *Each observable has a different dependence on HQE paramaters and quark masses*
- *Parameters determined by a Global Fit using:*
  - *12 mass moments (only ● are used in the fit, O agree well with the fit result)*
  - *13 E_ℓ moments*
  - *9 E_γ moments*

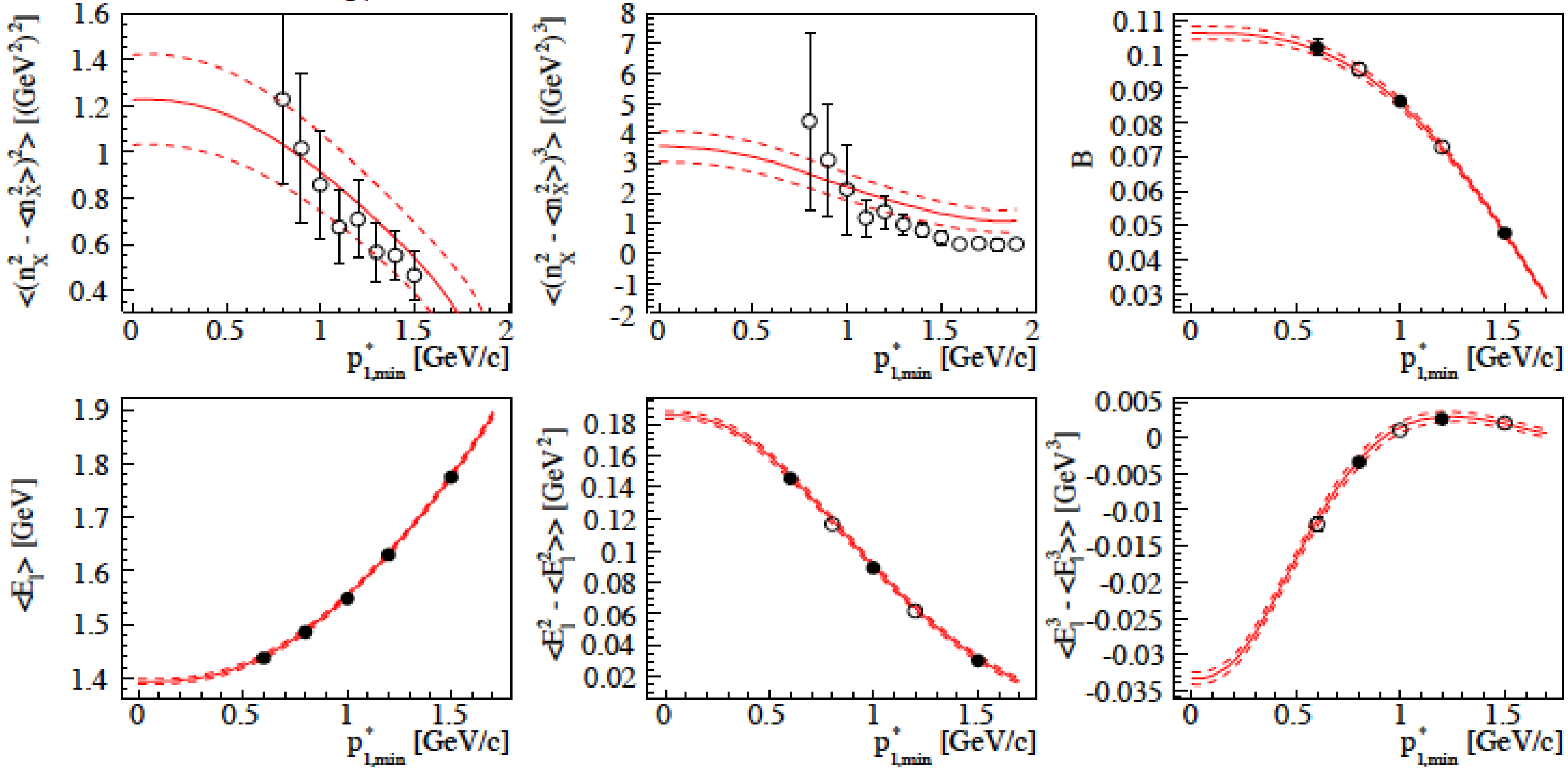


BaBar OPE fit



Fit prediction: good consistence with n_X^2 moments

- χ^2 = 11/28(ndf)



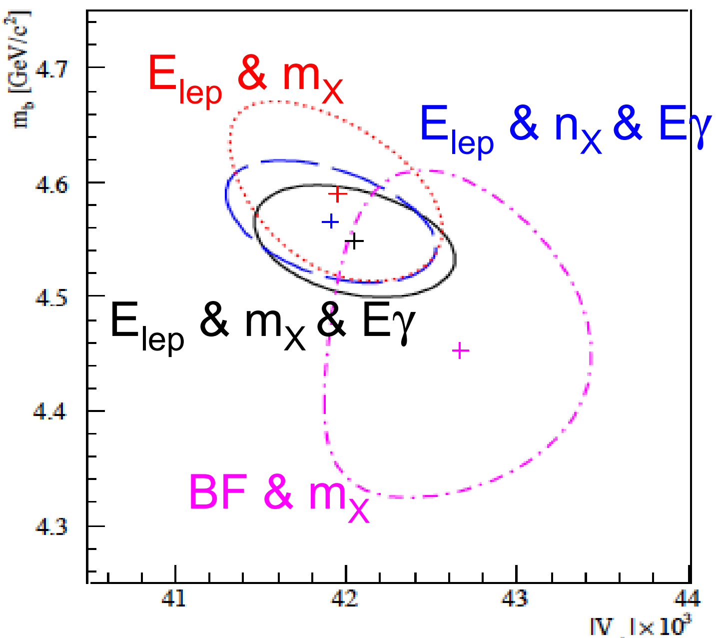
BaBar OPE fit

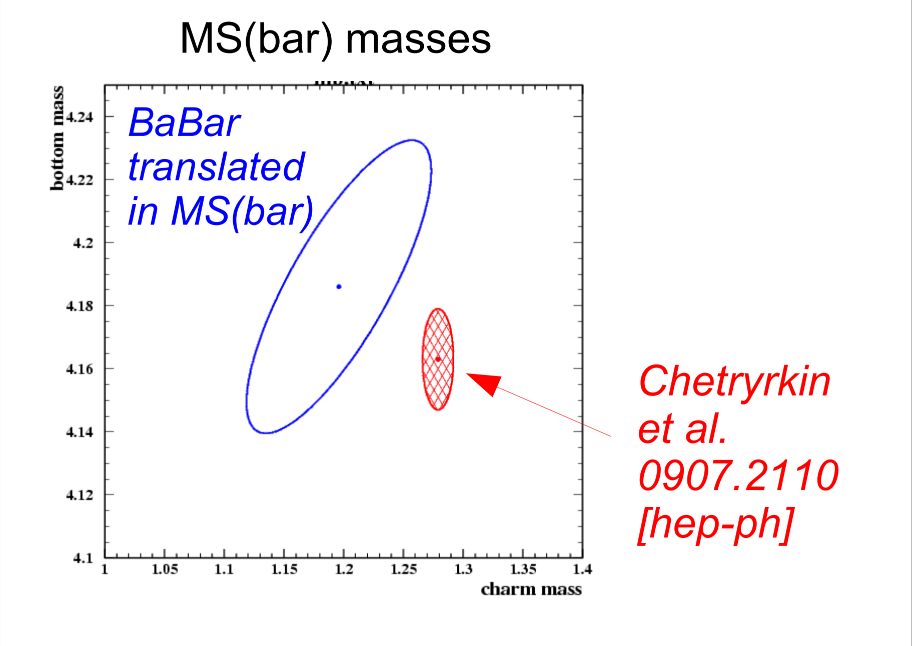
BaBar m_X
$\chi^2 = 11/28$

	$\lvert V_{cb}\rvert \times 10^3$	m_b [GeV/c^2]	m_c [GeV/c^2]	$\mathcal{B}$ [%]	μ_π^2 [GeV^2]	μ_G^2 [GeV^2]	ρ_D^3 [GeV^3]	ρ_{LS}^3 [GeV^3]
Results	42.05	4.549	1.077	10.642	0.476	0.300	0.203	-0.144
Δ_{exp}	0.45	0.031	0.041	0.165	0.021	0.044	0.017	0.075
Δ_{theo}	0.37	0.038	0.062	0.063	0.059	0.038	0.027	0.056
$\Delta\Gamma_{SL}$	0.59							
Δ_{tot}	0.83	0.049	0.074	0.176	0.063	0.058	0.032	0.094

BaBar n_X
kinetic
$\chi^2 = 8/28$

	$\lvert V_{cb}\rvert \times 10^3$	m_b [GeV/c^2]	m_c [GeV/c^2]	$\mathcal{B}$ [%]	μ_π^2 [GeV^2]	μ_G^2 [GeV^2]	ρ_D^3 [GeV^3]	ρ_{LS}^3 [GeV^3]
Results	41.91	4.566	1.101	10.637	0.452	0.304	0.190	-0.156
Δ_{exp}	0.48	0.034	0.045	0.166	0.023	0.047	0.013	0.079
Δ_{theo}	0.38	0.041	0.064	0.061	0.065	0.039	0.031	0.052
$\Delta\Gamma_{SL}$	0.59							
Δ_{tot}	0.85	0.053	0.078	0.176	0.069	0.061	0.034	0.095

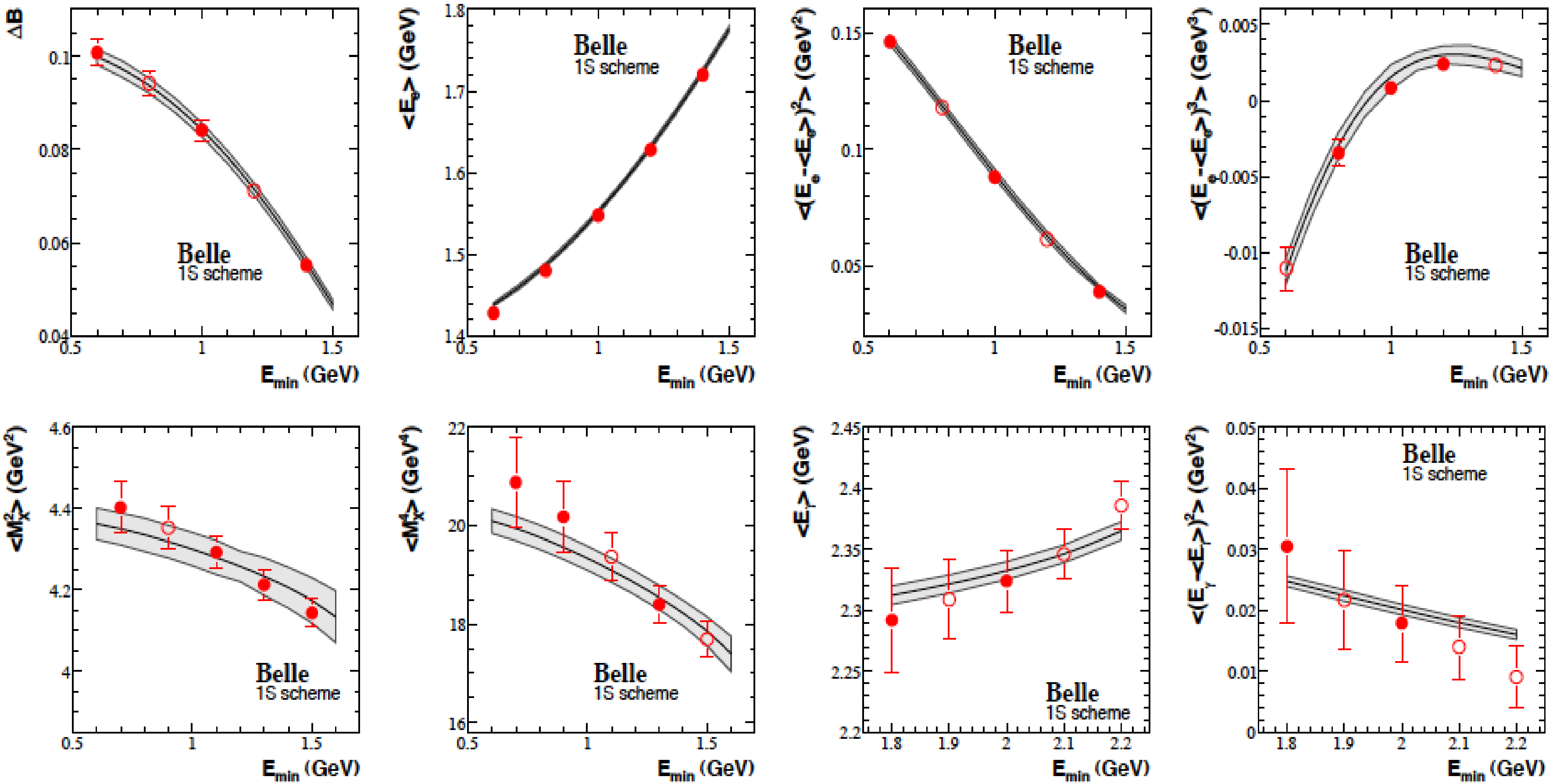




Belle OPE fit

- *7 m_x moments, 11 E_ℓ moments, 4 E_γ moments*
- *Both 1S scheme and Kinetic scheme*

Kinetic Scheme
Gambino, Uraltsev, Eur.Phys.J.C34,181(2004)
1S Scheme
Bauer, Ligeti, Luke, PRD64, 113004 ((2004)



Belle results

Belle m_X kinetic

$\chi^2 = 4.7/18$

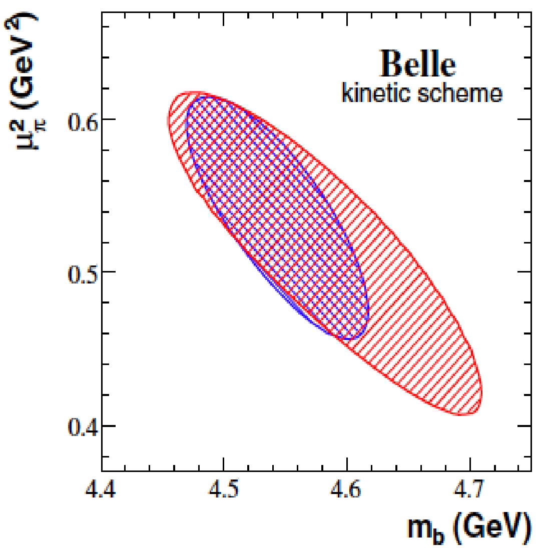
	$\lvert V_{cb}\rvert$ (10^{-3})	m_b (GeV)	m_c (GeV)	μ_π^2 (GeV^2)	$\tilde{\rho}_D^3$ (GeV^3)	μ_G^2 (GeV^2)	ρ_{LS}^3 (GeV^3)
Value	41.58	4.543	1.055	0.539	0.166	0.362	−0.153
σ(fit)	0.69	0.075	0.118	0.079	0.040	0.053	0.096
$\sigma(\tau_B)$	0.08						
σ(th)	0.58						

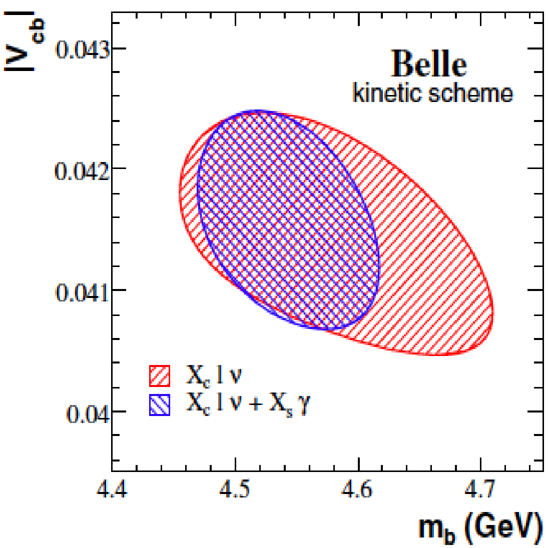
Belle m_X 1S

$\chi^2 = 7.3/18$

	$\lvert V_{cb}\rvert$ (10^{-3})	m_b (GeV)	λ_1 (GeV^2)	ρ_1 (GeV^3)	τ_1 (GeV^3)	τ_2 (GeV^3)	τ_3 (GeV^3)
Value	41.56	4.723	−0.303	0.067	0.125	−0.101	0.125
σ(fit)	0.68	0.055	0.046	0.030	0.005	0.056	0.005
$\sigma(\tau_B)$	0.08						

Different scheme are consistent (m_b compatible after proper scheme translation)





B-quark mass w/wo $b \rightarrow s\gamma$ are consistent

Comments & Questions

- *BaBar and Belle in good agreement: similar precision*
- *Moments are systematics dominated:*
  - *Limited by detector knowledge: neutral/track efficiency, PID*
  - *Signal composition: D** knowledge important*
- *Fit χ^2 is always too good:*
  - ***Are the correlations (theoretical) properly accounted?***
  - ***How sensitive is the fit result by assumptions on theoretical correlations?***
- *BaBar measures and fit n_X^2 moments:*
  - *Indicating that higher order corrections have been estimated correctly*
  - ***Are there other observables (combinations) that can be useful?***

Backup

Systematics: Belle

TABLE III. Breakup of the systematic error on $\langle M_X^2 \rangle$. Refer to the text for details.

	$\Delta\langle M_X^2\rangle$ (GeV$^2/c^4$)						
$E^*_{\rm min}$ (GeV)	0.7	0.9	1.1	1.3	1.5	1.7	1.9
Secondary/fake leptons	0.033	0.023	0.013	0.007	0.004	0.002	0.000
Combinatorial background	0.006	0.004	0.003	0.002	0.002	0.002	0.000
Continuum	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$B \to X_u \ell \nu$ background	0.004	0.004	0.004	0.004	0.006	0.007	0.009
$\mathcal{B}(D^{(*)}\ell\nu)$	0.008	0.007	0.007	0.007	0.006	0.005	0.003
$\mathcal{B}(D^{**}\ell\nu)$	0.022	0.014	0.006	0.000	0.000	0.008	0.006
$\mathcal{B}((D^{(*)}\pi)_{\text{non-res.}}\ell\nu)$	0.024	0.017	0.007	0.004	0.004	0.004	0.004
$D^{(*)}\ell\nu$ form factors	0.013	0.013	0.012	0.011	0.010	0.008	0.006
$D^{**}\ell\nu$ form factors	0.003	0.002	0.002	0.001	0.001	0.001	0.004
Unfolding	0.015	0.015	0.015	0.015	0.015	0.015	0.015
Binning	0.001	0.001	0.001	0.001	0.001	0.000	0.001
Efficiency	0.008	0.011	0.012	0.009	0.008	0.005	0.004
Total	0.052	0.041	0.029	0.024	0.022	0.022	0.021

Systematics: BaBar

k	$p^*_{\ell,\min}$ [GeV/c]	$\langle m_X^k \rangle$	σ_{stat}	σ_{sys}	MC statistics	simulation related	extraction method	back-groud	signal model
1	0.8	2.0906	±0.0063	±0.0166	0.0058	0.0099	0.0096	0.0047	0.0031
	0.9	2.0890	±0.0062	±0.0158	0.0048	0.0088	0.0103	0.0045	0.0028
	1.0	2.0843	±0.0061	±0.0153	0.0044	0.0076	0.0109	0.0044	0.0027
	1.1	2.0765	±0.0063	±0.0165	0.0044	0.0072	0.0127	0.0047	0.0026
	1.2	2.0671	±0.0064	±0.0160	0.0046	0.0073	0.0120	0.0045	0.0025
	1.3	2.0622	±0.0068	±0.0168	0.0048	0.0073	0.0131	0.0050	0.0023
	1.4	2.0566	±0.0073	±0.0183	0.0047	0.0069	0.0150	0.0054	0.0021
	1.5	2.0494	±0.0081	±0.0198	0.0036	0.0074	0.0168	0.0061	0.0019
	1.6	2.0430	±0.0092	±0.0221	0.0038	0.0082	0.0187	0.0070	0.0018
	1.7	2.0387	±0.0109	±0.0265	0.0047	0.0081	0.0232	0.0083	0.0015
	1.8	2.0370	±0.0143	±0.0337	0.0069	0.0097	0.0299	0.0098	0.0013
	1.9	2.0388	±0.0198	±0.0413	0.0082	0.0123	0.0355	0.0150	0.0008
2	0.8	4.429	±0.029	±0.070	0.027	0.047	0.030	0.018	0.008
	0.9	4.416	±0.027	±0.063	0.020	0.041	0.033	0.016	0.008
	1.0	4.394	±0.026	±0.058	0.020	0.033	0.035	0.015	0.008
	1.1	4.354	±0.026	±0.063	0.019	0.031	0.043	0.016	0.008
	1.2	4.308	±0.026	±0.058	0.019	0.030	0.039	0.015	0.007
	1.3	4.281	±0.027	±0.061	0.020	0.029	0.044	0.016	0.007
	1.4	4.253	±0.028	±0.066	0.021	0.028	0.051	0.018	0.006
	1.5	4.220	±0.031	±0.070	0.015	0.029	0.058	0.019	0.006
	1.6	4.183	±0.034	±0.078	0.015	0.032	0.065	0.022	0.005
	1.7	4.158	±0.040	±0.094	0.019	0.032	0.082	0.026	0.004
	1.8	4.145	±0.051	±0.120	0.026	0.036	0.107	0.031	0.004
	1.9	4.136	±0.069	±0.142	0.031	0.046	0.122	0.048	0.002