$B \to X_c \ell \nu$:
Status of $O(\alpha_s/m_b^2)$ calculations

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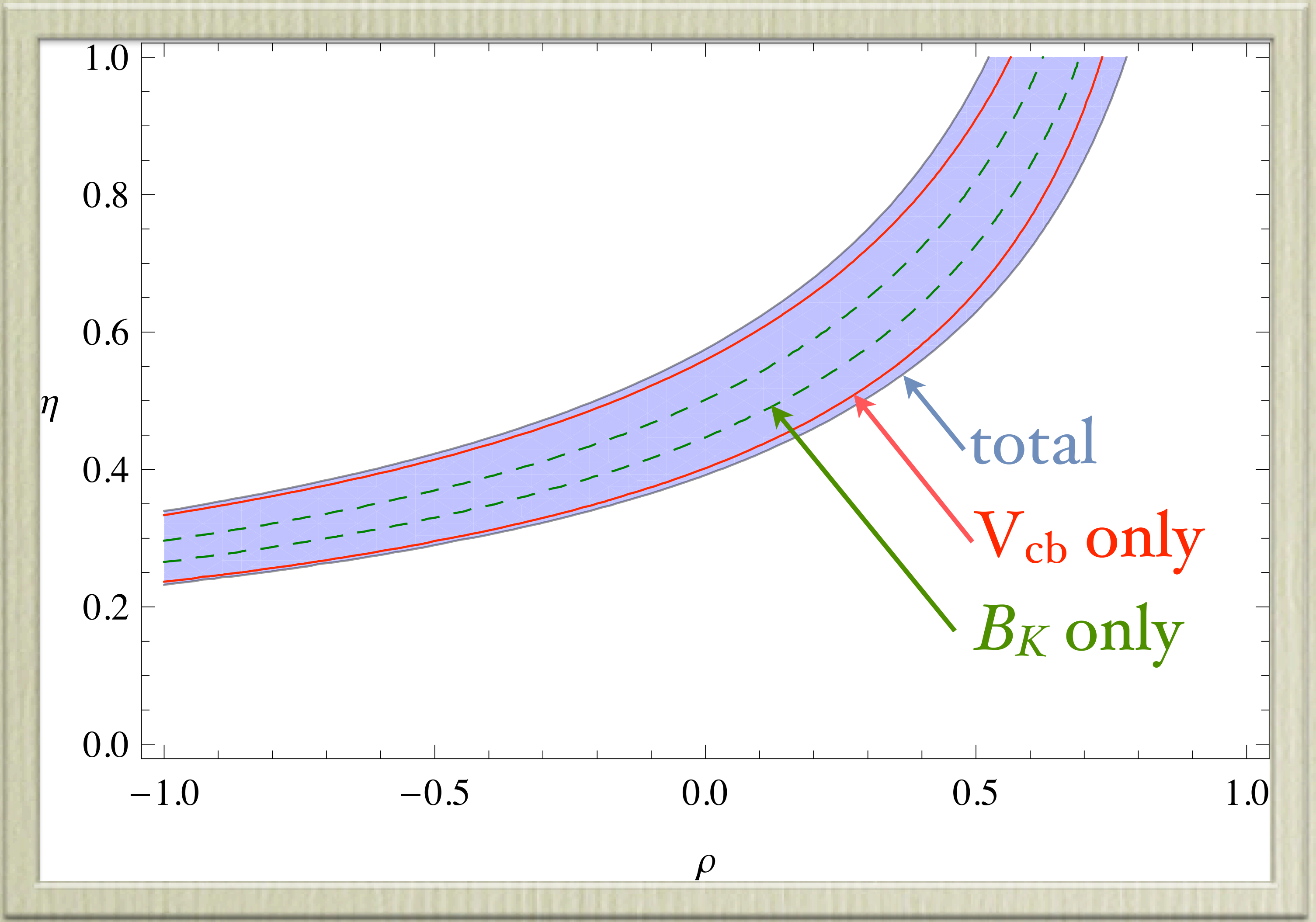
Outline

- Motivation
- Kinetic Corrections: method & results
- Chromo-magnetic corrections: status

The role of V_{ub} and V_{cb}

- V_{cb} is the dominant source of uncertainty on ε_K

$$|\varepsilon_K| = \kappa_\varepsilon C_\varepsilon \hat{B}_K |V_{cb}|^2 \lambda^2 \eta \Big(|V_{cb}|^2 (1-\bar{\rho}) \eta_{tt} S_0(x_t) + \eta_{ct} S_0(x_c, x_t) - \eta_{cc} x_c \Big)$$

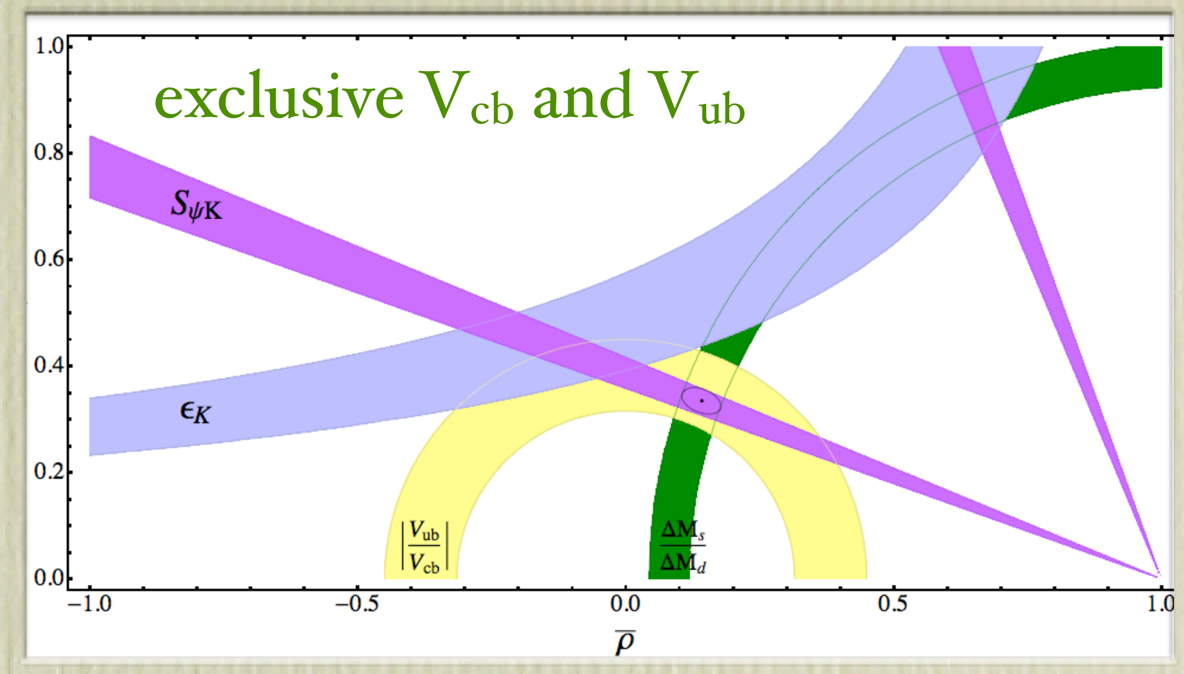


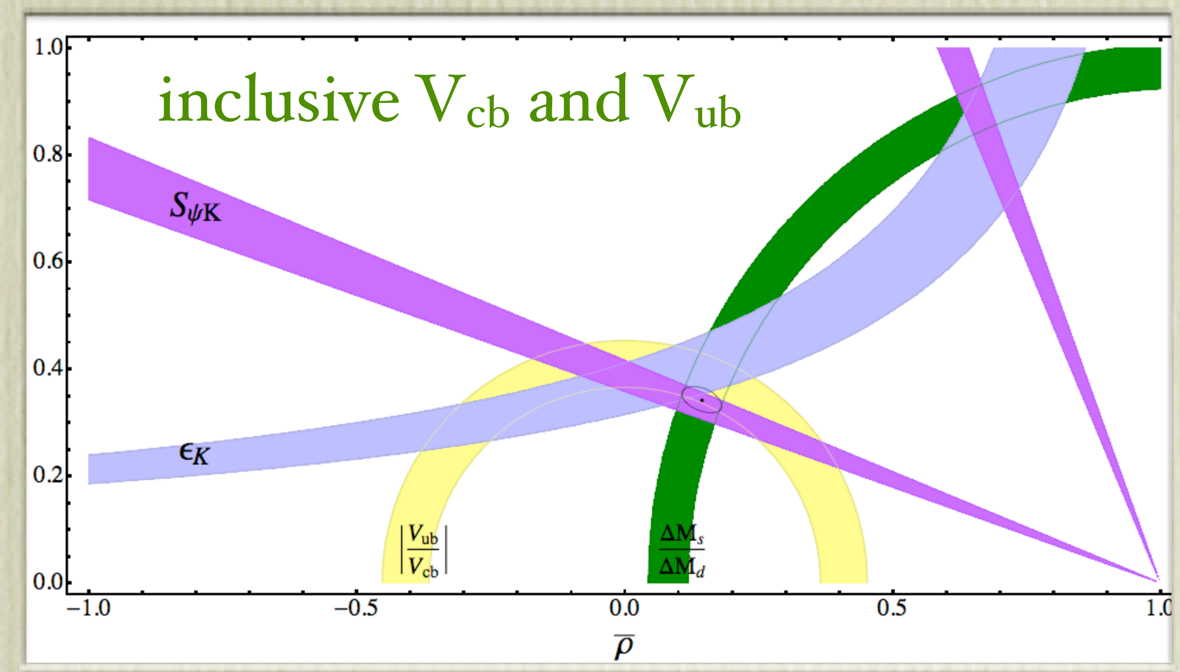
The role of V_{ub} and V_{cb}

- Inclusive and exclusive determinations of V_{cb} ***and*** V_{ub} are about 2σ apart.

- V_{cb} is critical to establish the presence of NP in the UT

- V_{ub} is measured in tree level processes that are not expected to be sensitive to NP. Its value is essential in order to decide whether we have effects in K or B mixing.





Operator Product Expansion

$$\Gamma = \frac{G_F |V_{cb}|^2 m_b^5}{192\pi^3} \left\{ f(\rho) + \kappa(\rho) \frac{\mu_\pi^2}{2m_b^2} + g(\rho) \frac{\mu_G^2}{2m_b^2} + d(\rho) \frac{\rho_D^3}{m_b^3} + l(\rho) \frac{\rho_{\rm LS}^3}{m_b^3}) \right\}$$

- Known perturbative orders ($\rho = m_c^2/m_b^2$):

$$f(\rho) = f^{(0)}(\rho) + \alpha_s\, f^{(1)}(\rho) + \alpha_s^2\, f^{(2)}(\rho)$$ → [Melnikov]

$$\kappa(\rho) = \kappa^{(0)}(\rho) + \alpha_s\, \kappa^{(1)}(\rho)$$ → [Becher, Boos, EL]

$$g(\rho) = g^{(0)}(\rho) + \alpha_s\, g^{(1)}(\rho)$$ → almost complete [Becher, Lange, EL]

$$d(\rho) = d^{(0)}(\rho) + \alpha_s\, d^{(1)}(\rho)$$ → possible with the techniques described in this talk

$$l(\rho) = l^{(0)}(\rho)$$

O($1/m_b^4$) also known at tree-level [Dassinger, Mannel, Turczyk]

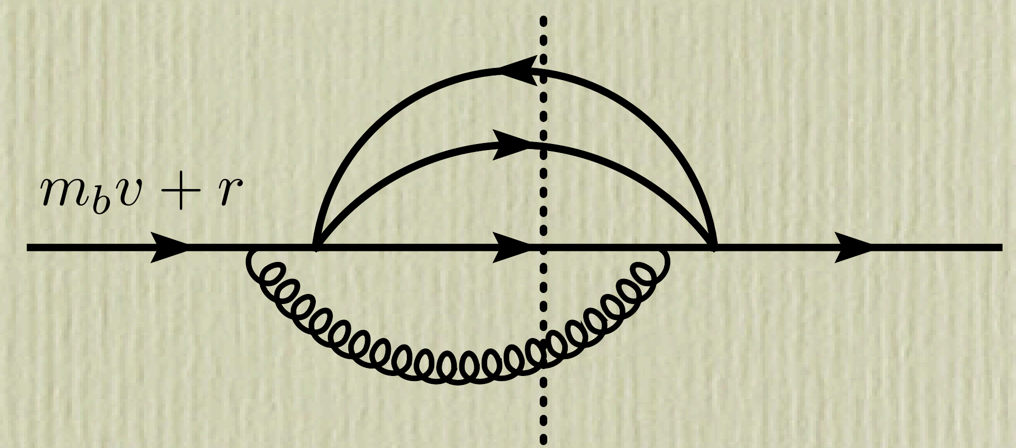
- Non-perturbative matrix elements definitions:

$$\mu_\pi^2 = -\frac{1}{2M_B} \langle \bar{B} | \bar{h}_v (iD)^2 h_v | \bar{B} \rangle \qquad \mu_G^2 = \frac{1}{2M_B} \langle \bar{B} | \frac{g_s}{2} \bar{h}_v \sigma_{\mu\nu} G^{\mu\nu} h_v | \bar{B} \rangle$$

Calculations at $O(\alpha_s/m_b^2)$

[Becher, Boos, EL; Becher, Lange, EL]

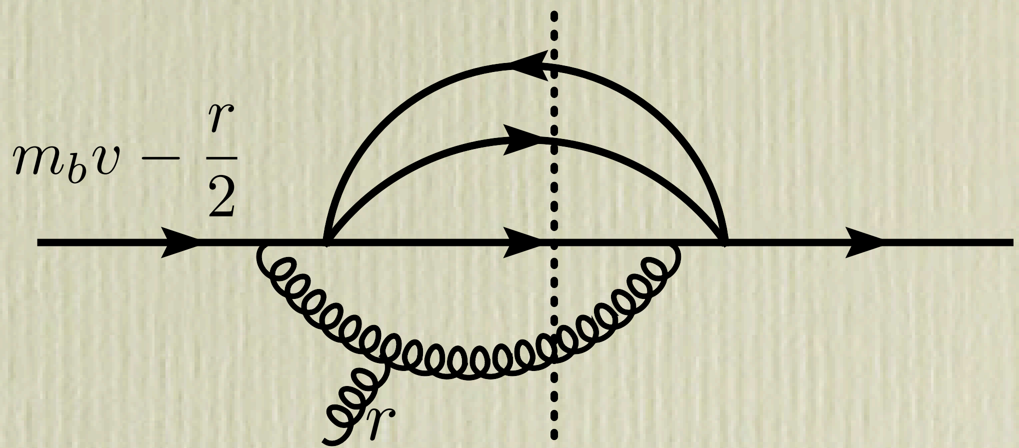
- Expected to be of the same order as NNLO corrections
- Corrections to the kinetic and chromo-magnetic operators:



Expand at $O(r^2)$

$$\bar{h}_v (iD)^2 h_v$$

[Complete]



Expand at $O(r)$

$$\frac{g_s}{2} \bar{h}_v \sigma_{\mu\nu} G^{\mu\nu} h_v$$

[Almost complete]

Kinetic corrections: methods

- Expand the 1-loop rate at second order in the residual momentum ($p_b^\mu = m_b v^\mu + r^\mu$)

- In the kinetic case we can choose whether to expand before or after loop integration

  - After: subtle but simpler. Requires only leading partonic rate at 1-loop and PS parametrization

  - Before: more complicated. Useful as check and as test ground to tackle magnetic corrections

Expansion after loop integration

- The hadronic rate can be written as: [Manohar, Wise]

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}\hat{q}^2} = \left[1 + \frac{\mu_\pi^2}{2m_b^2}\left(-1 + x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + \frac{1}{3}x^2\frac{\partial^2}{\partial x^2} + \frac{1}{3}y^2\frac{\partial^2}{\partial y^2} + \frac{2}{3}(xy - 2\hat{q}^2)\frac{\partial^2}{\partial x \partial y}\right)\right]\frac{\mathrm{d}\Gamma^{\mathrm{partonic}}}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}\hat{q}^2}$$

where $x = 2E_\nu/m_b$, $y = 2E_e/m_b$ and $\hat{q}^2 = (p_e + p_\nu)^2/m_b^2$

- Integrating by parts one gets (with a cut y_0 on E_e):

$$\left[x^n\, y^m\, (\hat{q}^2)^l\right]_{y_0} = \left[x^n\, y^m\, (\hat{q}^2)^l\right]_{y_0}^{\mathrm{partonic}}$$

$$+ \frac{\mu_\pi^2}{6m_b^2}\left[\left((n+m)^2 + 2m + 2n - 3\right) x^n\, y^m\, (\hat{q}^2)^l - 4\,m\,n\,x^{n-1}\,y^{m-1}\,(\hat{q}^2)^{l+1}\right]_{y_0}^{\mathrm{partonic}}$$

$$+ \frac{\mu_\pi^2}{6m_b^2}\left[\left((m + 2n + 1)xy_0 - 4n\hat{q}^2\right) x^{n-1} y_0^m (\hat{q}^2)^l\,\delta(y - y_0) + x^n y_0^{m+2} (\hat{q}^2)^l \delta'(y - y_0)\right]^{\mathrm{partonic}}$$

- Insert Phase Space parametrization (x and $\hat{q}^2$ are functions of y)
- Derivatives act on the whole integrand (including $x^n y^m (\hat{q}^2)^l$)

Expansion before loop integration

- We use on shell b-quarks: $p_b^2 = (m_b v + r)^2 = m_b^2 \quad \Rightarrow \quad v \cdot r = -r^2/(2m_b)$

- The partonic rate expanded in r and averaged over $r_\perp$ is:

$$d\Gamma^{\text{partonic}} = A - A_\mu \, v^\mu \frac{r^2}{2m_b^2} + A_{\mu\nu} \frac{r^2}{m_b^2} \frac{1}{d-1} (g_{\mu\nu} - v_\mu v_\nu)$$

- Replacing b-quark with B-meson matrix elements we get:

$$d\Gamma = A - \frac{\mu_\pi^2}{2m_b^2} \left[A - A_\mu \, v^\mu + A_{\mu\nu} \frac{2}{d-1} (g_{\mu\nu} - v_\mu v_\nu) \right]$$

- After the expansion the structure of the result is

$$d\Gamma \propto \frac{d^{d-1}p_e}{2E_e} \frac{d^{d-1}p_\nu}{2E_\nu} \left[f_0 \, \delta\left(p_c^2 - m_c^2\right) + f_1 \, \delta'\left(p_c^2 - m_c^2\right) + f_2 \, \delta''\left(p_c^2 - m_c^2\right) \right]$$

where f_i contain integration over Feynman parameters (virtual corrections) or gluon momentum (real corrections)

Expansion before loop integration

- Evaluation of delta functions require phase space parametrization with off-shell charm quark:

$$\Gamma \propto \int [\mathrm{d}\Pi]\, f_0\Big|_{p_c^2=m_c^2} - \frac{\mathrm{d}}{\mathrm{d}p_c^2}\int [\mathrm{d}\Pi]\, f_1\Big|_{p_c^2=m_c^2} + \frac{\mathrm{d}^2}{\mathrm{d}(p_c^2)^2}\int [\mathrm{d}\Pi]\, f_2\Big|_{p_c^2=m_c^2}\,,$$

where $[d\Pi]$ is the complete $b \to c\, e\, \nu\, (g)$ phase space with $p_c^2 \neq m_c^2$

- Requirements on the measure parametrization:
  - map both phase-space and loop integration on the unit hypercube
  - use lepton energy as variable for easy implementation of cut
  - restrict IR divergences to a single variable

Phase space parametrization

$$\int \left[\mathrm{d}\Pi_{b\to c+g+\ell+\bar\nu}\right] = \int \frac{dp_X^2}{2\pi} \int \left[\mathrm{d}\Pi_{b\to X+\ell+\nu}\right] \int \left[\mathrm{d}\Pi_{X\to c+g}\right]$$

- Three body phase space:

$$\int_{m_c^2}^{m_b^2} \frac{dp_x^2}{2\pi} \int \left[\mathrm{d}\Pi_{b\to x+\ell+\bar\nu}\right]$$
$$= \frac{\Omega_{d-1}\Omega_{d-2}\, m_b^{4-4\epsilon}}{2^{d+1}(2\pi)^{2d-2}} \int_0^{1-\rho} dy \int_{y_0}^{1} d\lambda_2 d\lambda_3 (1-\rho-y)^{2-2\epsilon} \kappa^{2\epsilon-2} \left(y(1-\lambda_2)\right)^{1-2\epsilon} \left((1-\lambda_3)\lambda_3\right)^{-\epsilon}$$

where $p_b = (m_b, 0, 0, 0) \quad p_\ell = (E_\ell, 0, 0, E_\ell) \quad p_\nu = (E_\nu, E_\nu \sin\theta_1, 0, E_\nu \cos\theta_1)$

$$E_\ell = m_b \frac{y}{2} \qquad E_\nu = m_b \frac{(1-\rho-y)(1-\lambda_2)}{2\kappa} \qquad \cos\theta_1 = 2\lambda_3 - 1$$

$$\rho = \frac{m_c^2}{m_b^2} \qquad \kappa = 1 - (1-\cos\theta_1)\, y/2 \qquad \Omega_d = \frac{2\pi^{d/2}}{\Gamma(d/2)}$$

- IR divergences in real corrections are controlled by $E_g \propto p_X^2 - \rho^2 = \lambda_2 (1-\rho-y)$

Phase space parametrization

$$\int \left[\mathrm{d}\Pi_{b\to c+g+\ell+\bar{\nu}}\right] = \int \frac{dp_X^2}{2\pi} \int [\mathrm{d}\Pi_{b\to X+\ell+\nu}] \int \left[\mathrm{d}\Pi_{X\to c+g}\right]$$

- Two body phase space:

$$\int \left[\mathrm{d}\Pi_{x\to c+g}\right] = \frac{1}{2(2\pi)^{d-2}} \int d\cos\theta_2 \, \frac{\sin^{d-4}\theta_2 \, E_g^{d-2}}{p_x^2 - m_c^2} \int d^{d-2}\vec{p}_\perp$$

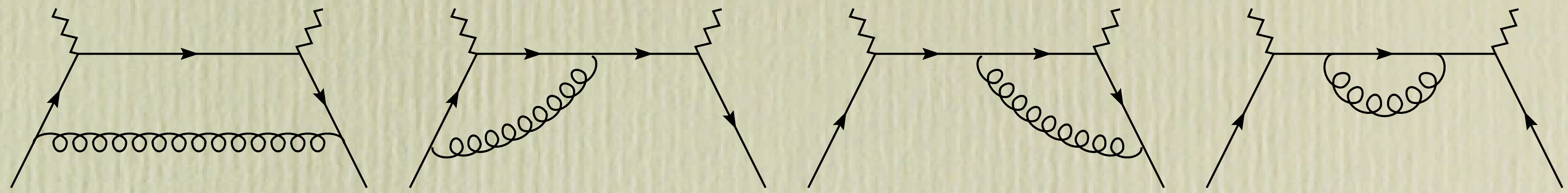
$$p_g = (E_g, 0, 0, 0) + E_g \cos\theta_2 \, (0, \frac{\vec{p}_x}{|\vec{p}_x|}) + E_g \sin\theta_2 \, (0, \vec{p}_\perp)$$

$$p_\perp = \sin\theta_3 \, (0, 0, 1, 0) + \cos\theta_3 \frac{1}{|\vec{p}_x|}(0, E_\nu c_1 + E_l, 0, -E_\nu s_1)$$

$$E_g = \frac{p_x^2 - m_c^2}{2(E_x - \cos\theta_2 |\vec{p}_x|)}$$

- All kin. quantities are expressed in terms of the integration variables
- IR divergences appear in the limit $E_g \propto \lambda_2 \to 0$: $\dfrac{1}{\lambda_2^{1+2\epsilon}} = -\dfrac{1}{2\epsilon}\delta(\lambda_2) + \left[\dfrac{1}{\lambda_2}\right]_+$

Summary of calculation

- Diagrams: 
- Expansion in the residual momentum ⇒ $\delta^{(n)}(p_c^2)$
- Phase space parametrization
- Evaluation of $\delta^{(n)}(p_c^2)$ for real and virtual contributions
- Isolation of IR divergences in real diagrams
- Renormalization (Larin convention for axial current in d dimensions)
- hurrah!

Kinetic corrections: results

- Moments with non vanishing leading power contributions ($E_\ell > 1$ GeV):

	1	$\frac{\alpha_s}{\pi}$	$\frac{\mu_\pi^2}{2m_b^2}$	$\frac{\alpha_s}{\pi}\,\frac{\mu_\pi^2}{2m_b^2}$	%
1	$0.5149(3)$	$-0.910(3)$	$-0.5692(6)$	$0.987(8)$	0.1
$\hat{E}_l$	$0.1754(1)$	$-0.314(1)$	$0.0109(3)$	$-0.024(3)$	$0.$
$\hat{E}_l^2$	$0.06189(5)$	$-0.1128(5)$	$0.1105(1)$	$-0.202(1)$	-0.2
$\hat{E}_l^3$	$0.02251(2)$	$-0.0418(2)$	$0.09269(5)$	$-0.1722(7)$	-0.6
$\hat{E}_x$	$0.2111(1)$	$-0.365(1)$	$-0.5694(2)$	$1.010(3)$	0.4
$\hat{E}_x^2$	$0.08917(7)$	$-0.1482(7)$	$-0.3378(1)$	$0.576(1)$	0.5
$\hat{E}_x^3$	$0.03867(4)$	$-0.0606(4)$	$-0.16898(6)$	$0.2639(7)$	0.5

- Effects are tiny as expected

- Impact on the extraction of V_{cb} should be small

Kinetic corrections: results

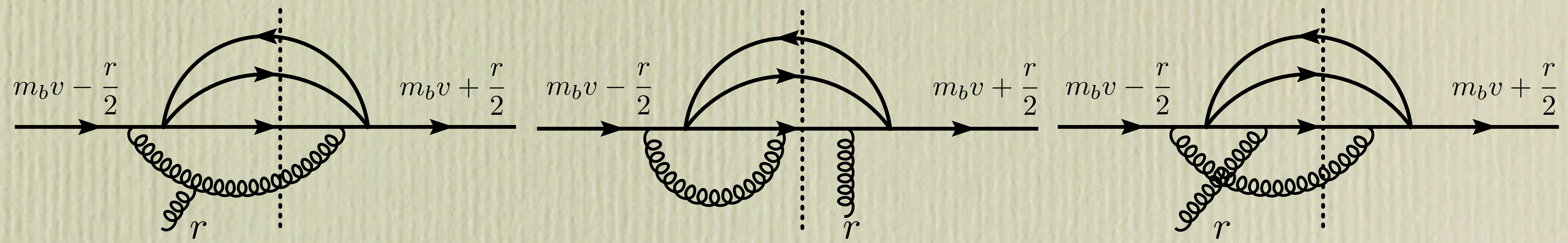
- Hadronic moments with no leading power contributions ($E_\ell > 1$ GeV):

	1	$\frac{\alpha_s}{\pi}$	$\frac{\mu_\pi^2}{2m_b^2}$	$\frac{\alpha_s}{\pi}\frac{\mu_\pi^2}{2m_b^2}$	%
$(\hat{p}_x^2-\rho)$	0	0.03618(2)	−0.6855(2)	1.213(2)	−25.5
$(\hat{p}_x^2-\rho)^2$	0	0.002808(2)	0.15198(4)	−0.4388(5)	−21.6
$(\hat{p}_x^2-\rho)^3$	0	0.0004053(3)	0	0.020998(4)	32.9
$\hat{E}_x(\hat{p}_x^2-\rho)$	0	0.01801(1)	−0.20707(6)	0.2961(8)	−39.2
$\hat{E}_x(\hat{p}_x^2-\rho)^2$	0	0.0015307(10)	0.06794(2)	−0.1897(3)	−20.1
$\hat{E}_x^2(\hat{p}_x^2-\rho)$	0	0.009147(6)	−0.05271(2)	0.0304(3)	12.4

- Impact on the extraction of μ_π^2 is expected to be $O(20\%)$
- Size of corrections as estimated in kinetic scheme ($\mu_\pi^2 \pm 20\%$) but they are different for the various moments
- Typical size of correction is $8 \times \frac{\alpha_s}{4\pi}\left(\frac{\Lambda_{QCD}}{m_b}\right)^2$: one order of magnitude larger than estimate in $1S$ fits

Magnetic corrections: solved issues

- Diagrams with external soft gluon:



- Perform the calculation off-shell ($p_b^2 = (m_b v \pm r/2)^2 \neq m_b^2$)
- Expansion in the residual momentum to linear order
- Phase space parametrization with off-shell gluon ($p_g^2 \neq 0$)
- The external gluon is slightly off-shell ⇒ use background field gauge

Magnetic corrections: solved issues

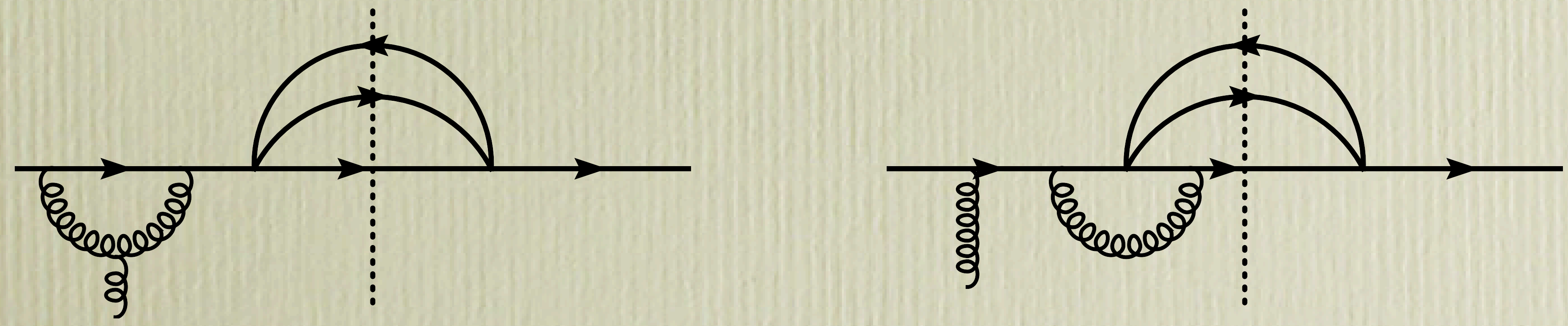
- Two operators have non vanishing projection onto $\bar{u}_v [\not{r}_\perp, \not{\epsilon}_\perp] u_v$:

$$O_{\rm mag} = \frac{g_s}{2} \bar{h}_v \sigma_{\mu\nu} G^{\mu\nu} h_v$$

$$O^{\rm EOM} = \frac{1}{m_b} \mathcal{L}_{\rm HQET} = \frac{1}{m_b} \bar{h}_v \, iv \cdot D \, h_v + \frac{1}{2m_b^2} \bar{h}_v \, (iD_\perp)^2 \, h_v + \frac{g_s}{4m_b^2} \bar{h}_v \, \sigma_{\mu\nu} G^{\mu\nu} \, h_v$$

$\Rightarrow$ calculate matrix elements with gluons polarized in both the transverse ($\epsilon_\perp \cdot v = 0$) and v^μ directions ($\epsilon_\parallel^\mu = (\epsilon_\parallel \cdot v) v^\mu$) and subtract

- External legs emissions contribute to the matching onto the chromomagnetic operator (because of the r expansion)



Phase space parametrization

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- Two body phase space:

$$\int \frac{dp_c^2}{2\pi} \int \frac{dp_g^2}{2\pi} \int \left[\mathrm{d}\Pi_{x\to c+g}\right] = \frac{1}{4(2\pi)^{d-2}} \int \frac{dp_c^2}{2\pi} \int \frac{dp_g^2}{2\pi} \int d\cos\theta_2 \, \frac{\sin^{d-4}\theta_2 \, |\vec{p}_g|^{d-2}}{E_x|\vec{p}_g| - \cos\theta_2 E_g \vec{p}_x} \int d^{d-2} p_\perp$$

$$p_g = (E_g, 0, 0, 0) + |\vec{p}_g| \cos\theta_2 \, (0, \frac{\vec{p}_x}{|\vec{p}_x|}) + |\vec{p}_g| \sin\theta_2 \, (0, \vec{p}_\perp)$$

$$|\vec{p}_g| = \frac{E_x \sqrt{\left(p_x^2 - p_c^2\right)^2 - 2\left(p_c^2 + p_x^2 + 2\sin^2\theta_2 \, \vec{p}_x^2\right) p_g^2 + p_g^4} + \cos\theta_2 \left(p_x^2 - p_c^2 + p_g^2\right) |\vec{p}_x|}{2\left(p_x^2 + \sin^2\theta_2 \vec{p}_x^2\right)}$$

- In the calculation of the diagrams we encounter terms proportional to $\delta(p_c^2 - m_c^2)\delta(p_g^2)$, $\delta'(p_c^2 - m_c^2)\delta(p_g^2)$ and $\delta(p_c^2 - m_c^2)\delta'(p_g^2)$

- There are some subtleties in the calculation of $\delta'(p_g^2)$ (boundary of the three body phase space integration is p_g^2 dependent)

Status and TODO

- Calculation of real and virtual corrections is completed
- Missing steps:
  - Renormalization in the full theory (UV divergencies of the diagrams)
  - Renormalization in the effective theory (IR divergencies of the diagrams): possible issue with the presence of O^{EOM} and its mixing with O_{mag}
- Implementation in global fit
- Calculation of Darwin corrections ($k^2(v \cdot k)$) not too complicated