Second order QCD effects in semileptonic B-decays

Kirill Melnikov

Johns Hopkins University

In collaboration with S. Biswas

Vxb Workshop, SLAC, October 30, 2009

Outline

- Introduction
- Techniques
- Results
- Conclusions

I use the following notations for radiative corrections

NLO = one-loop = $\mathcal{O}(\alpha_s)$

NNLO = two-loop = $\mathcal{O}(\alpha_s^2)$

Introduction

- Semileptonic B decays, B →Xc+ leptons, are benchmark processes for B factories – well-measured experimentally, well-understood theoretically
- Operator product expansion in the inverse mass of a heavy quark is the main theoretical tool to describe semileptonic B decays

  Shifman, Voloshin, Chay, Georgi, Grinstein etc.
- B → Xc + leptons provide information on Vcb, mb, mc, non-perturbative physics of B-mesons etc.

  Buchmuller, Flacher, Bauer, Ligeti, Luke, Manohar; BABAR
- Measured moments of kinematic invariants
  - lepton energy
  - hadron energy and/or hadron invariant mass

$$\mathcal{M}_n(s_i) = \int_{E_{\rm cut}} {\rm dE_l}\, (s_i)^n \frac{{\rm d}\Gamma}{{\rm d}E_l}$$

- Moments are measured in dependence of the lepton energy cut, to probe different kinematic regions

Introduction

- B-meson decays are intermediate-scale processes with energy release of about few GeV. Non-perturbative and perturbative physics go hand in hand
- Non-perturbative effects in semileptonic $B \to X_c$ decays are very well-understood
- It took a while to reach similar level of understanding for perturbative corrections. Even at NLO, radiative corrections became fully known only in 2004 and were used for consistent computation of hadronic moments Trott, Uraltsev, Aquila, Gambino, Ridolfi, Ossola
- It is important to carefully DEFINE what is meant by perturbative corrections, if we want to have convergent perturbative expansion Uraltsev, Hoang, Beneke

Introduction

- NNLO QCD corrections to $b \to X_c$ + leptons may be important because of
  - very high precision of existing measurements (few percent or better, for most moments),
  - aggressive estimates of theoretical uncertainties in V_{cb}, m_b, m_c, etc.
- Analyic calculations of NNLO QCD corrections to the total rate for $b \to X_c$ transition and moments, without lepton energy cut, exist

  Czarnecki, Pak, Dowling, Piclum

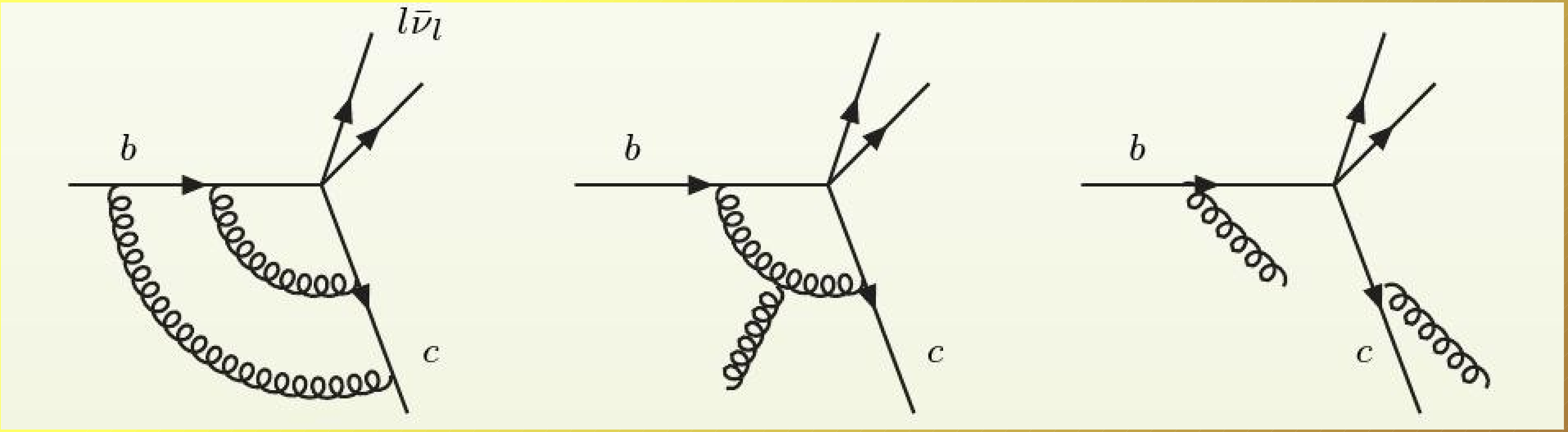
- Unclear if these results are sufficiently realistic for the comparison with experiment where lepton energy cuts of up to 1.5 GeV are applied

Techniques

- Want to have a computation of NNLO QCD corrections that is realistic and flexible. This means that the calculation must be able to
  - handle large number of different moments
  - deal with complicated nature of restrictions on final states
  - address observables which go beyond moments as used now
  - describe more complex final states (e.g. $B \to X_c$ + tau + neutrino)
- Similar requirements for perturbative computations exist in hadron collider physics, where realistic description of (partonic) final states is crucial.
- We apply techniques for two-loop computations developed in the context of hadron collider physics to compute NNLO QCD corrections to $b \to c$ transitions   Anastasiou, K.M., Petriello

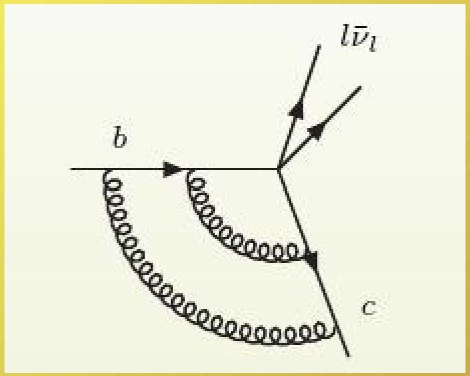
Techniques

- To compute NNLO QCD corrections we must deal with three types of processes with different number of gluons in the final state
- The three contributions have to be treated differently, yet they are not fully independent since only the sum of three is free of infra-red and collinear divergences

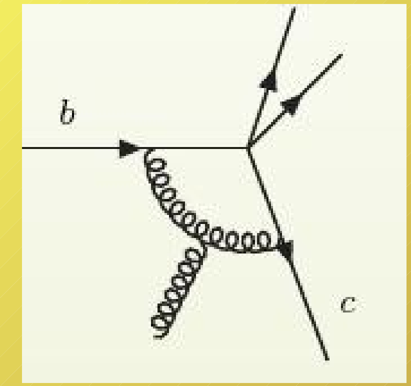


Techniques

- Two-loop virtual corrections are hard, as always
- We compute the two-loop virtual corrections by
  - Feynman parameterization
  - sequential integration over loop momenta
  - sector decomposition technique, to extract singularities from integrands over Feynman parameters Binoth, Heinrich
  - numerical integration of expressions, free of singularities
- Interestingly, massive particles (a nightmare for analytic computations!) and decay kinematics help a lot in numerical computations because we deal with less singular expressions



Techniques



- With gluons in the final state, the situation changes since we want to achieve two things
  - avoid integration over gluon energies and angles
  - extract singularities to ensure cancellation with the two-loop virtual corrections
  - this is accomplished by rewriting all singular integrals for real emissions as

$$\int_0^1 \frac{\mathrm{d}\omega}{\omega^{1+\epsilon}} F(\omega) = -\frac{1}{\epsilon} F(0) + \int_0^1 \mathrm{d}\omega \, \frac{F(\omega) - F(0)}{\omega}$$

- Further complication in one-loop corrections to $b \rightarrow c + g +$ lepton is the appearance of imaginary parts ; non-trivial to deal with in a fully numerical approach

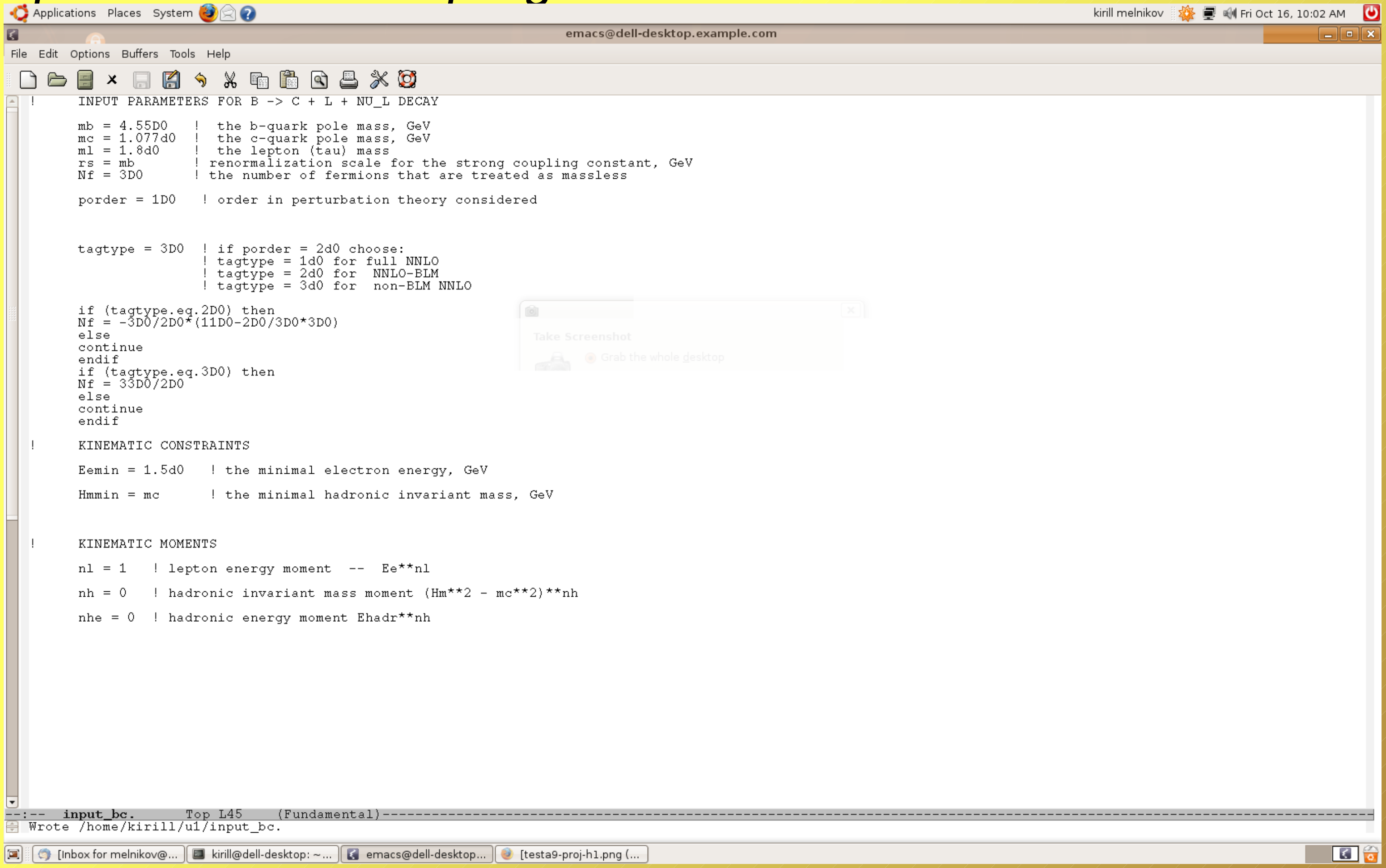
Anastasiou, Daleo, Lazopoulos, K.M., Petriello

Results

- These techniques enable the description of partonic decay $b \to X_c$+leptons through NNLO in QCD, keeping all the information about kinematics of all final state partons
- The calculation is implemented in the FORTRAN program
- The program includes the possibility to have a massive lepton in the final state; this covers semileptonic decays of $B \to X_c$ + tau-lepton
- The program can be used to compute any differential distribution which can be sensibly defined at the parton level. ARBITRARY cuts on the final state particles are allowed.

Results

- The input file for the program



Specify the input – and It is all numerical integration after that

Results

- For the discussion of the results, it is useful to separate BLM and non-BLM corrections

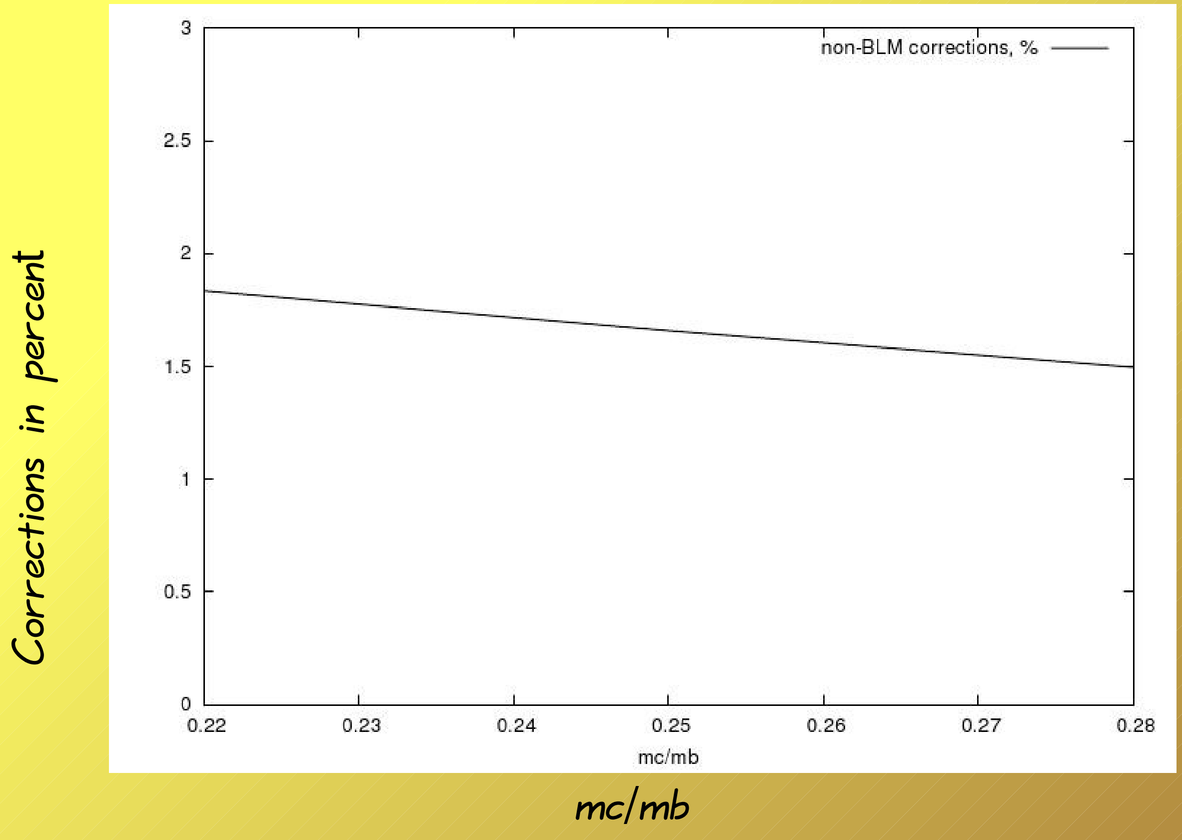
$$L_n(E_{\rm cut}) = \frac{\langle (E_l/m_b)^n \theta(E_l - E_{\rm cut}) {\rm d}\Gamma \rangle}{\langle {\rm d}\Gamma_0 \rangle}$$

$$L_n(E_{\rm cut}) = L_n^{(0)} + \frac{\alpha_s}{\pi} L_n^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \left(\beta_0 L_n^{(2),{\rm BLM}} + L_n^{(2)}\right)$$

- BLM corrections are special
  - they are naturally large
  - their interplay with non-perturbative corrections is non-trivial
  - they are technically simple and can be and are summed up to all orders. Uraltsev
  - their effect on moments is well-understood and it is taken into account in existing fits to semileptonic B-decays
- Non-BLM corrections are new and I'll focus on them in what follows

Results

- Non-BLM corrections to the total rate are small – about two percent (pole mass scheme) – and do not depend on the ratio of quark masses significantly.
- Two percent correction to the rate translates into one percent corrections to Vcb – comparable to stated Vcb uncertainty

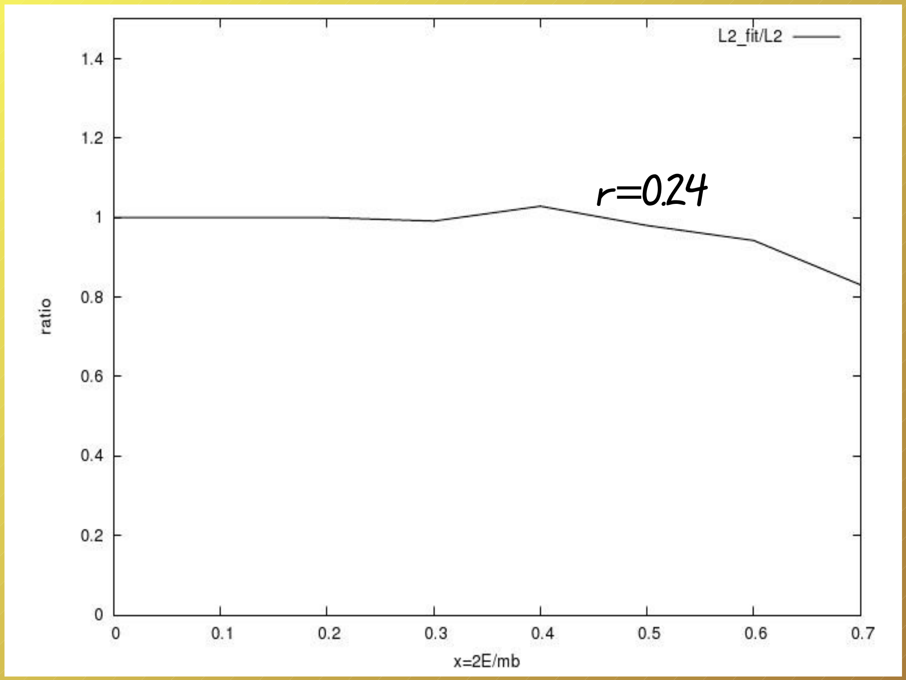


Results

- Dependence of non-BLM corrections to the moments on the lepton energy cut can be fitted by the energy dependence of the leading order moment, almost perfectly
- We illustrate this by comparing computed non-BLM corrections to the first moment with the results of the fit
- The fit gets worse for larger energy cuts and larger mc/mb

$$L_i^{(2)}(r,\chi)^{\rm fit} = \frac{L_i^{(2)}(r,0)}{L_i^{(0)}(r,0)} L_i^{(0)}(r,\chi)$$

$$r = \frac{m_c}{m_b}, \quad \chi = \frac{2E_{\rm cut}}{m_b}$$



x=2E/mb

Results

- Are non-BLM corrections to moments important for, say, b-mass determination? Lets look at the measured moment M1

$$\mathcal{M}_1(E_{\rm cut}) = \frac{\int\limits_{E_[{\rm cut}} E_l {\rm d}\Gamma}{\int\limits_{E_{\rm cut}} {\rm d}\Gamma} = \begin{cases} 1437.6(4.0)(5.7), & E_{\rm cut} = 0.6 \text{ GeV}; \\ 1773.7(1.9)(1.1), & E_{\rm cut} = 1.5 \text{ GeV}. \end{cases}$$

Babar

$$\mathcal{M}_1(E_{\rm cut}) = m_b F_{\rm th}, \quad F_{\rm th} = \frac{L_1(E_{\rm cut})}{L_0(E_{\rm cut})}.$$

- Including non-BLM corrections, extremely small shifts in the b quark mass is obtained. This is a consequence of enormous cancellations between non-BLM effects in L1 and L0

$$\delta m_b^{\rm nBLM} = \begin{cases} (6.6 + 0.7\delta_1 - 1.4\delta_0) \text{ MeV}, & E_{\rm cut} = 0.6 \text{ GeV}; \\ (6.4 + 2.6\delta_1 - 3\delta_0) \text{ MeV}, & E_{\rm cut} = 1.5 \text{ GeV} \end{cases}$$

Results

- Computed shifts in the b-quark mass look totally insignificant at the scale of 40-50 MeV uncertainties, as determined from fits to semileptonic moments
- However, I stress that non-BLM corrections to L1 and L0, when taken separately, are not small. For example, setting non-BLM corrections to L0 to zero AND repeating the same exercise, I get shifts in mb of about 100 MeV
- To claim that the uncertainty in the b-quark mass from semileptonic fits is indeed 40-50 MeV we need to assume high degree of correlation between higher order corrections to different moments. Note that NNLO non-BLM corrections are of the same order of magnitude as $\beta_0(\alpha_s/\pi)^3$ corrections!

Results

- Our calculation can be applied to describe QCD radiative effects in b-decays into a massive lepton: B → Xc + tau + neutrino

- Two experimental measurements of the branching fraction, thanks to ALEPH and OPAL. Taking the ALEPH result, I compute

$$\mathcal{R} = \frac{\Gamma(B \to X_c \tau \nu_\tau)}{\Gamma(B \to X_c e \nu_e)} = 0.237(31)$$

- Interesting ratio because
  - can be derived from first principles   Koyrakh, Falk, Neubert, Ligeti, Nir
  - can be used to constrain charged Higgs contributions to b → Xc + tau + neutrino

Results

- This ratio can be predicted in QCD. As always, perturbative and non-perturbative effects. Non-perturbative effects in the ratio are small, about -4 percent
- Perturbative QCD corrections are computed for mb = 4.6 GeV and mc = 1.15 GeV. We obtain

$$\Gamma(b \to c\tau\nu_\tau) = \Gamma_0(\tau)\left(1 + \frac{\alpha_s}{\pi}(-1.462) + \left(\frac{\alpha_s}{\pi}\right)^2(\beta_0(-1.82) + 3.16))\right)$$

$$\Gamma(b \to ce\nu) = \Gamma_0(e)\left(1 + \frac{\alpha_s}{\pi}(-1.777) + \left(\frac{\alpha_s}{\pi}\right)^2(\beta_0(-1.92) + 3.38)\right)$$

$$\mathcal{R}_{\rm pert} = \frac{\Gamma_0(\tau)}{\Gamma_0(e)}\left(1 + 0.0221_{\alpha_s} + 0.0044_{\rm BLM} + 0.0017_{\rm non-BLM}\right).$$

- We can use R to put constrain the charm mass. We find mc = 1040(200) MeV; compatible with fits but less precise

Conclusions

- We discussed NNLO QCD effects in semileptonic decays of b quarks into a charm quark and a lepton pair
- We are able to compute quantities of direct experimental interest and include constraints on the kinematics of final state particles exactly
- We are able to account for the mass of the lepton in the final state and describe decays $b \to Xc$ + tau + neutrino through NNLO QCD
- Non-BLM corrections are not very sensitive to the lepton energy cut, as long as it is not higher than O(1) GeV
- We find very strong correlations between non-BLM corrections to various moments; such correlations are crucial for justifying precision on various heavy quark parameters, including masses, claimed in semileptonic fits