Lattice QCD calculations of $B\to\pi\ell\nu$ form factors

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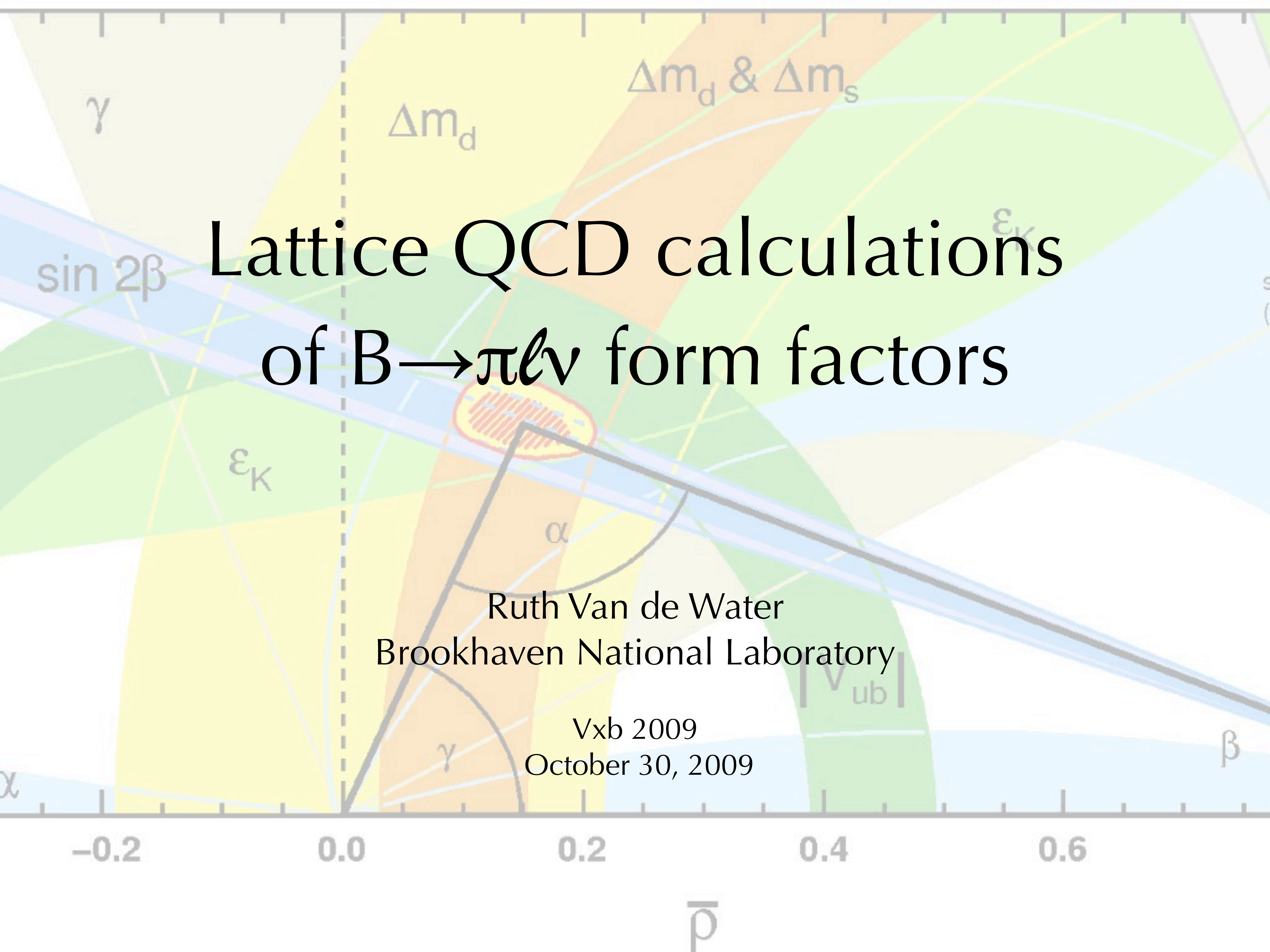
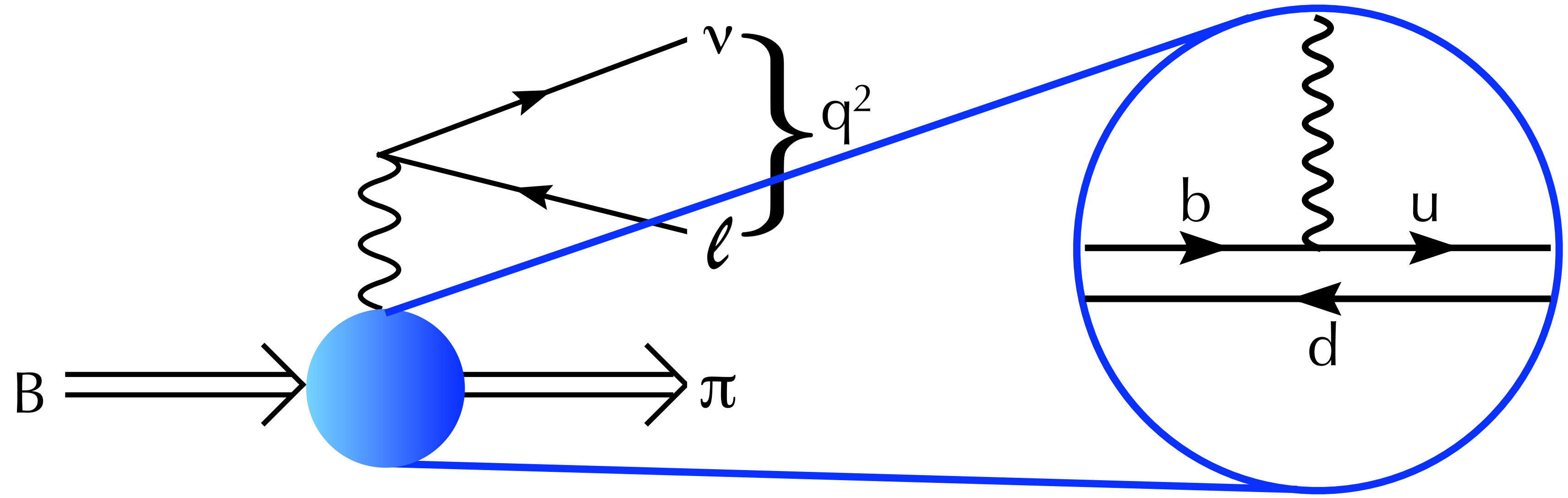


B→πℓν semileptonic decay



- Experiments can only measure the product $f_+(q^2) \times |V_{ub}|$

$$\frac{d\Gamma(B^0 \to \pi^- \ell^+ \nu)}{dq^2} = \frac{G_F^2}{192\pi^3 m_B^3}\left[(m_B^2 + m_\pi^2 - q^2)^2 - 4m_B^2 m_\pi^2\right]^{3/2} |V_{ub}|^2 |f_+(q^2)|^2$$

- Need lattice calculation of the B→πℓν form factor to determine $|V_{ub}|$

- Few percent determination of $|V_{ub}|$ difficult because errors in experimental branching fraction smallest at low q^2, whereas errors in lattice form factor determination smallest at high q^2

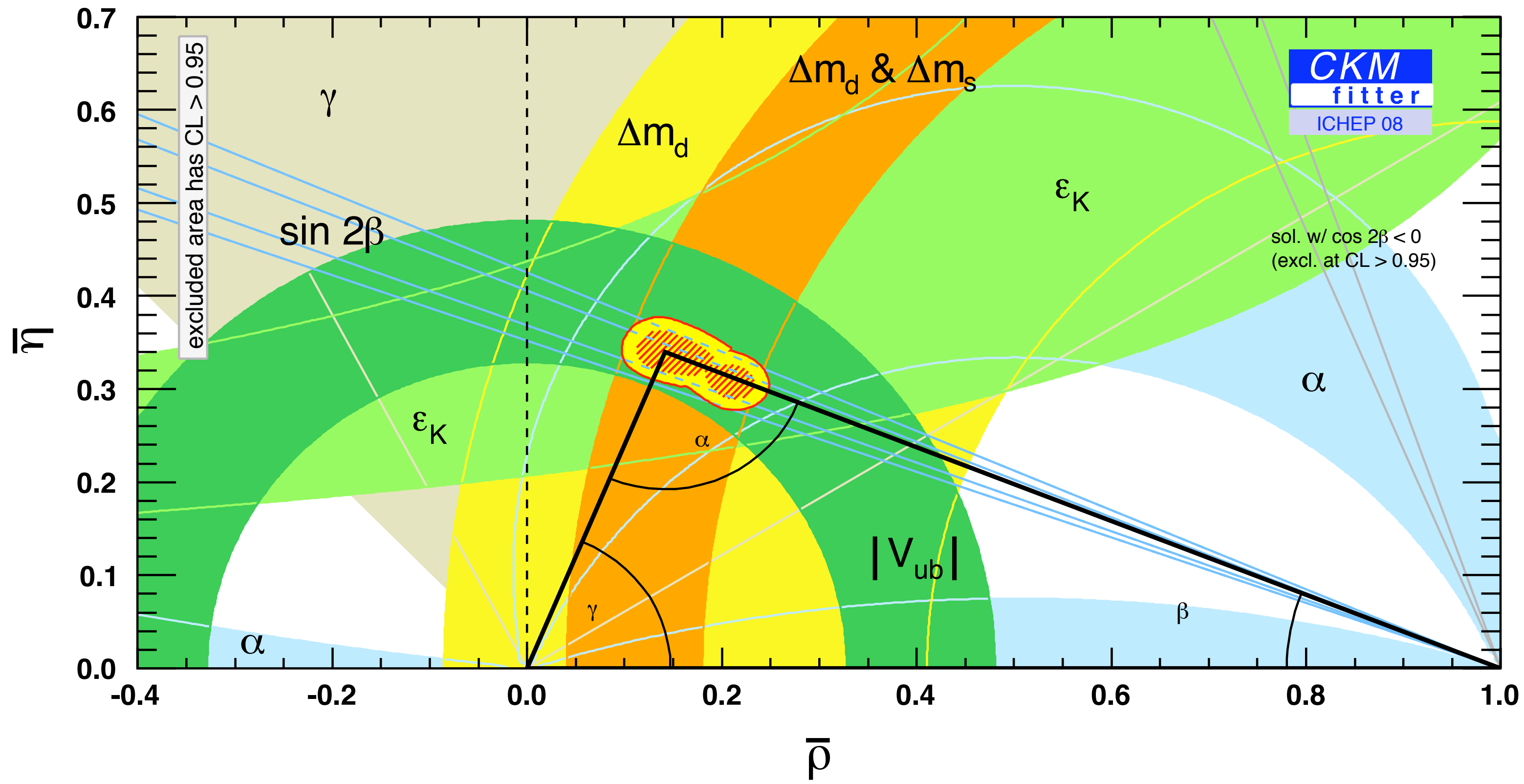
$|V_{ub}|$ and the CKM unitarity triangle

- $|V_{ub}|$ constrains the apex $(\bar\rho, \bar\eta)$ of the unitarity triangle:

$$\frac{|V_{ub}|}{|V_{cb}|} = \frac{\lambda}{1-\frac{\lambda^2}{2}}\sqrt{\bar\rho^2+\bar\eta^2}$$

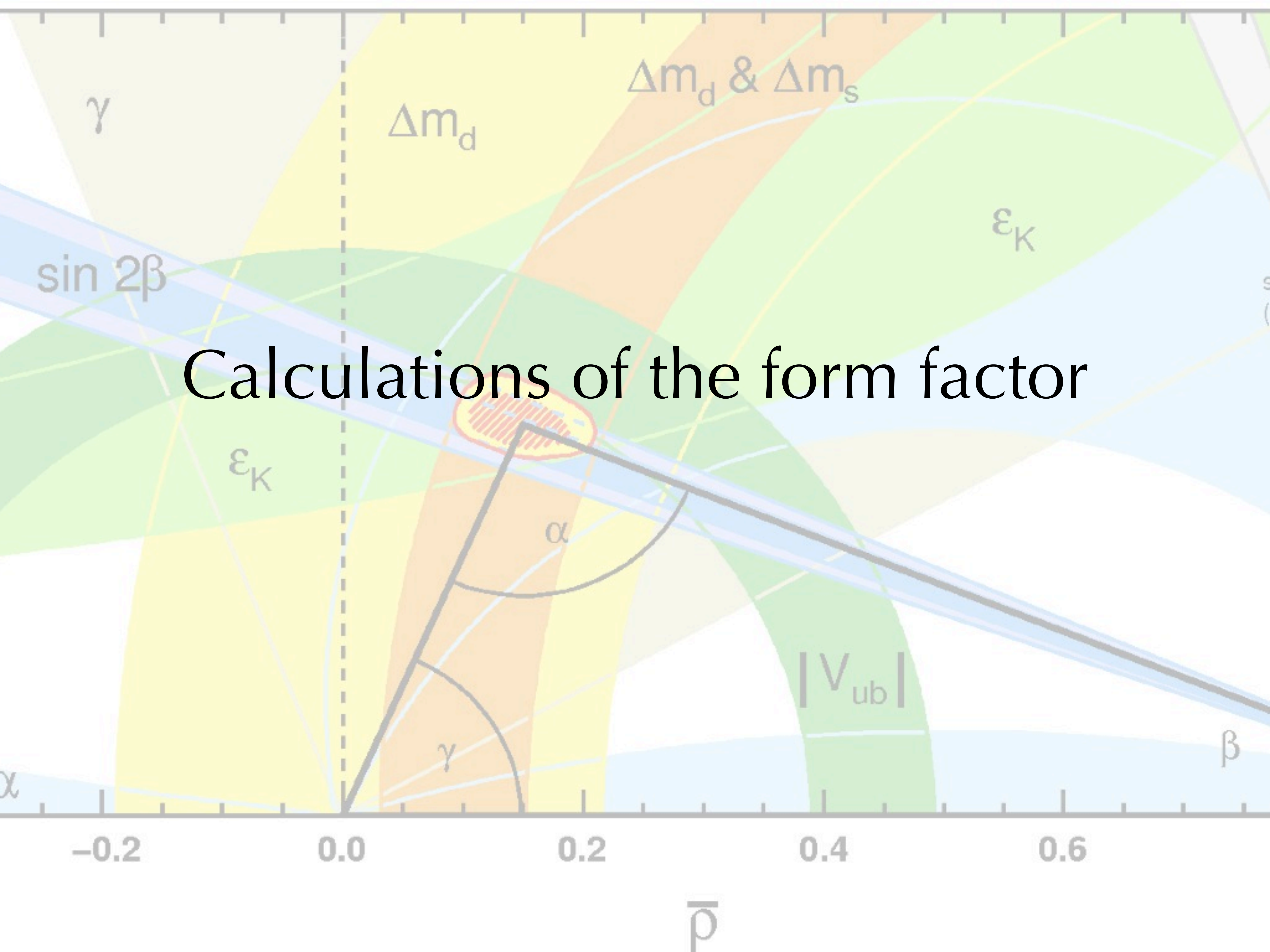
  - $\lambda = |V_{us}|$ known to ~0.5%
  - $|V_{cb}|$ known to ~2%

- Width of green error ring dominated by uncertainty in $|V_{ub}|$



- $\sin(2\beta)$ currently constrains the height to better than 4% and is still improving
- ∴ A **precise determination of $|V_{ub}|$ will allow a strong test of CKM unitarity**

Calculations of the form factor



Unquenched lattice calculations of $B \to \pi \ell \nu$

- Currently two groups calculating $B \to \pi \ell \nu$ with three dynamical quark flavors: Fermilab Lattice and MILC Collaborations & HPQCD Collaboration
- Both use the publicly available "2+1 flavor" **MILC configurations** [**Phys.Rev.D70:114501,2004**] which have three flavors of improved staggered quarks:
  - Two degenerate light quarks and one heavy quark ($\approx m_s$)
  - Light quark mass ranges from $m_s/10 \leq m_l \leq m_s$ (minimum $m_\pi \approx 260$ MeV)
- Five lattice spacings now available from $a \approx 0.045$ fm to $a \approx 0.15$ fm
  - 2007 HPQCD analysis uses the $a \approx 0.12$ fm ensembles (with a few data points at $a \approx 0.09$ fm to check discretization errors)
  - Recent Fermilab/MILC analysis obtains a continuum limit by using both the $a \approx 0.12$ fm and 0.09 fm ensembles to
- Primary difference is that groups use different heavy quark discretizations:
  - Fermilab/MILC uses Fermilab quarks
  - HPQCD uses nonrelativistic (NRQCD) heavy quarks

Heavy quarks on the lattice

PROBLEM: Generic lattice quark action will have discretization errors $\propto (am_q)^n$

SOLUTION: Use knowledge of the heavy quark/nonrelativistic quark limits of QCD to systematically eliminate HQ discretization errors order-by-order

FERMILAB METHOD
[Phys.Rev.D55:3933-3957,1997]

Continuum QCD

↓ (using HQET)

Lattice gauge theory

LATTICE NRQCD
[Phys.Rev.D46:4052-4067,1992]

Continuum QCD

↓

Nonrelativistic QCD

↓

Lattice gauge theory

- Both methods require tuning parameters of lattice action
- For heavy-light decays, must also match lattice *currents* to continuum
- Typically calculate matching coefficients in lattice perturbation theory [Phys.Rev.D48:2250-2264,1993]

Matching errors

- In principle, can remove errors of any order in heavy quark mass, but, in practice, becomes increasingly difficult at each higher order
- $\Longrightarrow$ Must estimate size of errors due to inexact matching

FERMILAB METHOD

QCD

$\downarrow$ "heavy quark discretization effects"

LGT

- Combine all errors associated with discretizing action
- Estimate errors using knowledge of short-distance coefficients and power-counting

LATTICE NRQCD

QCD

$\downarrow$ "relativistic errors", e.g. $O(\alpha_S \Lambda_{QCD}/m_Q)$ & $O(\Lambda_{QCD}^2/m_Q^2)$

NRQCD

$\downarrow$ " perturbation theory errors", e.g. $O(\alpha_S^2)$

LGT

- Estimate errors using power-counting

Extrapolation of lattice data with staggered χPT

- Because simulating light quarks at the physical up and down quark masses is prohibitively computationally expensive, use expressions derived in chiral perturbation theory to extrapolate to the physical quark masses in a controlled way

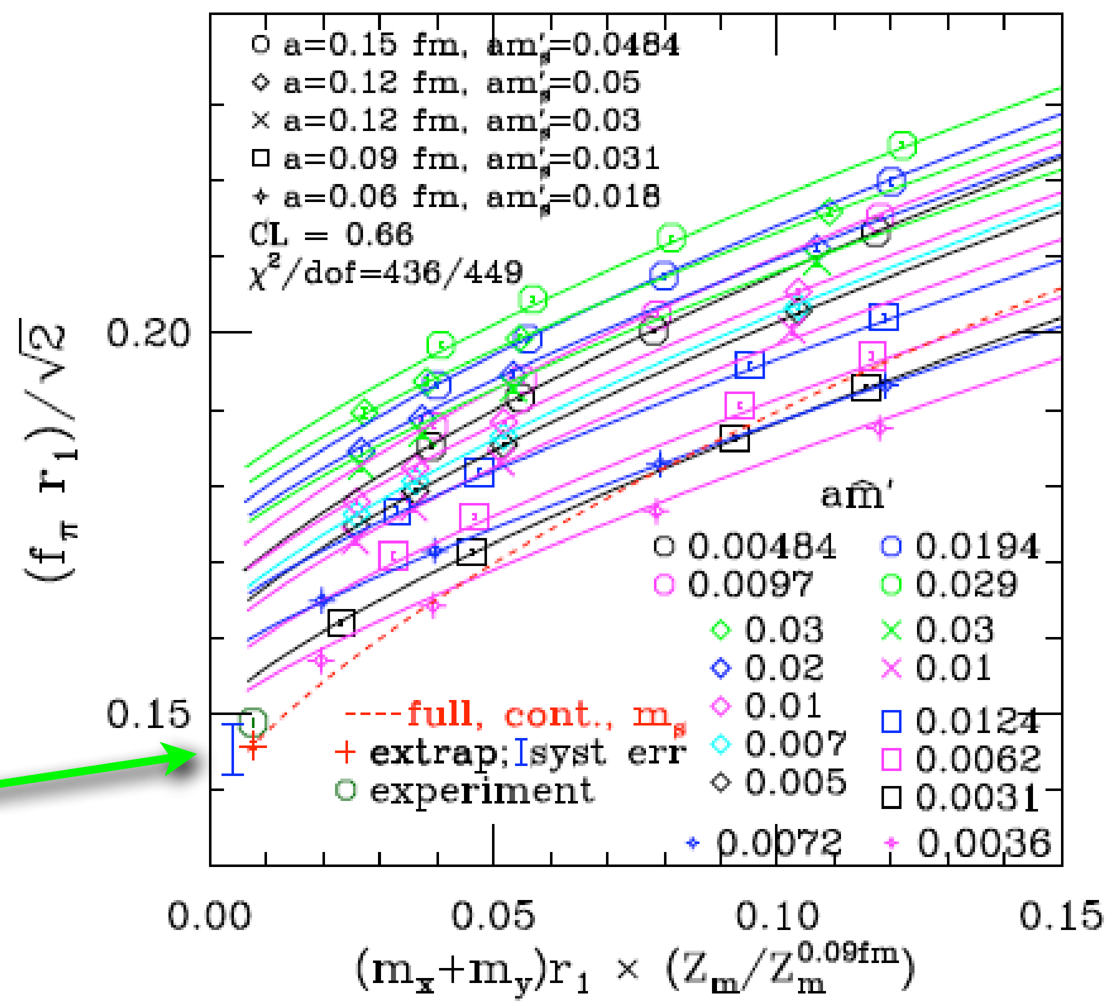
- Effective field theory of low-energy QCD in which degrees-of-freedom are pions and kaons

- For MILC lattices must use **staggered chiral perturbation theory** [**Lee & Sharpe, Aubin & Bernard, Sharpe & RV**]

  - All operators consistent with lattice symmetries are in chiral effective Lagrangian

  - Accounts for next-to-leading order light quark mass dependence and for light quark discretization effects through $O(\alpha_S^2 a^2 \Lambda_{QCD}^2)$

- Extremely successful for extrapolating light-light meson quantities such as f_π (**MILC Lat'07 arXiv:0710.1118 [hep-lat]**)



Staggered χPT for heavy-light form factors

- We work to zeroth order in $1/m_b$ and next-to-leading order in m_l, a^2, E_π, so the NLO HMSχPT expressions for $f_{||}$ and $f_\perp$ are [Aubin & Bernard]:

$$
\begin{aligned}
f_{\|}(m_l, E_\pi, a) &= \frac{c_{\|}^{(0)}}{f_\pi}\left[1 + \text{logs} + c_{\|}^{(1)} m_l + c_{\|}^{(2)}(2m_l + m_s) + c_{\|}^{(3)} E_\pi + c_{\|}^{(4)} E_\pi^2 + c_{\|}^{(5)} a^2\right] \\
f_\perp(m_l, E_\pi, a) &= \frac{c_\perp^{(0)}}{f_\pi}\left[\frac{1}{E_\pi + \Delta_B^* + \text{logs}} + \frac{1}{E_\pi + \Delta_B^*} \times \text{logs}\right] \\
&+ \frac{c_\perp^{(0)}/f_\pi}{E_\pi + \Delta_B^*}\left[c_\perp^{(1)} m_l + c_\perp^{(2)}(2m_l + m_s) + c_\perp^{(3)} E_\pi + c_\perp^{(4)} E_\pi^2 + c_\perp^{(5)} a^2\right]
\end{aligned}
$$

$$\Delta_B^* \equiv m_{B^*} - m_B$$

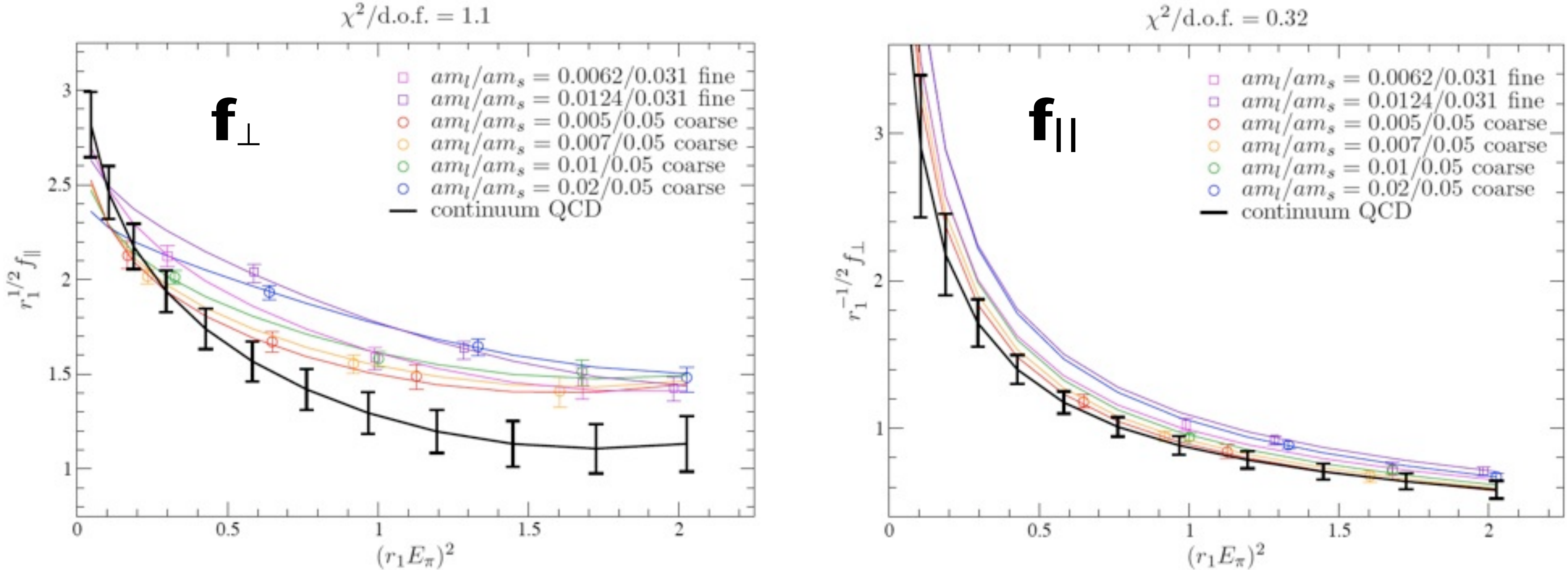
coefficient of some logarithms proportional to B^*-B-π coupling, $g_{B^*B\pi}$

- The leading $1/m_b$ corrections in the chiral and soft-pion limits are absorbed into the fitted values of the LO parameters ($c_{||}^{(0)} = f_B(m_B)^{1/2}$, $c_\perp^{(0)} = f_{B^*}(m_{B^*})^{1/2}$, and $g_{B^*B\pi}$) and the inclusion of the B^*-B mass-splitting (Δ_{B^*})

- The neglected logarithmic $1/m_b$ corrections are of $O[m_\pi^2/(4\pi f_\pi m_b), a^2/(4\pi f_\pi m_b), E_\pi/m_b]$, which are a few-percent effects that are within our current χPT errors but will have to be treated more carefully in future analyses

Chiral-continuum extrapolations of $f_{||}$ and $f_{\perp}$

[Fermilab/MILC, Phys.Rev.D79:054507,2009]



- Correlated fit to all $f_\perp$ (or $f_{||}$) data using NLO HMSχPT plus NNLO analytic terms
- **BLACK** curves show form factors extrapolated to physical quark masses and continuum with statistical errors
- Estimate the uncertainty due to omission of higher-order terms by adding NNLO and NNNLO analytic terms (with suitable priors) until the central values of the extrapolated form factors stabilize and the statistical errors reach a maximum

Typical error budget for $f_+(q^2)$

[HPQCD, Phys.Rev.D73:074502,2006, Erratum-ibid.D75:119906,2007]

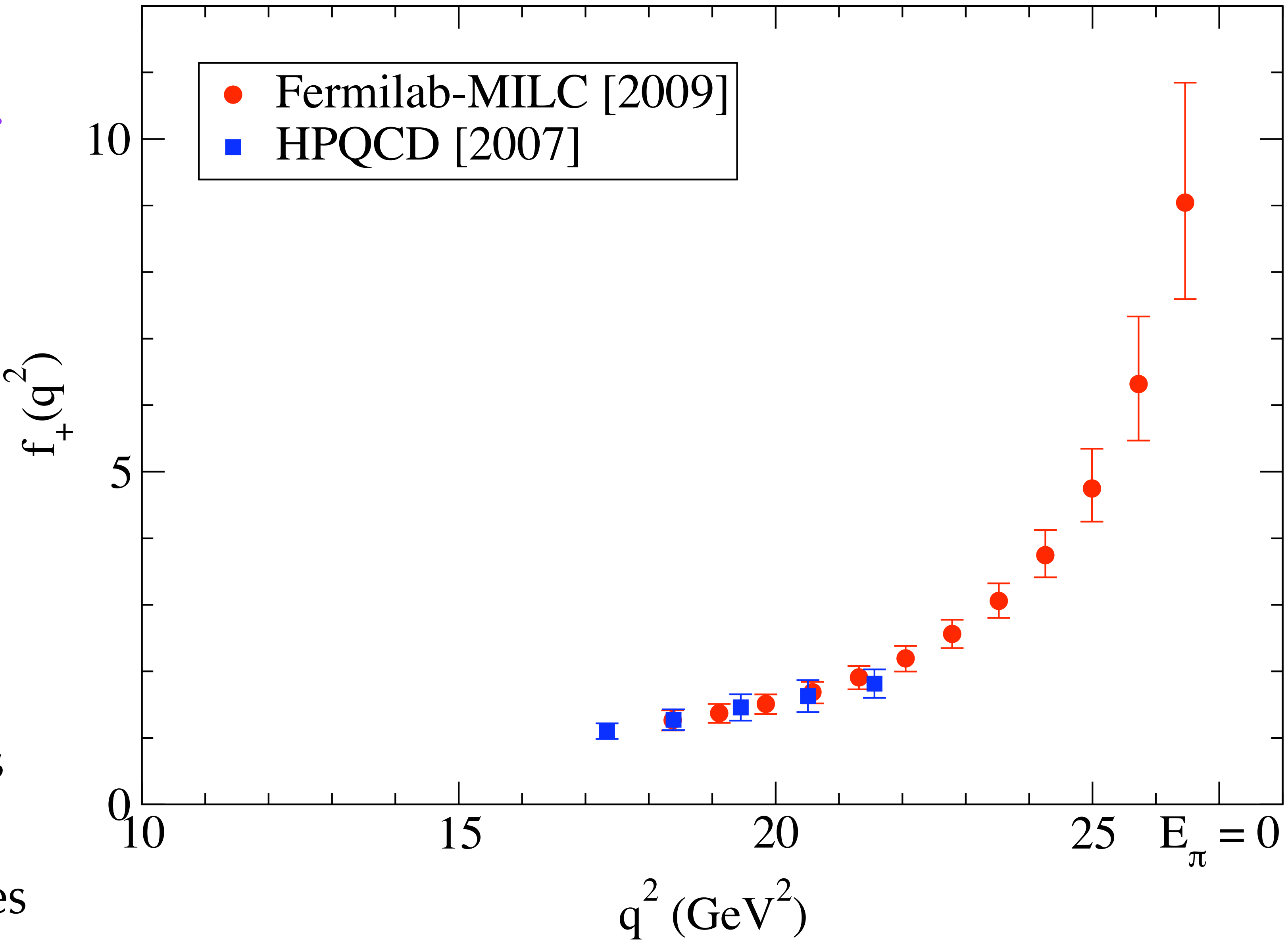
- **Dominant uncertainty from statistics and the chiral extrapolation** (the same is true for Fermilab/MILC)
- HPQCD also has large uncertainty from two-loop matching between NRQCD and the lattice theory

source of error	size of error (%)
statistics + chiral extrapolations	10
two-loop matching	9
discretization	3
relativistic	1
Total	14

- Fermilab/MILC has several smaller systematic errors of ~3% such as discretization errors, matching the lattice heavy-light current to the continuum, and the uncertainty in the B*-B-π coupling, $g_{B^*B\pi}$
- **Recent Fermilab/MILC result has smaller total errors in most q^2 bins** due to the analysis of more configurations, a more sophisticated chiral-continuum extrapolation procedure, and mostly nonperturbative operator matching

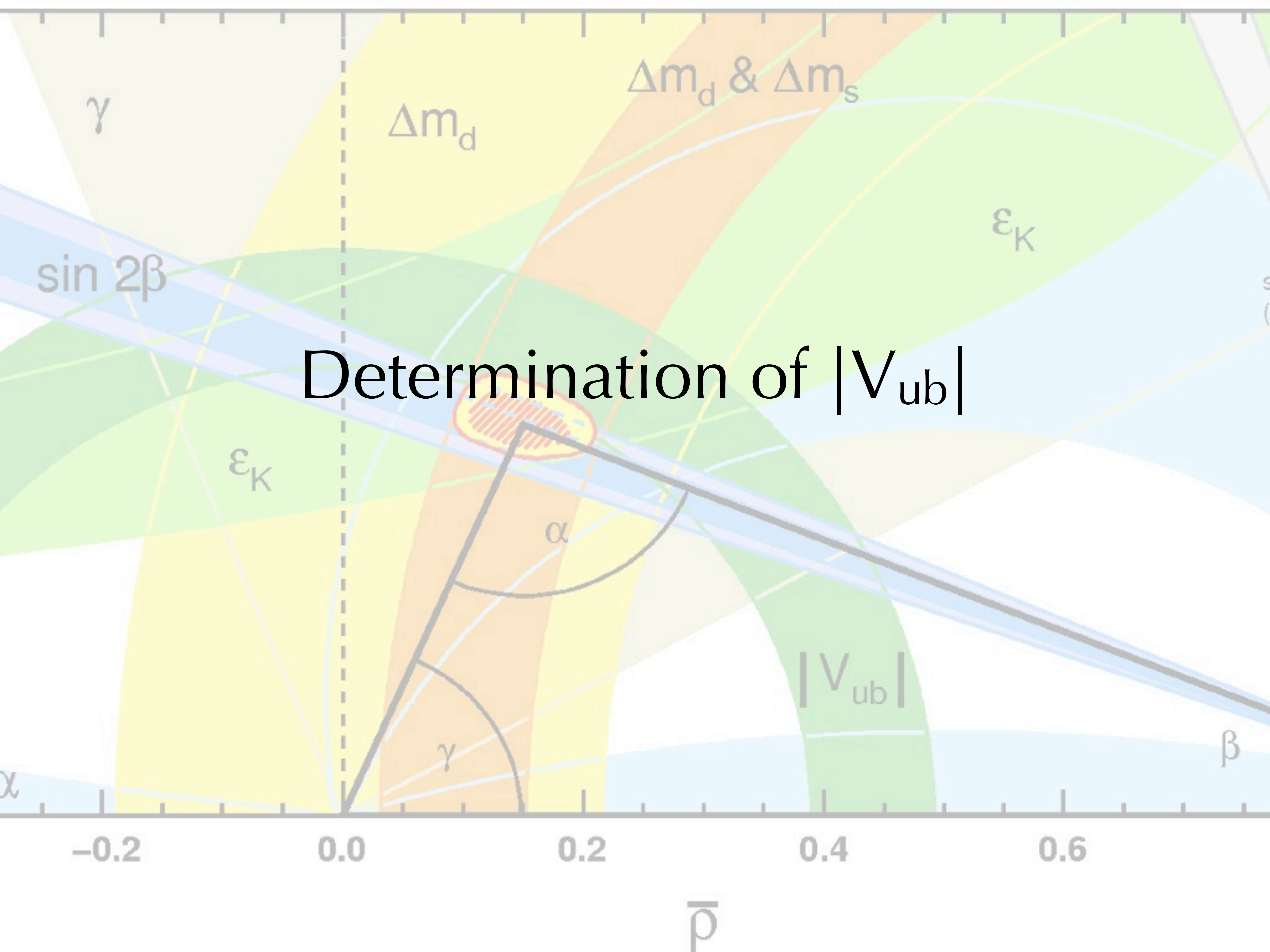
Results for the $B \to \pi \ell \nu$ form factor $f_+(q^2)$

- **2009 Fermilab/MILC result the only one with two lattice spacings and a continuum limit**
- Statistical errors are highly correlated between the two determinations because they use the same gauge configurations
- Systematic errors are largely uncorrelated since they use different heavy-quark actions and lattice-to-continuum operator matching procedures



- *Reasonable to treat the statistical errors as 100% correlated and the systematic errors as uncorrelated when averaging the results*, but will have to be more cautious about this treatment as the errors get smaller . . .

Determination of $|V_{ub}|$



Exclusive determination of $|V_{ub}|$ from $B\to\pi\ell\nu$

- Standard method is to combine lattice form factor experimentally-measured $B\to\pi\ell\nu$ branching fraction and B-meson lifetime and integrate over q^2:

$$\frac{\Gamma(q_{\rm min})}{|V_{ub}|^2} = \frac{G_F^2}{192\pi^3 m_B^3}\int_{q^2_{\rm min}}^{q^2_{\rm max}} dq^2 \left[(m_B^2+m_\pi^2-q^2)^2-4m_B^2m_\pi^2\right]^{3/2}|f_+(q^2)|^2$$

  - **Requires analytic parameterization** of lattice form factor $f_+(q^2)$

- Standard functional form used to interpolate/extrapolate form factor data is the Becirevic-Kaidalov parameterization:

$$f_+(q^2) = \frac{f(0)}{(1-q^2/m^2_{B^*})(1-\alpha\, q^2/m^2_{B^*})} \qquad f_0(q^2) = \frac{f(0)}{(1-\frac{1}{\beta}\, q^2/m^2_{B^*})}$$

properly incorporates B* pole

α and β parameterize physics above threshold (other poles and cuts)

  - Easy to use, but **introduces hard-to-estimate model dependence due to choice of fit ansatz**

z-expansion of semileptonic form factors

[**Arnesen *et. al.* Phys. Rev. Lett. 95, 071802 (2005) and refs. therein**]

- Consider mapping the variable q^2 onto a new variable, z, in the following way: $z = \dfrac{\sqrt{1-q^2/t_+} - \sqrt{1-t_0/t_+}}{\sqrt{1-q^2/t_+} + \sqrt{1-t_0/t_+}}$

- Choose the free parameter t_0 to make the maximum |z| in the region as small as possible -- choosing 0.65 t_- maps z in the $B\to\pi\ell\nu$ decay region onto $-0.34 < z < 0.22$

- In terms of z, semileptonic form factors have simple form:

$$P(t)\,\phi(t,t_0)\,f(t) = \sum_{k=0}^{\infty} a_k(t_0) z(t,t_0)^k$$

Accounts for subthreshold (e.g. B*) poles

"Arbitrary" analytic function -- choice only affects particular values of coefficients (a_k's)

- Unitarity constrains the size of the coefficients: $\sum_{k=0}^{N} a_k^2 \leq 1$

Constraint holds for any value of N

- Thus, in combination with the small range of |z|, **one needs only a small number of parameters** to obtain the form factors to a high degree of accuracy

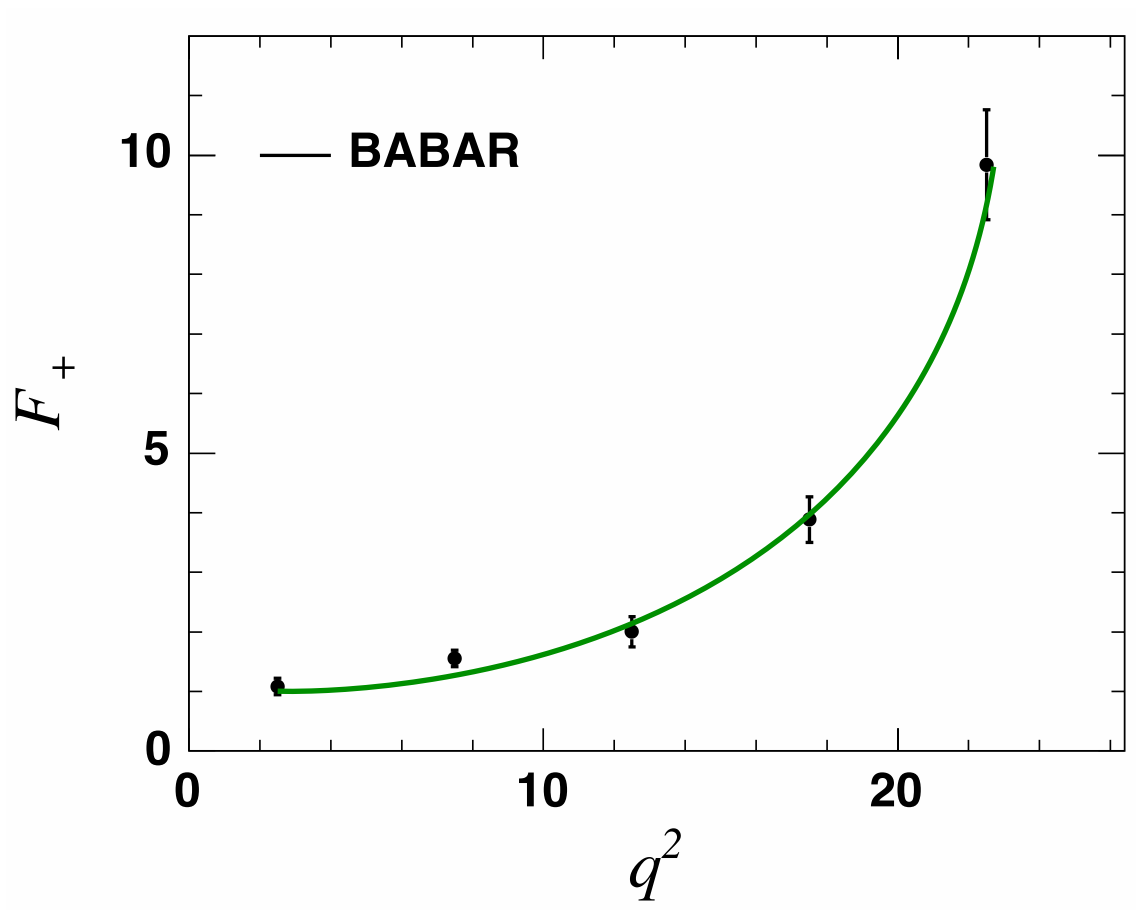
Heavy quark constraint on coefficients

- Unitarity bound on coefficients come from fact that the decay rate to the exclusive channel $B \to \pi \ell \nu$ must be less than the inclusive B-meson decay rate
- It is also true that, as the mass of B-meson increases, its branching fraction to any particular exclusive channel decreases
- The branching fraction for the semileptonic decay $B \to \pi \ell \nu$ as a power of Λ_{QCD}/m_B has been calculated by **Becher and Hill**
- It can be used to place an even **tighter constraint** on the coeffiecents of the z-expansion for the form factors:

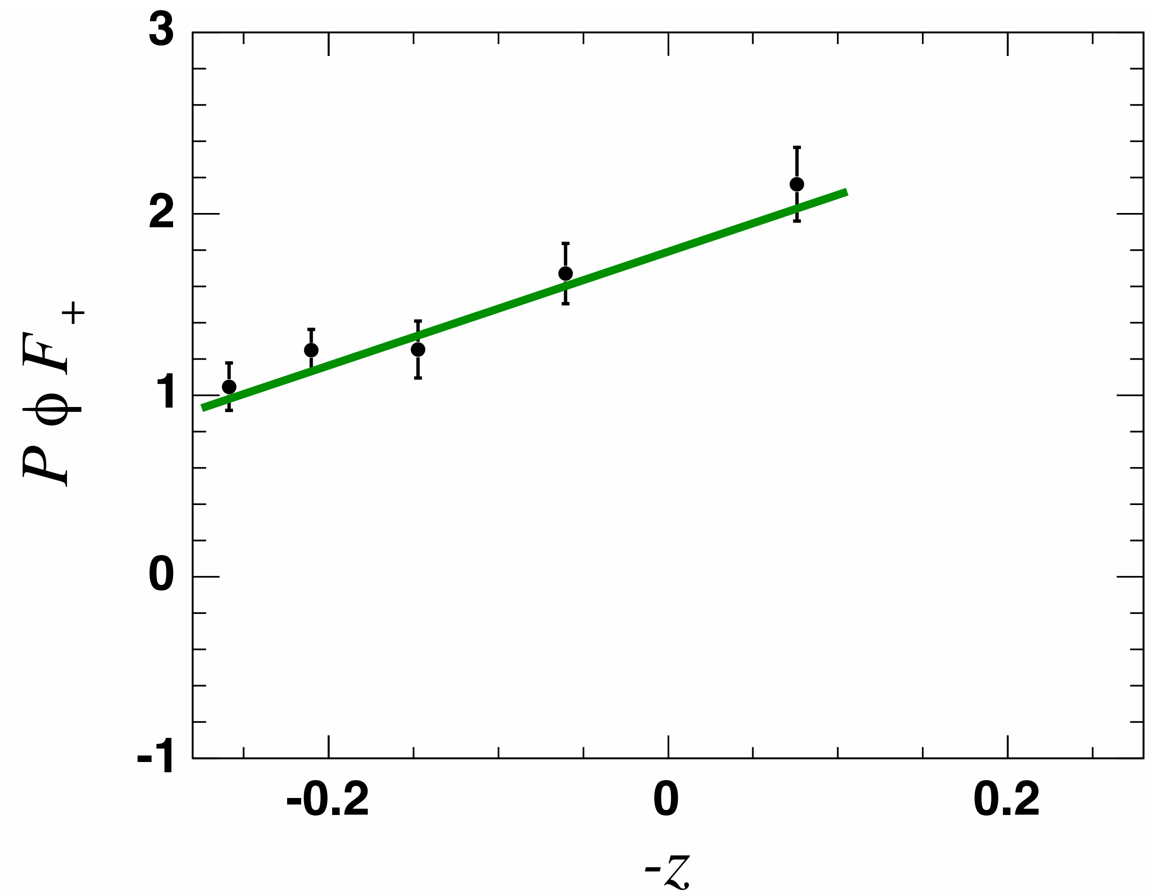
$$\sum_{k=0}^{N} a_k^2 \sim \left(\frac{\Lambda}{m_B}\right)^3 \approx 0.001$$

- Implies that the unitarity bound is far from saturated, i.e. that the coefficients will be much less than one

Effect of z-remapping on B→πℓν form factor



Striking curvature in B→πℓν form factor data versus q^2



No visible curvature after remapping

- Curvature in data due to well-understood perturbative QCD effects
- Data completely described by a *normalization and a slope*, and constrains the size of possible curvature

The program for lattice and experiment

(1) Fit experimental and lattice data in terms of z expansion

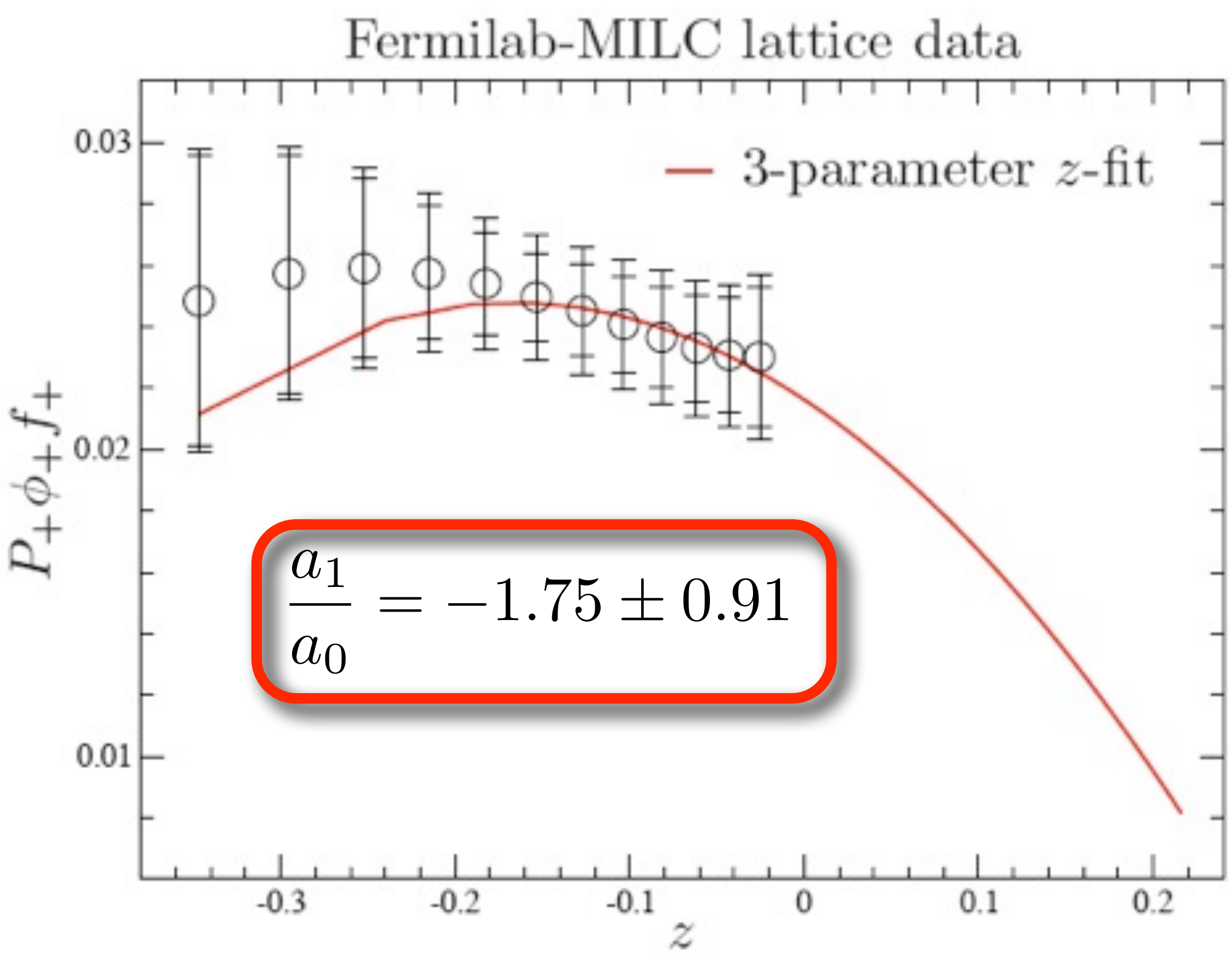
(2) Determine and compare the slopes (and curvature) in z

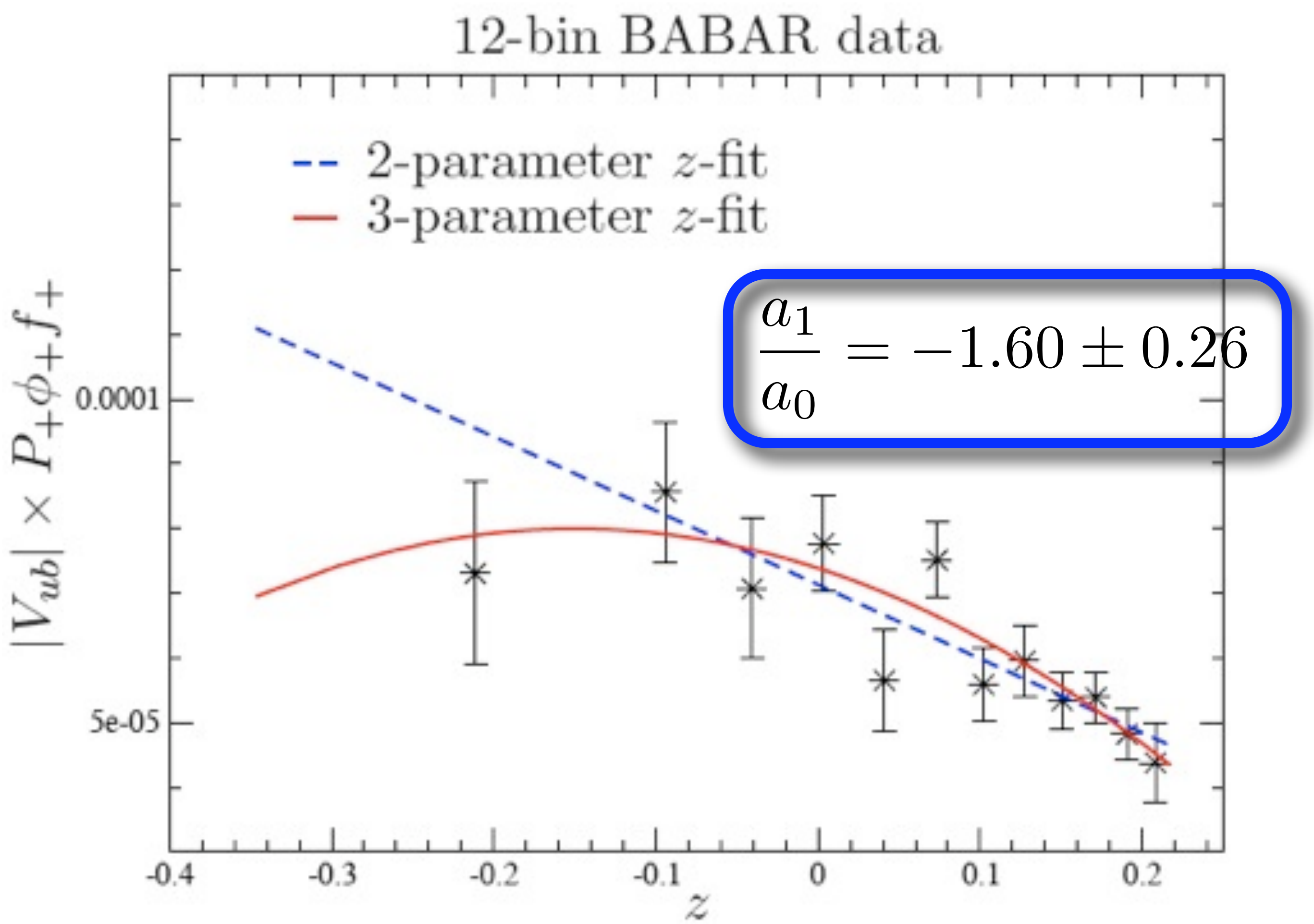
(3) If consistent, fit lattice and experimental data simultaneously with an unknown relative offset to determine $|V_{ub}|$

- This approach is:
  - **model-independent** (can quantify the agreement between lattice and experiment using slope measurements)
  - **systematically improvable** (as data gets more precise can add more terms in z)
  - and **minimizes the error in $|V_{ub}|$** by optimally combining lattice and experimental data
- Was recently implemented by the Fermilab Lattice and MILC collaborations with the hope that it will eventually adopted by other lattice theorists and experimentalists
- Although are other model-independent form factor parameterizations in the literature (e.g. **Flynn & Nieves**; **Bourrely, Caprini & Lellouch**), it is important to use a standard fit function to facilitate comparison between lattice and experiment
  - z-parameterization a good choice because it's easy to implement

Consistency check: separate z-fits

[Fermilab/MILC, Phys.Rev.D79:054507,2009]





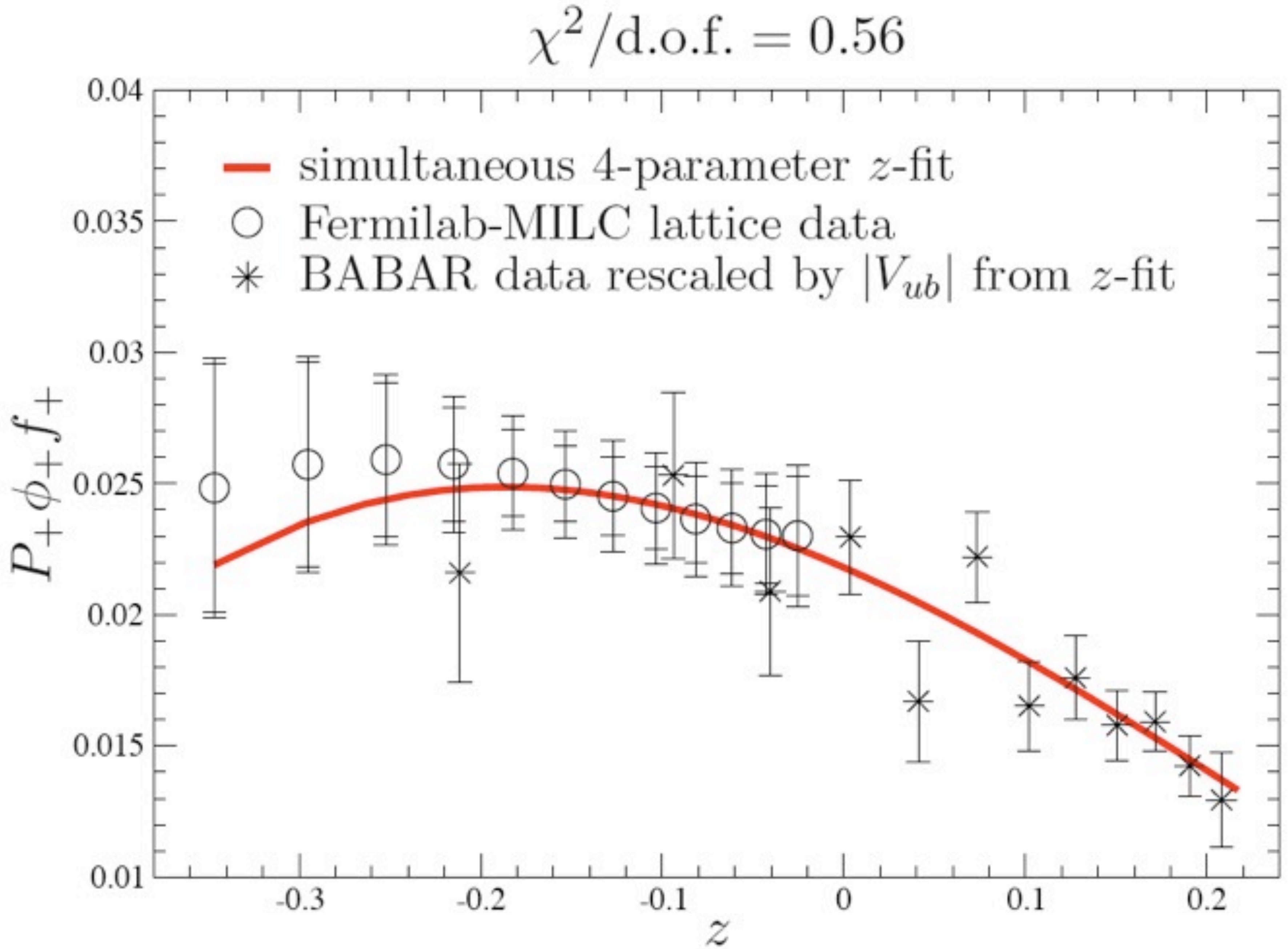
- ✦ Lattice data determines both the slope and curvature
- ✦ Experimental data consistent with zero curvature
- ✦ Lattice and experimental slope and curvature **agree within uncertainties**

⇒ Proceed to simultaneous fit of lattice and experimental data

Simultaneous z-fit to determine $|V_{ub}|$

[Fermilab/MILC, Phys.Rev.D79:054507,2009]

- Fit lattice and 12-bin BABAR experimental data [Phys. Rev. Lett. 98, 091801 (2007)] together to z-expansion leaving relative normalization factor ($|V_{ub}|$) as a free parameter



Fit results

[*alia et* **RV** *et al.* **Phys.Rev.D79:054507,2009**]

- The result of the 4-parameter combined z-fit is:

$$\boxed{\begin{aligned} |V_{ub}| \times 10^3 &= 3.38 \pm 0.36 \\ a_0 &= 0.0218 \pm 0.0021 \\ a_1 &= -0.0301 \pm 0.0063 \\ a_2 &= -0.059 \pm 0.032 \\ a_3 &= 0.079 \pm 0.068 \end{aligned}}$$

- Coefficients are much smaller than 1, as expected from heavy-quark power-counting:

$$\sum a_k^2 \sim 0.01$$

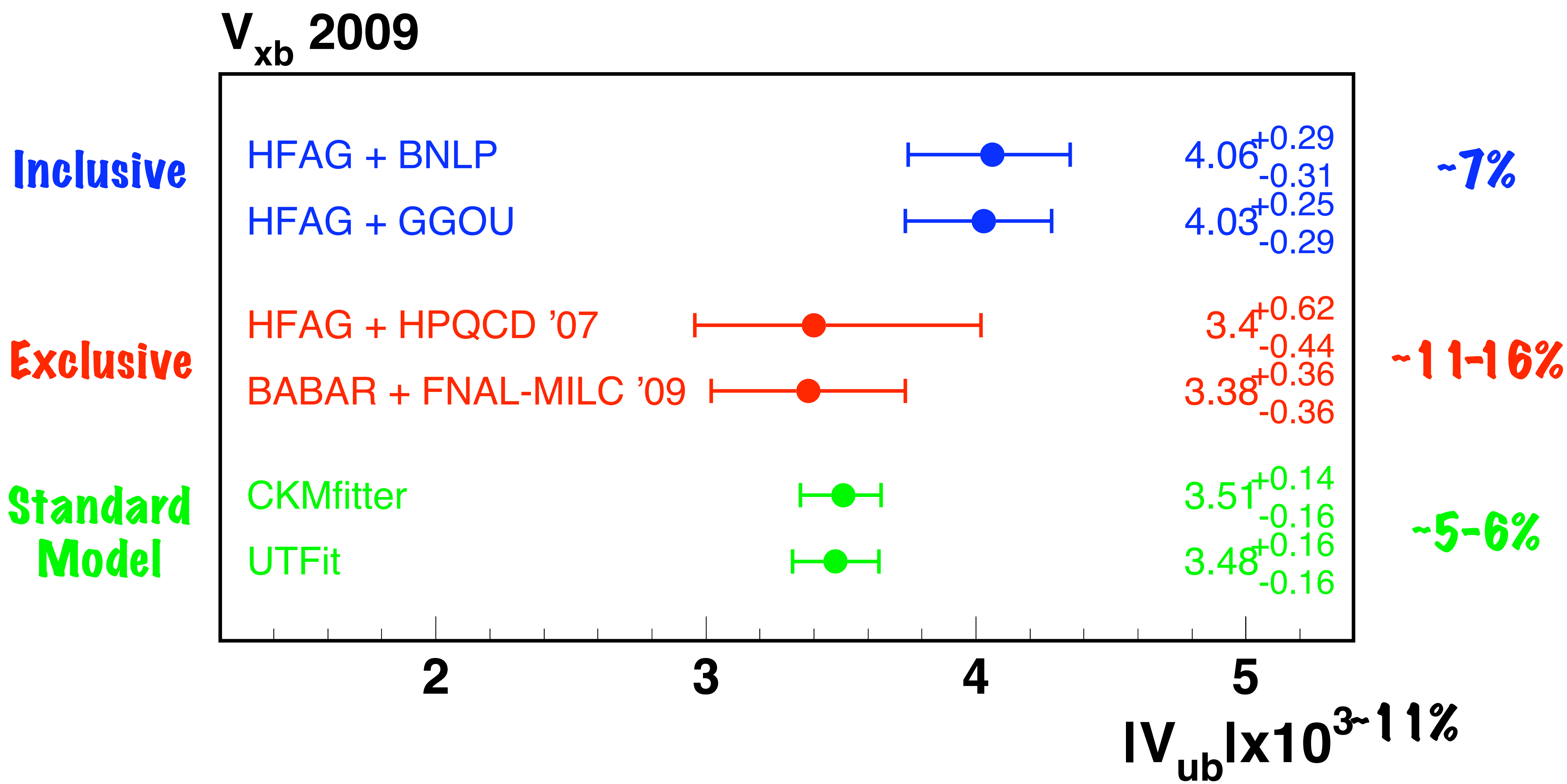
  - Result independent of constraint on coefficients

- $|V_{ub}|$ determined to ~**11% accuracy**

- improved uncertainty largely due to combined z-fit method

  - If perform separate z-fits of lattice and experimental data and take ratio of normalizations, only determine $|V_{ub}|$ to ~16%

Comparison with other determinations



- Exclusive $|V_{ub}|$ ~ **1-2 - σ below inclusive determinations**
- Consistent with preferred values from unitarity triangle analyses
- Inclusive $|V_{ub}|$ varies depending upon theoretical framework, and is highly sensitive to input m_b

How can lattice address tension in $|V_{ub}|$?

- The *discrepancy in $|V_{ub}|$ could be a sign of non-Standard Model (V+A) currents*
    - Could test this hypothesis using $|V_{ub}|$ determined from exclusive $B \to \rho \ell \nu$ decay
    - (Recall that $B \to \pi \ell \nu$ proceeds through the vector current while $B \to \rho \ell \nu$ proceeds through the both the vector and axial-vector currents)
- However, $B \to \rho \ell \nu$ more challenging for lattice QCD than $B \to \pi \ell \nu$ because the ρ-meson is unstable
    - Must first get the ρ−mass right before computing the form factor
    - Will be **difficult to control the chiral and continuum extrapolation of the form factor** because there will be a cusp at π-π threshold, but the ρ is not describable by chiral perturbation theory
    - Cautious approach is to wait for physical quark masses and a=0.045 fm lattices so the correct π-π threshold is apparent in the data; this is now only a few years away. . .

Improving current calculations of $B\to\pi\ell\nu$

- **Fermilab Lattice and MILC Collaborations**:
  - Still statistics limited, so currently generating data with 4× the gauge configurations
  - Also generating data on a third finer lattice spacing to reduce heavy-quark discretization errors and solidify the continuum extrapolation
  - Updated result available in 1-2 years?

- **HPQCD Collaboration** is studying new actions and strategies for b-quark simulations:
  - Combining NRQCD b-quarks with HISQ light quarks, but so far just for spectroscopy and decay constants
  - Relativistic HISQ b-quarks [**McNeile et al., arXiv:0910.2921**] (can simulate at close to the physical b-quark mass on the MILC $a \approx 0.045$ fm and $a \approx 0.06$ fm lattices)
  - Ultimately will have to decide which method looks more promising for $B\to\pi\ell\nu$; *improvements will potentially be significant, but results will not appear overnight*

Future calculations of $B \to \pi \ell \nu$

- **Other collaborations?**
  - Several groups (e.g. **ALPHA, ETMC, RBC/UKQCD, QCDSF Collaborations**) are calculating B-meson decay constants, mixing matrix elements, or $B \to D^{(*)} \ell \nu$ form factors with other heavy-quark and light quark actions (see **B. Blossier's talk** for more details)
  - Often in the quenched (or $N_f=2$) approximation, or with a static b-quark
  - No clear timeline for when these groups will address $B \to \pi \ell \nu$ with $N_f=2+1$ sea quarks and relativistic b-quarks . . .

General comments

- Studies of using a moving B-frame to obtain lattice data at lower q^2-values have shown that it is difficult to keep statistical noise under control [**McNiele *et al.* hep-lat/0611009**]
  - Method is still being pursued for rare B-decays by the Cambridge group using improved smearings [**Horgan *et al.* Phys.Rev.D80:074505,2009**]
  - Also **unclear how to control extrapolation to physical quark masses** since cannot use existing HMχPT expressions for the chiral extrapolation which are formulated in the rest frame of the B-meson and the pion energy in the rest frame is too large for χPT to apply
- The combined z-fit to lattice and experimental data requires full statistical and systematic correlation matrices from both lattice and experiment
  - *How do we properly account for correlations between the lattice results or experimental measurements in the z-fit?*
- When the statistical and discretization errors are smaller, **will be important to address $1/m_b$ logarithmic corrections, $g_{B^*B\pi}$ determination, dynamical charm contribution**