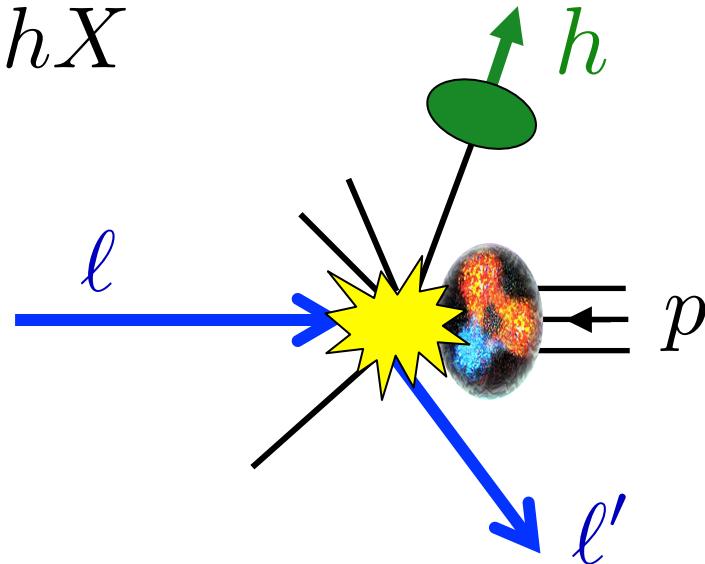


QCD corrections to high- p_T hadron production in ℓp scattering

Werner Vogelsang
Univ. Tübingen

ICAS, 09/20/2017

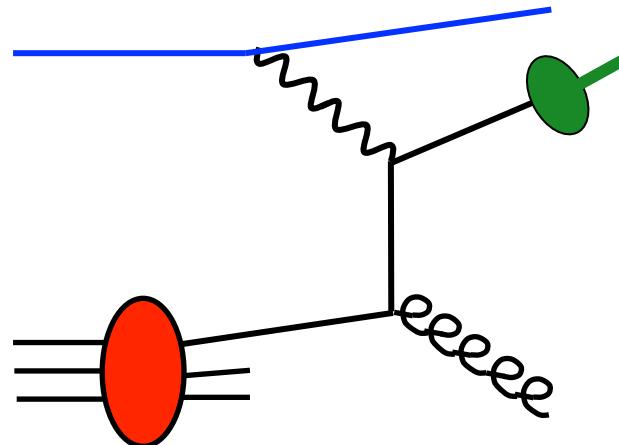
$$\ell p \rightarrow \ell' h X$$



fixed-target, EIC

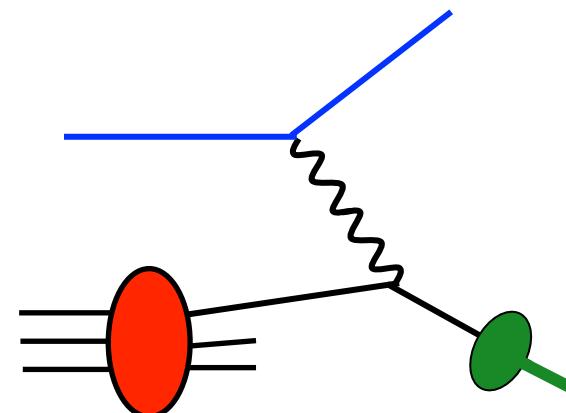
“photoproduction”

$$\gamma p \rightarrow h X$$



“single-inclusive”

$$\ell p \rightarrow h X$$



Outline:

$\gamma p \rightarrow hX$

- Introduction
- QCD threshold resummation
- Spin asymmetry A_{LL}

D. de Florian, M. Pfeuffer, A. Schäfer, WV

C. Uebler, A. Schäfer, WV

$\ell p \rightarrow hX$

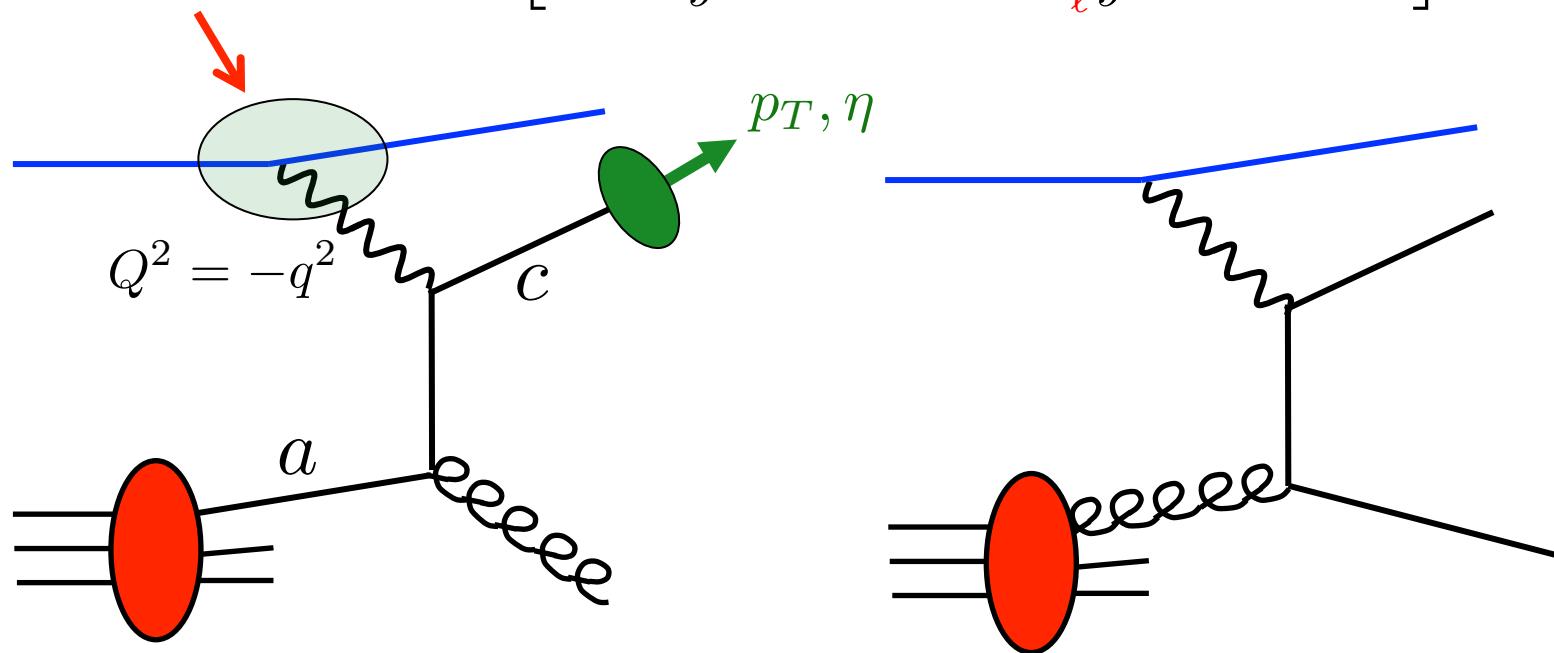
- Motivation
- NLO calculation for cross section
- Spin asymmetry A_N

P. Hinderer, M. Schlegel, WV

P. Hinderer, Y. Koike, WV

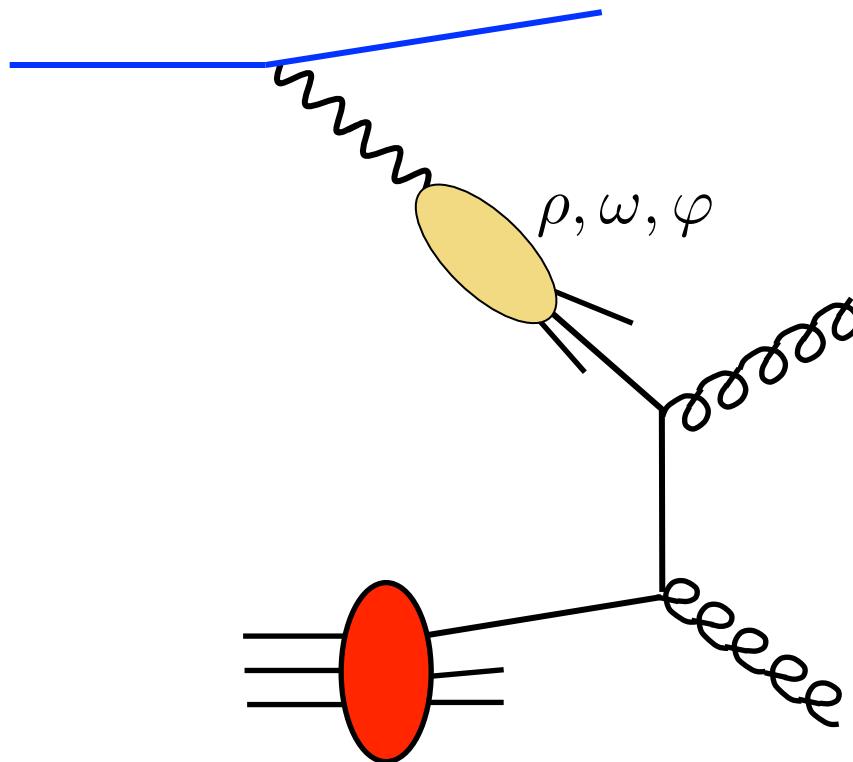
Photoproduction $\gamma p \rightarrow hX$

$$f^{\ell \rightarrow \gamma}(y) = \frac{\alpha_{\text{em}}}{2\pi} \left[\frac{1 + (1 - y^2)}{y} \ln \frac{Q_{\text{max}}^2(1 - y)}{m_\ell^2 y^2} + \dots \right]$$



$$\frac{d^2\sigma}{dp_T d\eta} = \sum_{a,c} \int dy f^{\ell \rightarrow \gamma}(y) \int dx_a f_a(x_a, \mu) \int dz_c D(z_c, \mu) \frac{d^2\hat{\sigma}_{\gamma a \rightarrow c X}}{dp_T d\eta}$$

However, photon may show partonic structure:

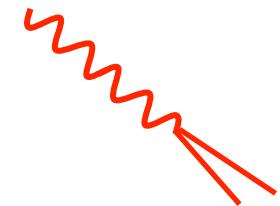


“Resolved photons” (vs. “direct”)

→ photon PDFs / structure functions $q^\gamma(x, \mu), g^\gamma(x, \mu)$

DGLAP evolution: (Witten '77, Bardeen, Buras, ... Glück, Reya, Vogt)

$$\frac{dq^\gamma(x, \mu)}{d \log \mu^2} = \frac{\alpha_s(\mu^2)}{2\pi} \left[P_{qq} \otimes q^\gamma + P_{qg} \otimes g^\gamma \right](x, \mu) + \frac{\alpha_{\text{em}}}{2\pi} P_{q\gamma}(x)$$



Find $q^\gamma \propto \mathcal{O}\left(\frac{\alpha_{\text{em}}}{\alpha_s}\right)$

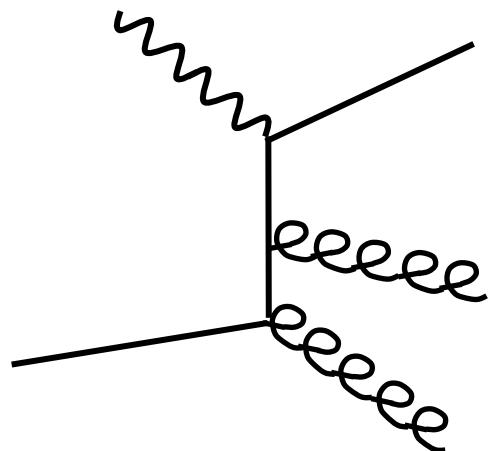
★ long-standing experience with photon structure,
 e^+e^- , HERA

★ recent study for EIC:

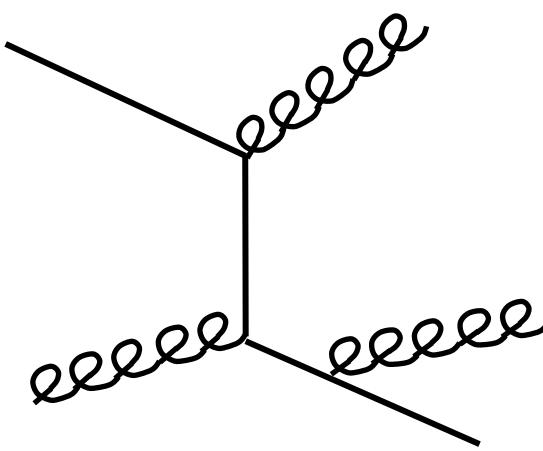
Chu, Aschenauer, Lee, Zheng, arXiv:1705.08831

★ NLO QCD corrections to $\gamma p \rightarrow hX$:

Aurenche et al.; Aversa et al.; Gordon, Storrow; Jäger, Stratmann, WV;
de Florian, WV



“direct”

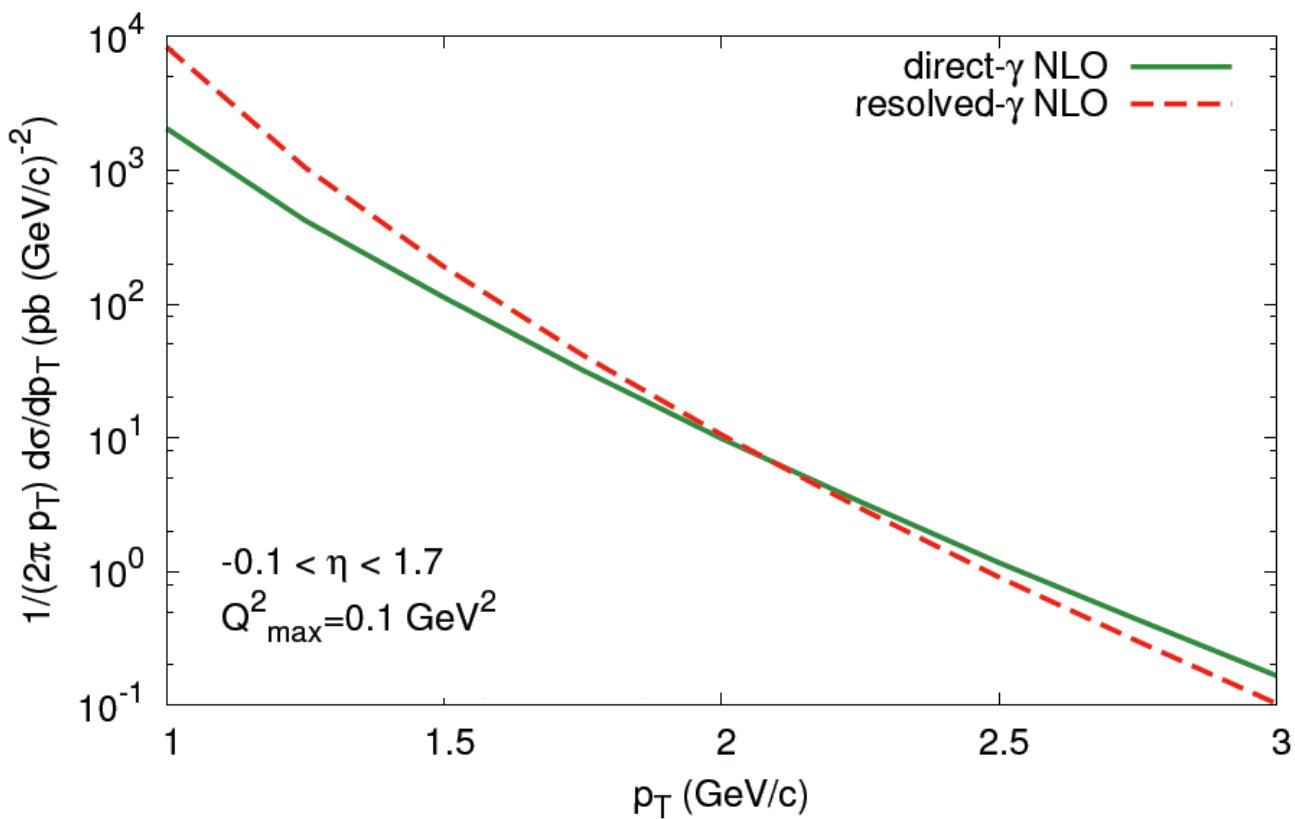


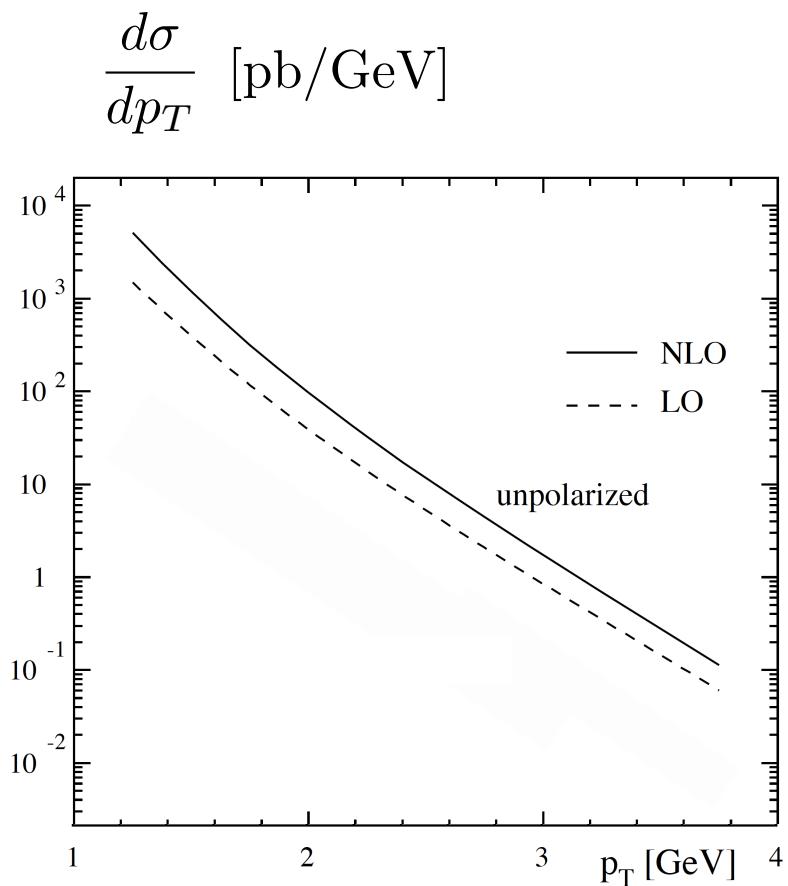
“resolved”

- ## ★ recent years: polarized scattering $\vec{\gamma} \vec{p} \rightarrow hX$ as probe of nucleon spin structure COMPASS @ CERN

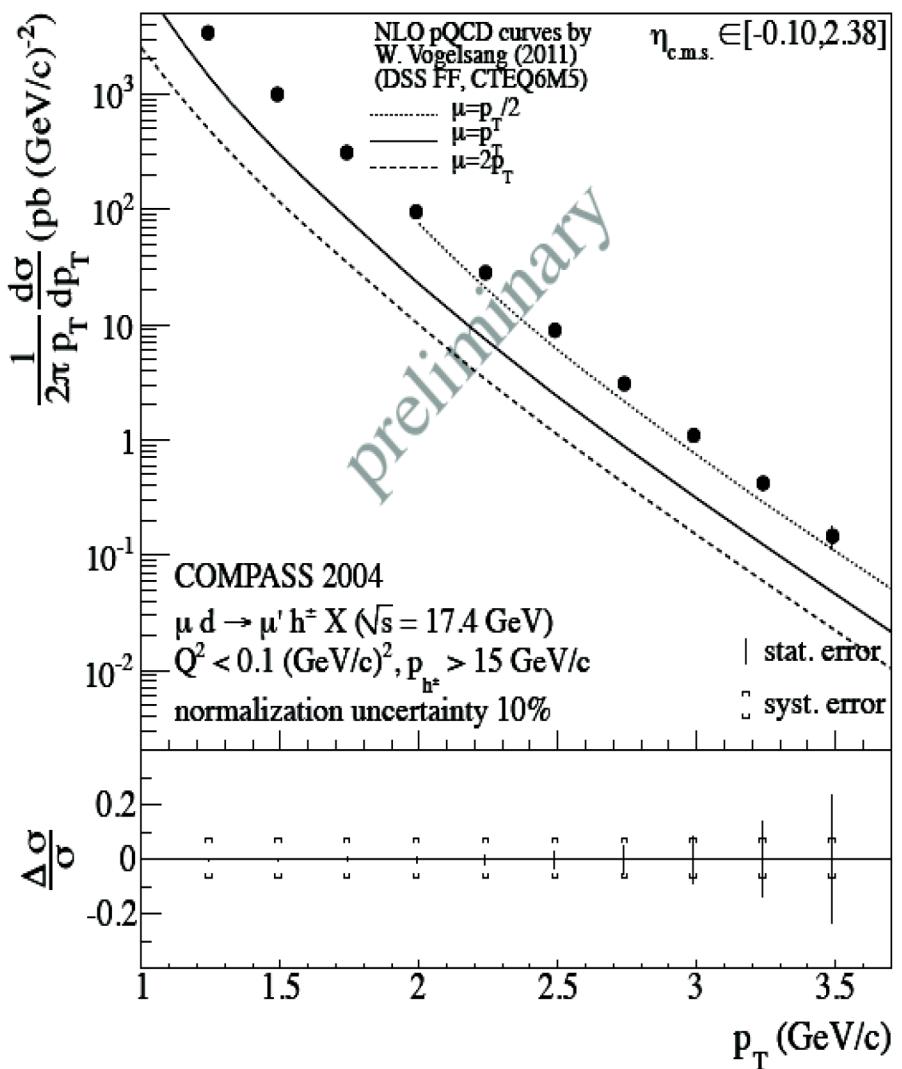
$$\mu + d \rightarrow \mu' h^\pm X$$

$$\sqrt{s} = 17.4 \text{ GeV}$$



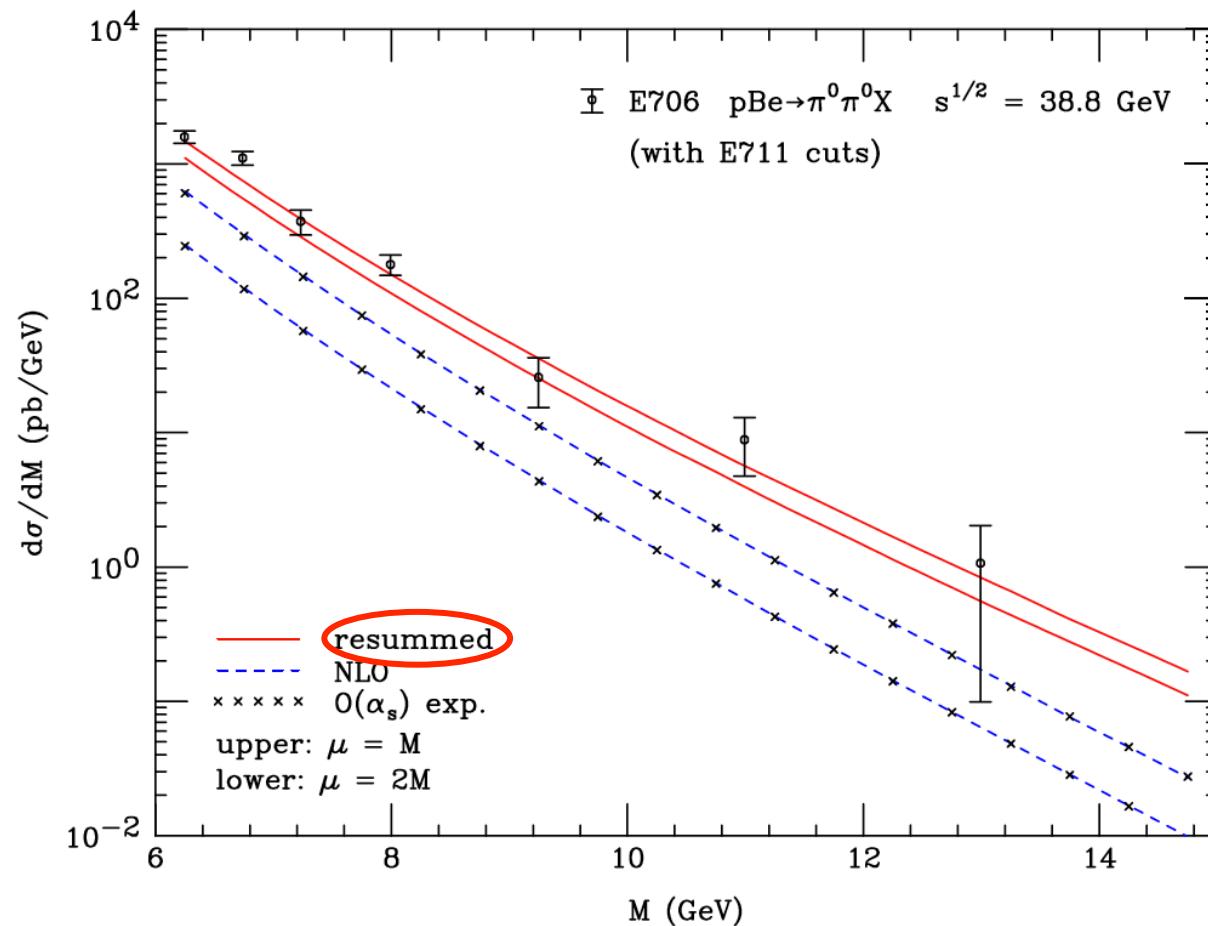


Jäger, Stratmann, WV



★ same trend as seen in pp scattering

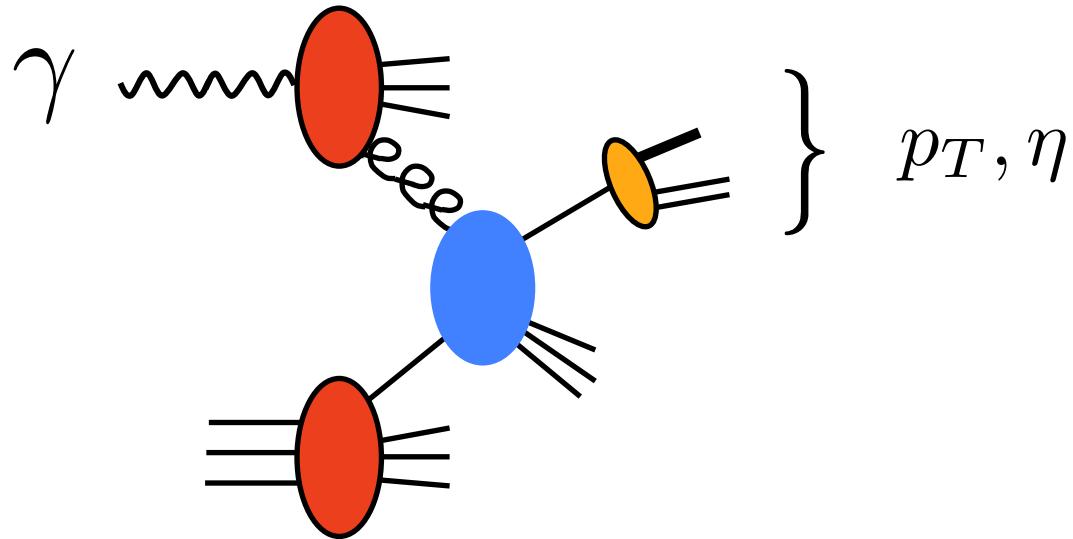
e.g. $pp \rightarrow h_1 h_2 X$



L. Almeida, G. Sterman, WV

QCD threshold resummation

One-particle inclusive (1PI) kinematics:

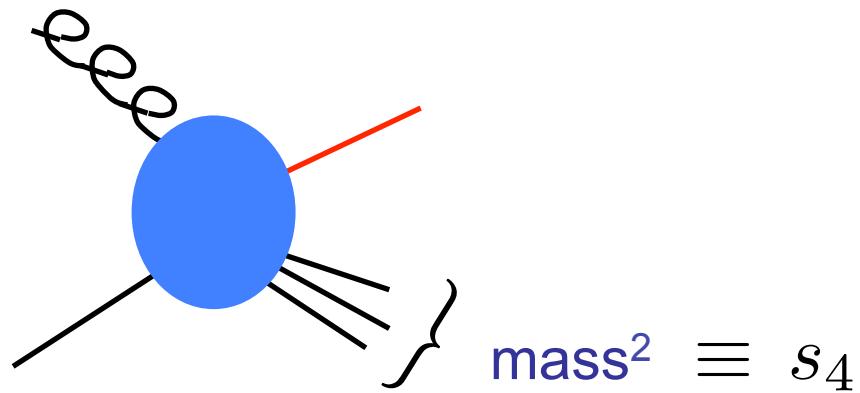


$$\frac{p_T^3 d\sigma}{dp_T d\eta} = \sum_{abc} \int_0^1 dx_a dx_b dz_c f_a(x_a) f_b(x_b) z_c^2 D_c^\pi(z_c)$$

$$\times \Omega_{ab \rightarrow cX} \left(\hat{x}_T^2, \hat{\eta}, \alpha_s(\mu), \frac{\mu^2}{\hat{s}} \right)$$

partonic variables:

$$\hat{x}_T = \frac{2p_T}{z_c \sqrt{\hat{s}}} \quad \hat{\eta} = \eta - \frac{1}{2} \ln \frac{x_a}{x_b}$$



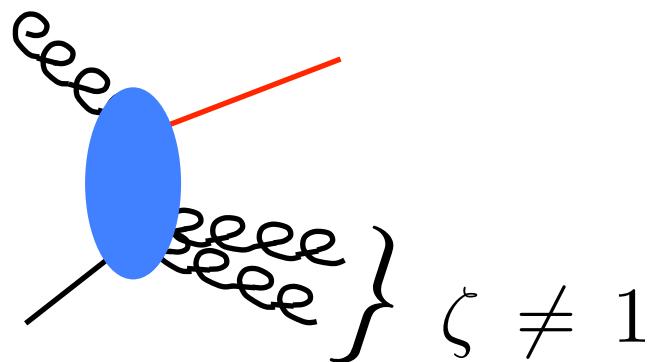
$$\zeta \equiv 1 - \frac{s_4}{\hat{s}} = \hat{x}_T \cosh \hat{\eta}$$

LO:

A Feynman diagram showing a blue circular vertex connected to three external lines labeled "eee" and one internal red line. Below the diagram is the equation $\hat{s}_4 = 0 \iff \zeta = 1$.

$$\Omega_{ab \rightarrow cX}^{(\text{LO})}(\zeta, \hat{\eta}) = \delta(1 - \zeta) \omega_{ab \rightarrow cd}^{(0)}(\hat{\eta})$$

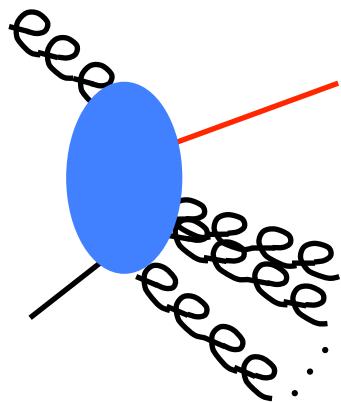
NLO:



$$\alpha_s \left(\frac{\log(1 - \zeta)}{1 - \zeta} \right)_+ + \dots$$

$$\zeta \neq 1$$

N^kLO:



$$\alpha_s^k \left(\frac{\log^{2k-1}(1 - \zeta)}{1 - \zeta} \right)_+ + \dots$$

- ★ soft and/or collinear radiation
- ★ threshold logarithms, may produce bulk of cross section

- resummation formulated in terms of suitable Mellin moments:

$$\tilde{\Omega}_{ab \rightarrow cX} \left(\textcolor{red}{N}, \hat{\eta}, \alpha_s(\mu), \frac{\mu^2}{\hat{s}} \right) \equiv \int_0^1 d\zeta \zeta^{\textcolor{red}{N}-1} \Omega_{ab \rightarrow cX} \left(\zeta, \hat{\eta}, \alpha_s(\mu), \frac{\mu^2}{\hat{s}} \right)$$

$$\alpha_s^k \left(\frac{\log^{2k-1}(1-\zeta)}{1-\zeta} \right)_+ \leftrightarrow \alpha_s^k \log^{2k}(N)$$

Fixed Order 

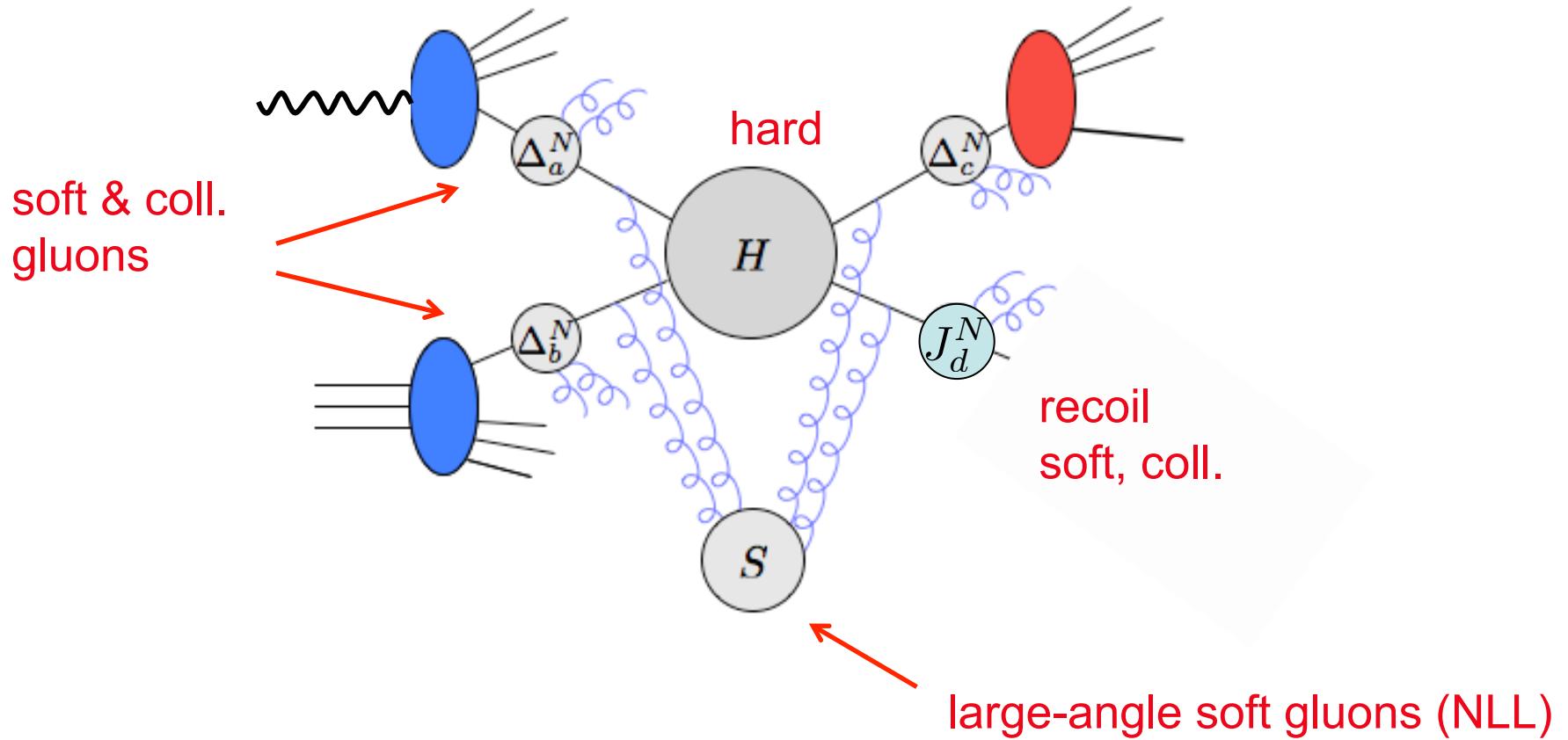
Resummation	LO	1				
	NLO	$\alpha_s L^2$	$\alpha_s L$	α_s		
	NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$	α_s^2

	$N^k LO$	$\alpha_s^k L^{2k}$	$\alpha_s^k L^{2k-1}$	$\alpha_s^k L^{2k-2}$	$\alpha_s^k L^{2k-3}$	$\alpha_s^k L^{2k-4}$



LL NLL NNLL

Factorization near threshold:



Becomes matrix problem in
color space:

$$\sum_{IL} H_{IL} S_{LI}$$

Kidonakis, Oderda, Sterman
Mert Aybat, Dixon, Sterman
Bonciani, Catani, Mangano, Nason

★ To NLL:

$$\begin{aligned}\tilde{\Omega}_{ab \rightarrow cX} \left(N, \hat{\eta}, \alpha_s(\mu), \frac{\mu^2}{s} \right) &= \Delta_a^{N_a} \Delta_b^{N_b} \Delta_c^N J_d^N \Delta_{(\text{int})}^N \\ &\times \tilde{\Omega}_{ab \rightarrow cX}^{\text{LO}} \left(1 + \frac{\alpha_s}{\pi} C_{ab \rightarrow cd}(\hat{\eta}) \right)\end{aligned}$$

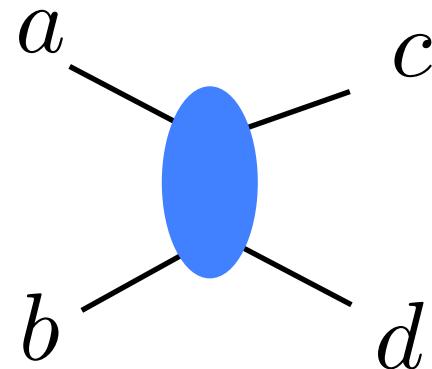
$$\Delta_i^N = \exp \left\{ \int_0^1 dz \frac{z^N - 1}{1 - z} \int_{\hat{s}}^{(1-z)^2 \hat{s}} \frac{dq^2}{q^2} A_i(\alpha_s(q)) \right\} \quad \begin{aligned}N_a &= \left(-\frac{u}{s} \right) N \\ N_b &= \left(-\frac{t}{s} \right) N\end{aligned}$$

$$A_q(\alpha_s) = C_F \left\{ \frac{\alpha_s}{\pi} + \left(\frac{\alpha_s}{\pi} \right)^2 \left[\frac{C_A}{2} \left(\frac{67}{18} - \zeta(2) \right) - \frac{5}{9} T_R n_f \right] + \dots \right\}$$

$$J_d^N = \exp \left\{ \int_0^1 dz \frac{z^N - 1}{1 - z} \left[\int_{(1-z)^2 \hat{s}}^{(1-z) \hat{s}} \frac{dq^2}{q^2} A_d(\alpha_s(q^2)) + \frac{1}{2} B_d(\alpha_s((1-z) \hat{s})) \right] \right\}$$

★ $C_{ab \rightarrow cd}(\hat{\eta})$ determined from comparison to NLO

Leading logarithms $\overline{(\text{MS})}$:



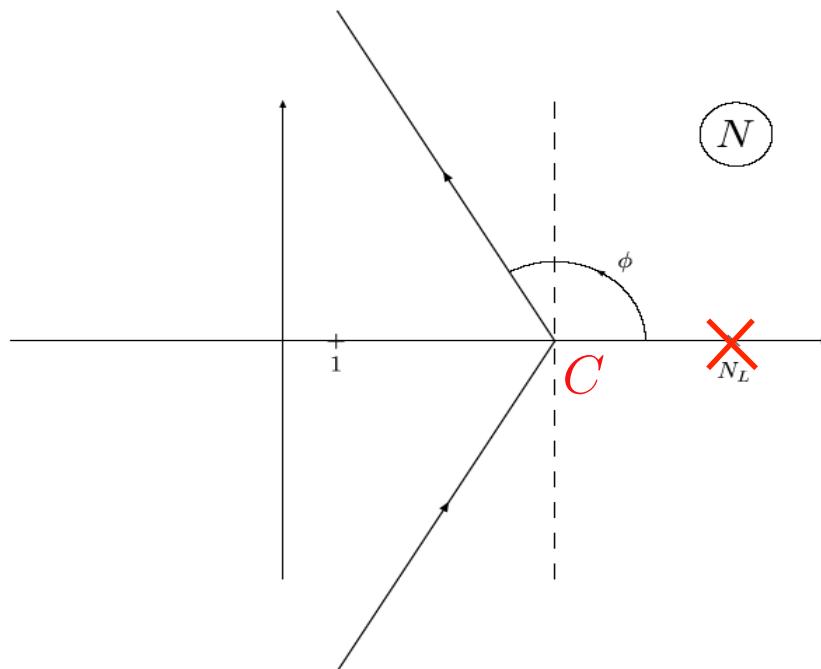
$$\tilde{\Omega}_{ab \rightarrow cX} \sim \exp \left[\left(C_a + C_b + \textcolor{red}{C_c} - \frac{1}{2} \textcolor{red}{C_d} \right) \frac{\alpha_s}{\pi} \ln^2 N \right]$$

$$(C_q = C_F, \ C_g = C_A)$$

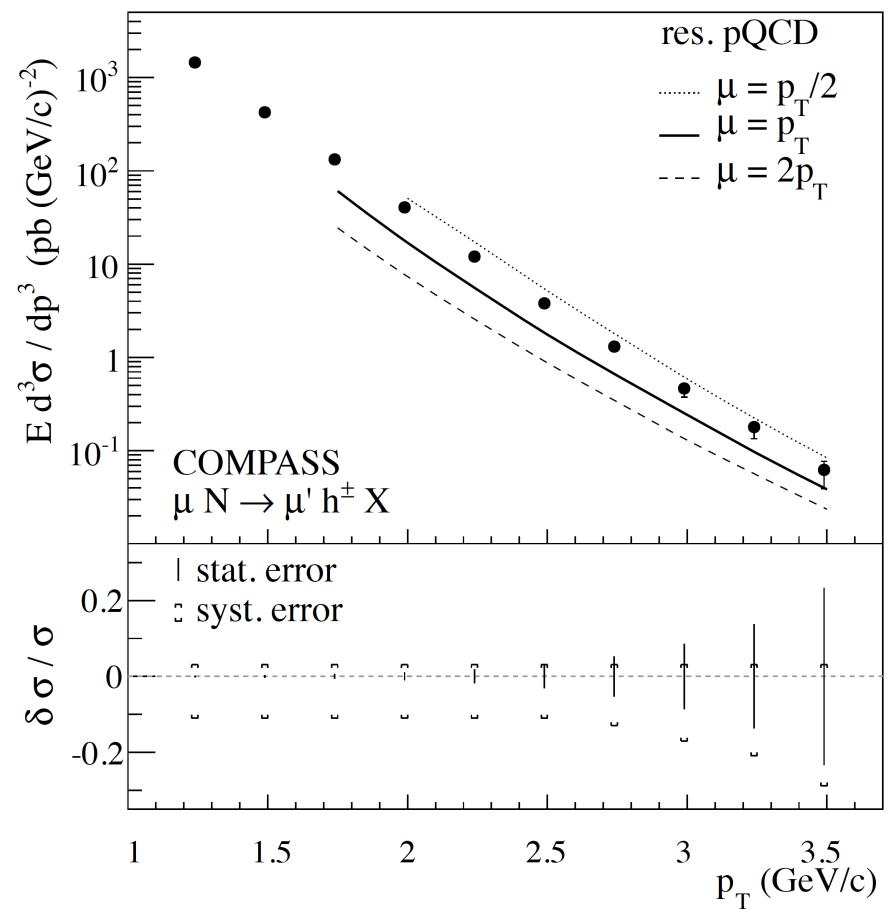
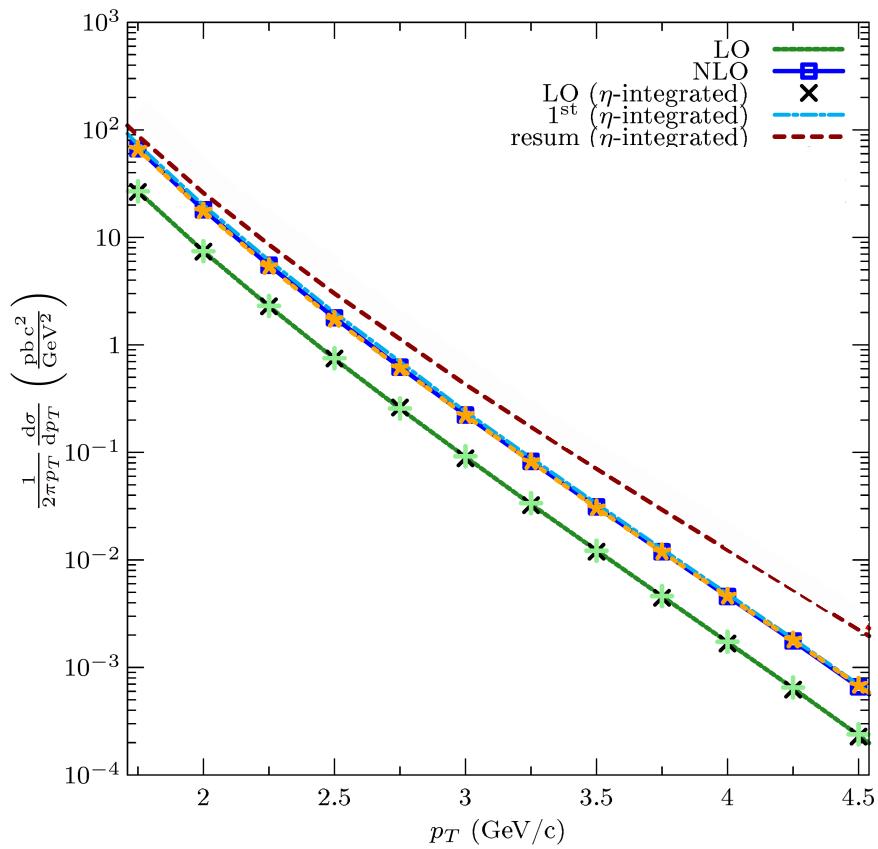
Inverse transform:

$$\Omega_{ab \rightarrow cX}(\zeta, \hat{\eta}) = \frac{1}{2\pi i} \int_{C-i\infty}^{C+i\infty} dN \zeta^{-N} \tilde{\Omega}_{ab \rightarrow cX}(N, \hat{\eta})$$

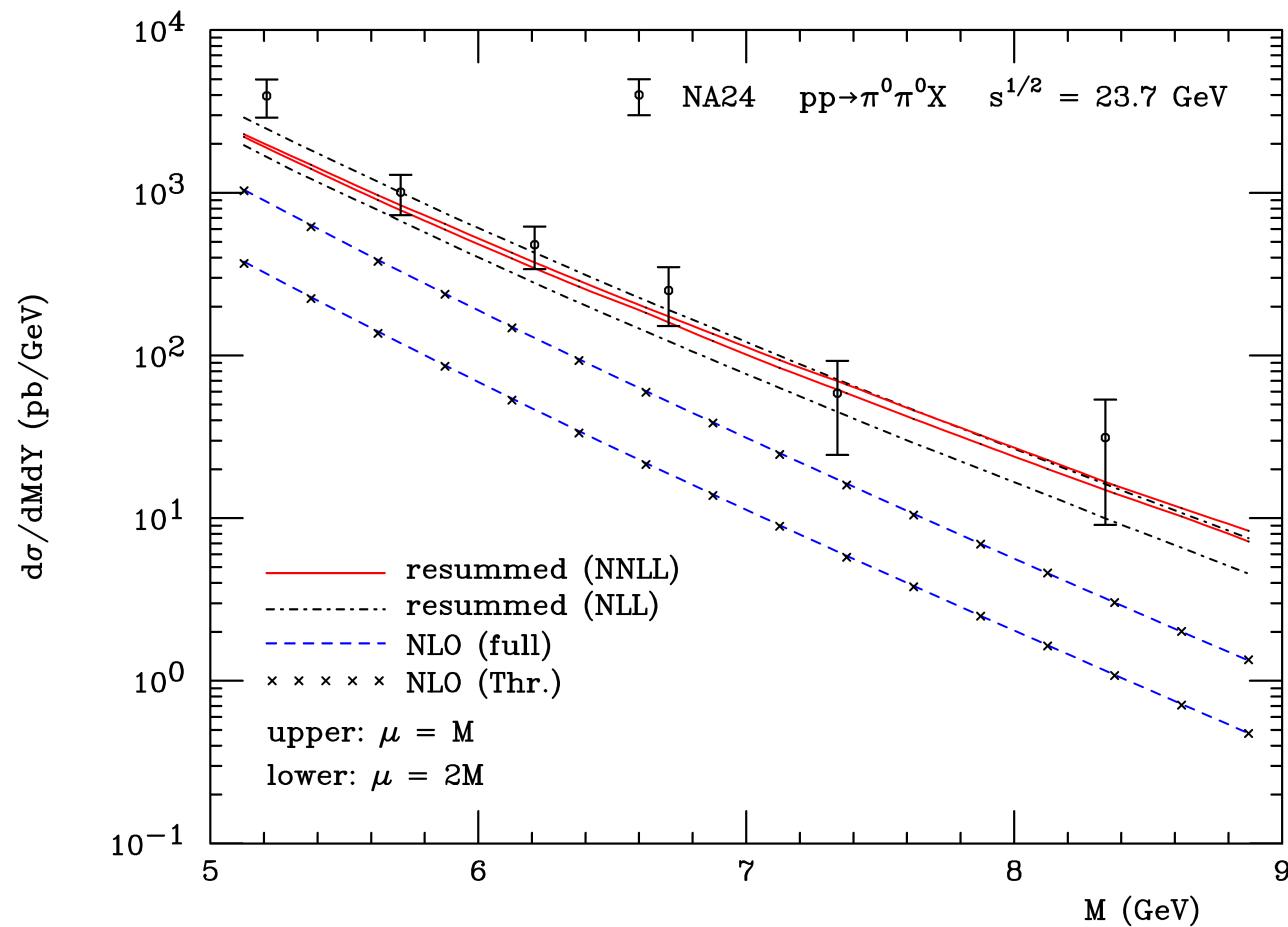
“Minimal prescription” Catani,Mangano,Nason,Trentadue



de Florian, Pfeuffer, Schäfer, WV



$pp \rightarrow h_1 h_2 X$ at NNLL:



P. Hinderer, F. Ringer, G. Sterman, WV

★ New: resummation for helicity case

Uebler, Schäfer, WV

$$\Delta\sigma \equiv \sigma^{++} - \sigma^{+-}$$

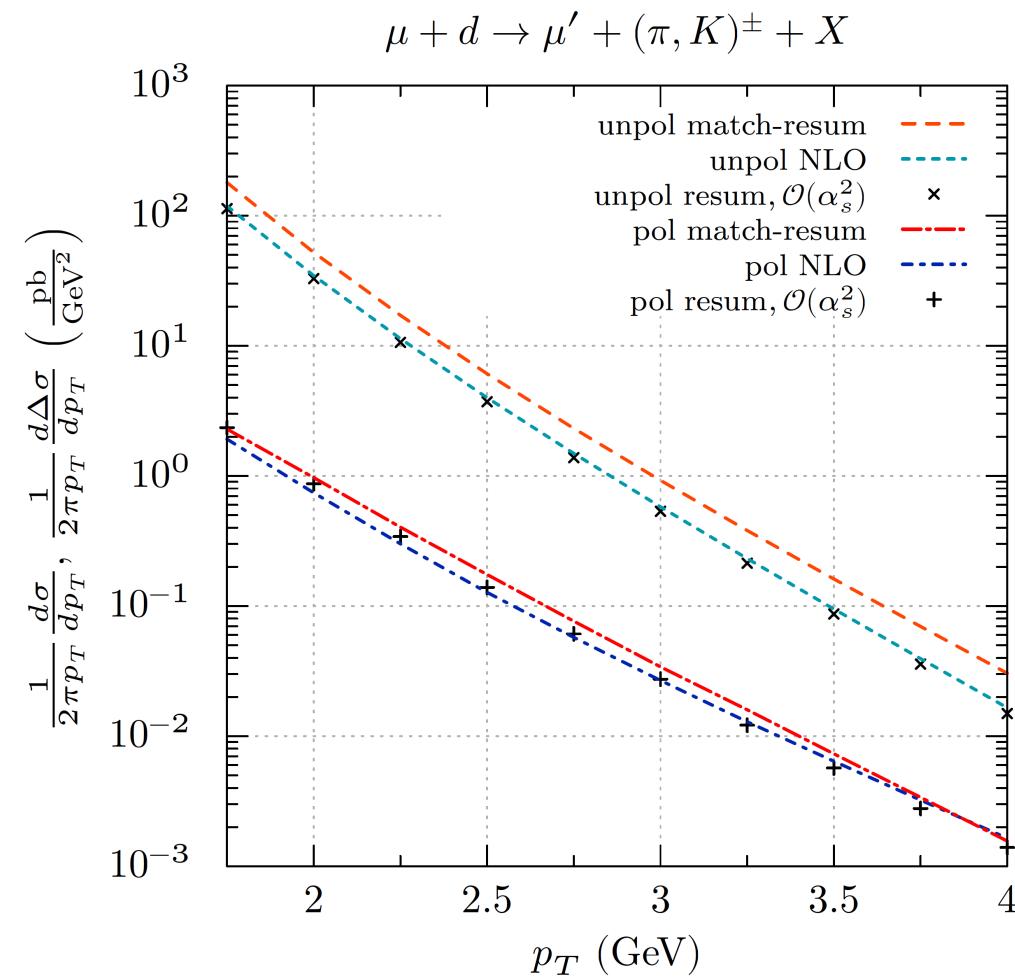
Only change:

$$\begin{aligned}\Delta \tilde{\Omega}_{ab \rightarrow cX} \left(N, \hat{\eta}, \alpha_s(\mu), \frac{\mu^2}{s} \right) &= \Delta_a^{N_a} \Delta_b^{N_b} \Delta_c^N J_d^N \Delta_{(int)}^N \\ &\times \Delta \tilde{\Omega}_{ab \rightarrow cX}^{\text{LO}} \left(1 + \frac{\alpha_s}{\pi} \Delta C_{ab \rightarrow cd}(\hat{\eta}) \right)\end{aligned}$$

and of course polarized PDFs, e.g.

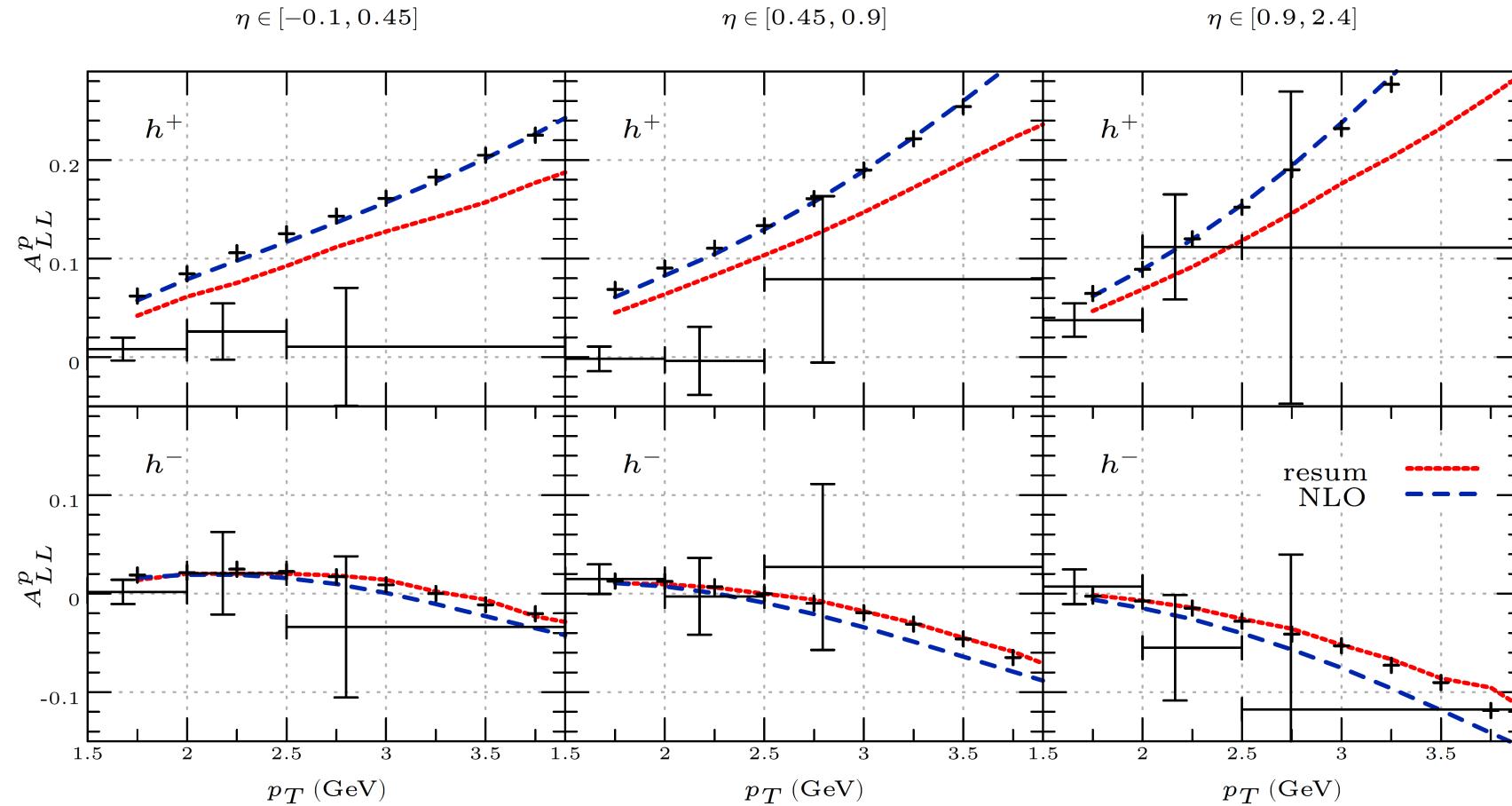
$$\Delta q(x) = \text{---} \quad \begin{array}{c} \text{---} \\ \text{---} \end{array} \quad - \quad \begin{array}{c} \text{---} \\ \text{---} \end{array} \quad \text{---}$$

with simple model for $\Delta q^\gamma, \Delta g^\gamma$: Uebler, Schäfer, WV



$$A_{LL} = \frac{\sigma_{++} - \sigma_{+-}}{\sigma_{++} + \sigma_{+-}}$$

COMPASS



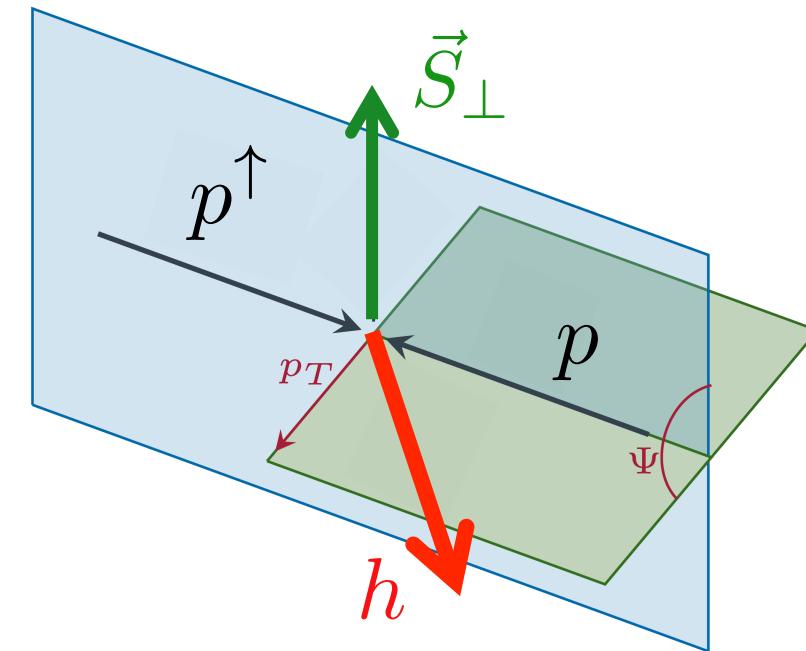
$\ell p \rightarrow hX$: Motivation

Single-inclusive scattering: $\ell p \rightarrow \text{jet } X$

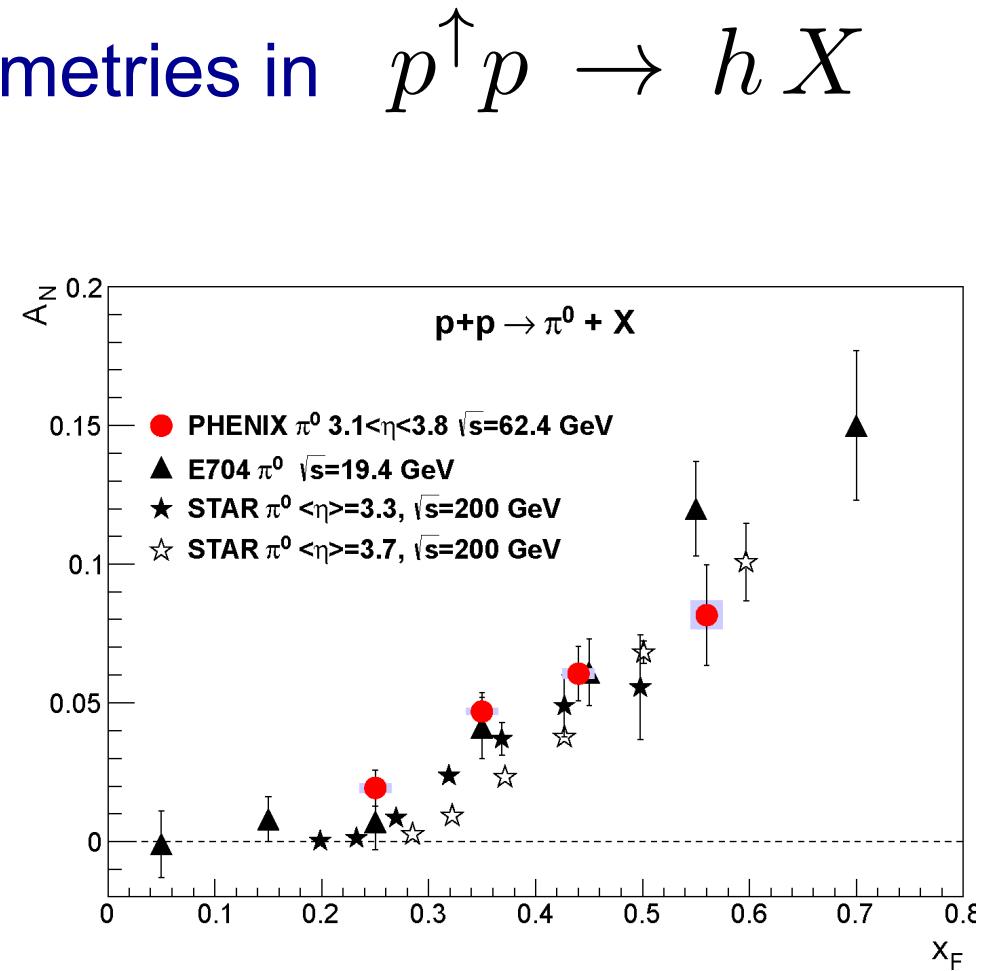
$\ell p \rightarrow h X$

Why care?

Large single-spin asymmetries in $p^\uparrow p \rightarrow h X$



$$A_N = \frac{\sigma^\uparrow - \sigma^\downarrow}{\sigma^\uparrow + \sigma^\downarrow}$$

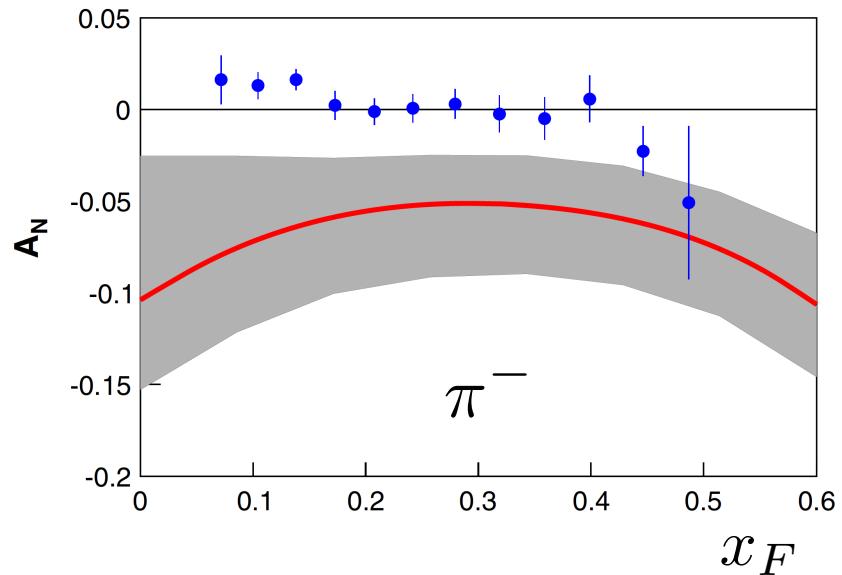
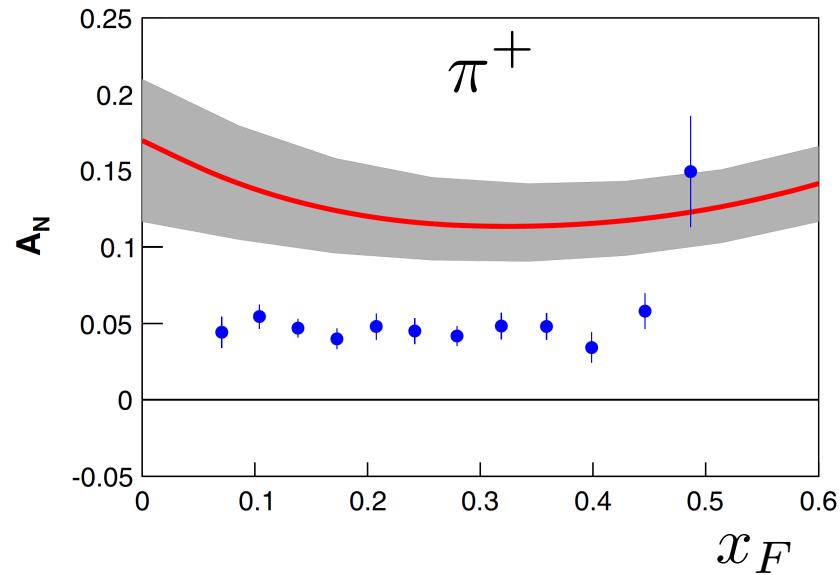


- A_N in $p^\uparrow p \rightarrow h X$ power-suppressed in QCD
- various mechanisms discussed:
 - ★ single hard scale p_T : collinear factorization
twist-3 proton matrix elements, twist-3 fragmentation
Qiu, Sterman; Efremov, Teryaev; Koike et al.; Kouvaris, Qiu, WV, Yuan;
Kanazawa, Koike, Metz, Pitonyak; ...
 - ★ a plethora of contributions, even at LO
 - ★ all studies LO so far, no proper evolution

- process $p^\uparrow \ell \rightarrow \text{jet } X$:
Anselmino, Boglione,
Hansson, Murgia '99; ...
Koike '00, '02
- ★ can choose same kinematics as for $p^\uparrow p$
 - ★ simpler -- fewer subprocesses, contributions
 - ★ no fragmentation (for jets)
 - ★ can shed light on mechanisms for $p^\uparrow p \rightarrow h X$
 - ★ may serve as testbed for theoretical calculations
especially: higher-order QCD corrections

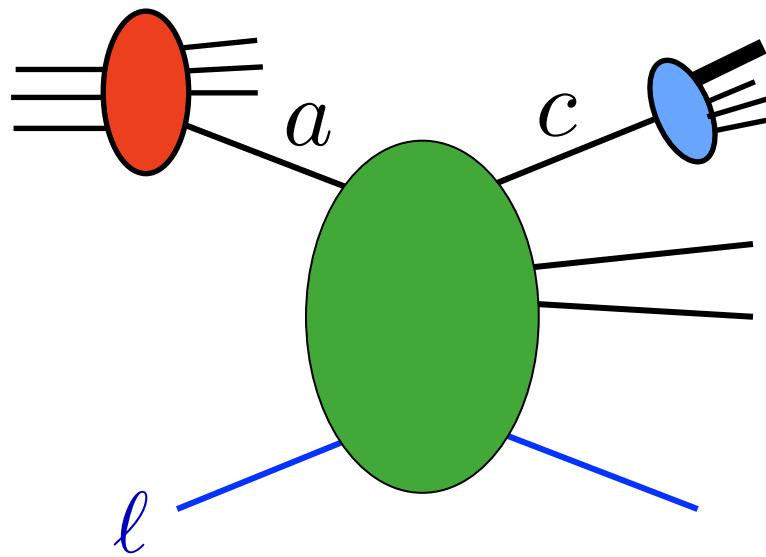
$p^\uparrow \ell \rightarrow \pi^\pm X$

Gamberg, Kang, Metz, Pitonyak, Prokudin

HERMES data $\sqrt{S} = 7.25$ GeV, $p_\perp \sim 1$ GeV

NLO cross section for $p\ell \rightarrow h X$

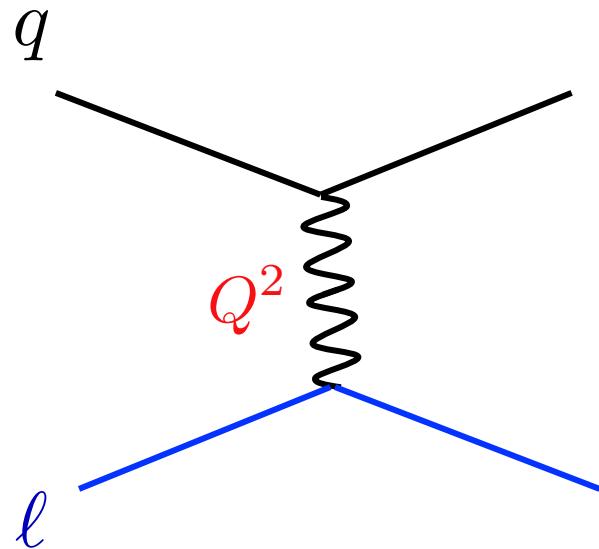
Hinderer, Schlegel, WV



$$\frac{E_h d^3 \sigma^{p\ell \rightarrow hX}}{d^3 P_h} = \frac{1}{\pi S} \sum_{a,c} \int \frac{dx}{x} \int \frac{dz}{z^2} f_a(x, \mu) D_c^h(z, \mu) \frac{d^2 \hat{\sigma}^{a\ell \rightarrow cX}}{v dv dw}$$

$$v = 1 + \frac{t}{s} \qquad \qquad w = -\frac{u}{t+s}$$

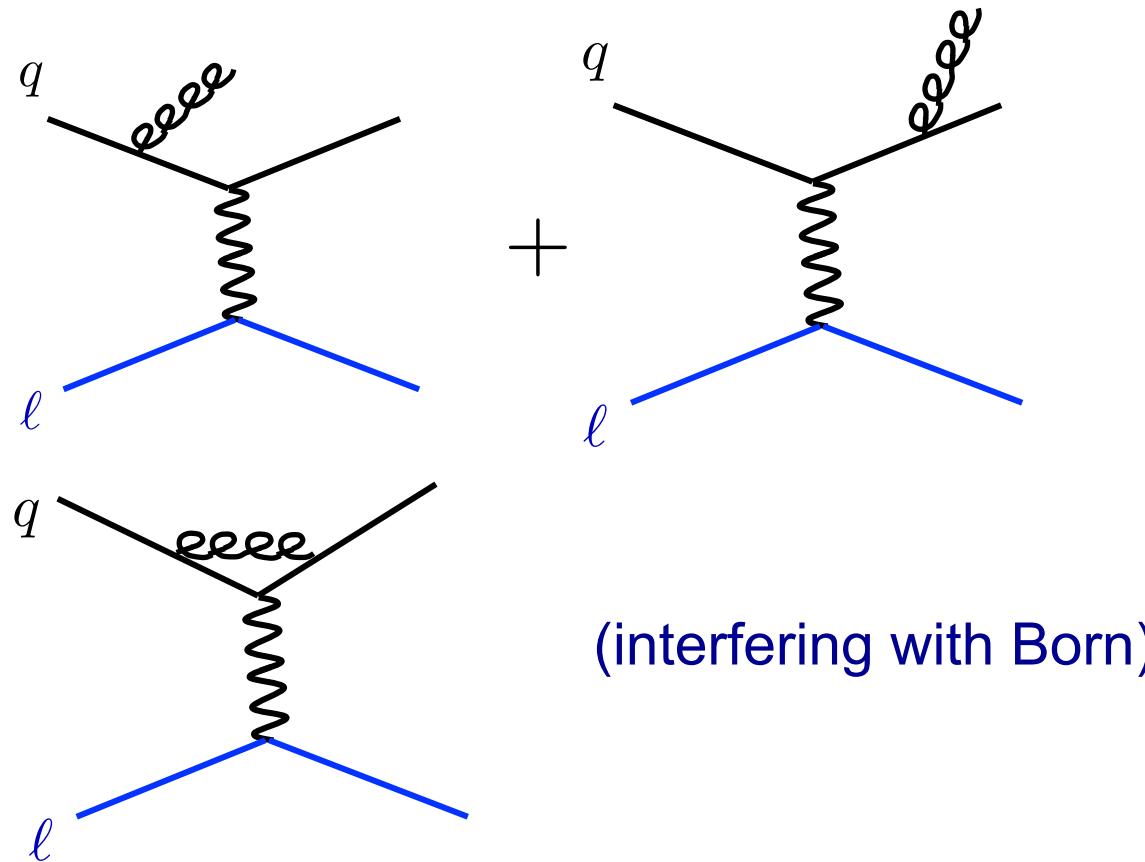
LO:



- always at large Q^2

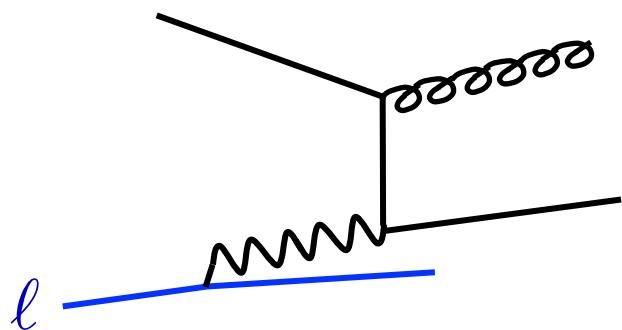
$$\frac{d^2 \hat{\sigma}_{\text{LO}}^{q\ell \rightarrow q\ell}}{v dv dw} \propto \alpha_{\text{em}}^2 \delta(1-w) \frac{1+v^2}{(1-v)^2}$$

NLO:



$$\frac{d\hat{\sigma}_{\text{NLO}}^{q \rightarrow q}}{v dv dw} \propto \alpha_{\text{em}}^2 \alpha_s \left[A_0^{q \rightarrow q}(v) \delta(1-w) + A_1^{q \rightarrow q}(v) \left(\frac{\ln(1-w)}{1-w} \right)_+ + \frac{1}{(1-w)_+} A_2^{q \rightarrow q}(v) + R(v, w) \right]$$

New configuration at NLO:



- almost real photon
(Weizsäcker-Williams)
- enhanced as m_ℓ small
- structure is

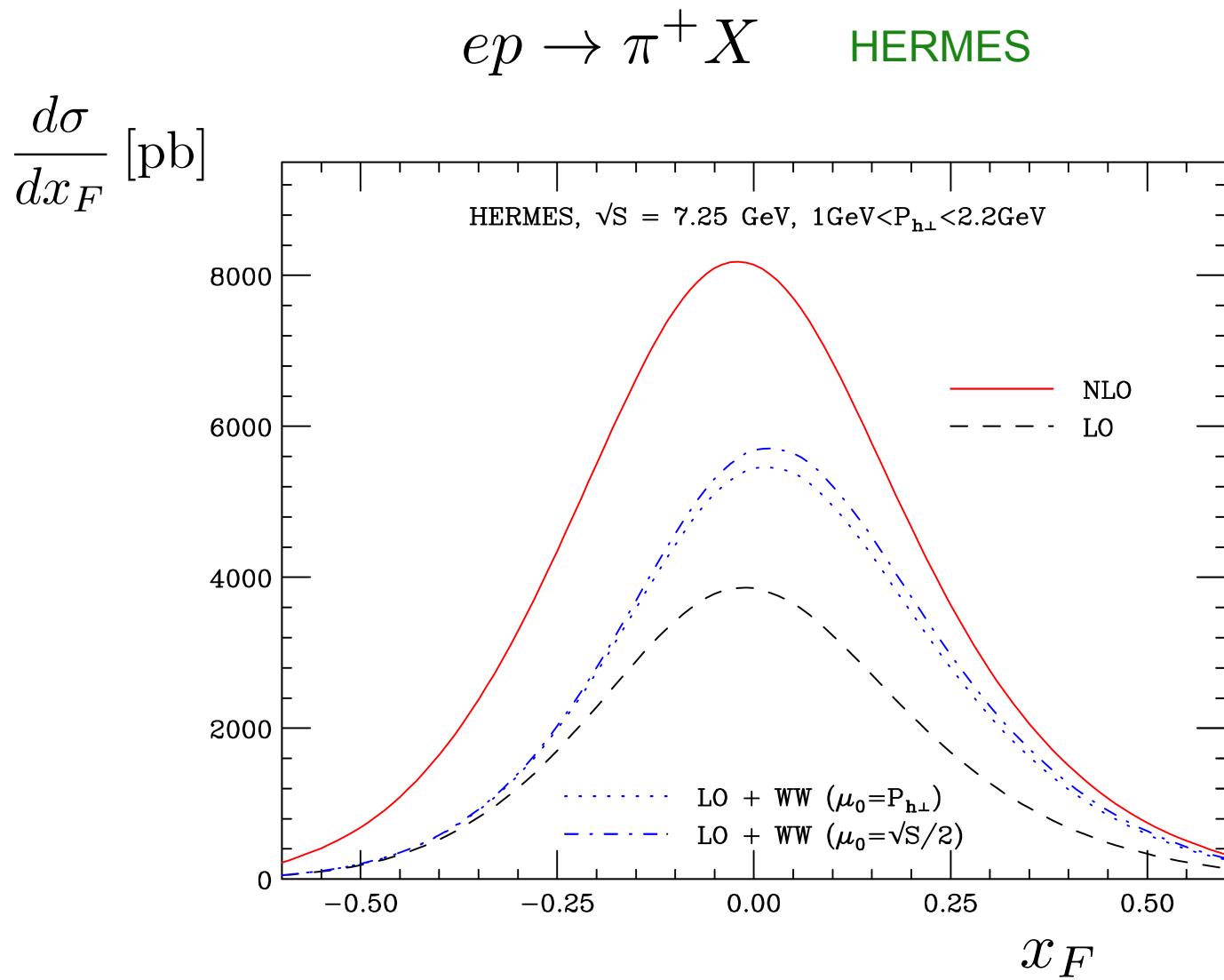
$$f^{\ell \rightarrow \gamma} \otimes \hat{\sigma}^{\gamma q \rightarrow qg}$$

- computed in two ways:
 - ★ keep lepton massive, but drop terms that vanish as $m_\ell \rightarrow 0$
 - ★ compute with massless lepton, subtract collinear singularity in $\overline{\text{MS}}$.
Define “photon parton distribution” in lepton:

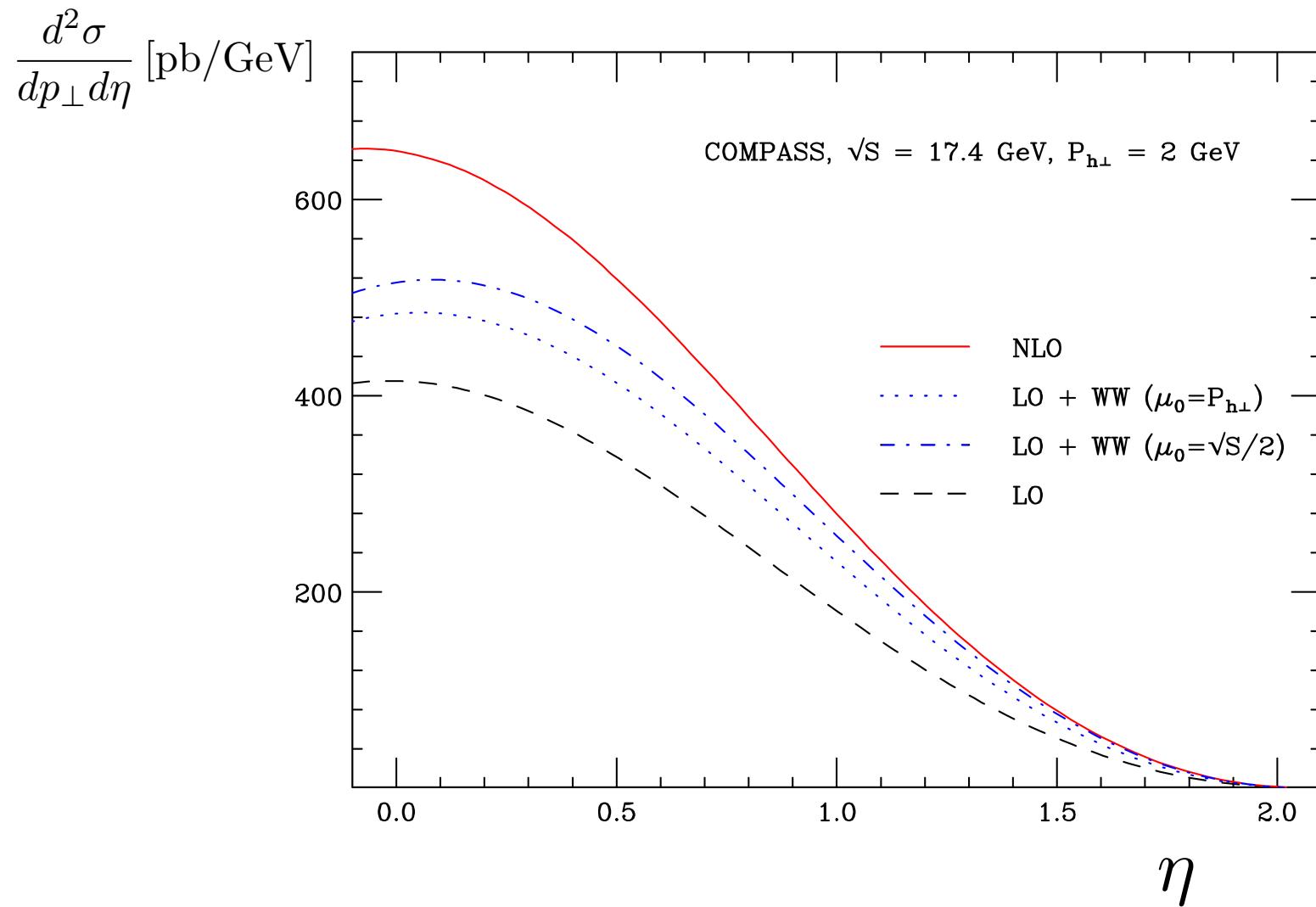
$$f_{\text{bare}}^{\ell \rightarrow \gamma}(y, \mu) = \frac{\alpha}{2\pi} P_{\gamma\ell}(y) \left[\frac{1}{\varepsilon} + \ln \left(\frac{\mu^2}{y^2 m_\ell^2} \right) - 1 \right]$$

- both results identical and lead to

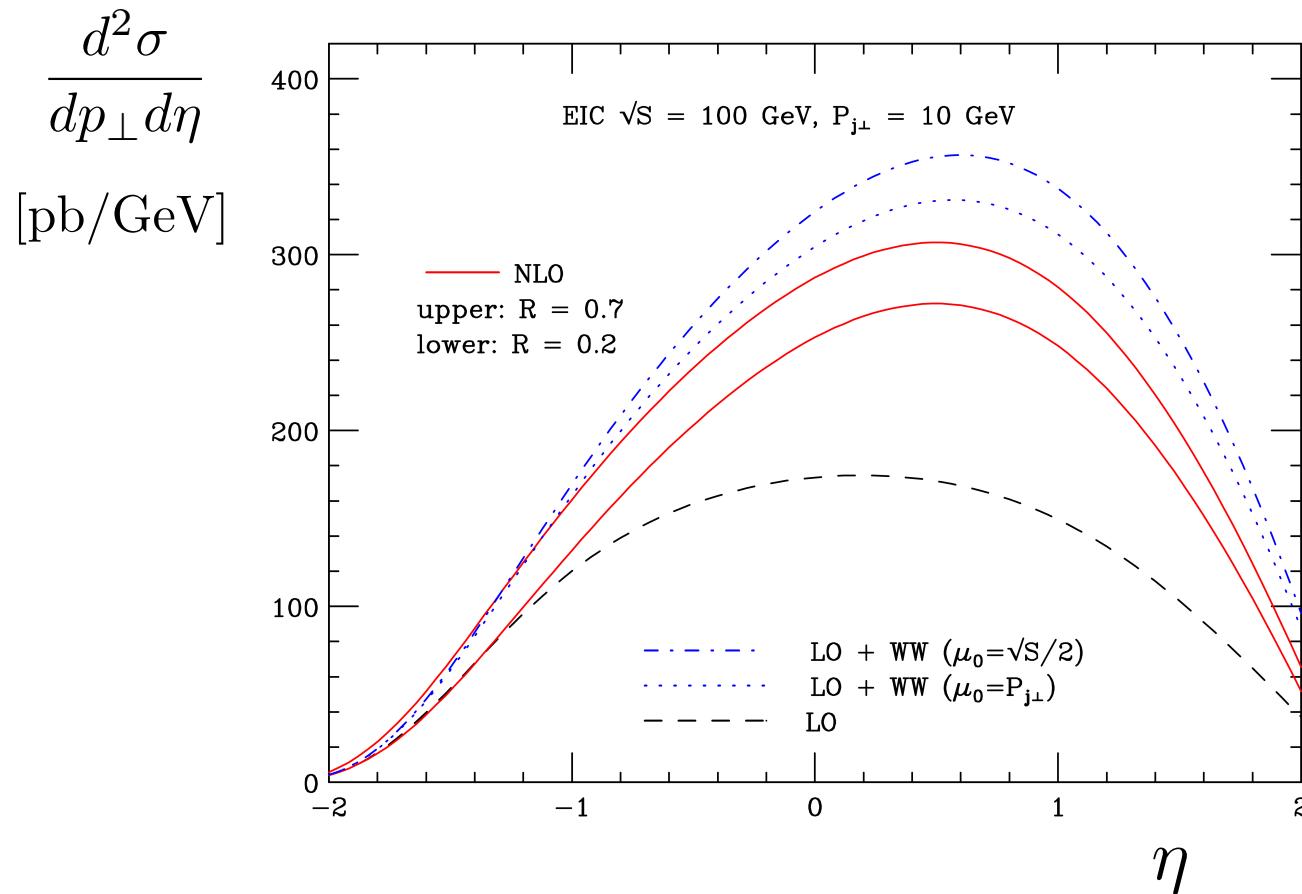
$$\begin{aligned} \frac{E_h d^3 \sigma^{p\ell \rightarrow hX}}{d^3 P_h} &= \frac{1}{\pi S} \sum_{a,c} \int \frac{dx}{x} \int \frac{dz}{z^2} f_a(x, \mu) D_c^h(z, \mu) \left[\frac{d^2 \hat{\sigma}_{\text{LO}}^{a\ell \rightarrow cX}}{v dv dw} + \frac{\alpha_s}{\pi} \frac{d^2 \hat{\sigma}_{\text{NLO}}^{a\ell \rightarrow cX}(\mu_0)}{v dv dw} \right. \\ &\quad \left. + f^{\ell \rightarrow \gamma}(z_\gamma, \mu_0) \frac{\alpha_s}{\pi} \frac{d^2 \hat{\sigma}_{\text{LO}}^{a\gamma \rightarrow cX}}{v dv dw} \right] \end{aligned}$$



$\mu p \rightarrow \pi^0 X$ COMPASS



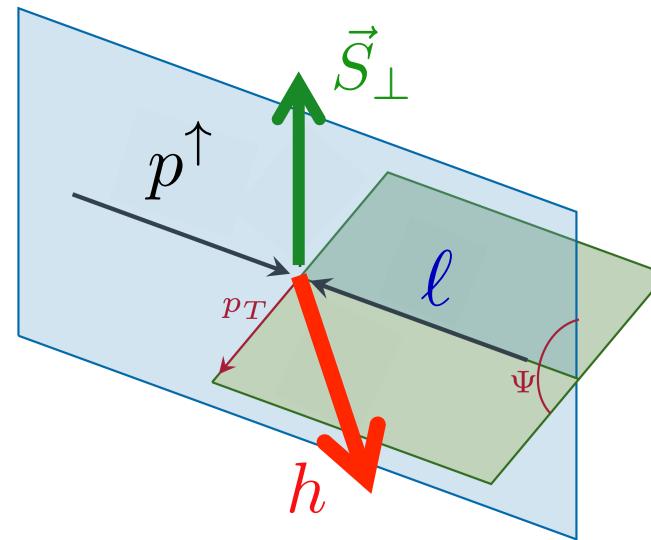
$ep \rightarrow \text{jet}X$ EIC



NLO for A_N

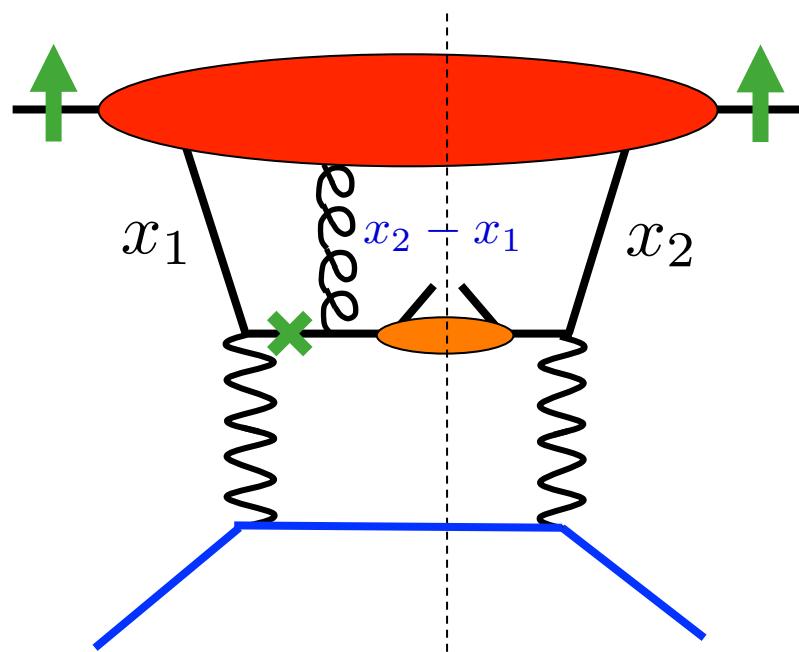
Hinderer, Koike, WV (in prep.)

$$p^\uparrow \ell \rightarrow h X$$

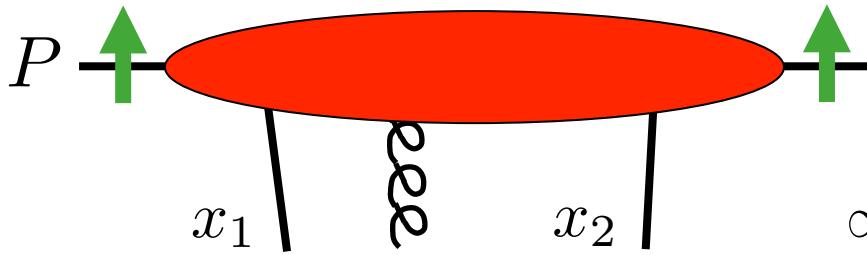


$$A_N = \frac{\sigma^\uparrow - \sigma^\downarrow}{\sigma^\uparrow + \sigma^\downarrow}$$

Qiu, Sterman; Kanazawa, Koike; ...

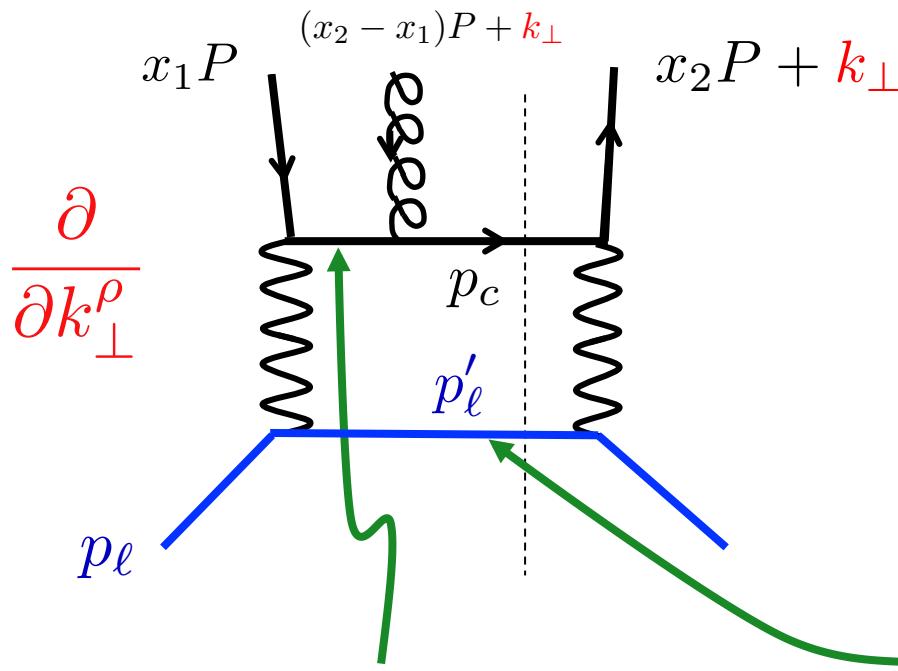


- phase in hard-scattering



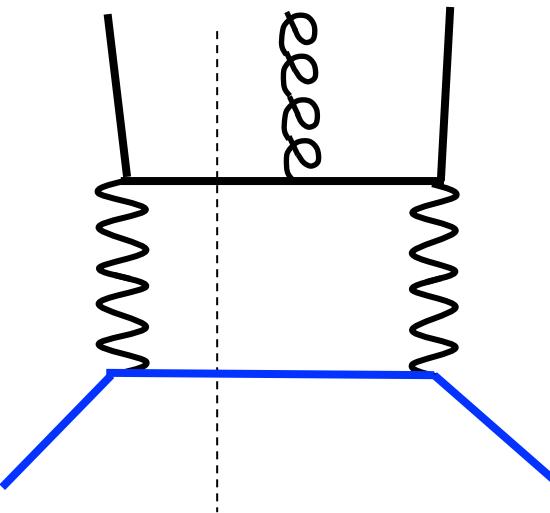
$$G_F(x_1, x_2)$$

$$\propto \epsilon^{S_\perp \alpha n \bar{n}} \mathcal{FT} \left[\langle P, S | \bar{\psi} \gamma^+ F_\alpha^+ \psi | P, S \rangle \right]$$



$$\frac{1}{(p_c - (x_2 - x_1)P - k_\perp)^2 + i\varepsilon}$$

$$\rightarrow -\frac{i\pi}{t} \delta \left(x_2 - x_1 - \frac{2p_c \cdot k_\perp}{t} \right)$$



$$\delta((p'_\ell)^2)$$

$$\propto \delta \left(x_2 - \frac{-u}{s+t} - \frac{2p_c \cdot k_\perp}{s+t} \right)$$

eventually: LO formula

$$\begin{aligned} \frac{E_h d^3 \Delta \sigma^{p\ell \rightarrow hX}}{d^3 P_h} &= \frac{\epsilon^{S_\perp p_c n \bar{n}}}{\pi S} \sum_{a,c} \int \frac{dx}{x} \int \frac{dz}{z^2} D_c^h(z, \mu) \\ &\times \left[G_{F,a}(x, x) - x \frac{d}{dx} G_{F,a}(x, x) \right] \\ &\times \frac{s}{tu} \frac{d^2 \Delta \hat{\sigma}_{\text{LO}}^{a\ell \rightarrow cX}}{v dv} \delta(1-w) \end{aligned}$$

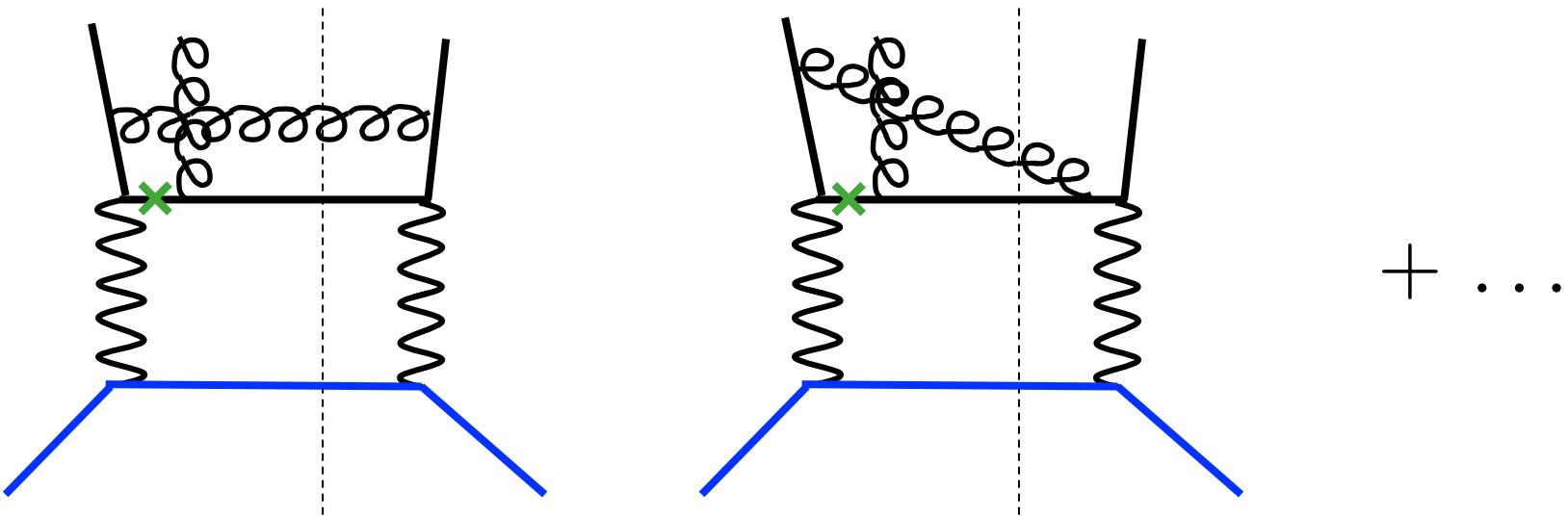
“Soft-gluon pole”

Qiu, Sterman '98

Kouvaris, Qiu, WV, Yuan '06
Koike, Tanaka
Kang, Metz, Qiu, Zhou

Some features of NLO calculation:

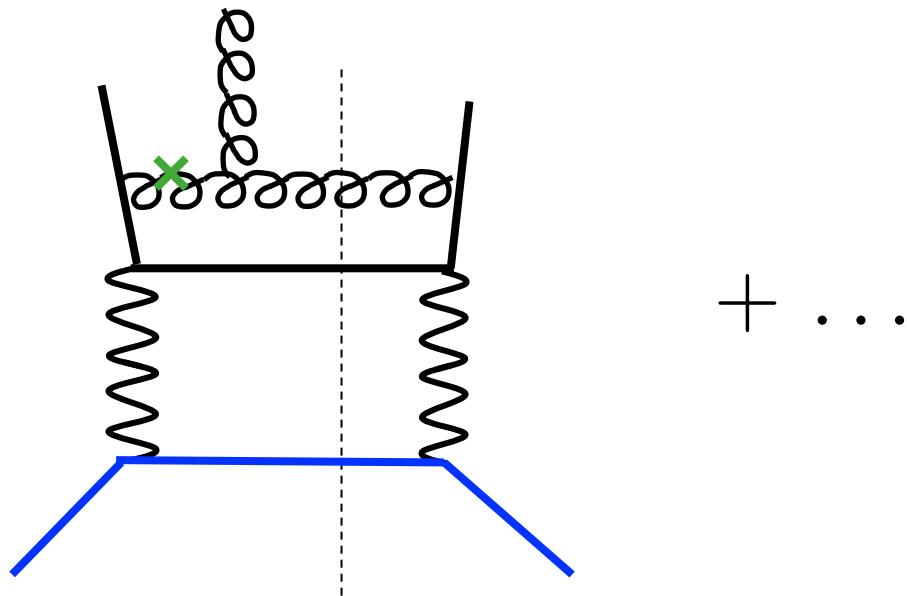
- ★ coupling to fragmenting quark line:



contributions $G_F(x, x)$ and $x \frac{d}{dx} G_F(x, x)$

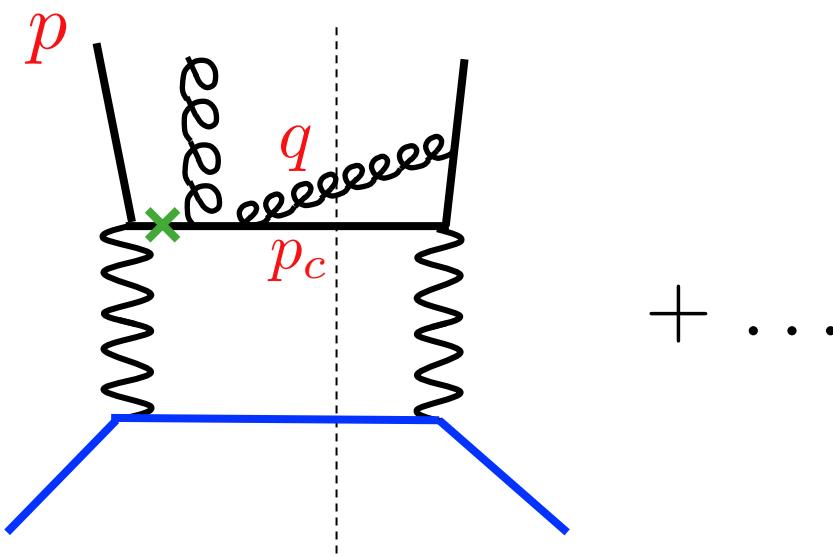
with distributions $\left(\frac{\ln(1-w)}{1-w} \right)_+$ etc.

★ coupling to non-fragmenting gluon:



even contributes derivative piece $x \frac{d}{dx} G_F(x, x)$

★ hard-pole contributions:



makes x_1, x_2 kinematics-dependent, e.g.

$$G_F \left(x, x + \frac{(q + p_c)^2}{(p - p_c)^2 + (p - q)^2} \right)$$

requires care when extracting $\frac{1}{\varepsilon}$ poles

★ factorization of collinear singularities:

$$\frac{\partial}{\partial \log \mu^2} G_F(x, x, \mu^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} \left\{ G_F(z, z, \mu^2) \textcolor{red}{P}_{qq}(\hat{x}) - N_c \delta(1-\hat{x}) G_F(z, z, \mu^2) \right.$$

$$\hat{x} = \frac{x}{z} \quad \left. + \frac{N_c}{2} \left[\frac{1+\hat{x}}{(1-\hat{x})_+} G_F(z, \textcolor{red}{x}, \mu^2) - \frac{1+\hat{x}^2}{(1-\hat{x})_+} G_F(z, z, \mu^2) \right] \right\}$$

Braun, Manashov, Pirnay; Kang, Qiu; Yuan, WV; Ma, Wang; Yoshida

★ find:

$$\frac{\partial}{\partial \log \mu^2} (G_F(x, x) - x G'_F(x, x)) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} \left\{ (G_F(z, z) - z G'_F(z, z)) P_{qq}(\hat{x}) \right.$$

$$- \frac{N_c}{2} \delta(1-\hat{x}) (G_F(z, z) - z G'_F(z, z))$$

$$- \frac{N_c}{2} \frac{G_F(z, z) - G_F(z, x)}{1-\hat{x}}$$

$$\left. + \frac{N_c}{2} (1+\hat{x}) x \frac{d}{dx} \left(\frac{G_F(z, z) - G_F(z, x)}{1-\hat{x}} \right) \right\}$$

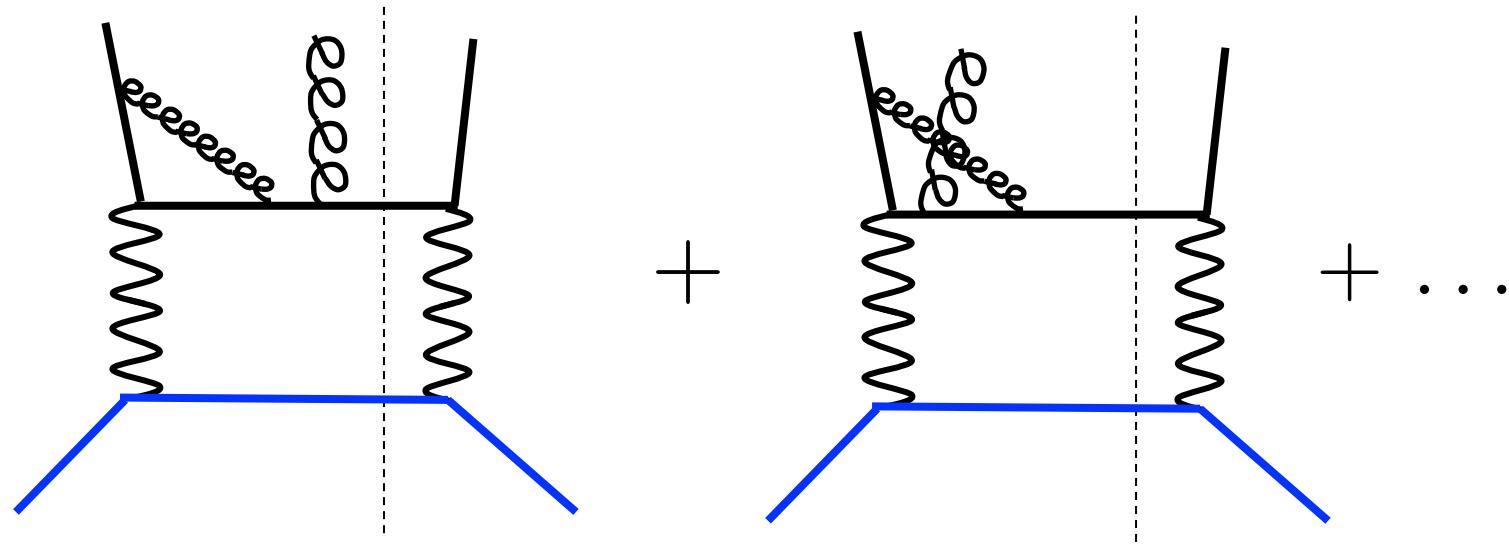
★ cancelation of singularities for $w \neq 1$ after substantial amount of integration by parts

★ leading terms:

$$\frac{d^2 \hat{\sigma}_{\text{NLO}}^{q\ell q X}}{v dv dw} = \frac{\alpha^2 e_q^2 C_F}{sv} \left[8 \frac{1+v^2}{(1-v)^2} \left(\frac{\log(1-w)}{1-w} \right)_+ + \frac{A(v)}{(1-w)_+} + \dots \right]$$

$$\frac{d^2 \Delta \hat{\sigma}_{\text{NLO}}^{q\ell q X}}{v dv dw} = \frac{\alpha^2 e_q^2 C_F}{sv} \left[8 \frac{1+v^2}{(1-v)^2} \left(\frac{\log(1-w)}{1-w} \right)_+ + \frac{A'(v)}{(1-w)_+} + \dots \right]$$

★ final step: virtual corrections $\propto \delta(1 - w)$



★ do not have LO structure $G_F(x, x) - xG'_F(x, x)$

★ numerical studies will be major challenge

Conclusions:

- much interest in processes

$$\gamma p \rightarrow hX \qquad \ell p \rightarrow hX$$

- very relevant for EIC (also jets!)
- NLO QCD corrections sizable, resummation
- NLO QCD calculations for A_N in single-inclusive high- p_T scattering feasible
→ $\ell p \rightarrow hX$ as template for $pp \rightarrow hX$