



# **One-dimensional** *p*-wave superconductor toy-model for Majorana fermions in multiband semiconductor nanowires

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# Abstract

Majorana fermions are particles identical to their antiparticles proposed theoretically in 1937 by Ettore Majorana as real solutions of the Dirac equation. Alexei Kitaev suggested that Majorana particles should emerge in condensed matter systems as zero modes excitations in onedimensional p-wave superconductors, with possible applications in quantum computation due to their non-abelian statistics. The search for Majorana zero modes in condensed matter systems led to one of the first realistic models based in a semiconductor nanowire with high spinorbit coupling and induced superconducting s-wave pairing and Zeeman splitting. Soon, it was realized that sizequantization effects should generate subbands in these systems that could even allow the emergence of more than one Majorana mode in each edge, one of the reasons the zero bias signature of these modes were not detected as predicted theoretically. In this work, we provide a connection between a finite-size nanowire with two occupied subbands and a 2-band Kitaev chain and discuss the advantage of an one-dimensional model to understand the phenomenology of the system, including the presence of a hidden chiral symmetry and its similarity with a spinfull Kitaev chain under a magnetic field.

# **Effective Hamiltonian**

#### Assumptions:

- Spin orbit can be added in first order perturbation theory;
- Zeeman field is high enough to neglect one of the spin channels;

Chiral symmetry and topological classification

Two possible chiral symmetries

$$\mathcal{S}_{DIII} = \tau_{\phi+\pi/2} \otimes \rho_0;$$
$$\mathcal{S}_{BDI} = \tau_{\phi+\pi/2} \otimes \vec{d} \cdot \vec{\rho}$$

**General Hamiltonian** 

Nanowire Hamiltonian:

 $\mathcal{H}_0 = \mathcal{H}_{SM} + \mathcal{H}_R + \mathcal{H}_Z,$ (1) $M_{-} = \frac{1}{(\Pi^{2} + \Pi^{2})} = \mu + V(r L \cdot \mu L)$ 11

• The effective superconducting coupling can be added as the mean value of the electron-hole coupling.

#### Hamiltonian:

 $\mathcal{H}_{eff} = \epsilon(p_z)\tau_z \otimes \rho_0 + \mu_{12}(p_z)\tau_0 \otimes \rho_y$  $- \tau_{\phi} \otimes e^{i\phi} \vec{
ho} \cdot \vec{d} \sin p_z.$ (7)

$$\vec{d} := \frac{\lambda}{B} (-\Delta_{-}, \Delta_{+}, \Delta_{12}) \tag{8}$$

$$\epsilon(p_z) := \left(\frac{p_z^2}{2m} - \mu\right)$$

$$\mu_{12}(p_z) := E_{bm} \left[1 - \left(\frac{\lambda p_z}{2B}\right)^2\right]$$
(9)
(10)

# **Two band Kitaev chain!**

Chiral condition: symmetry  $\{\mathcal{H}_{eff}, S\} = 0$ • DIII class:  $\mu(p_z) = 0$ • BDI class:  $\mu(p_z) = 0$  or  $d^y = 0$ 

Analogy with spinfull Kitaev chain

 $\mathcal{H}_{eff} = \epsilon(p_z)\tau_z \otimes \rho_0 + \tau_0 \otimes \vec{\mu}_{12}(p_z) \cdot \vec{\rho}$  $-\tau_{\phi} \otimes e^{i\phi} \vec{\rho} \cdot \vec{d} \sin p_z. \tag{11}$ BDI chiral condition:  $\vec{d} \perp \vec{\mu}_{12}(p_z)$ 

**Topological invariant** 

$$w = \left| \oint_{BZ} \frac{dk}{4\pi i} \operatorname{tr} \left[ \mathcal{S}_{BDI} H_k^{-1} \partial_k H_k \right] \right|. \quad (12)$$





$$\mathcal{H}_{SM} = \frac{1}{2m^*} (\Pi_x + \Pi_y) - \mu + V(x, L_x; y, L_y),$$
(2)
$$\mathcal{H}_R = \lambda \left( \vec{\sigma} \wedge \vec{\Pi} \right) \cdot \hat{z},$$
(3)
$$\mathcal{H}_Z = \frac{1}{2} \vec{\sigma} \cdot \vec{B}.$$
(4)

**Proximity-induced** *s*-wave supercon-

ducting order parameter:

 $\mathcal{H} = \begin{pmatrix} \mathcal{H}_0 & \Delta \\ \Delta^* \ \mathcal{T} \mathcal{H}_0 \mathcal{T}^{-1} \end{pmatrix}$ 

(5)

Size-quantization on the nanowire

#### Assumptions:

• Only two subbands occupied;

•  $L_x \to \infty;$ 

• Finite  $L_y$ ;

Figure 1: Eigenvalues (in units of t) for a system described by the Hamiltonian (15) with 100 lattice positions as a function of  $\gamma$  with  $\mu = 0$ , m = 0.5t and d = 0.75t. (a)  $\rho_i = \rho_x, \rho_j = \rho_y$ ; (b)  $\rho_i = \rho_x, \rho_z = \rho_y$ ; (c)  $\rho_i = \rho_y, \rho_j = \rho_z$ . It is visible that unless d lies on the x - z plane, minigap states can emerge, indicating chiral symmetry breaking.

# Numerical implementation

### References

Simplified Hamiltonian:

$$\mathcal{H}_{eff} = \epsilon(p_z)\tau_z \otimes \rho_0 + \tau_0 \otimes \vec{\mu}_{12}(p_z) \cdot \vec{\rho} - \tau_y \otimes d\rho_\gamma \sin p_z,$$
(13)  
$$\rho_\gamma = \rho_i \sin \gamma + \rho_j \cos \gamma.$$
(14)

$$\rho_{\gamma} = \rho_i \sin \gamma + \rho_j \cos \gamma.$$

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Sumanta Tewari.

Hidden-symmetry decoupling of majorana



#### Hamiltonian:

$$\mathcal{H} = \tau_z \otimes \left[ \left( \frac{p_x^2}{2m} - \mu \right) \sigma_0 \otimes \rho_0 + \lambda \sigma_y k_x \otimes \rho_0 + \frac{1}{2} \sigma_x B_x \otimes \rho_0 + \sigma_0 \otimes \frac{E_{sb}}{2} (\rho_0 - \rho_z) \right] \\ - \tau_\phi \otimes \sigma_y \otimes (\rho_x |\Delta_{12}| + \rho_0 \Delta_+ + \rho_z \Delta_-) \\ - E_{bm} \tau_0 \otimes \sigma_x \otimes \rho_y.$$
(6)



Figure 2: Winding number calculated for the Hamiltonian in Eq. 15 varying  $\mu$  and m and with d = 0.75t,  $\rho_{\gamma} = \rho_x$ . We see that three distinct topological phases are possible, with winding number 0, 1, and 2.

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[3] Christoph W Groth, Michael Wimmer, Anton R Akhmerov, and Xavier Waintal. Kwant: a software package for quantum transport.

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