Lepton CP violation in ν2HDM$\otimes S_3$

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In collaboration with...

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- **M. Zeleny-Mora**

On the lepton CP violation in a ν2HDM with flavor
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BUAP

BUAP CIFFU Centro Internacional de Física Fundamental
"Ciencia de frontera desde México"

FCE

FCFM
Facultad de Ciencias Físico Matemáticas

The Standard Model

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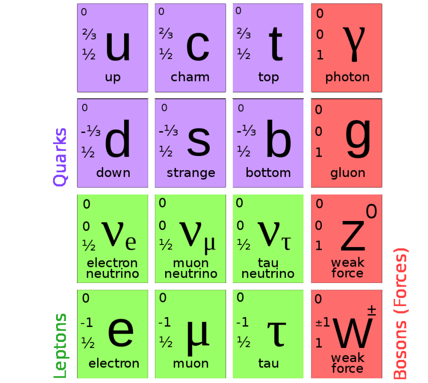

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- ♠ Now that we know the Higgs boson and its mass, we can see that particle fits properly into the Standard Model.
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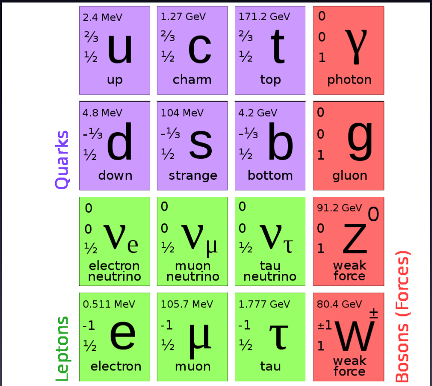

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  - ♯ The electron effective magnetic moment reported by the Borexino Collaboration [M. Agostini $et\ al$. Phys. Rev. D 96 (2017) 091103]

$$\mu_{\nu_e} \leqslant 2.8 \times 10^{-10} \mu_B \quad (90\% \text{ C.L.})\,.$$

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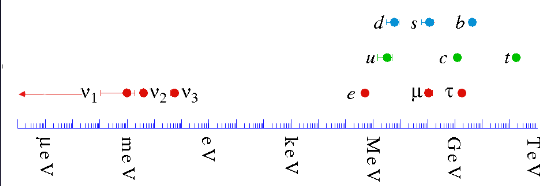

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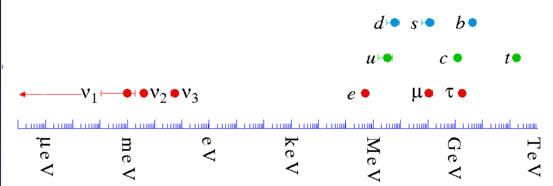
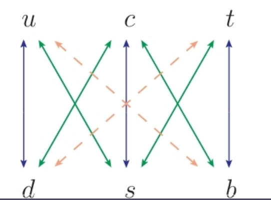


◇ Flavor-changing neutral currents (FCNC) problem: Why do we not observe "horizontal" inter-generation transitions?



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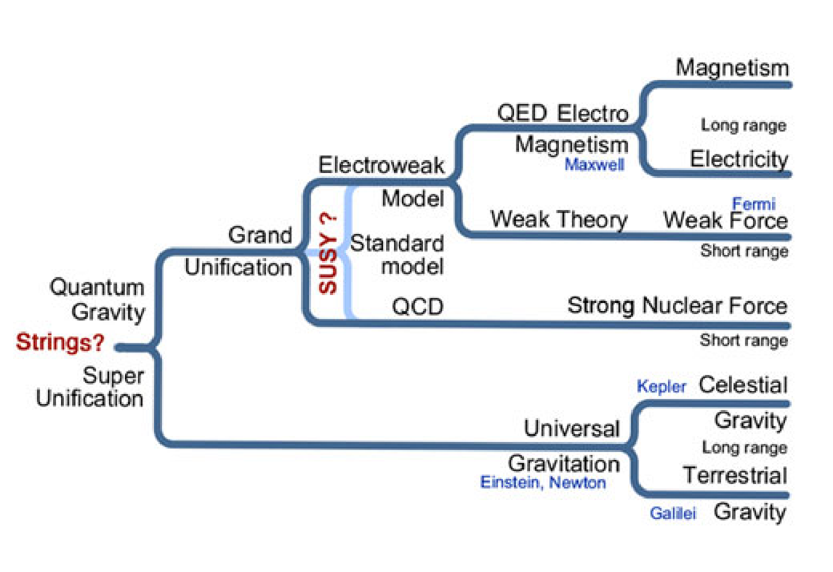

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  In this context, we can reproduce the small value of neutrino mass and the large value for the mixing angles.

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- The value of neutrino magnetic moment is several orders of magnitude smaller than the present experimental limits if to account for the existed constraints on neutrino masses.
- We can improve the value of neutrino magnetic moment if we increase the number of particles in the Higgs sector.

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- $$\underbrace{\begin{array}{ll} \widehat{m}_e \sim 10^{-6}, & \widehat{m}_\mu \sim 10^{-3}, \\ \widehat{m}_u \sim 10^{-5}, & \widehat{m}_c \sim 10^{-3}, \\ \widehat{m}_d \sim 10^{-3}, & \widehat{m}_s \sim 10^{-2}, \end{array}}_{\mathbf{2}} \qquad \underbrace{\begin{array}{l} \widehat{m}_\tau \sim 1, \\ \widehat{m}_t \sim 1, \\ \widehat{m}_b \sim 1. \end{array}}_{\mathbf{1}}$$

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- Under the action of S_3 flavor symmetry group the right-handed neutrinos as well as the two Higgs fields transform as singlets.
- The active neutrinos are considered as Majorana particles and their masses are generated through type-I seesaw mechanism.

The symmetry group S_3

- The permutations of symmetry group S_3 can be represented on the reducible triplet as:

$$\mathbf{D}^{(3)}(E) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{D}^{(3)}(A_1) = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{D}^{(3)}(A_2) = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix},$$

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- In this representation the projection operators take the form:

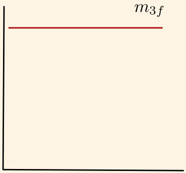
Symmetric singlet, $\quad \mathbf{P_1} = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} = |v_1\rangle\langle v_1|.$

Anti-symmetric singlet, $\quad \mathbf{P_{1'}} = 0.$

Doublet, $\quad \mathbf{P_2} = \frac{1}{3}\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} = |v_{\mathbf{2A}}\rangle\langle v_{\mathbf{2A}}| + |v_{\mathbf{2S}}\rangle\langle v_{\mathbf{2S}}|.$

The explicit sequential breaking of S_3

S_3 exact



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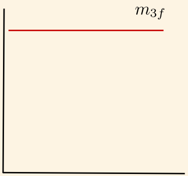

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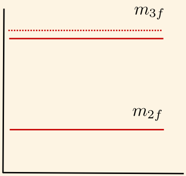

$$\frac{m_{3f}}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

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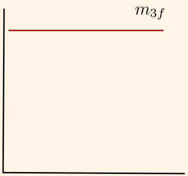

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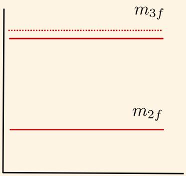

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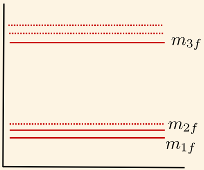

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$$+\begin{pmatrix} \epsilon & i\rho & \epsilon - i\rho \\ -i\rho & \epsilon & -\epsilon + i\rho \\ \epsilon + i\rho & -\epsilon - i\rho & 0 \end{pmatrix}$$

Dirac fermion mass matrices in 2HDM$\otimes S_3$

In the weak basis and under the explicit sequential breaking of flavor symmetry according to the chain;

$$S_{3L}^{\mathrm{j}} \otimes S_{3R}^{\mathrm{j}} \supset S_{3}^{\mathrm{diag}} \supset S_{2}^{\mathrm{diag}}.$$

The Yukawa matrices which produce three massive fermions are:

$$\mathbf{Y}_{k}^{\mathrm{w,j}} = \mathbf{Y}_{k}^{\mathrm{j3}} + \mathbf{Y}_{k}^{\mathrm{j2}} + \mathbf{Y}_{k}^{\mathrm{j1}} = \alpha_{k}^{\mathrm{j}}\, \mathbf{P_1} + \beta_{k}^{\mathrm{j}}\, \mathbf{T}_{z1}^{+} + \gamma_{k}^{\mathrm{j}}\, \mathbf{T}_{z2}^{+} + \epsilon_{k}^{\mathrm{j}}\, \mathbf{T}_{x}^{+} + \rho_{k}^{\mathrm{j}}\, \mathbf{T}_{x}^{-},$$

$$\mathbf{Y}_{k}^{\mathrm{w,j}} = \begin{pmatrix} e_{k}^{\mathrm{w,j}} & a_{k}^{\mathrm{w,j}} & f_{k}^{\mathrm{w,j}} \\ a_{k}^{\mathrm{w,j}\,*} & b_{k}^{\mathrm{w,j}} & c_{k}^{\mathrm{w,j}} \\ f_{k}^{\mathrm{w,j}\,*} & c_{k}^{\mathrm{w,j}\,*} & d_{k}^{\mathrm{w,j}} \end{pmatrix},$$

In this same basis, now the fermion mass matrices $\mathbf{M}_{\mathrm{j}}^{\mathrm{w}}$ take the form:

$$\mathbf{M}_{\mathrm{j}}^{\mathrm{w}} = \frac{1}{\sqrt{2}} \sum_{k=1}^{2} v_k \mathbf{Y}_{k}^{\mathrm{w,j}} = \frac{1}{\sqrt{2}} \sum_{k=1}^{2} v_k \left(\alpha_{k}^{\mathrm{j}}\, \mathbf{P_1} + \beta_{k}^{\mathrm{j}}\, \mathbf{T}_{z1}^{+} + \gamma_{k}^{\mathrm{j}}\, \mathbf{T}_{z2}^{+} + \epsilon_{k}^{\mathrm{j}}\, \mathbf{T}_{x}^{+} + \rho_{k}^{\mathrm{j}}\, \mathbf{T}_{x}^{-} \right).$$

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The active neutrino mass matrix is given by the type-I seesaw mechanism

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In the flavor adapted basis the active neutrinos mass matrix is

$$\mathbf{M}_{\nu}^{\mathrm{s}} = \mathbf{P}_{\nu}^{\dagger} \begin{pmatrix} 0 & a_{\nu} & 0 \\ a_{\nu} & |b_{\nu}| & |c_{\nu}| \\ 0 & |c_{\nu}| & d_{\nu} \end{pmatrix} \mathbf{P}_{\nu}^{\dagger},$$

where $\mathbf{P}_{\nu} = e^{i\phi_{\nu}} \mathrm{diag}\left(1, e^{-i2\phi_{\nu}}, e^{-i\phi_{\nu}}\right)$ with $\phi_{\nu} = \arg\{C_{\nu}\}$, and $\arg\{C_{\nu}\} = 2\arg\{B_{\nu}\}$,

$$a_{\nu} = \frac{\left|A_{\nu_D}\right|^2}{A_R}, \quad b_{\nu} = \frac{C_{\nu_D}^2}{D_R} + \frac{2B_{\nu_D}A_{\nu_D}^*}{A_R},$$

$$c_{\nu} = \frac{C_{\nu_D}D_{\nu_D}}{D_R} + \frac{C_{\nu_D}A_{\nu_D}^*}{A_R}, \quad d_{\nu} = \frac{D_{\nu_D}^2}{D_R}.$$

Theoretical lepton mixing matrix

In the theoretical framework of ν2HDM$\otimes S_3$, the unitary matrices that diagonalize the mass matrices of charged leptons and active neutrinos are defined as

$$\mathbf{U}_{\ell}^{\mathrm{n[i]}} = \mathbf{U}_{\mathrm{s}} \mathbf{P}_{\ell} \mathbf{O}_{\ell}^{\mathrm{n[i]}}, \qquad \ell = l, \nu.$$

Theoretical lepton mixing matrix

In the theoretical framework of ν2HDM$\otimes S_3$, the unitary matrices that diagonalize the mass matrices of charged leptons and active neutrinos are defined as

$$\mathbf{U}_\ell^{\mathrm{n[i]}} = \mathbf{U}_{\mathrm{s}} \mathbf{P}_\ell \mathbf{O}_\ell^{\mathrm{n[i]}}, \qquad \ell = l, \nu.$$

where

$$\mathbf{O}_f^{\mathrm{n[i]}} = \begin{pmatrix} \sqrt{\frac{\widehat{m}_{f2[1]}\xi_{f1[3]}}{\mathcal{D}_{f1[3]}}} & -\sqrt{\frac{\widehat{m}_{f1[3]}\xi_{f2[1]}}{\mathcal{D}_{f2[1]}}} & \sqrt{\frac{\widehat{m}_{f1[3]}\widehat{m}_{f2[1]}\delta_f}{\mathcal{D}_{f3[2]}}} \\ \sqrt{\frac{\widehat{m}_{f1[3]}\left(1-\delta_f\right)\xi_{f1[3]}}{\mathcal{D}_{f1[3]}}} & \sqrt{\frac{\widehat{m}_{f2[1]}\left(1-\delta_f\right)\xi_{f2[1]}}{\mathcal{D}_{f2[1]}}} & \sqrt{\frac{\delta_f\left(1-\delta_f\right)}{\mathcal{D}_{f3[2]}}} \\ -\sqrt{\frac{\widehat{m}_{f1[3]}\delta_f\xi_{f2[1]}}{\mathcal{D}_{f1[3]}}} & -\sqrt{\frac{\widehat{m}_{f2[1]}\delta_f\xi_{f1[3]}}{\mathcal{D}_{f2[1]}}} & \sqrt{\frac{\xi_{f1[3]}\xi_{f2[1]}}{\mathcal{D}_{f3[2]}}} \end{pmatrix}.$$

In this orthogonal matrix we have

$$\begin{aligned} \mathcal{D}_{f1[3]} &= (1-\delta_f)\left(\widehat{m}_{f1[3]} + \widehat{m}_{f2[1]}\right)\left(1 - \widehat{m}_{f1[3]}\right), \\ \mathcal{D}_{f2[1]} &= (1-\delta_f)\left(\widehat{m}_{f1[3]} + \widehat{m}_{f2[1]}\right)\left(1 + \widehat{m}_{f2[1]}\right), \\ \mathcal{D}_{f3[2]} &= (1-\delta_f)\left(1 - \widehat{m}_{f1[3]}\right)\left(1 + \widehat{m}_{f2[1]}\right). \end{aligned}$$

Theoretical lepton mixing matrix

$\mathbf{P}_{\mathrm{j}} = \mathrm{diag}\left(1, e^{-i\phi_{\mathrm{j}}}, e^{-i\phi_{\mathrm{j}}}\right)$ with $\phi_{\mathrm{j}} = \arg\{A_{\mathrm{j}}\}$, and

$$\mathbf{U}_{\mathrm{s}} = \frac{1}{\sqrt{6}} \begin{pmatrix} \sqrt{3} & 1 & \sqrt{2} \\ -\sqrt{3} & 1 & \sqrt{2} \\ 0 & -2 & \sqrt{2} \end{pmatrix}.$$

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The lepton flavor mixing matrix is defined as $\mathbf{U}_{\mathrm{PMNS}} = \mathbf{U}_l^{\dagger}\mathbf{U}_{\nu}$. In this context the PMNS matrix takes the form

$$\mathbf{U}_{\mathrm{PMNS}} = \mathbf{O}_l^{\top}\mathbf{P}_l^{\dagger}\mathbf{U}_{\mathrm{s}}^{\dagger}\mathbf{U}_{\mathrm{s}}\mathbf{P}_{\nu}\mathbf{O}_{\nu}^{\mathrm{n[i]}} = \mathbf{O}_l^{\top}\mathbf{P}^{(\nu-l)}\mathbf{O}_{\nu}^{\mathrm{n[i]}},$$

where $\mathbf{P}^{(\nu-l)} = \mathrm{diag}\left(1, e^{i\phi_{\ell 1}}, e^{i\phi_{\ell 2}}\right)$ with $\phi_{\ell 1} = \phi_l - 2\phi_{\nu}$ and $\phi_{\ell 2} = \phi_l - \phi_{\nu}$.

Cheng-Sher Parameters

The Yukawa matrices in the mass states basis can be expressed in terms of geometric mean of Dirac fermion masses normalized with respect of electroweak scale,

$$\left(\widetilde{\mathbf{Y}}_{k}^{\mathrm{j}}\right)_{\mathrm{rt}} = \frac{\sqrt{m_{\mathrm{jr}} m_{\mathrm{jt}}}}{v} \left(\widetilde{\chi}_{k}^{\mathrm{j}}\right)_{\mathrm{rt}} \qquad (\mathrm{r,t} = 1, 2, 3).$$

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The lepton-lepton-higgs coupling $h \to \ell_i \ell_j$:

$$\mathcal{L}_{Y}^{l} = \frac{g}{2} \bar{l}_i \left[\left(\frac{m_{l_i}}{m_W}\right) \frac{cos\alpha}{cos\beta} \delta_{ij} + \frac{sen(\alpha - \beta)}{\sqrt{2} cos\beta} \left(\frac{\sqrt{m_{l_i} m_{l_j}}}{m_W} \right) \tilde{\chi}_{ij}^{l} \right] l_j H^0$$

$$+ \frac{g}{2} \bar{l}_i \left[-\left(\frac{m_{l_i}}{m_W}\right) \frac{sen\alpha}{cos\beta} \delta_{ij} + \frac{cos(\alpha - \beta)}{\sqrt{2} cos\beta} \left(\frac{\sqrt{m_{l_i} m_{l_j}}}{m_W} \right) \tilde{\chi}_{ij}^{l} \right] l_j h^0$$

$$+ \frac{ig}{2} \bar{l}_i \left[-\left(\frac{m_{l_i}}{m_W}\right) tan\beta \delta_{ij} + \frac{1}{\sqrt{2} cos\beta} \left(\frac{\sqrt{m_{l_i} m_{l_j}}}{m_W} \right) \tilde{\chi}_{ij}^{l} \right] \gamma^5 l_j A^0$$

Likelihood test χ^2

In the symmetric parametrization of lepton flavor mixing matrix,

$$\sin^2\theta_{13} \equiv |\mathbf{U}_{e3}|^2,\ \sin^2\theta_{12} \equiv \frac{|\mathbf{U}_{e2}|^2}{1-|\mathbf{U}_{e3}|^2},\ \sin^2\theta_{23} \equiv \frac{|\mathbf{U}_{\mu 3}|^2}{1-|\mathbf{U}_{e3}|^2}.$$

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The lepton Jarlskog invariant is defined as:

$$\mathcal{J}_{CP} = \mathcal{I}m\left\{\mathbf{U}_{e1}^{*}\mathbf{U}_{\mu 3}^{*}\mathbf{U}_{e3}\mathbf{U}_{\mu 1}\right\},$$

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The phase factors associated with the CP violation can be written as:

$$\sin(\delta_{CP}) = \frac{\mathcal{J}_{\mathrm{CP}}\left(1-|\mathbf{U}_{e3}|^2\right)}{|\mathbf{U}_{e1}||\mathbf{U}_{e2}||\mathbf{U}_{e3}||\mathbf{U}_{\mu 3}||\mathbf{U}_{\tau 3}|},$$

$$\sin\left(-2\phi_{12}\right) = \frac{\mathcal{I}_1}{|\mathbf{U}_{e1}|^2|\mathbf{U}_{e2}|^2}, \quad \sin\left(-2\phi_{13}\right) = \frac{\mathcal{I}_2}{|\mathbf{U}_{e1}|^2|\mathbf{U}_{e3}|^2}.$$

Likelihood test χ^2

We make a likelihood test where the χ^2 function is defined as:

$$\chi^2 = \sum_{i<j}^{3} \frac{\left(\sin^2\theta_{ij}^{\rm exp} - \sin^2\theta_{ij}^{\rm th}\right)^2}{\sigma_{\theta ij}^2}.$$

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The χ^2 function depends of five free parameters

$$\chi^2 = \chi^2\left(\Phi_{\ell 1}, \Phi_{\ell 2}, \delta_\ell, \delta_\nu, m_{\nu_{1[3]}}\right).$$

Since

$$m_{\nu_{3[2]}} = \sqrt{m_{\nu_{1[3]}}^2 + \Delta m_{31[23]}^2}, \quad \text{and} \quad m_{\nu_{2[1]}} = \sqrt{m_{\nu_{1[3]}}^2 + \Delta m_{21[31]}^2}.$$

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The neutrino oscillation parameters have the following numerical values (at BFP$\pm 1\sigma$ and 3σ)

$$\begin{aligned}
\Delta m_{21}^2\ \left(10^{-5}\ \text{eV}^2\right) &= \quad 7.50_{-0.17}^{+0.19},\ 7.03 - 8.09, \\
\Delta m_{31}^2\ \left(10^{-3}\text{eV}^2\right) &= \quad 2.524_{-0.040}^{+0.039},\ 2.407 - 2.643,\ \text{for NH}, \\
\Delta m_{23}^2\ \left(10^{-3}\text{eV}^2\right) &= \quad 2.514_{-0.041}^{+0.038},\ 2.399 - 2.635,\ \text{for IH}.
\end{aligned}$$

Likelihood test χ^2

$$\sin^2\theta_{12}^{\text{exp}}(10^{-1}) = 3.06 \pm 0.12, \quad \sin^2\theta_{23}^{\text{exp}}(10^{-1}) = \begin{cases} 4.41^{+0.27}_{-0.21}, \\ 5.87^{+0.20}_{-0.24}, \end{cases}$$

$$\sin^2\theta_{13}^{\text{exp}}(10^{-2}) = \begin{cases} 2.166 \pm 0.0075, \\ 2.179 \pm 0.0076, \end{cases}$$

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If simultaneously we consider $\Phi_{\ell 1}$, $\Phi_{\ell 2}$, δ_ℓ, δ_ν, and $m_{\nu_{1[3]}}$ as free parameters in the likelihood test, we can only determine the values of these parameters in the best fit point (BFP).

	Δm_{ij}^2 at	$\Phi_{\ell 1}$ [°]	$\Phi_{\ell 2}$ [°]	$m_{\nu_{\text{lightest}}}$ [eV]	δ_ℓ	δ_ν	$\chi^2_{\min}$
NH	3σ	270	195	2.57×10^{-3}	0.20460	0.63519	4.63×10^{-4}
	BFP$\pm 1\sigma$	270	195	2.57×10^{-3}	0.22256	0.64507	3.13×10^{-4}
	BFP	270	195	2.57×10^{-3}	0.21492	0.64008	8.75×10^{-2}
IH	3σ	290	187	2.49×10^{-2}	0.59943	0.01999	2.30×10^{-2}
	BFP$\pm 1\sigma$	290	187	2.49×10^{-2}	0.59888	0.01995	4.56×10^{-2}
	BFP	290	187	2.49×10^{-2}	0.59798	0.01971	1.08×10^{-1}

Likelihood test χ^2

	$\theta_{12}^{\mathrm{th}}$ [°]	$\theta_{23}^{\mathrm{th}}$ [°]	$\theta_{13}^{\mathrm{th}}$ [°]	δ_{CP} [°]	ϕ_{12} [°]	ϕ_{13} [°]	χ^2_{min}
NH	33.58	41.60	8.47	-68.65	-5.86	14.77	4.63×10^{-4}
	33.59	41.61	8.46	-70.74	-5.79	14.67	3.13×10^{-4}
	33.80	41.63	8.45	-69.85	-5.80	14.73	8.75×10^{-2}
IH	33.67	50.08	8.48	-80.90	-5.25	-2.18	2.30×10^{-2}
	33.74	50.05	8.49	-80.88	-5.25	-2.18	4.56×10^{-2}
	33.83	49.99	8.49	-80.83	-5.25	-2.19	1.08×10^{-1}

We perform a new χ^2 analysis for the case when the oscillation parameters Δm^2_{ij} take the values at BFP and where we fix $m_{\nu_{1[3]}}$, $\Phi_{\ell 1}$ and $\Phi_{\ell 2}$ parameters to the values given in above tables.

Likelihood test χ^2 results

From our analysis we obtain the following values for the three mixing angles, at BFP$\pm 1\sigma$ C.L.:

$$\sin^2\theta_{12}^{\rm th}(10^{-1}) = \begin{cases} 3.09^{+0.066}_{-0.065}, \\ 3.10^{+0.011}_{-0.011}, \end{cases} \quad \sin^2\theta_{23}^{\rm th}(10^{-1}) = \begin{cases} 4.41^{+0.10}_{-0.14}, \\ 5.87^{+0.224}_{-0.223}, \end{cases}$$

$$\sin^2\theta_{13}^{\rm th}(10^{-2}) = \begin{cases} 2.160 \pm 0.14, \\ 2.177 \pm 0.12. \end{cases}$$

We also obtained the following allowed value ranges at BFP$\pm 1\sigma$ for the "Dirac-like" phase $\delta_{\rm CP}$, as well as for the two Majorana phase factors ϕ_{12} and ϕ_{13}:

$$\begin{gathered} \delta_{CP}(^\circ) = \begin{cases} -69.8^{+5.508}_{-6.110}, \\ -80.83^{+0.652}_{-0.709}, \end{cases} \\ \phi_{12}(^\circ) = \begin{cases} -5.800^{+0.170}_{-0.150}, \\ -5.24^{+0.153}_{-0.148}, \end{cases} \quad \phi_{13}(^\circ) = \begin{cases} 14.744^{+1.266}_{-1.366}, \\ -2.190^{+0.0030}_{-0.0005}. \end{cases} \end{gathered} \tag{1}$$

We can conclude that values for the $\delta_{\rm CP}$ phase obtained in our scheme are consistent with a maximal CP violation.

Likelihood test χ^2 results

Finally, as a immediate result of the above likelihood analysis, the entries magnitude of $\mathbf{U}_{\mathrm{PMNS}}$ mixing matrix can numerically computed. So, at 3σ C.L., we have that $\mathbf{U}_{\mathrm{PMNS}}$ matrix takes the form:

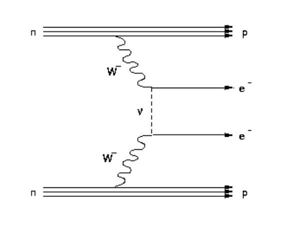
$$\begin{pmatrix} 0.822^{+0.0044}_{-0.0045} & 0.550^{+0.0055}_{-0.0054} & 0.147^{+0.0047}_{-0.0048} \\ 0.395^{+0.0181}_{-0.0154} & 0.642^{+0.0008}_{-0.0001} & 0.657^{+0.0082}_{-0.0111} \\ 0.410^{+0.0056}_{-0.0089} & 0.534^{+0.0045}_{-0.0056} & 0.739^{+0.0088}_{-0.0064} \end{pmatrix}, \qquad \text{Normal Hierarchy,}$$

$$\begin{pmatrix} 0.822^{+0.0012}_{-0.0012} & 0.551^{+0.0007}_{-0.0006} & 0.147^{+0.0041}_{-0.0041} \\ 0.355^{+0.0072}_{-0.0071} & 0.547^{+0.0144}_{-0.0149} & 0.758^{+0.0138}_{-0.0141} \\ 0.446^{+0.0077}_{-0.0080} & 0.630^{+0.0120}_{-0.0122} & 0.636^{+0.0174}_{-0.0178} \end{pmatrix}, \qquad \text{Inverted Hierarchy.}$$

Phenomenological Implications

- The neutrinoless double beta decay $0\nu\beta\beta$.
- The CP violation in neutrino oscillations in matter.

The neutrinoless double beta decay



- The $0\nu\beta\beta$ is a rare second order weak process $(A, Z) \rightarrow (A, Z+2) + e^- + e^-$.
- The observation of this process would establish that neutrino are Majorana particles and that total lepton number is not a conserved symmetry in nature.

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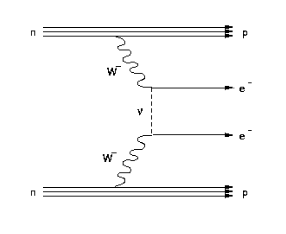

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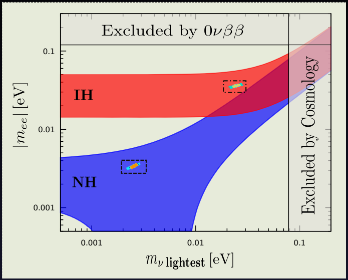
- The observation of this process would establish that neutrino are Majorana particles and that total lepton number is not a conserved symmetry in nature.

In the symmetric parametrization of lepton mixing matrix the effective mass parameter have the shape:

$$|m_{ee}| = \left| m_{\nu_1}\cos^2\theta_{12}\cos^2\theta_{13} + m_{\nu_2}\sin^2\theta_{12}\cos^2\theta_{13}e^{-i2\phi_{12}} + m_{\nu_3}\sin^2\theta_{13}e^{-i2\phi_{13}} \right|,$$

where ϕ_{12} and ϕ_{13} are the Majorana phases.

- Red and blue bands correspond to the experimental data on neutrino oscillations at 3σ.
- From the combination of EXO-200 and KamLAND-ZEN results, $|m_{ee}| < 0.120$ eV.
- From the results reported by Planck Collaboration $\sum_i m_i < 0.230$ eV
- The allowed regions for $|m_{ee}|$ obtained at 95% C.L. in the context of ν2HDM$\otimes S_3$.



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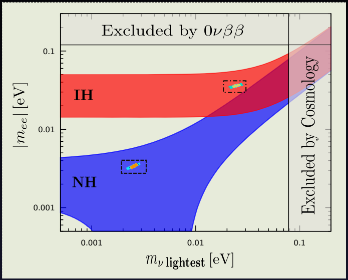

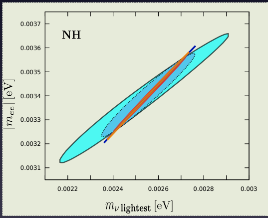

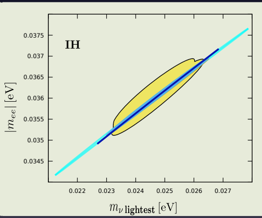

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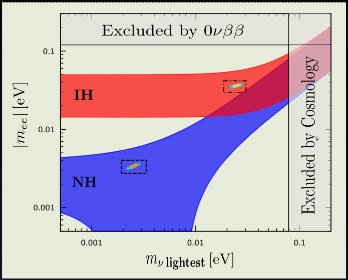

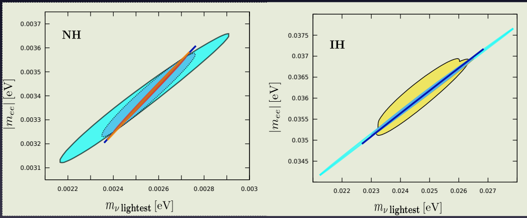

Each one of these regions was obtained by setting the values of some of the five free parameters in the $\chi\left(\Phi_{\ell 1}, \Phi_{\ell 2}, \delta_{\ell}, \delta_{\nu}, m_{\nu_{1[3]}}\right)$ function.

The allowed numerical ranges, at 95% C.L., for the effective mass parameter magnitude m_{ee} and the lightest neutrino mass $m_{\nu_{\rm lightest}}$.

	Fixed parameters	$m_{\nu_{\rm lightest}}$ $[10^{-2}$ eV]	$\|m_{ee}\|$ $[10^{-2}$ eV]
NH	$\phi_{\ell 1}$, $\phi_{\ell 2}$, δ_e, δ_ν	[0.2360 , 0.2768]	[0.3204 , 0.3608]
	$\phi_{\ell 2}$, δ_e, δ_ν	[0.2374 , 0.2735]	[0.3215 , 0.3583]
	$\phi_{\ell 1}$, δ_e, δ_ν	[0.2404 , 0.2711]	[0.3251 , 0.3563]
	$\phi_{\ell 1}$, $\phi_{\ell 2}$, δ_ν	[0.2349 , 0.2761]	[0.3229 , 0.3577]
	$\phi_{\ell 1}$, $\phi_{\ell 2}$, δ_e	[0.2164 , 0.2908]	[0.3121 , 0.3659]
IH	$\phi_{\ell 1}$, $\phi_{\ell 2}$, δ_e, δ_ν	[2.268 , 2.685]	[3.491 , 3.717]
	$\phi_{\ell 2}$, δ_e, δ_ν	[2.317 , 2.635]	[3.511 , 3.694]
	$\phi_{\ell 1}$, δ_e, δ_ν	[2.311 , 2.650]	[3.515 , 3.696]
	$\phi_{\ell 1}$, $\phi_{\ell 2}$, δ_ν	[2.301 , 2.648]	[3.512 , 3.695]
	$\phi_{\ell 1}$, $\phi_{\ell 2}$, δ_e	[2.123 , 2.787]	[3.416 , 3.767]

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From these results it is easy conclude that for the normal hierarchy $m_{\nu_1} \sim 2 \times 10^{-3}$ eV and $|m_{ee}| \sim 3 \times 10^{-3}$ eV, while for the inverted hierarchy $m_{\nu_3} \sim 2 \times 10^{-2}$ eV and $|m_{ee}| \sim 3 \times 10^{-2}$ eV.

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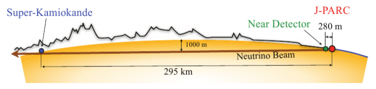
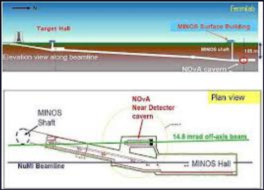
The CP violation in neutrino oscillations in matter.

- ⋆ In the recent years, we have entered into a precision era in the determination of flavor leptonic mixing angles.
- ⋆ However, it is not the same situation for the CP violation in this sector, since has yet to be determined experimentally the numerical value of CP violation phase.
- ⋆ But we have a hunch of where to look: *the neutrino oscillations with matter effects* .
  - Long-Baseline (LBL) Neutrino Oscillation Experiments.
  - One of the aims of the LBL neutrino experiments such as: T2K and NOνA, as well as the proposed experiment DUNE, it is determination of the "Dirac-like" CP violation phase and other parameters that rule the neutrino oscillations

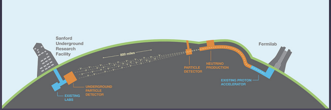
$$\nu_\mu \to \nu_e \quad \text{and} \quad \bar{\nu}_\mu \to \bar{\nu}_e .$$



- The T2K neutrino oscillation experiment has a LBL of 295 km, while the energy of its neutrino beam has a peak around to 0.6 GeV and width of ∼0.3 GeV.



- The NOνA neutrino oscillation experiment has a LBL of 810 km, while the energy of its neutrino beam has a peak around to 2 GeV.



- Finally, the future neutrino oscillation experiment DUNE will have a LBL of ~ 1300 km, while the energy of its neutrino beam will have a peak around to $2.5 - 3.0$ GeV.

The transition probability in matter for the oscillations $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$, have the form

$$P(\nu_\mu \to \nu_e) \simeq P_{\text{atm}} + P_{\text{sol}} + 2\sqrt{P_{\text{atm}}}\sqrt{P_{\text{sol}}}\cos(\Delta_{32} + \delta_{\text{CP}}),$$
$$P(\bar{\nu}_\mu \to \bar{\nu}_e) \simeq \mathcal{P}_{\text{atm}} + P_{\text{sol}} + 2\sqrt{\mathcal{P}_{\text{atm}}}\sqrt{P_{\text{sol}}}\cos(\Delta_{32} - \delta_{\text{CP}}),$$

where

$$\sqrt{P_{\text{sol}}} = \cos\theta_{23}\sin 2\theta_{12}\frac{\sin aL}{aL}\Delta_{21},$$
$$\sqrt{P_{\text{atm}}} = \sin\theta_{23}\sin 2\theta_{13}\frac{\sin(\Delta_{31}-aL)}{(\Delta_{31}-aL)}\Delta_{31},$$
$$\sqrt{\mathcal{P}_{\text{atm}}} = \sin\theta_{23}\sin 2\theta_{13}\frac{\sin(\Delta_{31}+aL)}{(\Delta_{31}+aL)}\Delta_{31}.$$

In the above expressions, L is the Base-Line,

$$\Delta_{ij} = \frac{\Delta m_{ij}^2 L}{4E}, \quad \Delta m_{ij}^2 = m_i^2 - m_j^2 \quad \text{and} \quad a = \frac{G_F N_e}{\sqrt{2}}.$$

Here, E is the energy of neutrino beam, G_F is the Fermi constant and N_e is the density of electrons. The a parameter is $a \approx (3500\ \text{km})^{-1}$ for the Earth crust.

The asymmetry between $P\left(\nu_\mu \to \nu_e\right)$ and $P\left(\bar{\nu}_\mu \to \bar{\nu}_e\right)$ in matter is

$$\begin{aligned}
\mathcal{A}_{\mu e} &= \frac{P(\nu_\mu \to \nu_e) - P(\bar{\nu}_\mu \to \bar{\nu}_e)}{P(\nu_\mu \to \nu_e) + P(\bar{\mu}_\nu \to \bar{\nu}_e)} \\
&= \frac{(P_{\text{atm}} - \mathcal{P}_{\text{atm}}) + 2\sqrt{P_{\text{sol}}}\left(\sqrt{P_{\text{atm}}}\cos(\Delta_{32}+\delta_{\text{CP}}) - \sqrt{\mathcal{P}_{\text{atm}}}\cos(\Delta_{32}-\delta_{\text{CP}})\right)}{(P_{\text{atm}} + \mathcal{P}_{\text{atm}}) + 2\sqrt{P_{\text{sol}}}\left(\sqrt{P_{\text{atm}}}\cos(\Delta_{32}+\delta_{\text{CP}}) + \sqrt{\mathcal{P}_{\text{atm}}}\cos(\Delta_{32}-\delta_{\text{CP}})\right) + 2P_{\text{sol}}}.
\end{aligned}$$

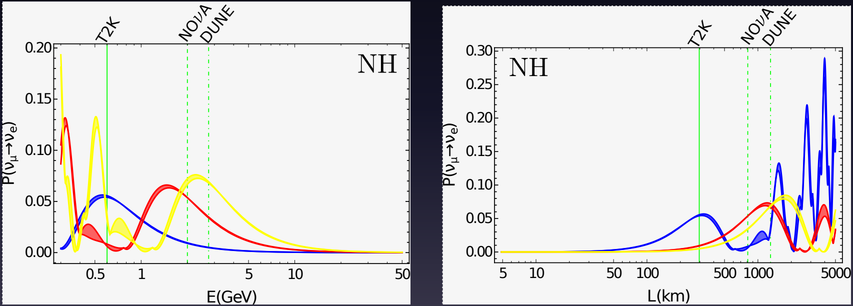
The above asymmetry $\mathcal{A}_{\mu e}$ is basically due to the absence of positrons in the journey of neutrino (anti-neutrino) through the earth. Hence, a neutrino experiment with a LBL would be more sensitive to measure this asymmetry.

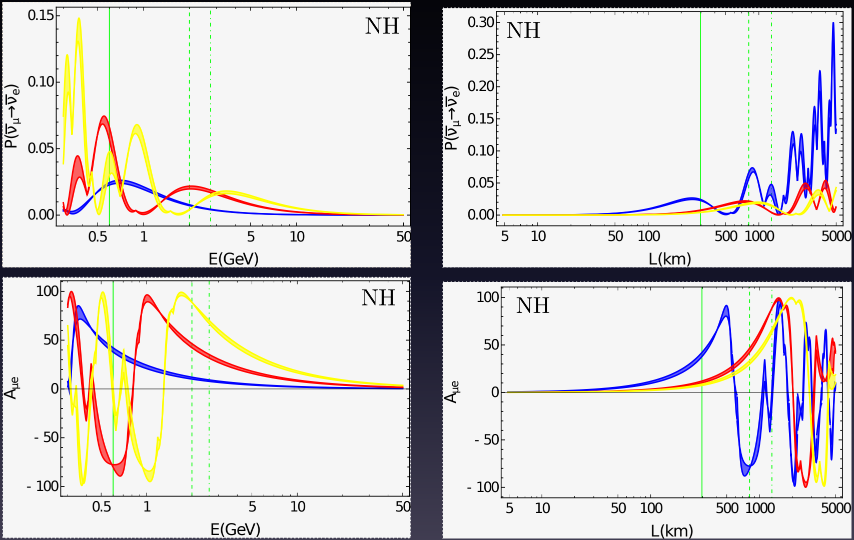
Here, we consider that δ_{CP} phase takes values within 1σ C.L.

$$\delta_{CP}(^\circ) = \left\{ \begin{array}{l} -69.8^{+5.508}_{-6.110}, \\ -80.83^{+0.652}_{-0.709}, \end{array} \right.$$

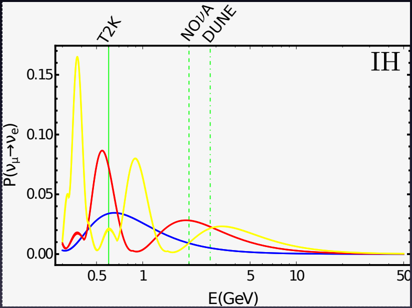
The remaining parameters are fixed to the values obtained at BFP.

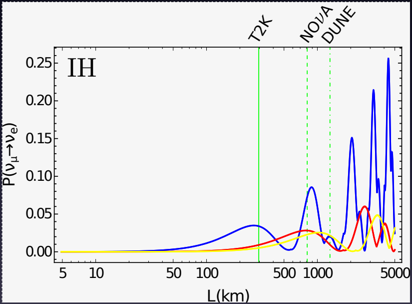
The blue, red and yellow bands are obtained for a Base-Line energy L of 295, 810 and 1300 km for left-panels. For the right-panels these bands belong to a neutrino energy E of 0.3, 2 and 2.8 GeV, which correspond to the T2K, NOνA and DUNE experiment, respectively.

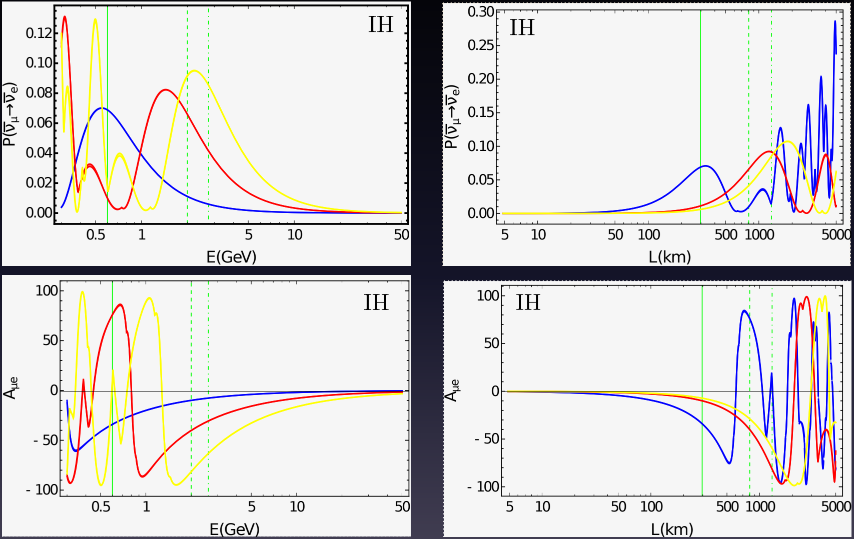



NH
$P(\overline{\nu}_\mu \to \overline{\nu}_e)$
E(GeV)
0.00
0.05
0.10
0.15
0.5
1
5
10
50
NH
$P(\overline{\nu}_\mu \to \overline{\nu}_e)$
L(km)
0.00
0.05
0.10
0.15
0.20
0.25
0.30
5
10
50
100
500
1000
5000
NH
$A_{\mu e}$
E(GeV)
100
50
0
- 50
- 100
0.5
1
5
10
50
NH
$A_{\mu e}$
L(km)
100
50
0
- 50
- 100
5
10
50
100
500
1000
5000

The blue, red and yellow bands are obtained for a Base-Line energy L of 295, 810 and 1300 km for left-panels. For the right-panels these bands belong to a neutrino energy E of 0.3, 2 and 2.8 GeV, which correspond to the T2K, NOνA and DUNE experiment, respectively.






IH
$P(\overline{\nu}_\mu \to \overline{\nu}_e)$
E(GeV)
IH
$P(\overline{\nu}_\mu \to \overline{\nu}_e)$
L(km)
IH
$A_{\mu e}$
E(GeV)
IH
$A_{\mu e}$
L(km)

Summary:

In this ν2HDM$\otimes S_3$ extension of Standard Model:

- The explicit sequential breaking of flavor symmetry according the chain $S_{3L}^{\mathrm{j}} \otimes S_{3R}^{\mathrm{j}} \supset S_3^{\mathrm{diag}} \supset S_2^{\mathrm{diag}}$, allow us to represent in the flavor basis the Yukawa matrices with an Hermitian matrix with two texture zeroes. Consequently, we obtained an unified treatment for all fermion mass matrices in the model, which are represented through of a matrix with two texture zeroes.
- We make a likelihood test where we compare the theoretical expressions of the flavor mixing angles with the current experimental data on masses and flavor mixing of leptons. The results obtained in this χ^2 analysis are in very good agreement with the current experimental data.
- We also analyzed the phenomenological implications of the above numerical values of the CP-violation phases on the neutrinoless double beta decay, as well as for LBL neutrino oscillation experiments such as T2K, NOνA, and DUNE.