The Muon g-2 Theory Initiative

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(University of Illinois
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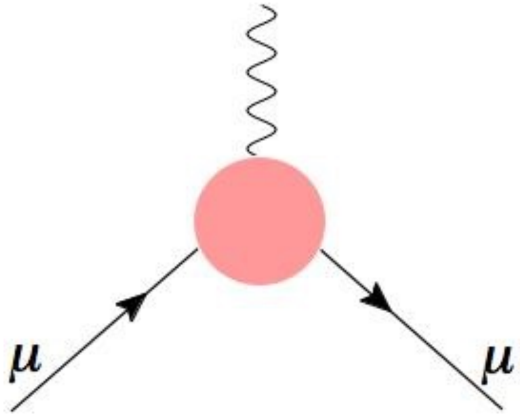
2018 Aspen Winter Conference: The Particle Frontier
25-31 March 2018

Outline

- Introduction
  - anomalous magnetic moment
  - theory initiative: goals and plans
- Hadronic Vacuum Polarization
  - Dispersive methods
  - Lattice methods
    - Introduction to lattice QCD
    - Status of LQCD calculations
- Hadronic Light-by-Light contribution
  - Dispersive methods
  - Lattice methods
- Summary and Outlook

Introduction



$$= (-i\,e)\,\bar{u}(p')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right]u(p)$$

muon anomalous magnetic moment: $a_\mu = F_2(0)$

- ✦ is generated by quantum effects (loops).
- ✦ receives contributions from QED, EW, and QCD effects in the SM.
- ✦ is a sensitive probe of new physics.

Introduction

Anomalous magnetic moment: $$a \equiv \frac{g-2}{2} = F_2(0)$$

τ lepton $-0.052 < a_\tau < 0.013$ exp.

$a_\tau = 1.17721(5)\,10^{-3}$ SM

muon: $10^{11}\,a_\mu =$ $116592089\,(63)$ exp.
$116591803\,(49)$ SM

electron: $10^{14}\,a_e =$ $115965218091\,(26)$ exp.
$115965218173\,(77)$ SM

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3.6 σ difference

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115965218173 (77) SM

➠ determine $\alpha_{\rm QED}$

Sensitivity to new physics: $a_\ell^{\rm NP} \sim \frac{m_\ell^2}{\Lambda^2}$

$(m_\mu/m_e)^2 \sim 4 \times 10^4$

Introduction

SM contribution	10^{11} × (value ± error)		Refs and notes
QED (5 loops)	116584718.951	± 0.080	[Ayoma et al, 2012, Laporta'17]
EW (2 loops)	153.6	± 1.0	[Gnendiger et al, 2013]
HVP (LO)	6923	± 42	[DHMZ'11, see also HLMNT'11,JS'11,…]
HVP (NLO)	-98.4	± 1.0	[Hagiwara et al, 2011]
HVP (NNLO)	12.4	± 0.1	[Kurz et al, 2014]
HLbL	105	± 26	[Prades et al, 2014] ``Glasgow consensus"
HLbL (NLO)	3	± 2	[Colangelo et al, 2014]
Total	116591803	± 49	[Davier et al, 2011]
Experiment	116592089	± 63	[Bennet at al, 2006]
Diff (Exp. - SM):	286	± 80	

The difference is large: ~ 2 × (EW contribution)

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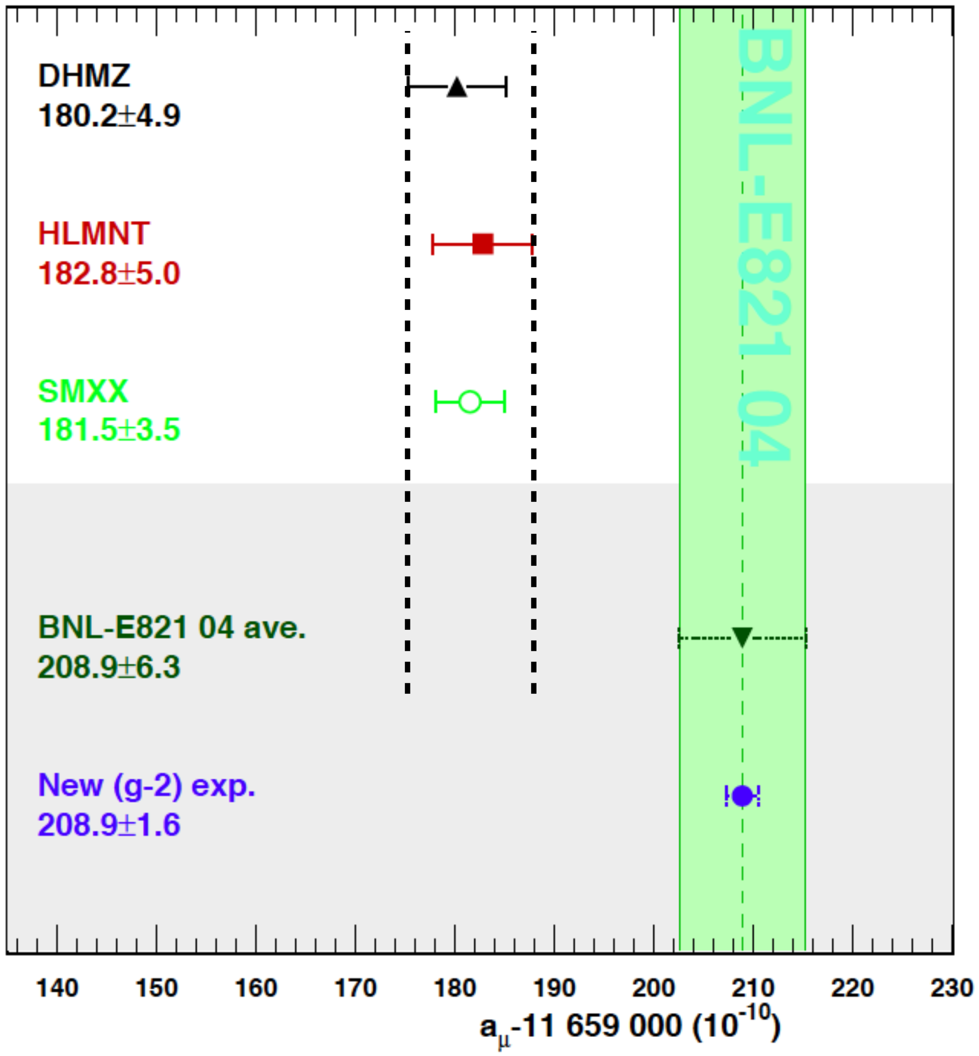
Introduction

Experiment vs SM theory

T. Blum et al. (arXiv:1311.2198)



Fermilab g-2 experiment:

- ✦ reduce exp. error by a factor of 4
- ✦ first result with "Brookhaven level" statistics expected in early 2019.

J-PARC experiment:

- ✦ completely different experimental method (ultra-cold muons)
- ✦ expect measurement at 0.3-0.4 ppm level

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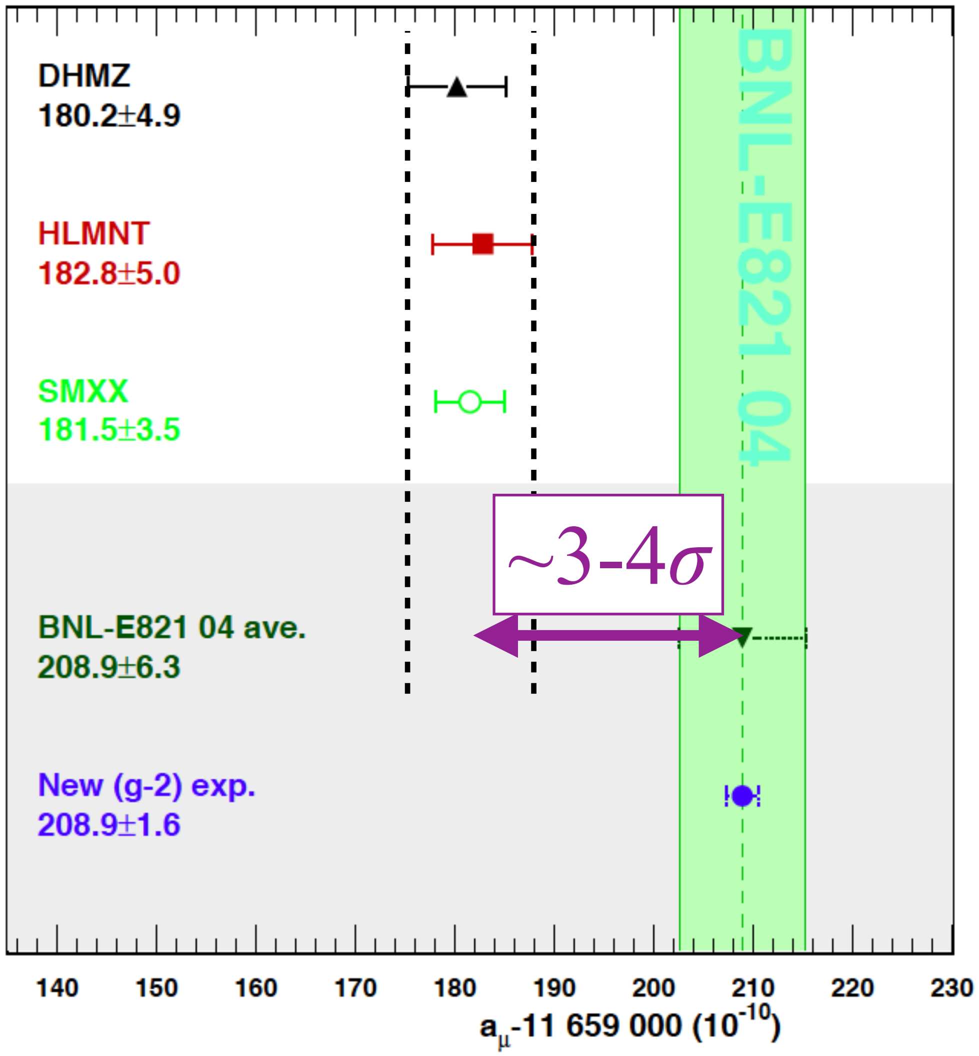

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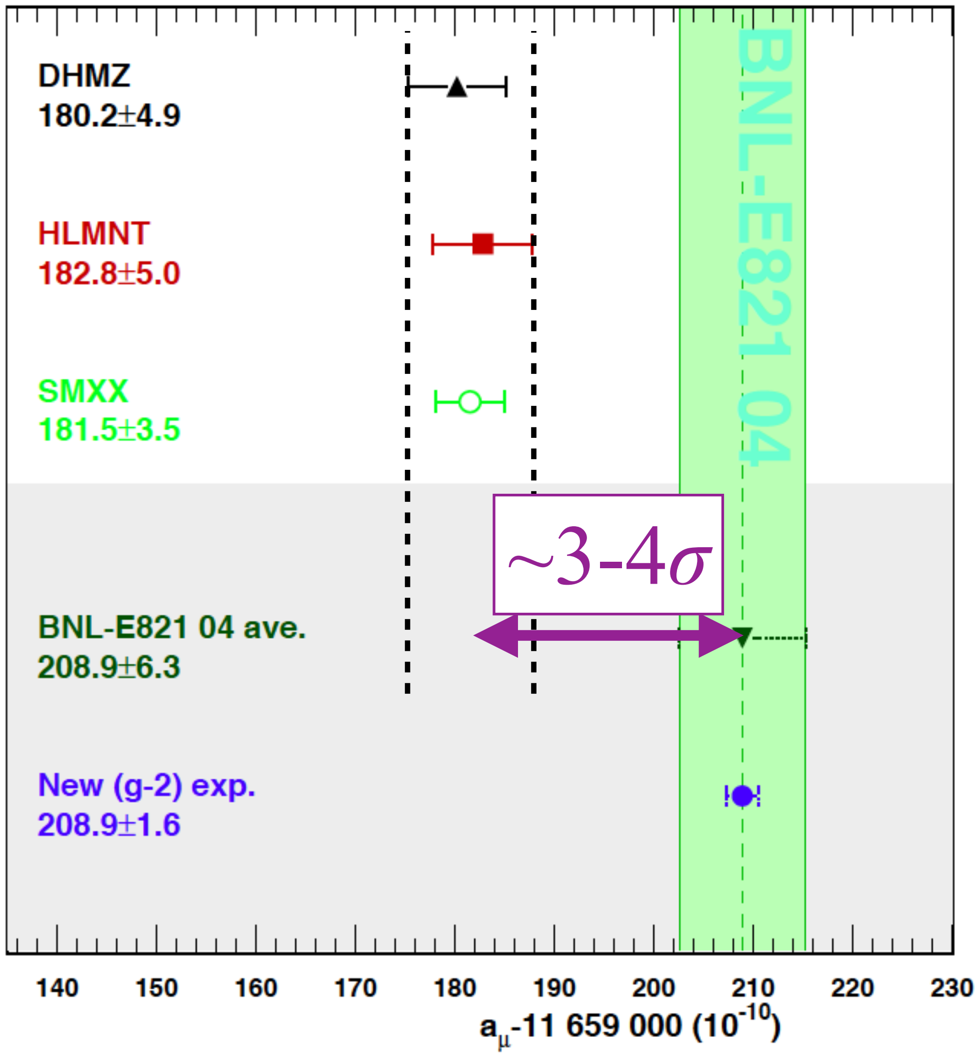

The Fermilab g-2 experiment is up and running!
(see Tammy Walton talk,Tuesday PM)

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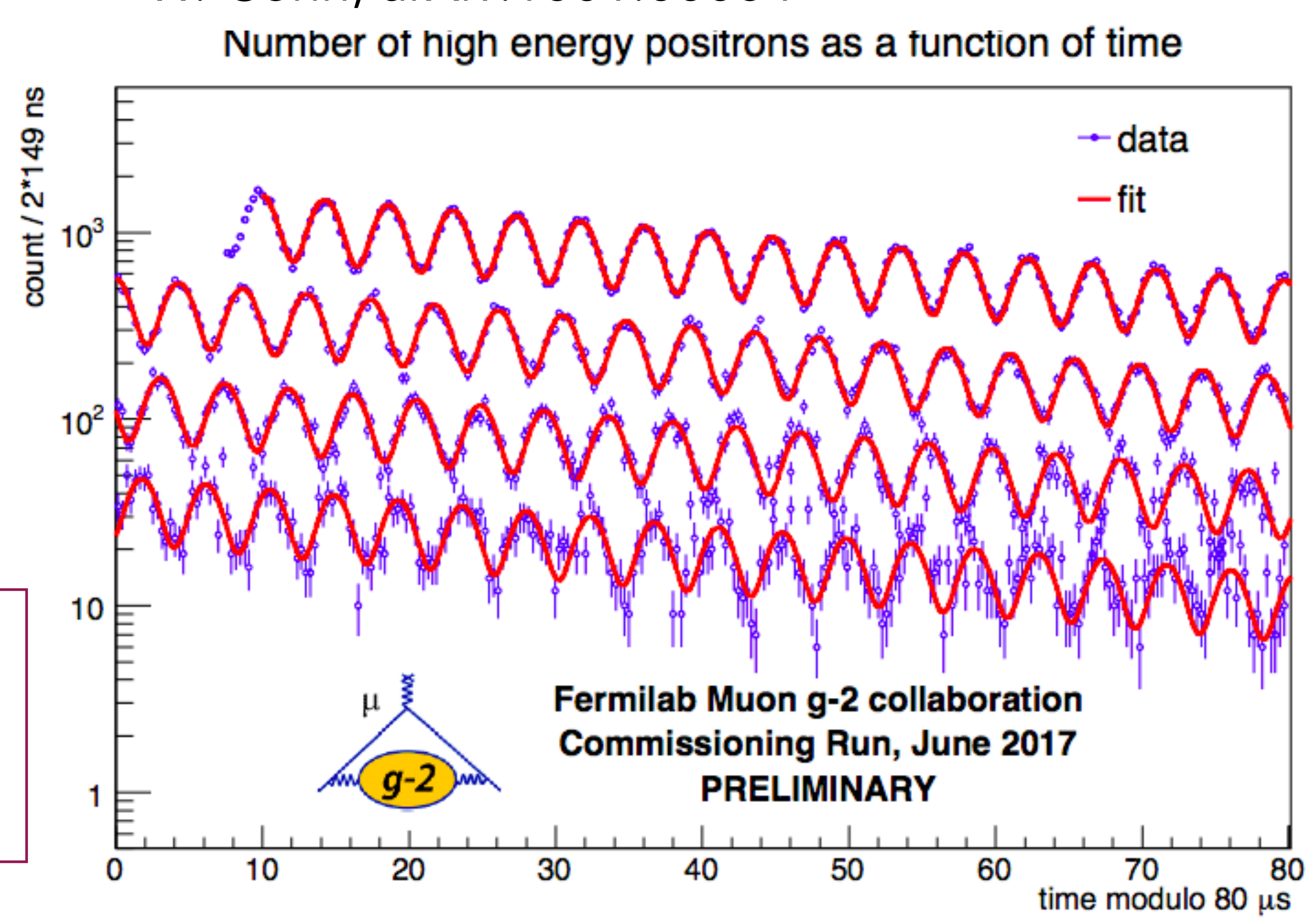
W. Cohn, arXiv:1801.00084



Introduction

Assuming the same central values, we expect ….

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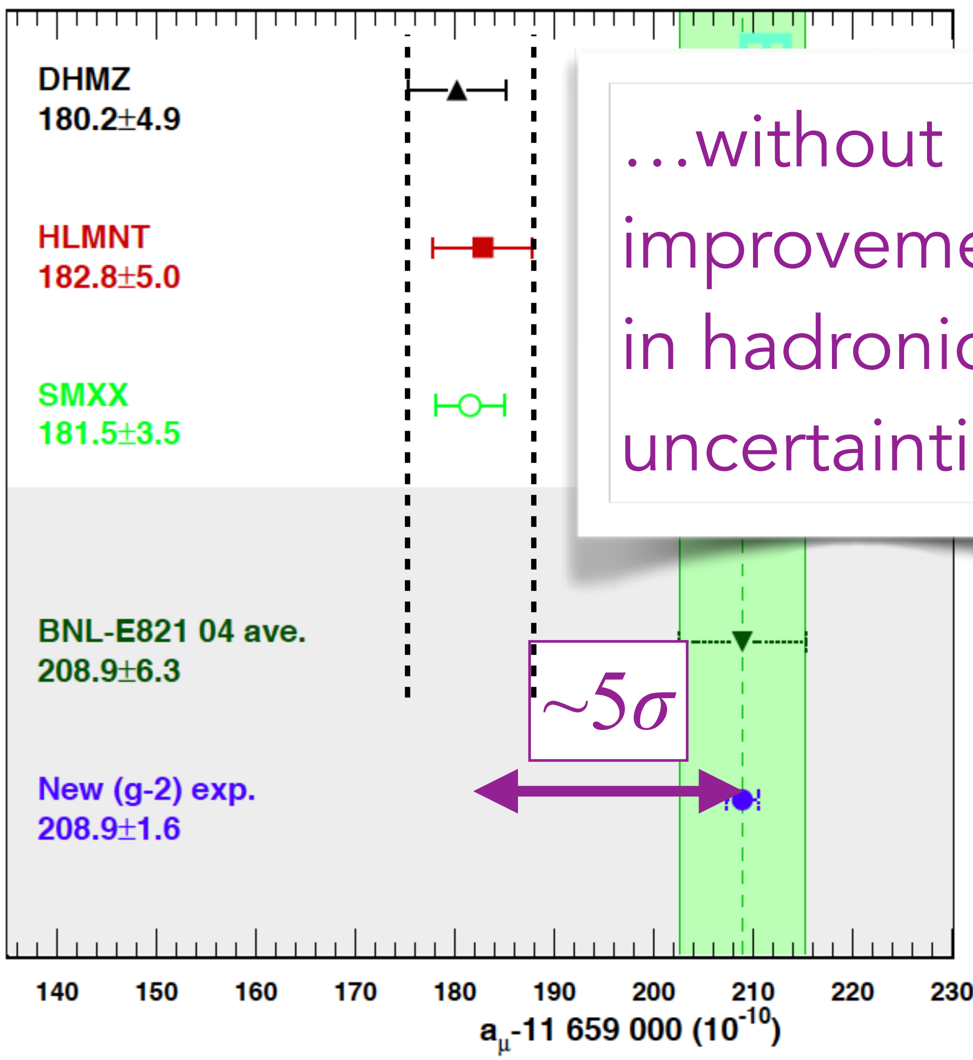

…without improvements in hadronic uncertainties:

Fermilab g-2 experiment:

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- first result with "Brookhaven level" statistics expected in early 2019.

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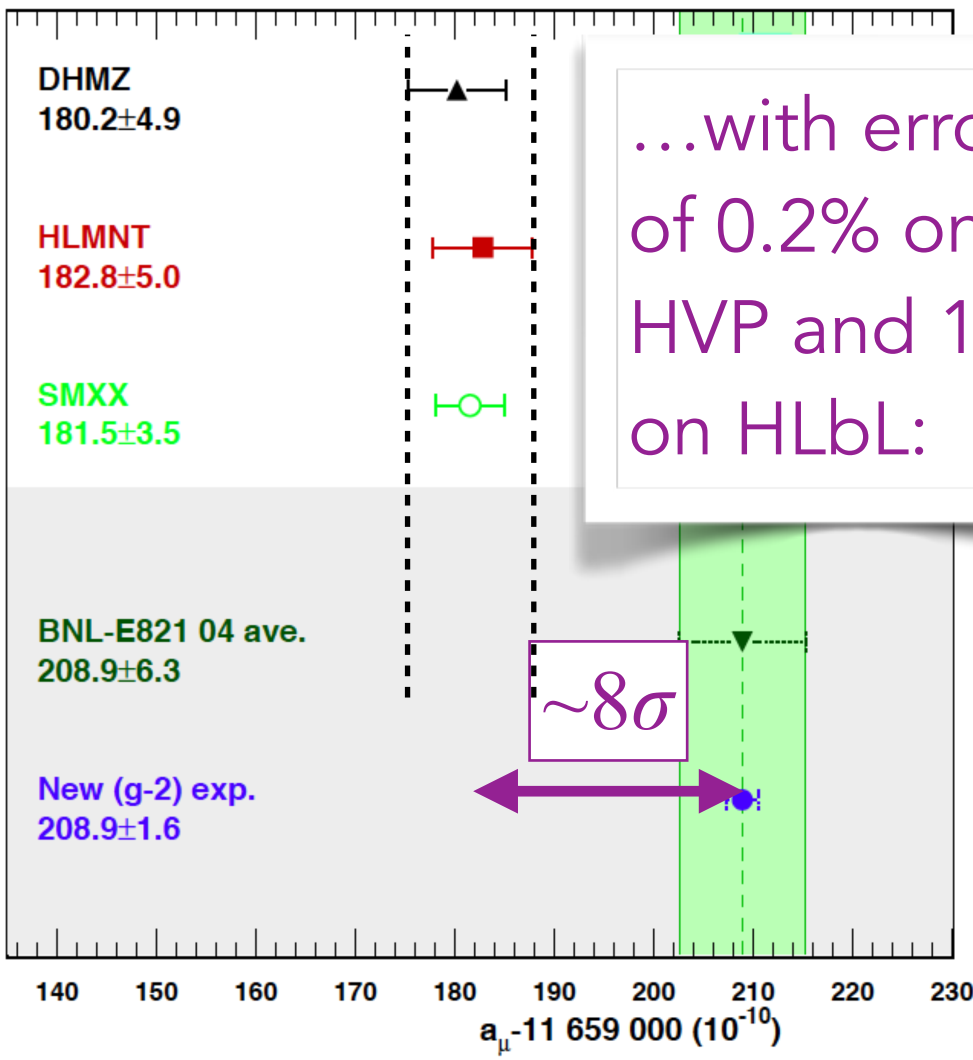

...with errors of 0.2% on HVP and 10% on HLbL:

Fermilab g-2 experiment:

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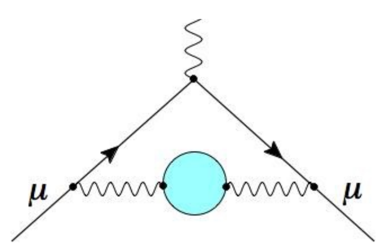
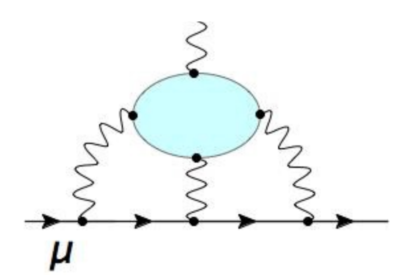
Need to reduce and better control the errors on the hadronic corrections:

Hadronic Vacuum Polarization



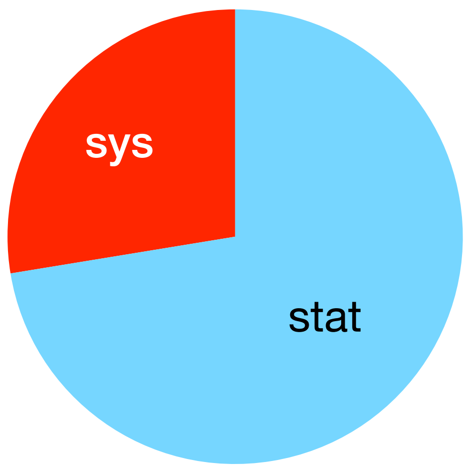
Hadronic Light-by-Light



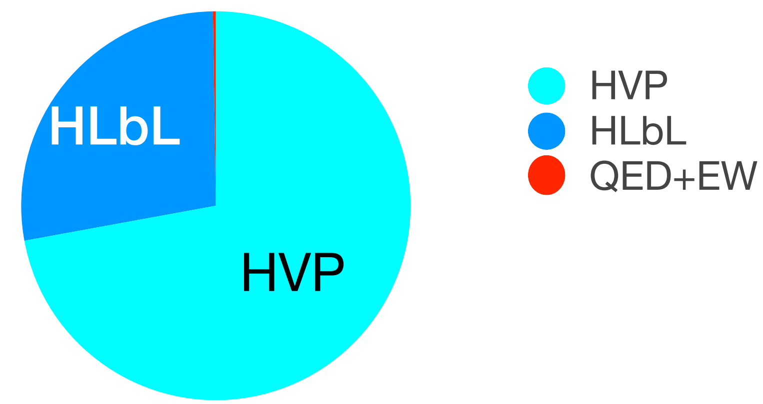
Introduction

Error budget comparison

BNL E821

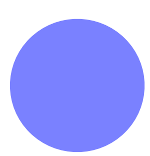


SM theory



coordinated effort

FNAL E989



Muon g-2 Theory Initiative: Goals

- theory support to the Fermilab and J-PARC experiments to maximize their impact
  ➠ work towards reducing and quantifying the uncertainties on the hadronic corrections
- summarize the theoretical calculations of the hadronic corrections to the muon g-2
  ➠ comparisons of intermediate quantities between the different approaches. For example, lattice vs R-ratio
  ➠ assess reliability of uncertainty estimates
- write a report **before** the Fermilab and J-PARC experiments announce their first results; first target date: December 2018

Muon g-2 Theory Initiative

Steering Committee:

- Gilberto Colangelo (Bern) gilberto@itp.unibe.ch
- Michel Davier (Orsay) davier@lal.in2p3.fr
- Simon Eidelman (Novosibirsk) eidelman@cern.ch
- Aida El-Khadra (UIUC & Fermilab) axk@illinois.edu
- Christoph Lehner (BNL) clehner@bnl.gov
- Tsutomu Mibe (KEK) mibe@post.kek.jp
  J-PARC E34 experiment
- Andreas Nyffeler (Mainz) nyffeler@uni-mainz.de
- Lee Roberts (Boston) roberts@bu.edu
  Fermilab E989 experiment
- Thomas Teubner (Liverpool) thomas.teubner@liverpool.ac.uk

Muon g-2 Theory Initiative: Plan

- Organize ``plenary" workshops to bring the different communities together

    First workshop: held near Fermilab, June 2017:

    kick-off

First Workshop of the Muon g-2 Theory Initiative

3-6 June 2017 *Q Center*
US/Central timezone

Search

66 registered participants, 40 talks, 15 discussion sessions (525 minutes)

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    - First workshop: held near Fermilab, June 2017:
      kick-off
    - Second workshop: Mainz, 18-22 June 2018:
      organize first report
    - Third workshop: in summer 2019?
- Form two working groups, one for HVP and one for HLbL:

Muon g-2 Theory Initiative: WGs

- HVP WG coordinators:
  - Michel Davier davier@lal.in2p3.fr
  - Simon Eidelman eidelman@cern.ch
  - Aida El-Khadra axk@illinois.edu
  - Thomas Teubner thomas.teubner@liverpool.ac.uk
- HLbL WG coordinators:
  - Gilberto Colangelo gilberto@itp.unibe.ch
  - Christoph Lehner clehner@bnl.gov
  - Andreas Nyffeler nyffeler@uni-mainz.de

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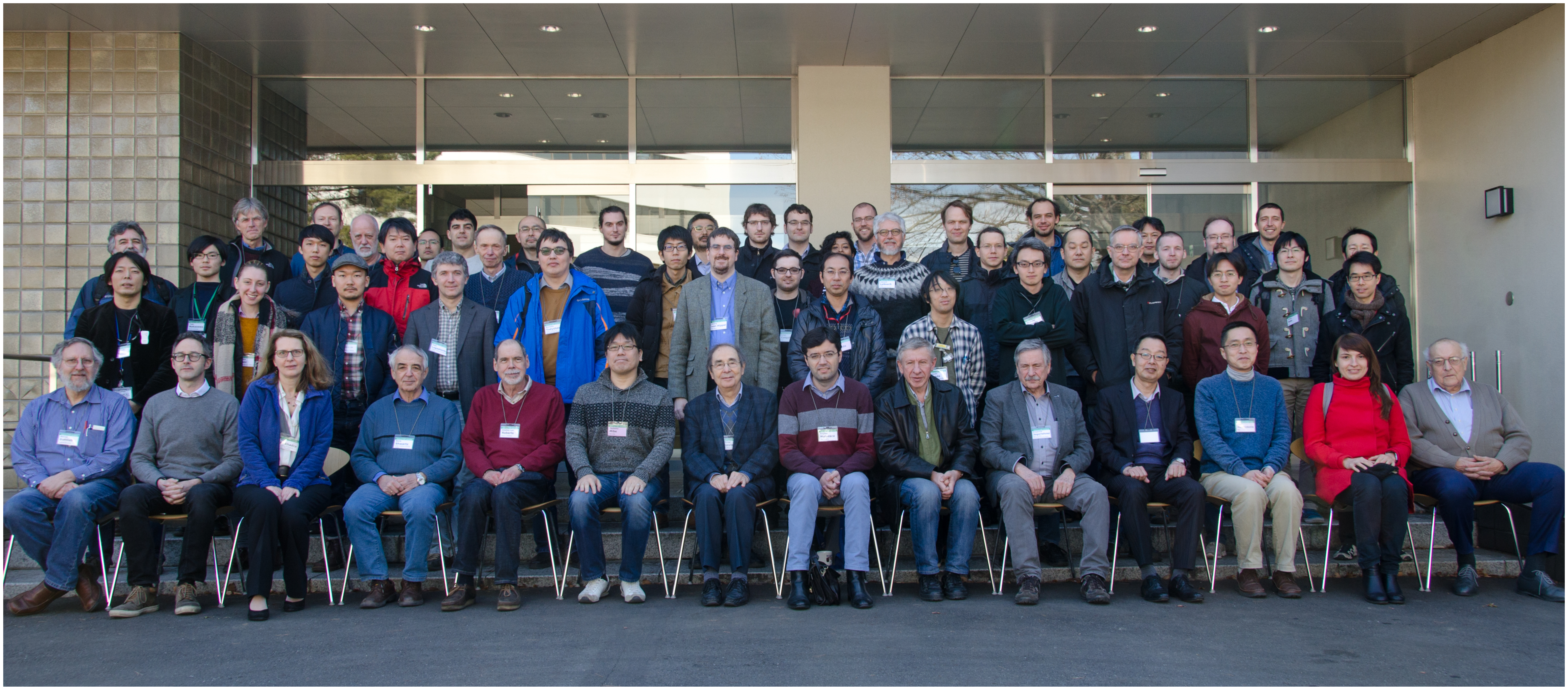
  - invite community participation: 53 in HVP WG, 33 in HLbL WG
  - organize focused workshops to advance the work:
    - HVP workshop @ KEK: 12-14 February 2018
    - HLbL workshop @ U Connecticut: 12-14 March 2018

Workshop on hadronic vacuum polarization contributions to muon g-2

February 12-14, 2018
KEK, Tsukuba, Japan

http://www-conf.kek.jp/muonHVPws/index.html



70 registered participants, 28 talks, 6 discussion sessions (330 minutes)

Muon g-2 Theory Initiative Hadronic Light-by-Light working group workshop

https://indico.phys.uconn.edu/event/1/

12-14 March 2018
UConn Physics Department

21 registered participants, 22 talks, 4 discussion sessions (160 minutes)

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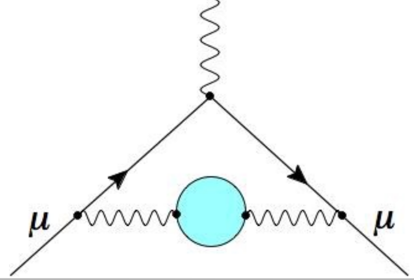
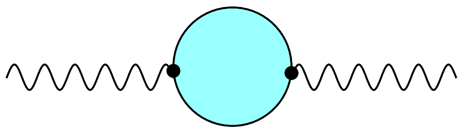


Hadronic vacuum polarization



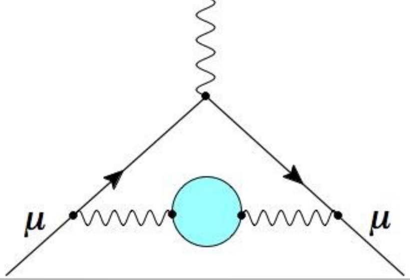
$$\hat{\Pi}(q^2) = \Pi(q^2) - \Pi(0)$$

$$\Pi_{\mu\nu} = \int d^4x e^{iqx} \langle j_\mu(x) j_\nu(0) \rangle = (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi(q^2)$$

Leading order HVP correction: $$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 \omega(q^2)\, \hat{\Pi}(q^2)$$

- Use optical theorem and dispersion relation to rewrite the integral in terms of the hadronic *e+e-* cross section:

$$a_\mu^{\mathrm{HVP,LO}} = \frac{m_\mu^2}{12\pi^3} \int ds \frac{\hat{K}(s)}{s} \sigma_{\mathrm{exp}}(s)$$



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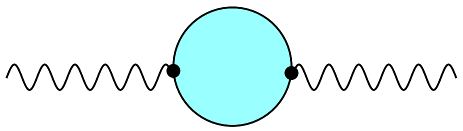

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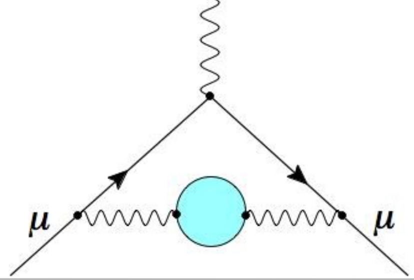
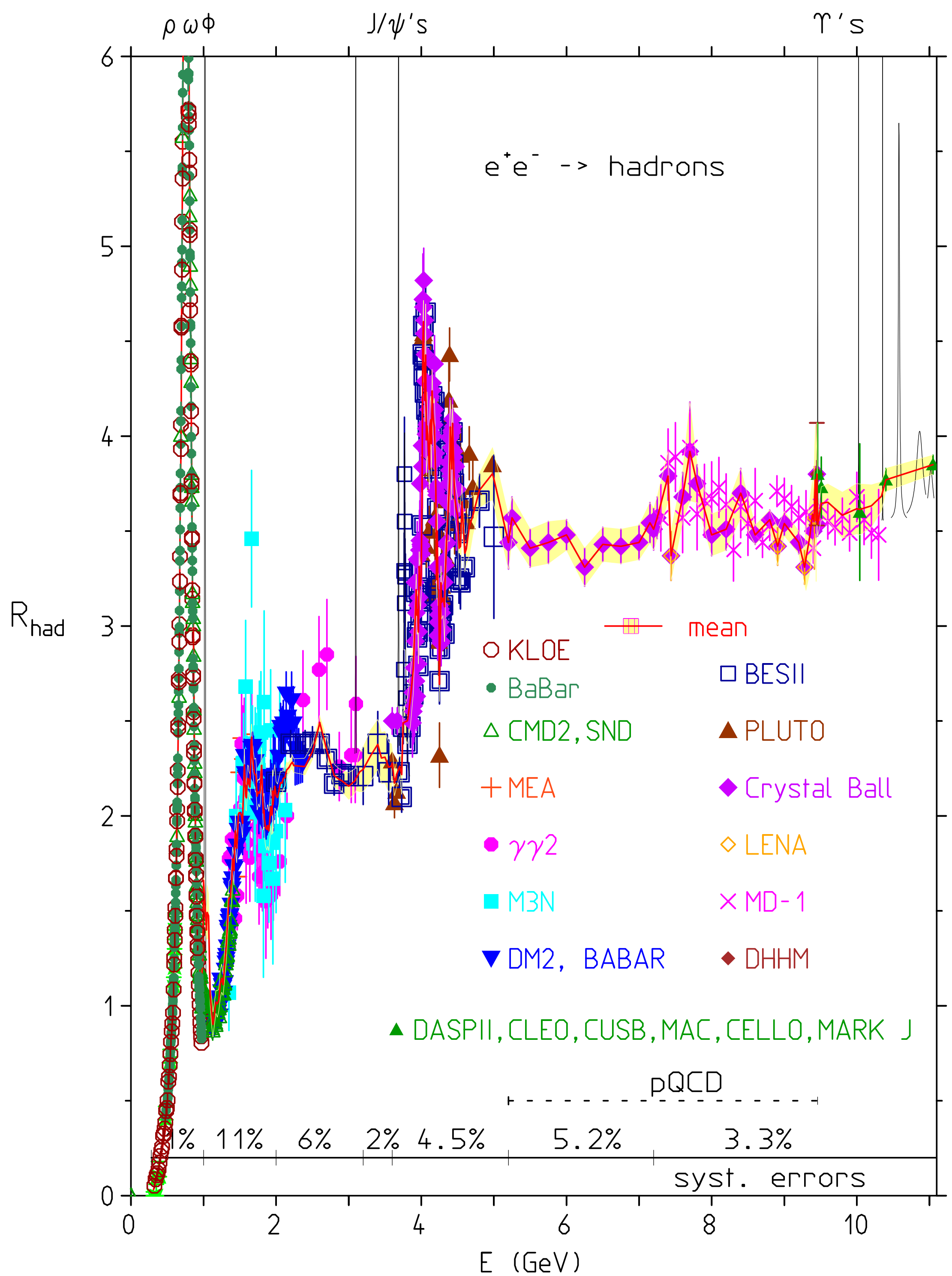
Dominant contributions from low energies

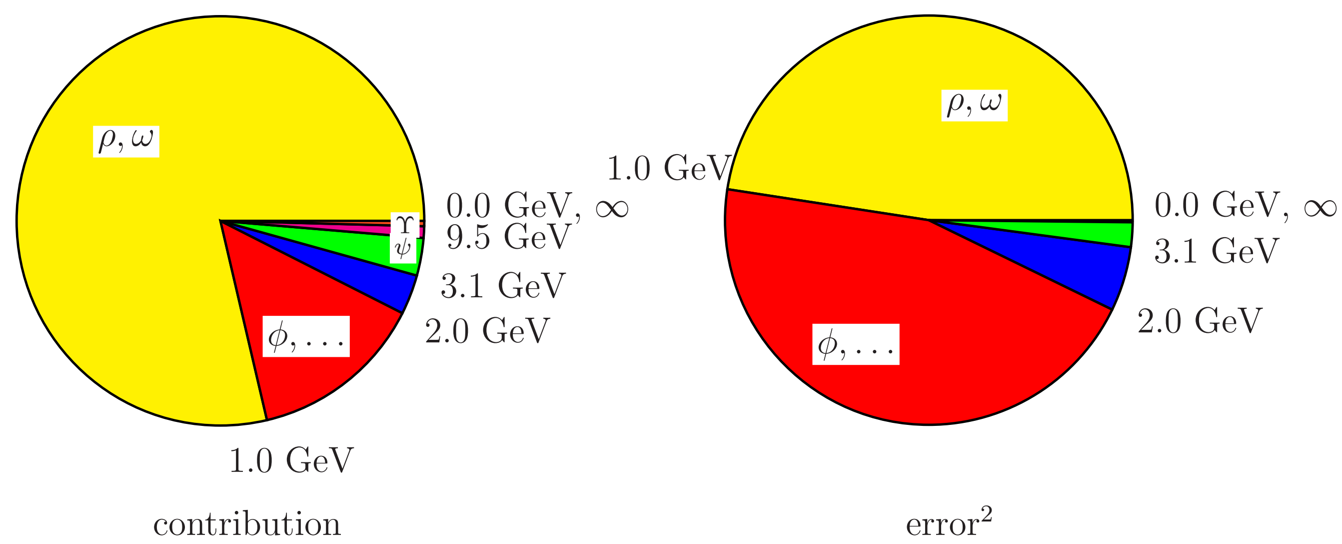
$\pi^+\pi^-$ channel: 73% of total $a_\mu^{\mathrm{HVP,LO}}$



Hadronic vacuum polarization

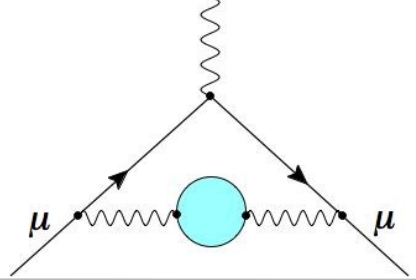
F. Jegerlehner @ HVP KEK 2018:





$$a_\mu^{\mathrm{had}(1)} = (686.99 \pm 4.21)[687.19 \pm 3.48]\ 10^{-10}$$

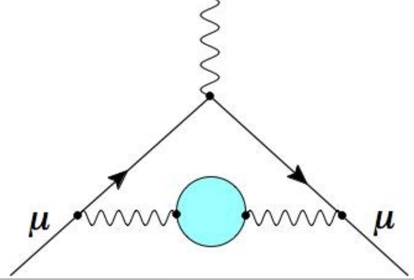
e^+e^-–data based [incl. τ]



Hadronic vacuum polarization

Challenges:

- below ~2 GeV:
  sum ~30 exclusive channels: 2π, 3π, 4π, 5π, 6π, 2K, 2Kπ, 2K2π, ηπ,….
  (use isospin relations for missing channels)
- above ~1.8 GeV:
  inclusive, pQCD (away from flavor thresholds)
  + narrow resonances (J/ψ, ϒ,..)
- Combine data from different experiments:
  energy scan vs. radiative return
  different energy bins
  understanding correlations
  different sources of sys. error
- include FS radiative corrections



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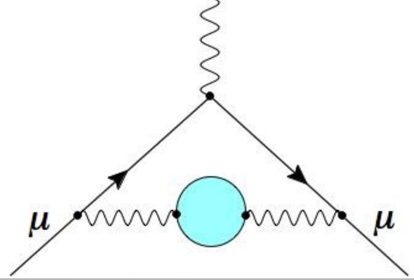
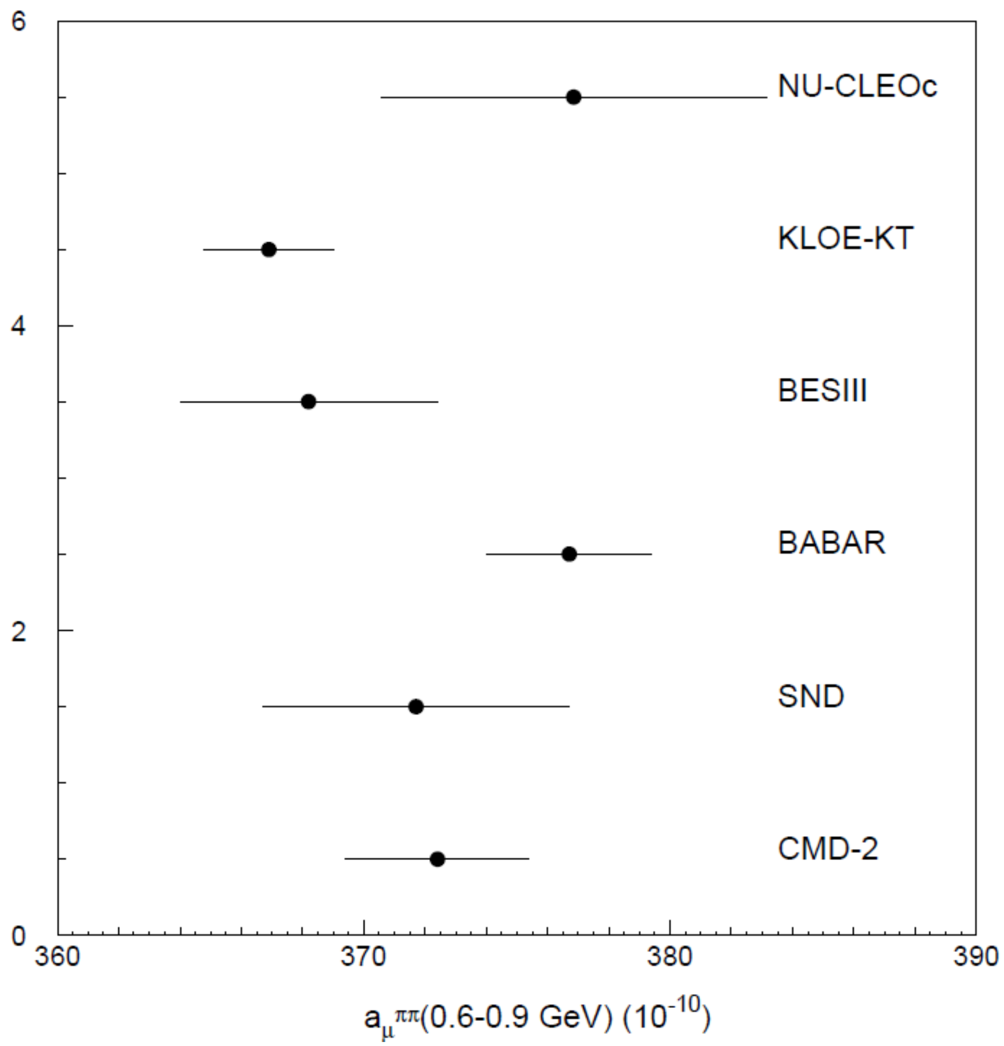
- current uncertainty ~0.4-0.5%
- can be improved with more precise experimental data
- new experimental measurements expected/ongoing at BaBar, BES-III, Belle-II, CMD-3, SND, KLOE,….

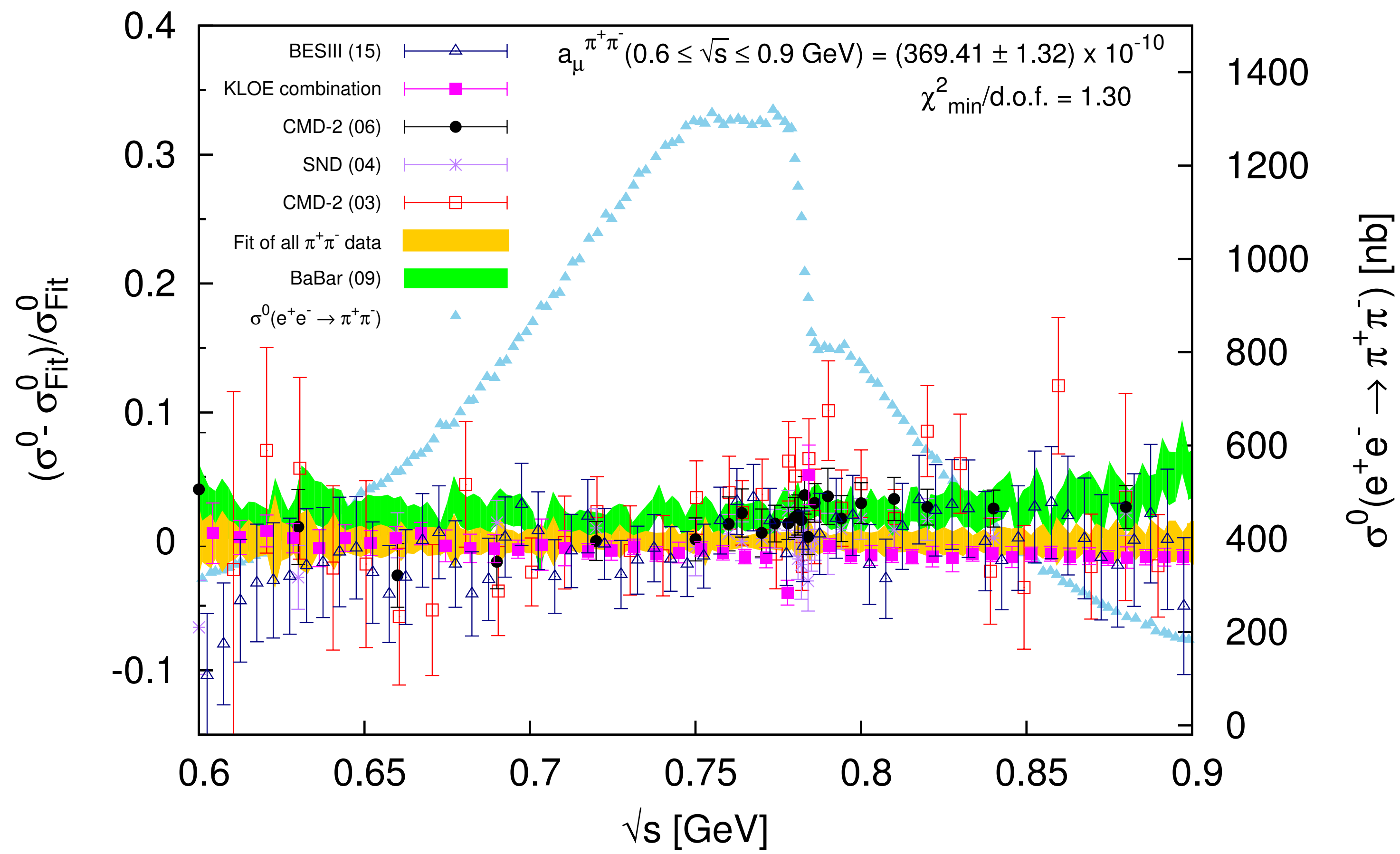


Hadronic vacuum polarization

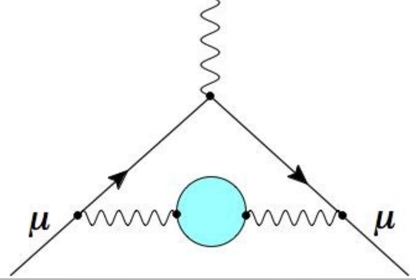
M. Davier @ HVP KEK 2018



A. Keshavarzi @ HVP KEK 2018



Some tension between different data sets



Hadronic vacuum polarization

Leading order HVP correction: $$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 \omega(q^2)\, \hat{\Pi}(q^2)$$

- Calculate a_μ^{HVP} in Lattice QCD:
  - ✦ Calculate $\hat{\Pi}(q^2)$ and evaluate the integral (Blum,PRL 03, Lautrup et al, 71)
  - ✦ Time-momentum representation: reorder the integrations and compute $C(t) = \frac{1}{3}\sum_{i,x}\langle j_i(x,t) j_i(0,0)\rangle$

$$a_\mu^{\mathrm{HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int dt\, \tilde{\omega}(t)\, C(t)$$

(Bernecker & Meyer, EPJ 12)

  - ✦ Time-moments: (Chakraborty et al, PRD 14)

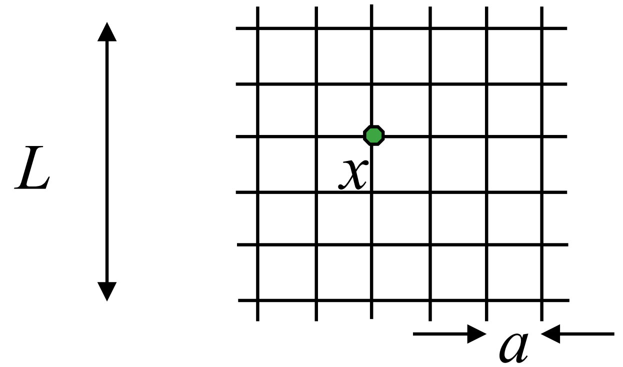
Taylor expand $\hat{\Pi}(q^2) = \sum_k q^{2k}\Pi_k$

and compute Taylor coefficients from time moments:

$$C_{2n} = a\sum_t t^{2n} C(t)$$

Lattice QCD Introduction

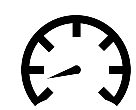
$$\mathcal{L}_{\rm QCD} = \sum_f \bar{\psi}_f (\not{D} + m_f)\psi_f + \frac{1}{4}{\rm tr}F_{\mu\nu}F^{\mu\nu}$$



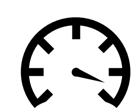
- ✦ discrete Euclidean space-time (spacing a)
  derivatives ➙ difference operators, etc…
- ✦ finite spatial volume (L)
- ✦ finite time extent (T)

adjustable parameters

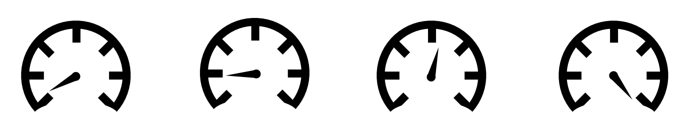
- ❖ lattice spacing: $a \to 0$



- ❖ finite volume, time: $L \to \infty$, $T > L$



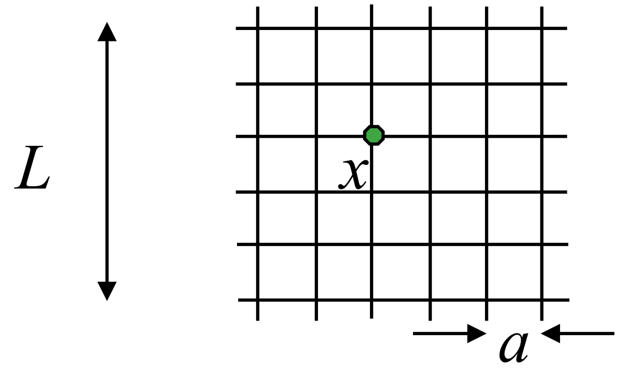
- ❖ quark masses (m_f): $M_{H,\rm lat} = M_{H,\rm exp}$
  tune using hadron masses $m_f \to m_{f,\rm phys}$
  extrapolations/interpolations



m_{ud} m_s m_c m_b

Lattice QCD Introduction

$$\mathcal{L}_{\rm QCD} = \sum_f \bar\psi_f(\not{D} + m_f)\psi_f + \frac{1}{4}{\rm tr}F_{\mu\nu}F^{\mu\nu}$$

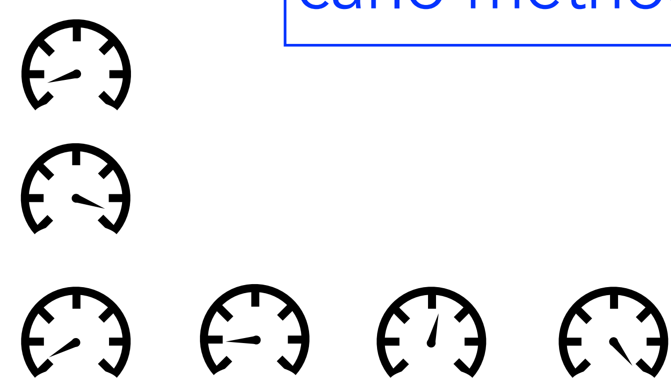


- ✦ discrete Euclidean space-time (spacing a)
  derivatives ➙ difference operators, etc…
- ✦ finite spatial volume (L)
- ✦ finite time extent (T)

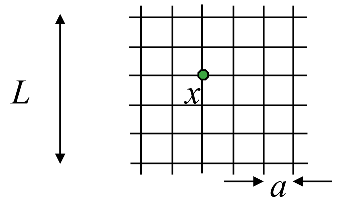
Integrals are evaluated numerically using monte carlo methods.

adjustable parameters

- ❖ lattice spacing: $a \to 0$
- ❖ finite volume, time: $L \to \infty,\ T > L$
- ❖ quark masses (m_f): $M_{H,\rm lat} = M_{H,\rm exp}$
  tune using hadron masses $m_f \to m_{f,\rm phys}$
  extrapolations/interpolations



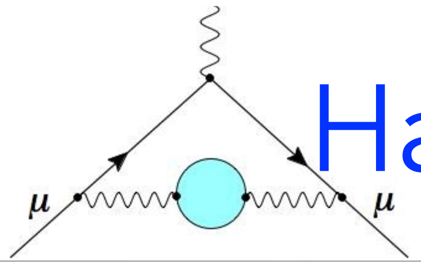
m_{ud} m_s m_c m_b



Lattice QCD Introduction

The State of the Art

- ☆ Lattice QCD calculations of simple quantities (with at most one stable meson in initial/final state) that **quantitatively account for all systematic effects** (discretization, finite volume, renormalization,…) , in some cases with
  - sub percent precision.
  - total errors that are commensurate (or smaller) than corresponding experimental uncertainties.
- ☆ Scope of LQCD calculations is increasing due to continual development of new methods:
  - nucleons and other baryons
  - nonleptonic decays ($K \to \pi\pi$, …)
  - resonances, scattering, long-distance effects, …
  - QED effects
  - …

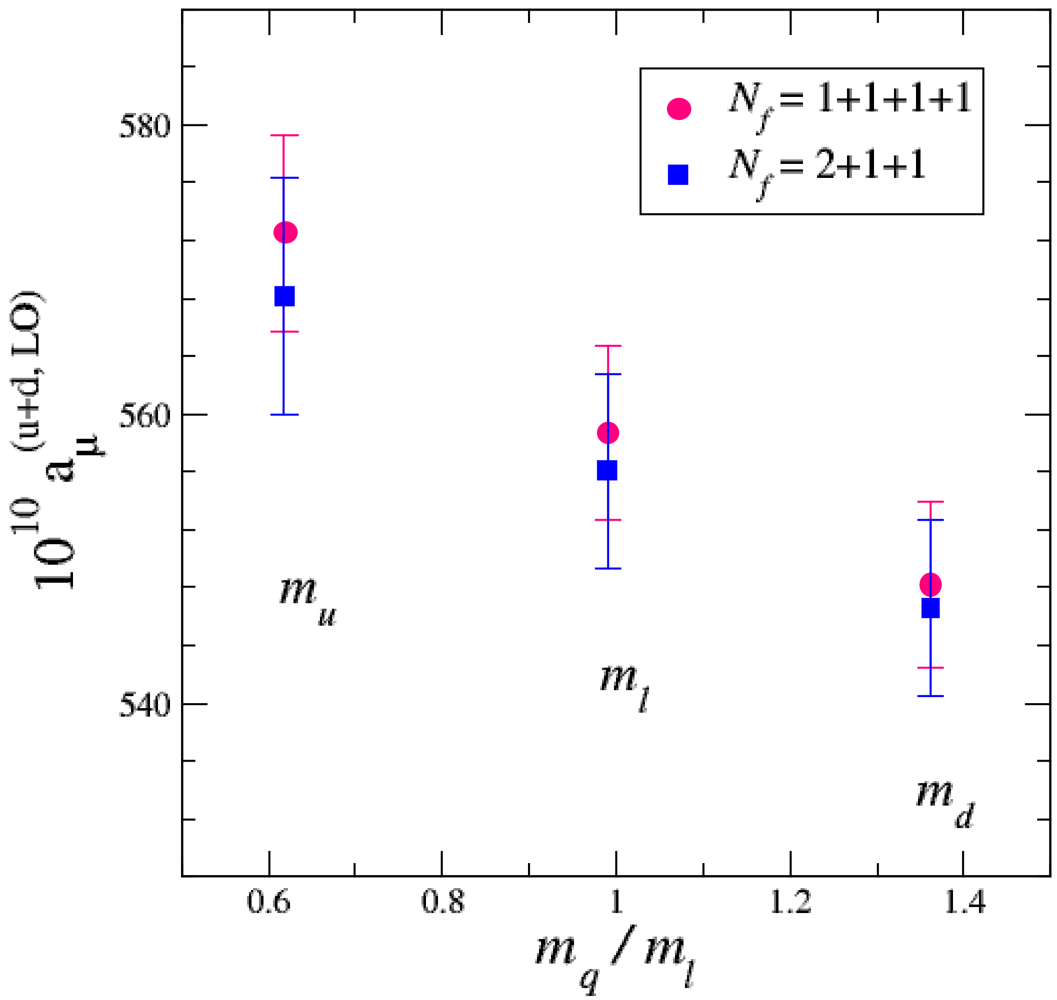


Hadronic vacuum polarization

- ✦ Target: ~0.2% total error
- ✦ Dispersion relation + experimental data for $e^+e^- \to \mathrm{hadrons}$ (and τ data)
  - current uncertainty ~0.4-0.5%
  - can be improved with more precise experimental data
  - new experimental measurements expected/ongoing at BaBar, BES-III, Belle-II, CMD-3, SND, KLOE,….
- ✦ Complete lattice QCD results by several groups.
  A complete LQCD result …
  - is based on physical mass ensembles
  - includes disconnected contributions
  - includes QED and strong isospin breaking corrections ($m_u \neq m_d$)
  - includes finite volume corrections, continuum extrapolation

Isospin corrections

R. Van de Water @ HVP KEK 2018 for FNAL/MILC/HPQCD [arXiv:1710.11212]:



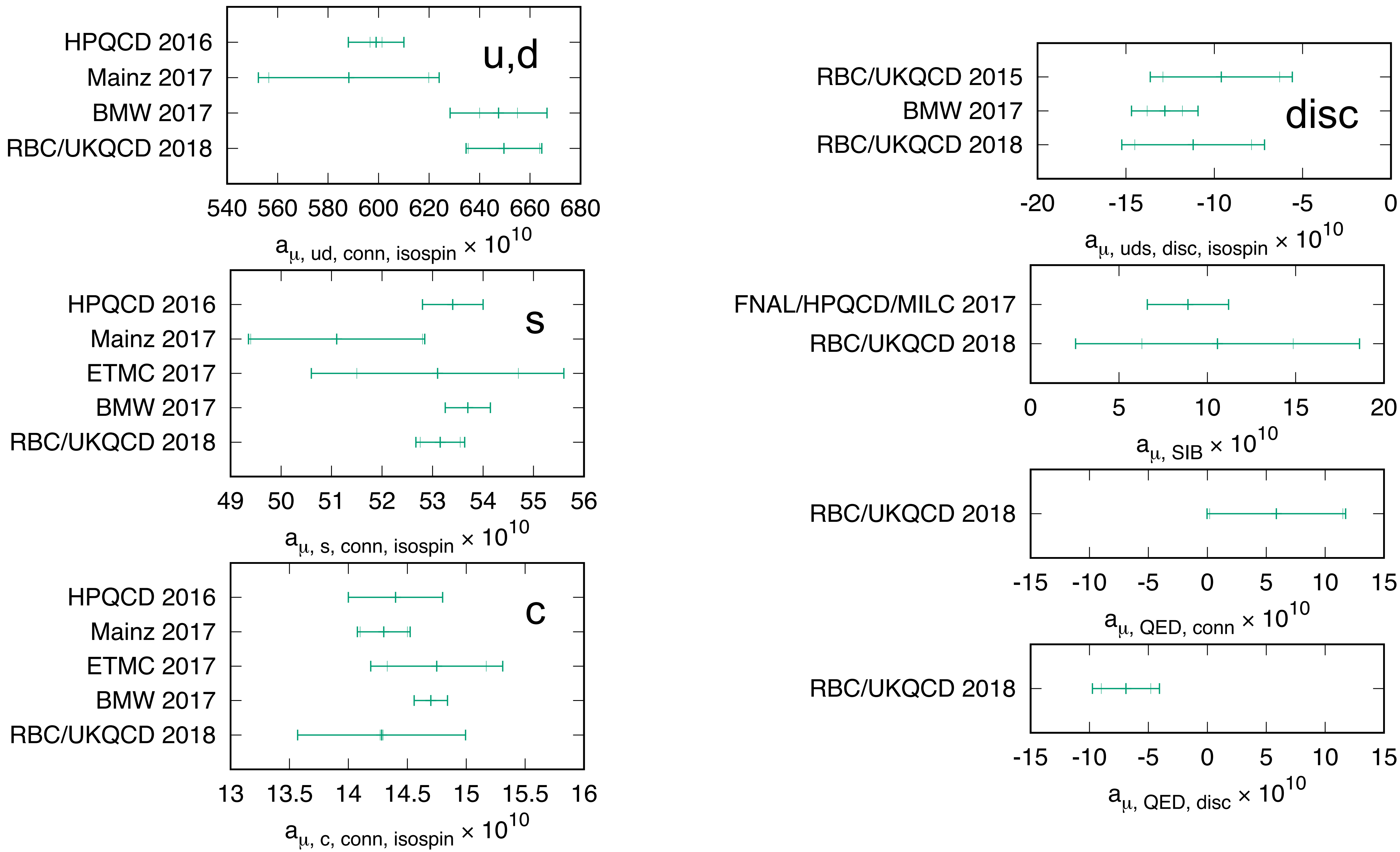
First LQCD calculation at physical pion mass.

Consistent with a recent calculation by RBC [T. Blum et al, arXiv:1801.07224] and preliminary results by ETM [D. Giusti @ Lattice 2017; arXiv:1710.06240]

$$\delta a_\mu^{\mathrm{HVP}, m_u \neq m_d} = (1.5 \pm 0.7)\%$$

Comparison of recent lattice HVP results

C. Lehner @ HVP KEK 2018 (from T. Blum et al, arXiv:1801.07224)



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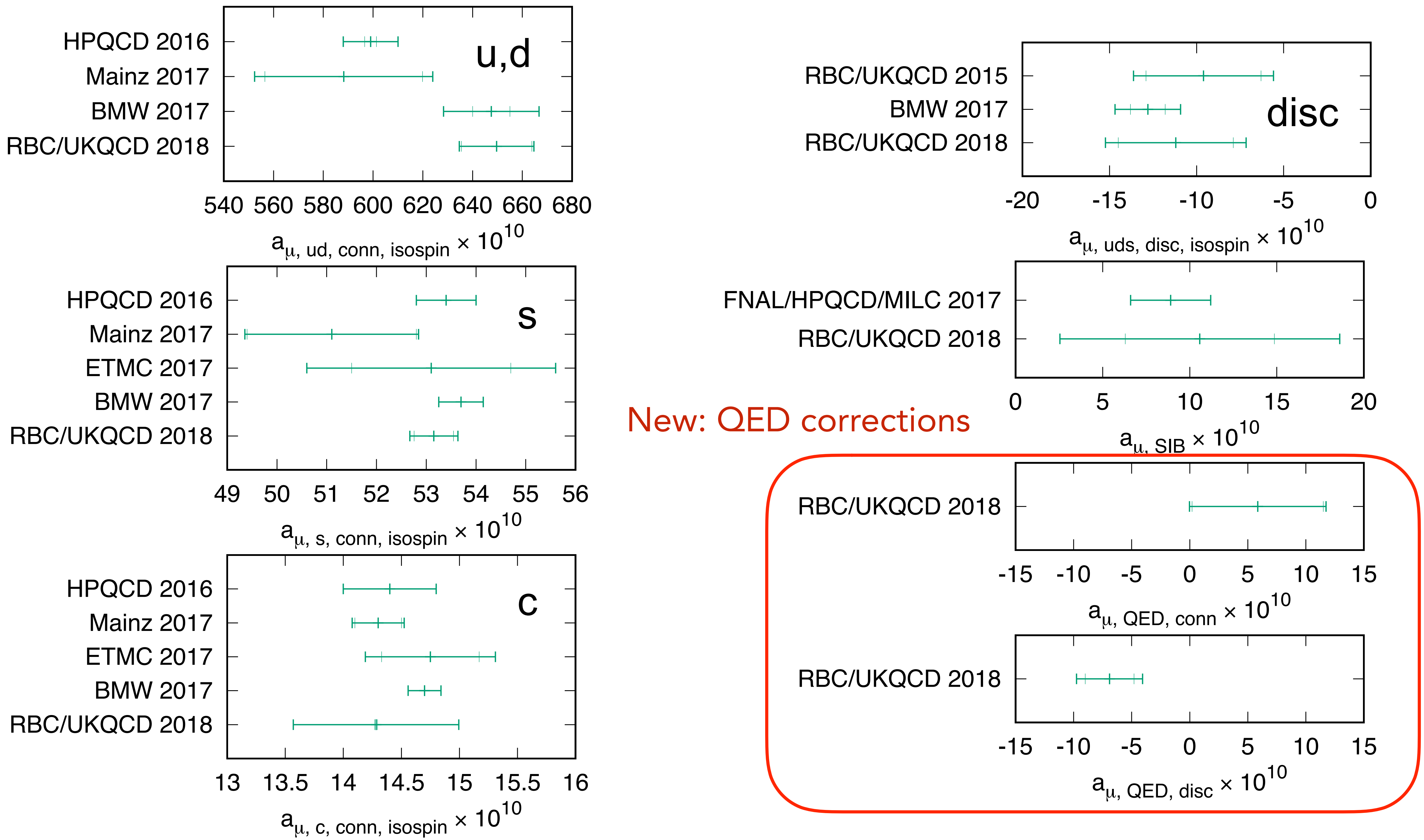

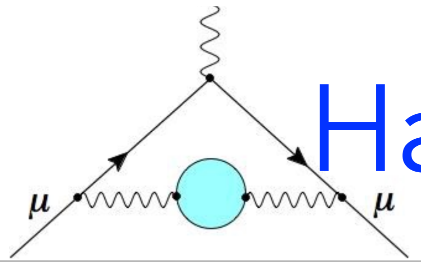

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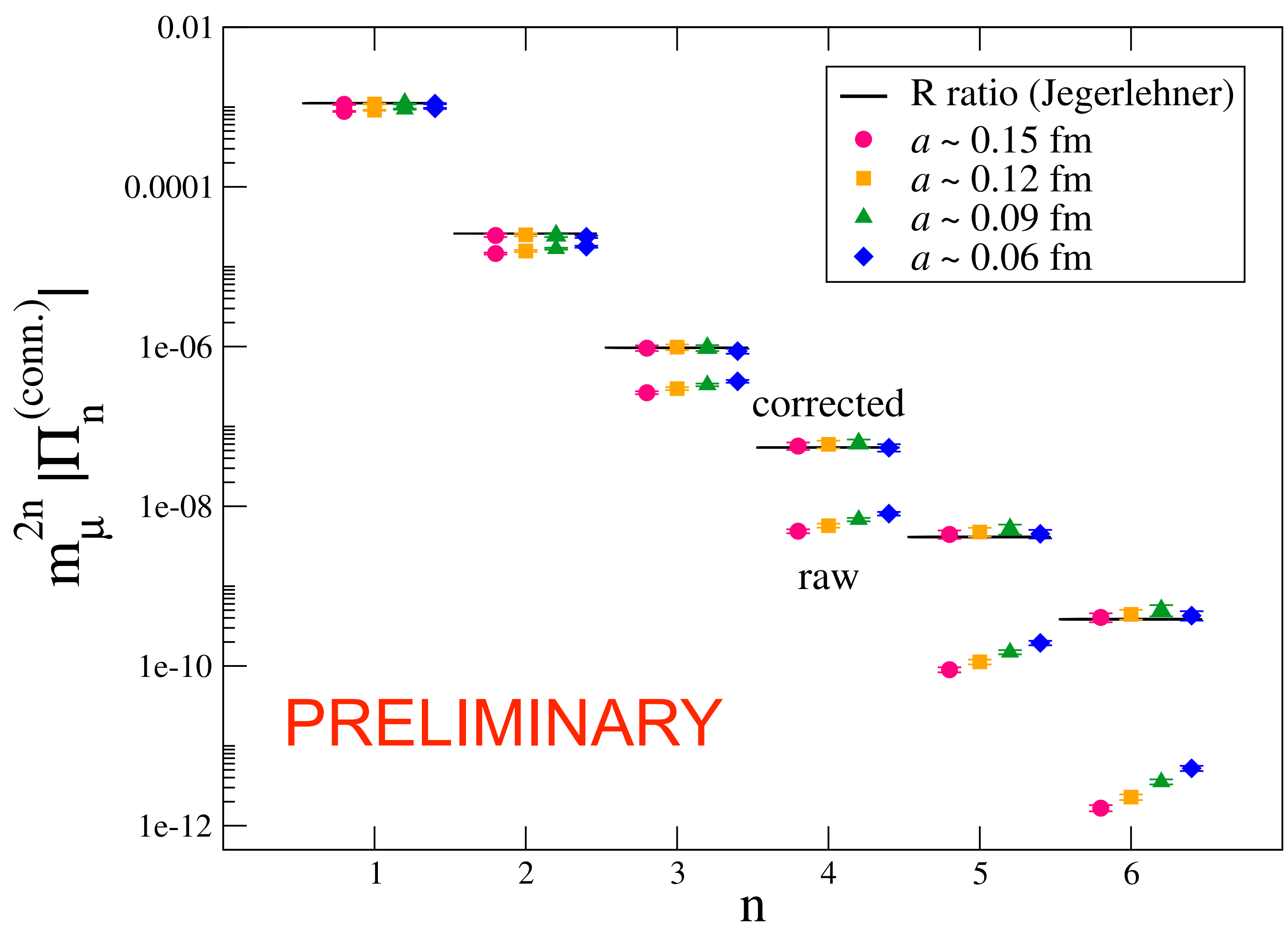
  - includes disconnected contributions
  - includes QED and strong isospin breaking corrections ($m_u \neq m_d$)
  - includes finite volume corrections, continuum extrapolation
- ✦Compare intermediate quantities (Taylor coefficients,…) with R-ratio data.

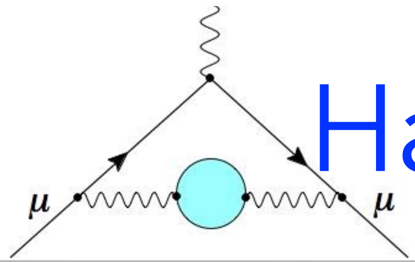
Compare HPQCD results to R-ratio data

R. Van de Water @ HVP KEK 2018 (for FNAL/MILC/HPQCD):

Compare Taylor coefficients before and after (finite volume, discretization,..) corrections with R-ratio data:



Lowest moments make the largest contributions to a_μ.



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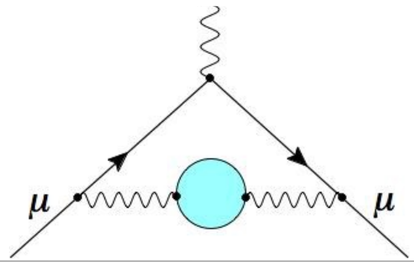
- ✦ Compare intermediate quantities (Taylor coefficients, …) with R-ratio data
- ✦ Hybrid method: combine LQCD with R-ratio data



Hadronic vacuum polarization

Hybrid method: combine LQCD with R-ratio data

C. Lehner (RBC/UKQCD) @ Lattice 2017

Direct LQCD calculations of HVP are still less precise than dispersive methods.
But comparisons between R-ratio and lattice data are already useful.

- Convert R-ratio data to Euclidean correlation function (via the dispersive integral).

C. Lehner @ HVP KEK 2018 (from T. Blum et al, arXiv:1801.07224)

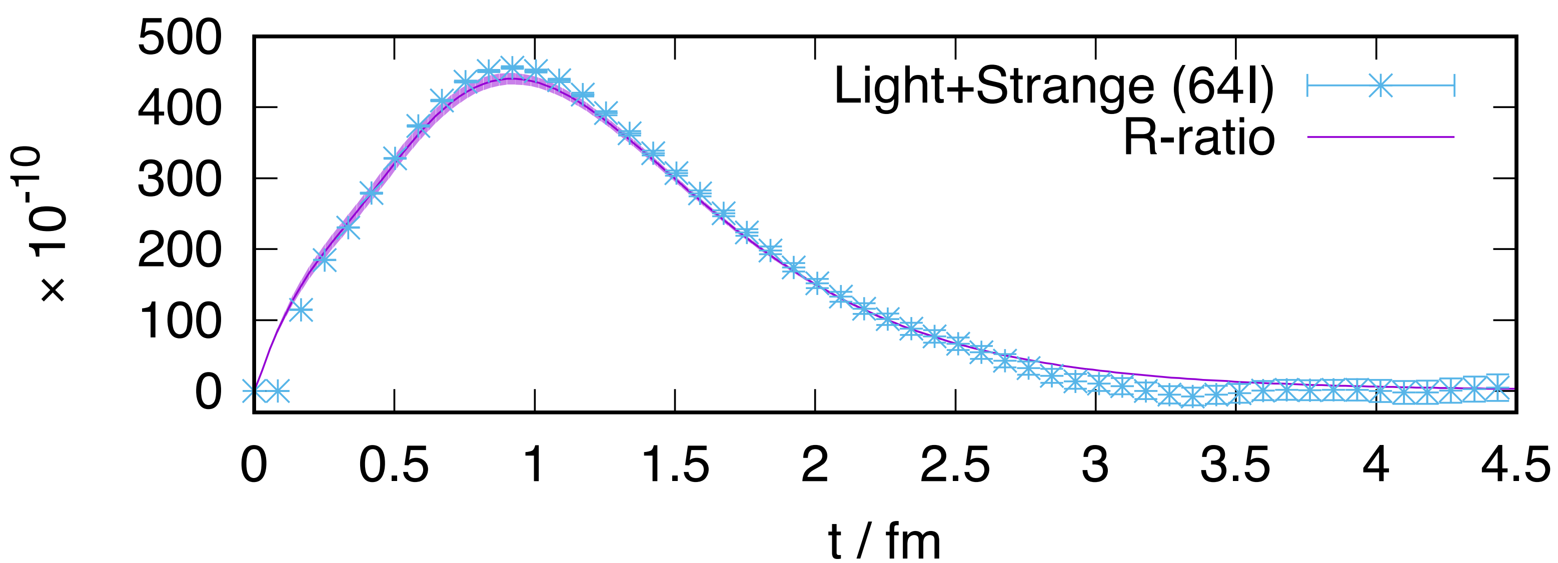
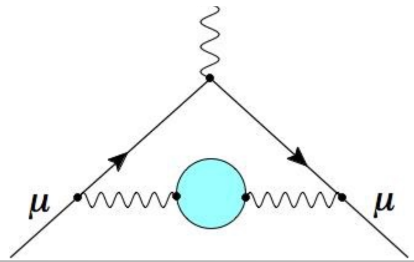


FIG. 4. Comparison of $w_t C(t)$ obtained using R-ratio data [1] and lattice data on our 64I ensemble.



Hadronic vacuum polarization

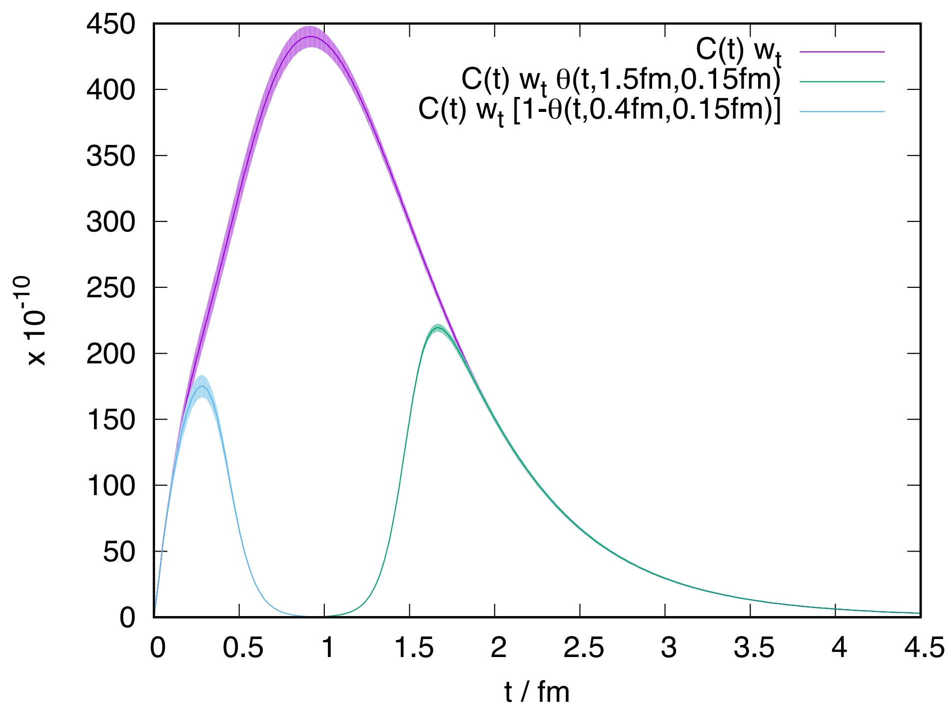
Hybrid method: combine LQCD with R-ratio data

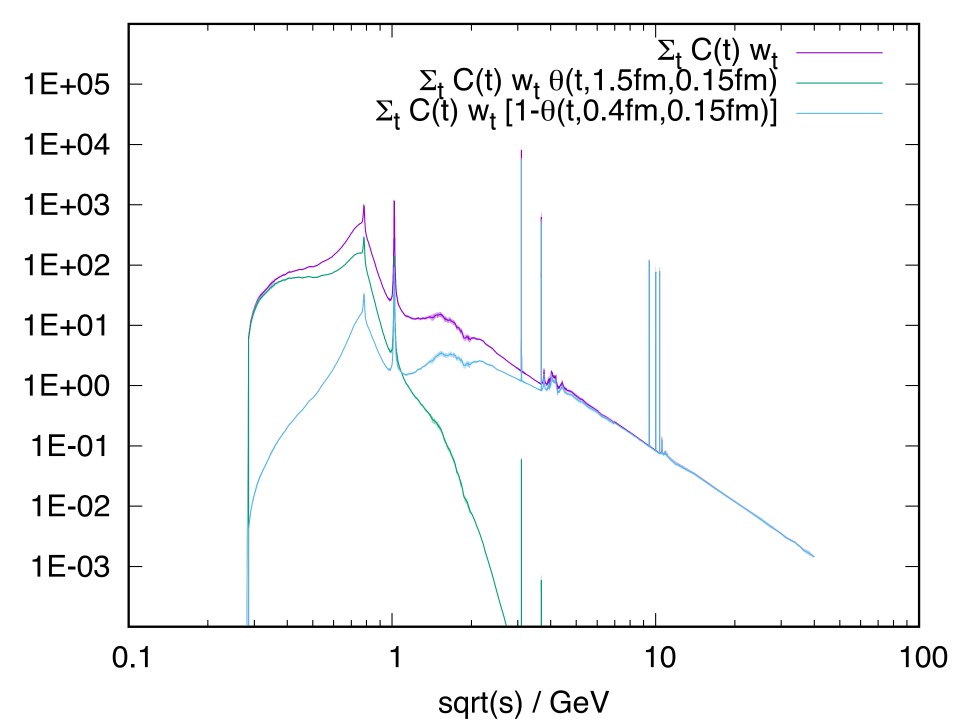
C. Lehner (RBC/UKQCD) @ Lattice 2017

Direct LQCD calculations of HVP are still less precise than dispersive methods.
But comparisons between R-ratio and lattice data are already useful.

- Convert R-ratio data to Euclidean correlation function (via the dispersive integral).
- Compare lattice/R-ratio data (after adding all the corrections and extrapolating to continuum, infinite volume).
- Use R-ratio data where LQCD errors are large and vice versa.

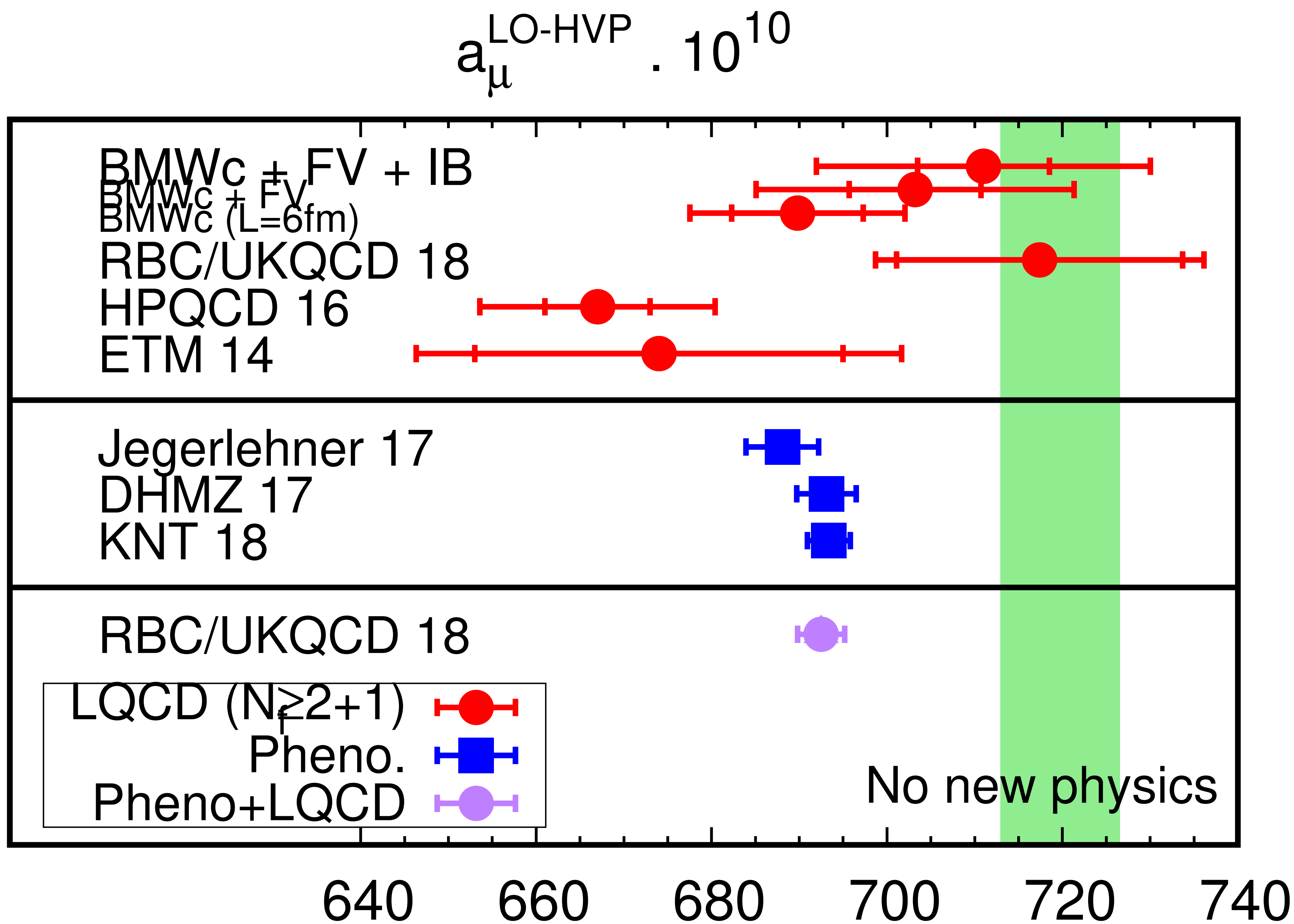
C. Lehner @ HVP KEK 2018 (from T. Blum et al, arXiv:1801.07224)

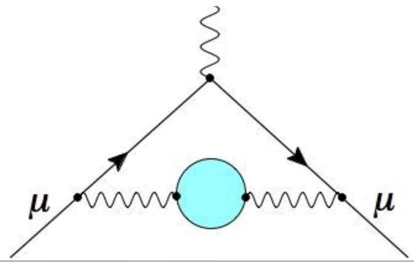




Summary of recent HVP results

L. Lellouch @ HVP KEK 2018 (for BMW collaboration)

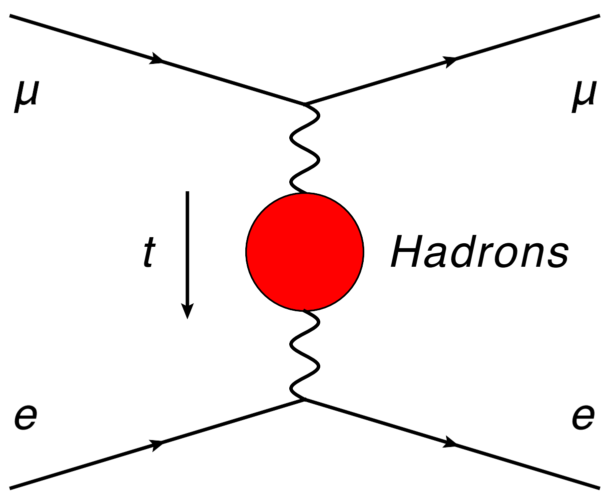




Hadronic vacuum polarization

μ-e elastic scattering to measure a_μ^{HVP}

M. Passera @ HVP KEK 2018 (A. Abbiendi et al, arXiv:1609.08987, EPJC 2017)

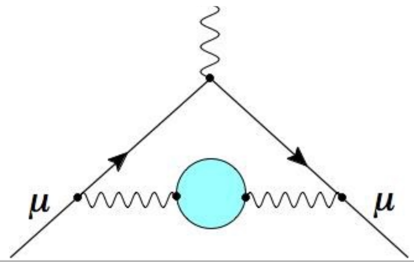


$$a_\mu^{\mathrm{HLO}} = \frac{\alpha}{\pi} \int_0^1 dx\,(1-x)\,\Delta\alpha_{\mathrm{had}}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

$\Delta\alpha_{\mathrm{had}}(t)$ is the hadronic contribution to the running of α in the space-like region. It can be extracted from scattering data!

- use CERN M2 muon beam (150 GeV)
- test detector prototype in August 2018
- LOI planned for 2018-2019
- Physics beyond colliders program @ CERN

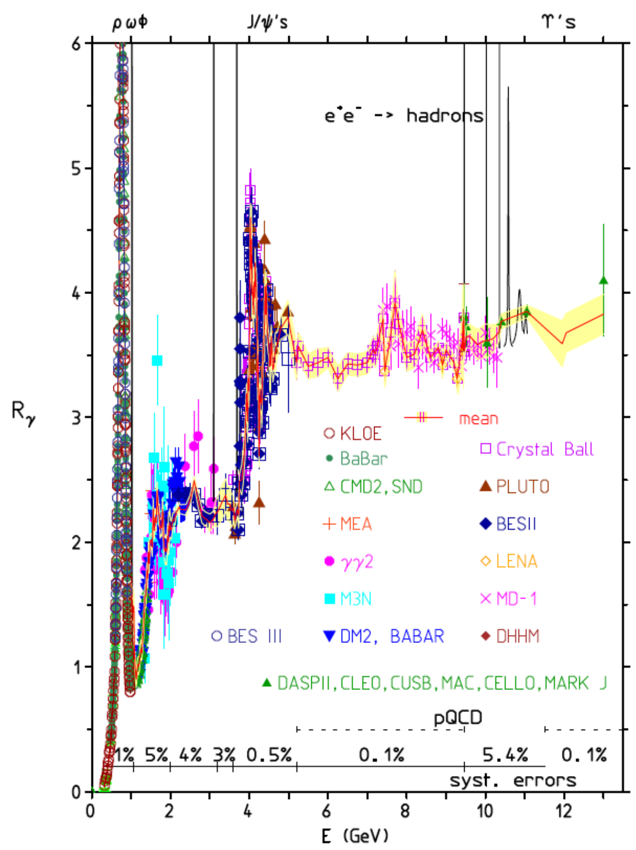


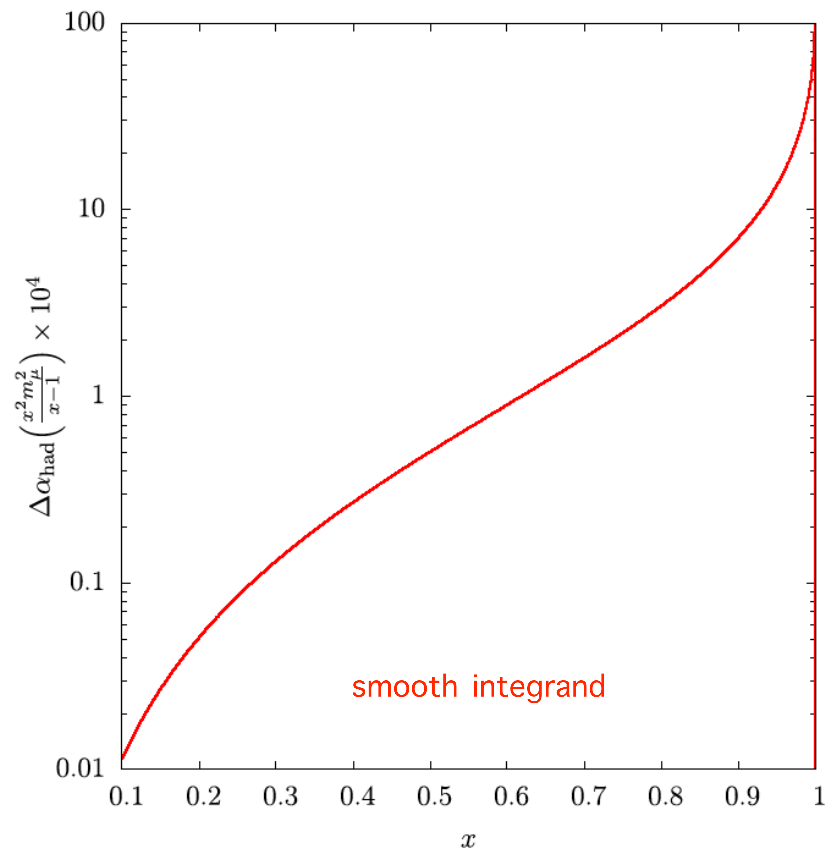
Hadronic vacuum polarization

μ-e elastic scattering to measure a_μ^{HVP}

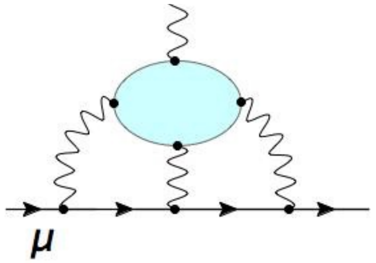
M. Passera @ HVP KEK 2018 (A. Abbiendi et al, arXiv:1609.08987, EPJC 2017)

Time-like → **Space-like**





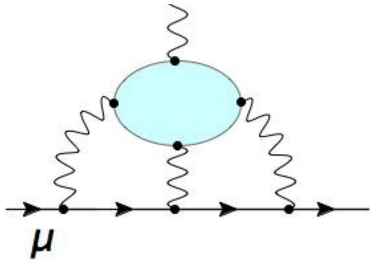
- complement region not accessible to experiment with LQCD calculation
- requires NNLO QED calculation, …



Hadronic Light-by-light

Hadronic light-by-light:

- ✦ Target: ≲ 10% total error
- ✦ current estimate "Glasgow consensus" based on different QCD models
- ✦ theory error not well determined and not improvable



Hadronic Light-by-light

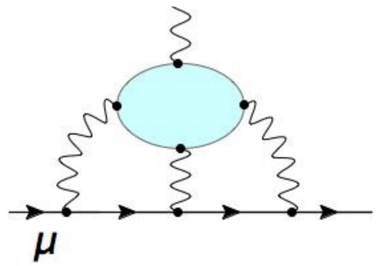
G. Colangelo @ Lattice 2017:

Jegerlehner-Nyffeler 2009

Contribution	BPaP(96)	HKS(96)	KnN(02)	MV(04)	BP(07)	PdRV(09)	N/JN(09)
π^0, η, η'	85±13	82.7±6.4	83±12	114±10	–	114±13	99±16
π, K loops	−19±13	−4.5±8.1	–	–	–	−19±19	−19±13
" " + subl. in N_c	–	–	–	0±10	–	–	–
axial vectors	2.5±1.0	1.7±1.7	–	22± 5	–	15±10	22± 5
scalars	−6.8±2.0	–	–	–	–	−7± 7	−7± 2
quark loops	21± 3	9.7±11.1	–	–	–	2.3	21± 3
total	83±32	89.6±15.4	80±40	136±25	110±40	105±26	116±39

Legenda: B=Bijnens Pa=Pallante P=Prades H=Hayakawa K=Kinoshita S=Sanda Kn=Knecht N=Nyffeler M=Melnikhov V=Vainshtein dR=de Rafael J=Jegerlehner

- large uncertainties (and differences among calculations) in individual contributions
- pseudoscalar pole contributions most important
- second most important: pion loop, *i.e.* two-pion cuts (*K s are subdominant*)
- heavier single-particle poles decreasingly important (unless one models them to resume the high-energy tail)



Hadronic Light-by-light

Hadronic light-by-light:

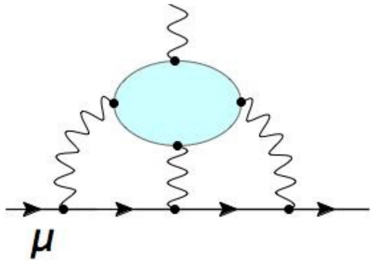
- ✦ Target: ≲ 10% total error
- ✦ current estimate "Glasgow consensus" based on different QCD models
- ✦ theory error not well determined and not improvable

Dispersive approach:

- ✦ significantly more complicated than for HVP
  (Colangelo et al, arXiv:1702.07347; Kubis et al, 2012, 2014; Hoferichter et al, 2012, 2014; Hanhardt et al, 2013; Pascalutsa et al, Pauk et al, Danilkin et al,…)
- ✦ combine with exp. data and/or LQCD calculations

Direct lattice QCD calculations:

- ✦ QCD + $\mathrm{QED_L}$ (finite volume)
  (Jin et al, arXiv:1610.04603, 2016 PRL; arXiv:1705.01067)
- ✦ QCD + QED (infinite volume & continuum)
  (Asmussen @Lattice 2017; Green et al, arXiv:PRL 2015; T. Blum et al, arXiv:1705.01067, 2017 PRD)
- ✦ dominant contribution from pion pole (transition form factors)
  (Gerardin et al, arXiv:1607.08174, 2016 PRD; Lattice 2017)



Hadronic Light-by-light

Breakthrough (RBC/UKQCD):

First LQCD calculation of connected and leading disconnected contribution with good statistical significance (T. Blum et al, arXiv:1610.04603, 2017 PRL).

$$a_\mu^{\rm HLbL} = (5.35 \pm 1.35) \times 10^{-10}$$

- ✦ a = 0.11 fm, L = 5.5 fm, physical pion mass, statistical error only.
- ✦ uses QCD + QED_L (finite volume)
- ✦ systematic error analysis (finite volume, continuum limit, …) in progress.
- ✦ Mainz group:
  LbL forward scattering amplitude (Gerardin @ HLbL UConn 2018)
  pion transition form factor((Gerardin et al, arXiv:1607.08174, 2016 PRD; Lattice 2017)

QCD + QED (infinite volume):

- ✦ RBC/UKQCD:
  calculation in progress (can reuse QCD part from QCD+QED_L calculation)
- ✦ Mainz group:
  work in progress (Asmussen @ Lattice 2017, HLbL UConn 2018)

Summary

- ☆ The First Workshop of the Muon g-2 Theory Initiative kicked off the activities.
- ☆ Two Working Groups (HVP and HLbL) have been formed.
  Invited community participation.
- ☆ Three workshops in 2018:
  HVP WG workshop @ KEK: 12-14 February
  HLbL WG workshop @ U Connecticut: 12-14 March
  Plenary workshop in Mainz: 18-22 June
- ☆ Rapid progress in lattice QCD calculations (HVP + HLbL), development of the dispersive method for HLbL
- ☆ Plan to coordinate with other working groups/efforts, for example Radio MonteCarLow and FLAG.

Outlook

Amala Willenbrock

Outlook

☆ HVP: target of ~0.2% precision appears reachable

new R-measurements

significant progress in lattice QCD calculations thanks to:

- ➢ the availability of gauge ensembles with **physical mass light quarks.**
- ➢ in progress: including QED and strong isospin breaking effects
- ➢ many talks at Lattice 2017, HVP workshop

☆ HLbL: target of ~10% precision also reachable

- ✦ breakthrough calculation by RBC/UKQCD. Systematic error analysis in progress.
- ✦ progress also reported by Mainz group
- ✦ development of dispersive methods in progress
- ✦ new results broadly consistent with model estimates: models (Glasgow consensus) unlikely the cause of the difference between experiment & SM expectations.

☆ GREAT discovery potential for New Physics by g-2 experiments:

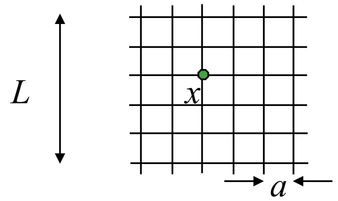
non-minimal SUSY, 2HDM, dark photon, light scalar, Leptoquarks, light Z', …

Amala Willenbrock

Thank you!

Farah Willenbrock

Appendix:
Introduction to Lattice QCD



Lattice QCD Introduction

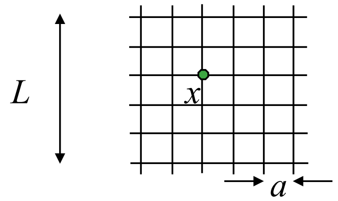
$$\langle \mathcal{O} \rangle \sim \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A \, \mathcal{O}(\psi, \bar{\psi}, A) \, e^{-S} \qquad S = \int d^4x \left[\bar{\psi}(\not{D} + m)\psi + \frac{1}{4}(F^a_{\mu\nu})^2 \right]$$

use monte carlo methods (importance sampling) to evaluate the integral.

Note: Integrating over the fermion fields leaves $\det(\not{D} + m)$ in the integrand. The correlation functions, O, are then written in terms of $(\not{D}+m)^{-1}$ and gluon fields.

steps of a lattice QCD calculation:

1. generate gluon field configurations according to $\det(\not{D}+m)\, e^{-S}$
2. calculate quark propagators, $(\not{D}+m_q)^{-1}$, for each valence quark flavor and source point
3. tie together quark propagators into hadronic correlation functions (usually 2 or 3-pt functions)
4. statistical analysis to extract hadron masses, energies, hadronic matrix elements, …. from correlation functions
5. systematic error analysis



Lattice QCD Introduction

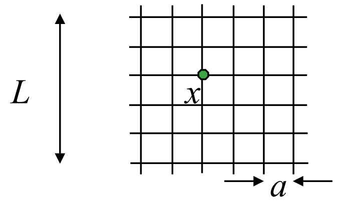
systematic error analysis

...of lattice spacing, chiral, heavy quark, and finite volume effects is based on EFT (Effective Field Theory) descriptions of QCD

➙ ab initio

- finite a: Symanzik EFT
- light quark masses: Chiral Perturbation Theory
- heavy quarks: HQET
- finite L: finite volume EFT

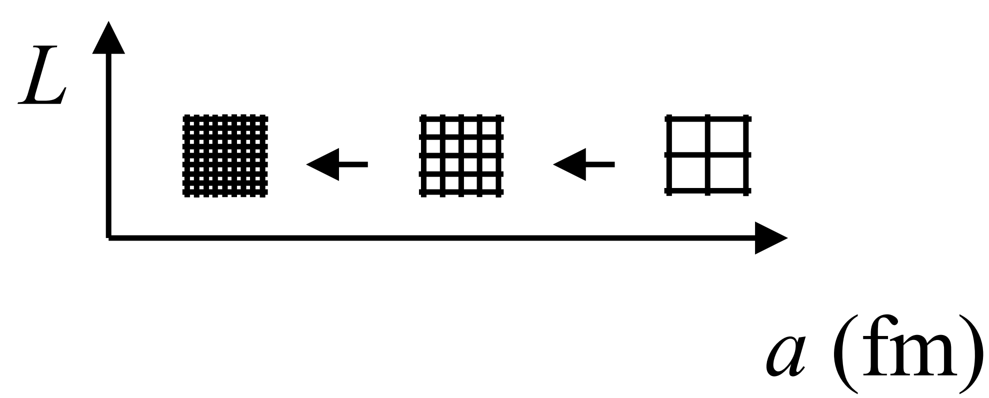
- need large enough L and small enough a and simulations with several a, L, ...

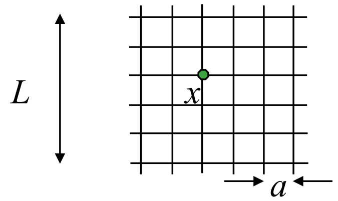


Lattice QCD Introduction

discretization effects — continuum extrapolation

- typical momentum scale of quarks gluons inside hadrons: $\sim\Lambda_{\rm QCD}$
- make a small to separate the scales: $\Lambda_{\rm QCD} \ll 1/a$
- Symanzik EFT: $\langle\mathcal{O}\rangle^{\rm lat} = \langle\mathcal{O}\rangle^{\rm cont} + O(a\Lambda)^n,\ n \geq 2$
  - provides functional form for extrapolation (depends on the details of the lattice action)
  - can be used to build improved lattice actions
  - can be used to anticipate the size of discretization effects
- to control and reliably estimate the error, repeat …





Lattice QCD Introduction

light quark mass effects

Simulations with $m_{\rm light} = 1/2\ (m_u + m_d)$ **at the physical u/d quark masses are now available,** but they are computationally expensive and many still have

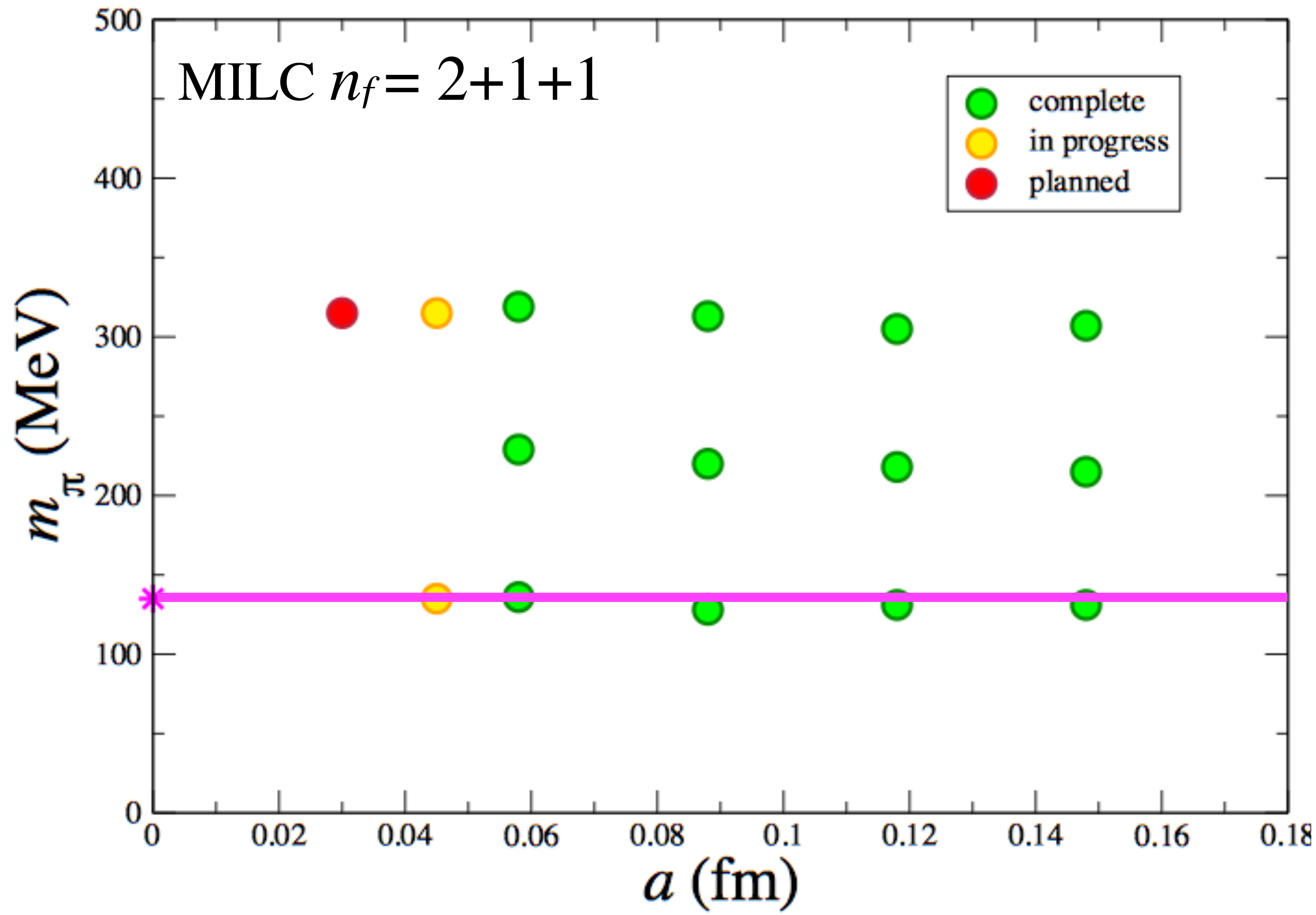
$$m_{\rm light} > 1/2\ (m_u + m_d)_{\rm phys}$$

Chiral Perturbation Theory (χPT) can be used to extrapolate/interpolate to the physical point.

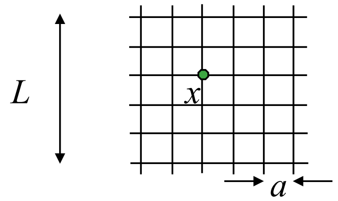
- Can include discretization effects
- It is now common practice to perform a combined continuum-chiral extrapolation/interpolation

Lattice set-up

Example: Set of ensembles by MILC collaboration



Five collaborations have now generated sets of ensembles that include sea quarks with physical light-quark masses: PACS-CS, BMW, MILC, RBC/UKQCD, ETM



finite volume effects

One stable hadron (meson) in initial/final state:

If L is large enough, FV error $\sim e^{-m_\pi L}$

- keep $m_\pi L \gtrsim 4$

To quantify residual error:

- include FV effects in χPT
- compare results at several Ls (with other parameters fixed)

The story changes completely with two or more hadrons in initial/final state or if there are two or more intermediate state hadrons.

➠ "simple quantities":
no more than one stable hadron in initial/final state

If QED is included, FV effects also become more complicated…

Appendix:
more g-2 slides

FNAL/MILC/HPQCD g-2 group

Subgroup of members of the three collaborations actively engaged in the g-2 project:

Fermilab Lattice Collaboration

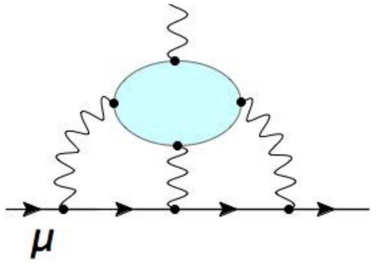
- Aida El Khadra (Illinois)
- Andreas Kronfeld (Fermilab)
- Ethan Neil (Colorado)
- Ruth Van de Water (Fermilab)

HPQCD

- Bipasha Chakraborty (JLAB)
- Daniel Hatton (Glasgow)
- Christine Davies (Glasgow)
- Jonna Koponen (INFN, Rome)
- Peter Lepage (Cornell)
- Andrew Lytle (Glasgow)
- Craig McNeile (Plymouth)

MILC Collaboration

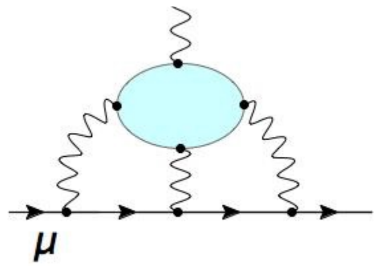
- Carleton DeTar (Utah)
- Steve Gottlieb (Indiana)
- Jack Laiho (Syracuse)
- Yuzhi Liu (Indiana)
- Doug Toussaint (Arizona)
- Alejandro Vaquero (Utah)



Hadronic Light-by-light

G. Colangelo @ Lattice 2017:

- The HLbL contribution to $(g-2)_\mu$ can be expressed in terms of measurable quantities in a dispersive approach
- master formula: HLbL contribution to a_μ as triple-integral over scalar functions which satisfy dispersion relations
- the relevant measurable quantity entering the dispersion relation depends on the intermediate state:
  - single-pion contribution: pion transition form factor
  - pion-box contribution: pion vector form factor
  - 2-pion rescattering: $\gamma^*\gamma^{(*)} \to \pi\pi$ helicity amplitudes
- I have presented results for the pion-box and the *S*-wave pion-rescattering contributions:
  model independence $=$ much reduced uncertainties

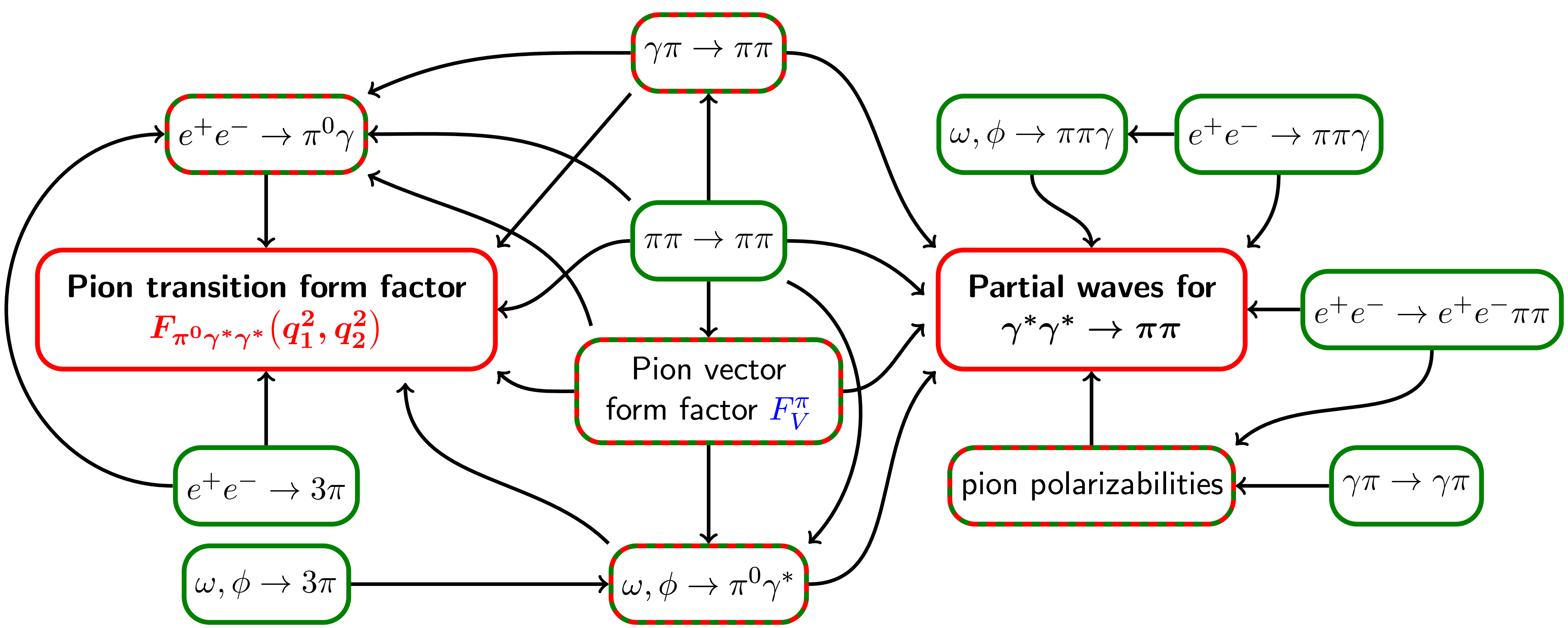


Hadronic Light-by-light

G. Colangelo @ Lattice 2017:

Hadronic light-by-light: a roadmap

GC, Hoferichter, Kubis, Procura, Stoffer arXiv:1408.2517 (PLB '14)

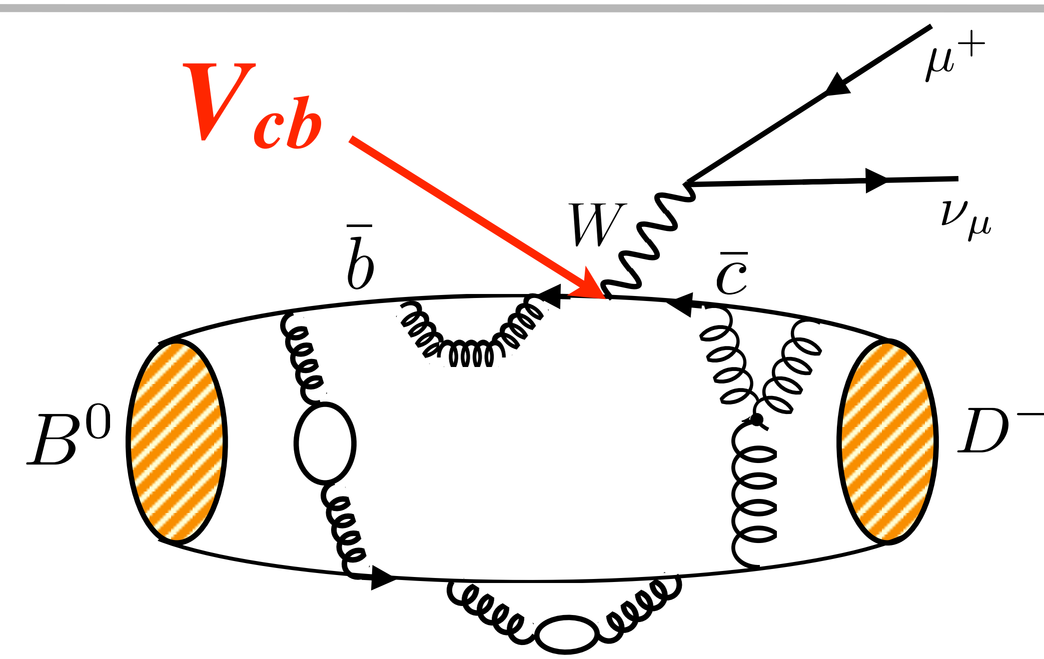


Artwork by M. Hoferichter

Appendix:
quark flavor physics
and lattice QCD

Introduction and Motivation

example: $B^0 \to D^- \mu^+ \nu_\mu$



Experiment vs. SM theory:

(experiment) = (known) x (**CKM factors**) x (had. matrix element)

$$\frac{d\Gamma(B \to \pi \ell \nu)}{dq^2}, \frac{d\Gamma(B \to K \ell^+ \ell^-)}{dq^2}, \ldots$$

$$\frac{d\Gamma(B \to D \ell \nu)}{d\omega}, \frac{d\Gamma(B \to D \tau \nu)}{d\omega}, \ldots$$

$$\Delta m_{d(s)}$$

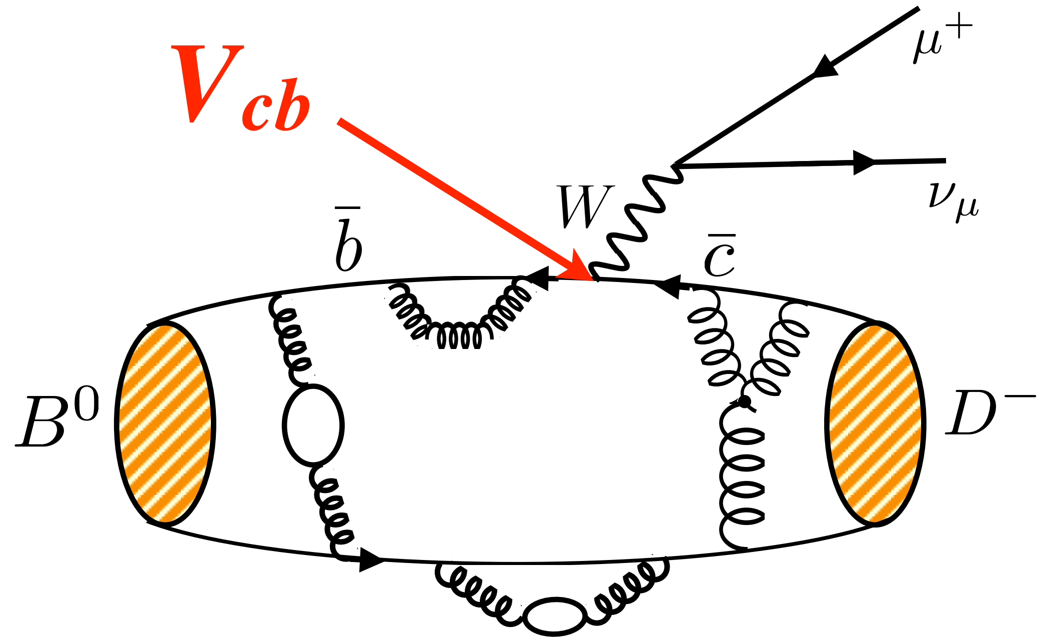
$$\vdots$$

Lattice QCD

parameterize the MEs in terms of form factors, decay constants, bag parameters, ...

Introduction and Motivation

example: $B^0 \to D^- \mu^+ \nu_\mu$



Experiment vs. SM theory:

(experiment) = (known) x (**CKM factors**) x (had. matrix element)

Two main purposes:

- ✦ combine experimental measurements with LQCD results to determine CKM parameters.
- ✦ confront experimental measurements of rare processes or lepton flavor (universality) violating observables with SM theory using LQCD inputs.

⋮

Lattice QCD

parameterize the MEs in terms of form factors, decay constants, bag parameters, ...

The CKM Matrix

$$
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
\pi \to \mu\nu & K \to \pi \ell\nu,\ K \to \mu\nu & B \to \pi \ell\nu,\ B_s \to K \ell\nu,\ \Lambda_b \to p \ell\nu \\
V_{cd} & V_{cs} & V_{cb} \\
D \to \pi \ell\nu,\ D \to \ell\nu & D \to K \ell\nu,\ D_s \to \ell\nu & B_{(s)} \to D_{(s)}, D^*_{(s)} \ell\nu \\
V_{td} & V_{ts} & V_{tb} \\
B^0 - \overline{B^0},\ B \to \pi \ell\ell & B_s^0 - \overline{B_s^0},\ B \to K \ell\ell &
\end{pmatrix}
$$

(ρ, η) $K^0 - \overline{K^0}$

The CKM Matrix

Precise Lattice QCD results **with complete systematic error budgets** now exist for all these processes ➠ improved determinations of the corresponding CKM elements

V_{ud} $\pi \to \mu\nu$	V_{us} $K \to \pi \ell\nu$ $K \to \mu\nu$	V_{ub} $B \to \pi \ell\nu,\ B_s \to K \ell\nu$ $\Lambda_b \to p\ \ell\nu$
V_{cd} $D \to \pi \ell\nu$ $D \to \ell\nu$	V_{cs} $D \to K \ell\nu$ $D_s \to \ell\nu$	V_{cb} $B_{(s)} \to D_{(s)}, D^*_{(s)}\ \ell\nu$
V_{td} $B^0 - \overline{B^0}$ $B \to \pi \ell\ell$	V_{ts} $B^0_s - \overline{B^0_s}$ $B \to K \ell\ell$	V_{tb}

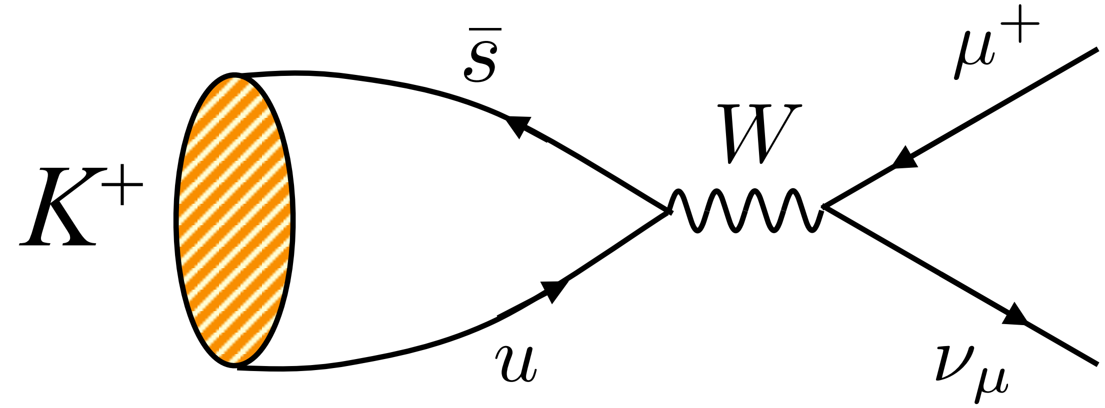
(ρ, η) $K^0 - \overline{K^0}$

Outline

- Motivation and Introduction
- Leptonic, Semileptonic Decays, Mixing
  - ⋆ Kaons
  - ⋆ D mesons
  - ⋆ B mesons
- Implications
  - ⋆ CKM determinations
- Summary

Leptonic Decay

example: $K^+ \to \mu^+ \nu_\mu$



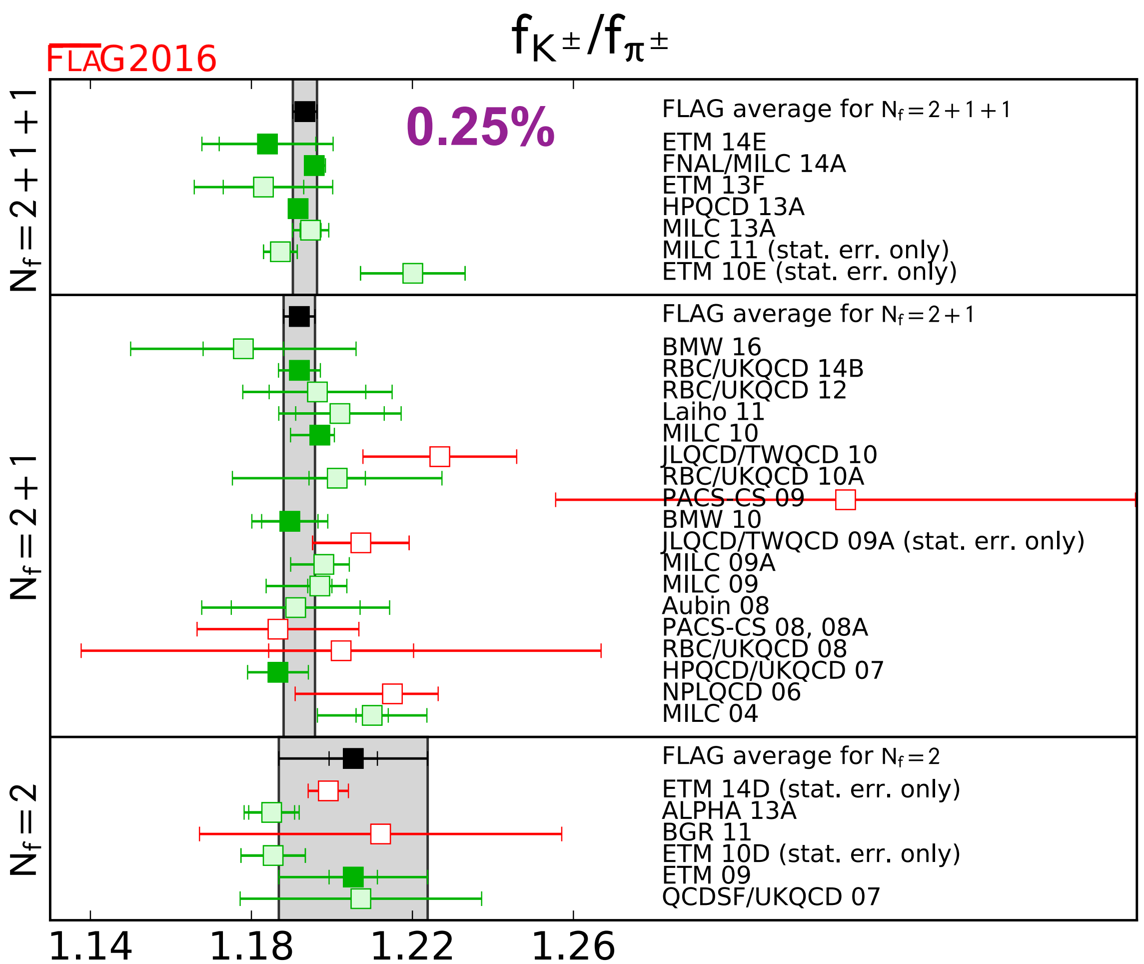
$$\Gamma\left(K^+ \to \ell^+ \nu_\ell(\gamma)\right) = (\text{known}) \times (1 + \delta_{\text{EM}}^{\ell}) \times |V_{us}|^2 \times f_{K^+}^2$$

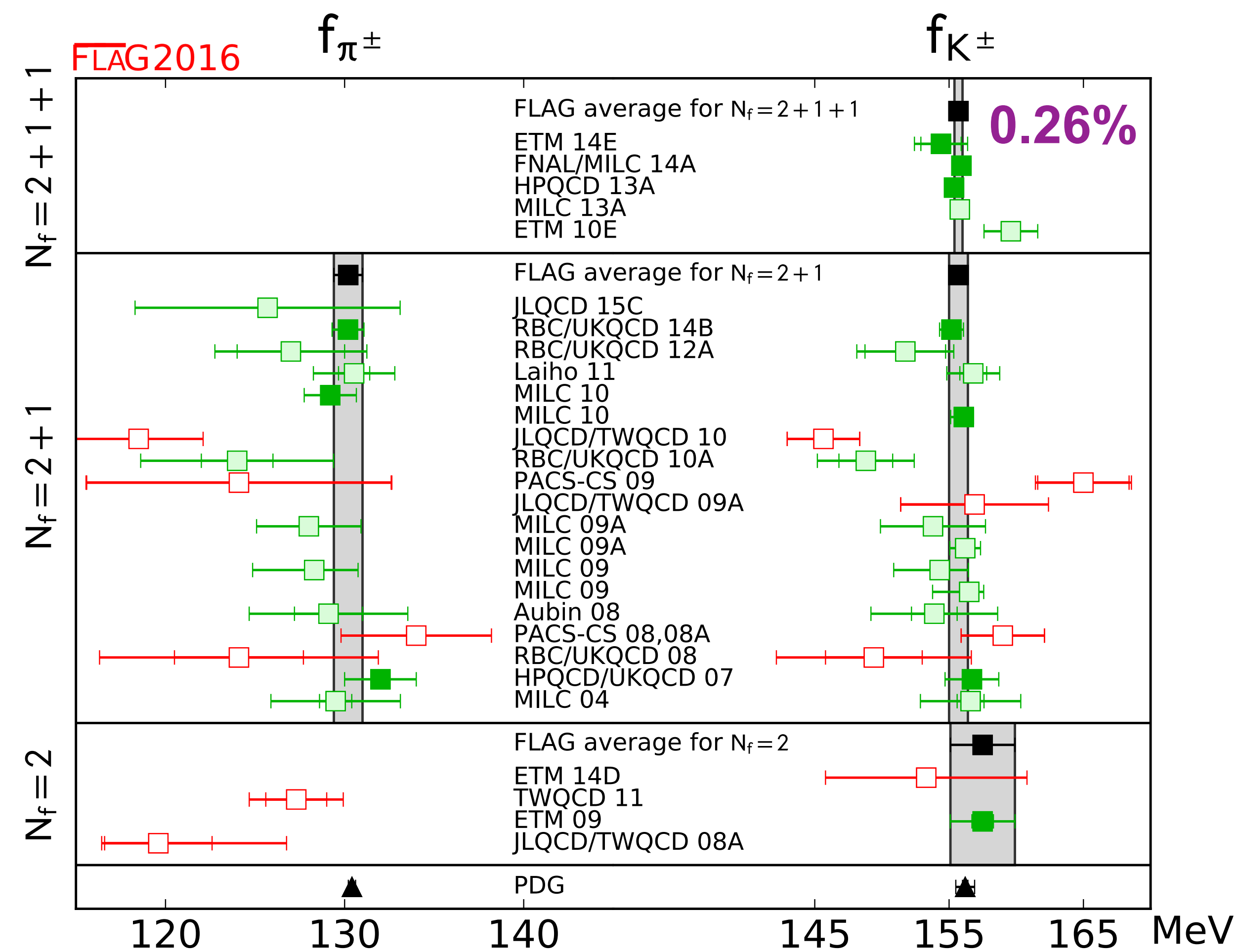
- use experiment + LQCD input for determination of CKM element
- similar for B ($|V_{ub}|$) and $D_{(s)}$ ($|V_{cd(s)}|$) mesons
- **ratios** such as f_{K^+}/f_{π^+} : reduced statistical and systematic errors.
- $\delta_{\text{EM}}^{\ell}$ includes structure dependent EM corrections, needed to relate the "pure QCD" decay constant to experimental decay rate. It is currently estimated phenomenologically within SU(3) ChPT

[Cirigliano et al, arXiv:1107.6001, RMP 2012].

FLAG review of Kaon quantities

S. Aoki et al [FLAG-3 review, arXiv:1607.00299, EPJC 17, web update]



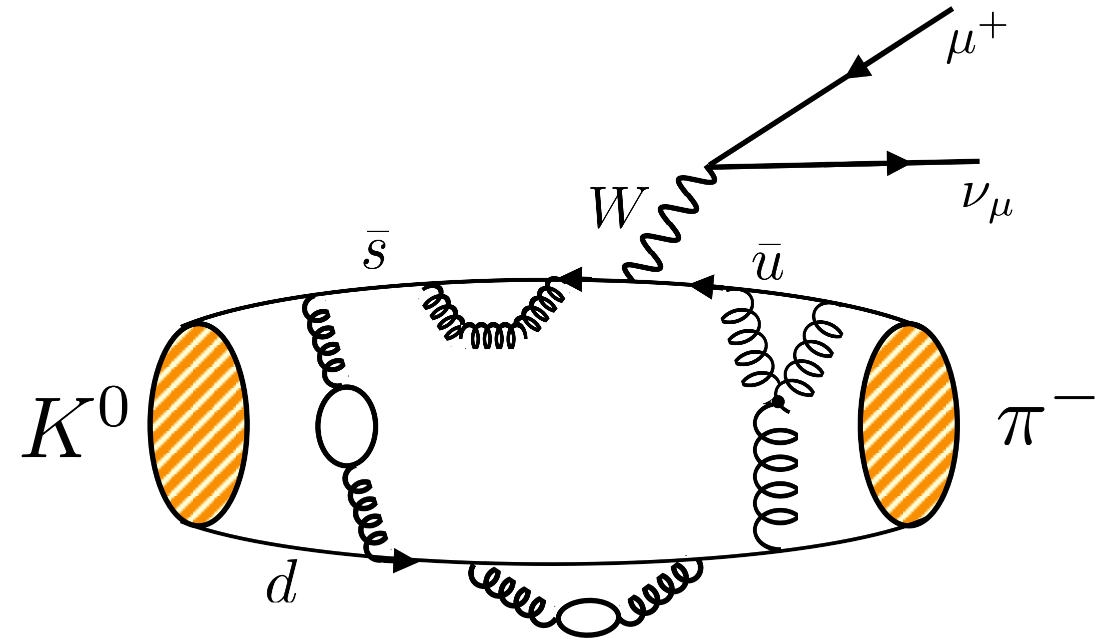


small errors due to

- ✦ **physical light quark masses**
- ✦ improved light-quark actions
- ✦ NPR or no renormalization

Semileptonic Decay

example: $K^0 \to \pi^- \ell^+ \nu_\ell$

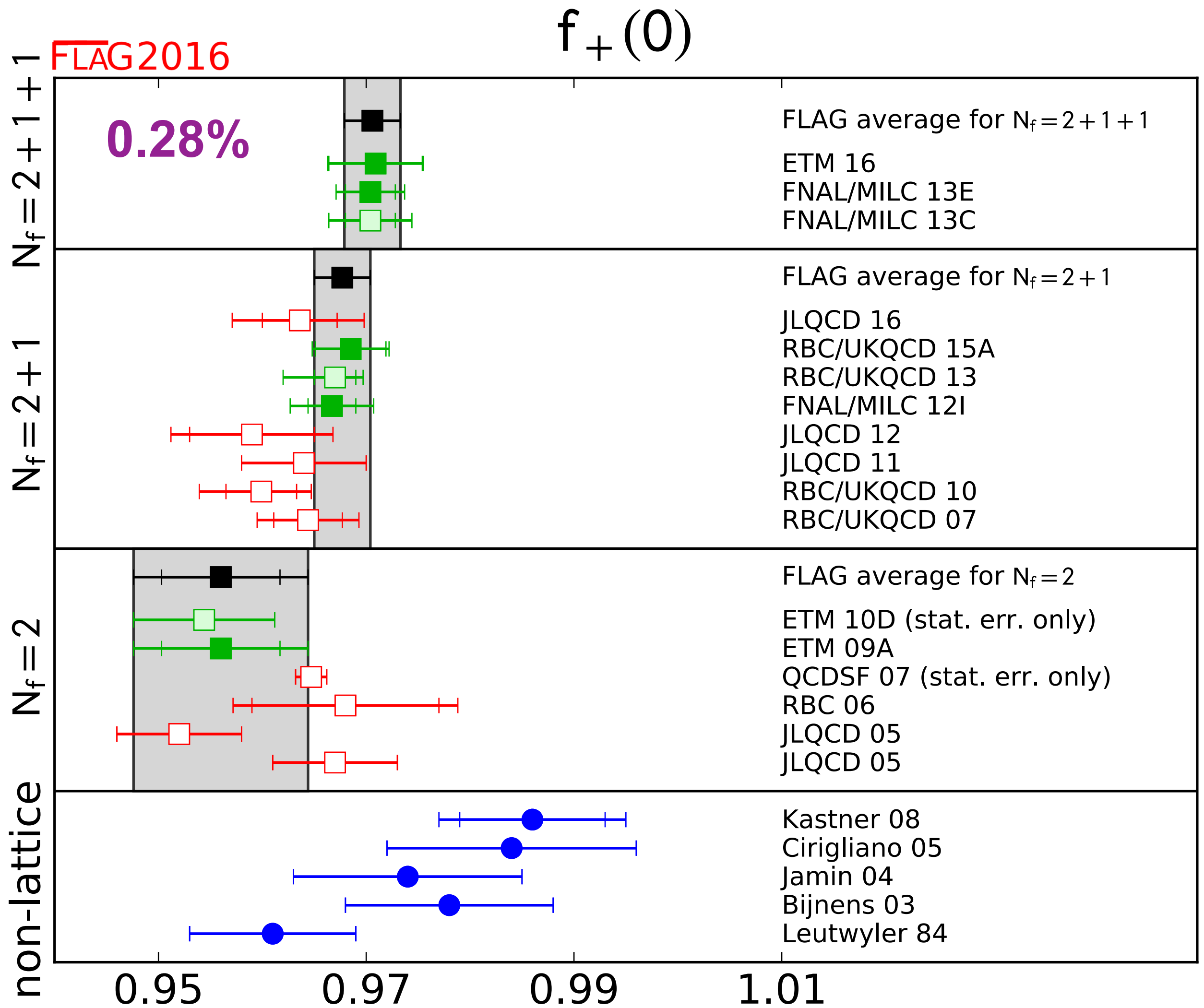


$$\Gamma_{K\ell 3} = (\text{known}) \times \begin{pmatrix} \text{phase} \\ \text{space} \end{pmatrix} \times (1 + \delta_{\text{EM}}^{K\ell} + \delta_{\text{SU(2)}}^{K\pi}) \times |V_{us}|^2 \times |f_+^{K^0\pi^-}(0)|^2$$

Needed to relate "pure QCD" form factor to experiment. Currently estimated phenomenologically [Cirigliano et al, arXiv:1107.6001, RMP 2012].

FLAG review of Kaon quantities

S. Aoki et al [FLAG-3 review, arXiv:1607.00299, EPJC 17, web update]

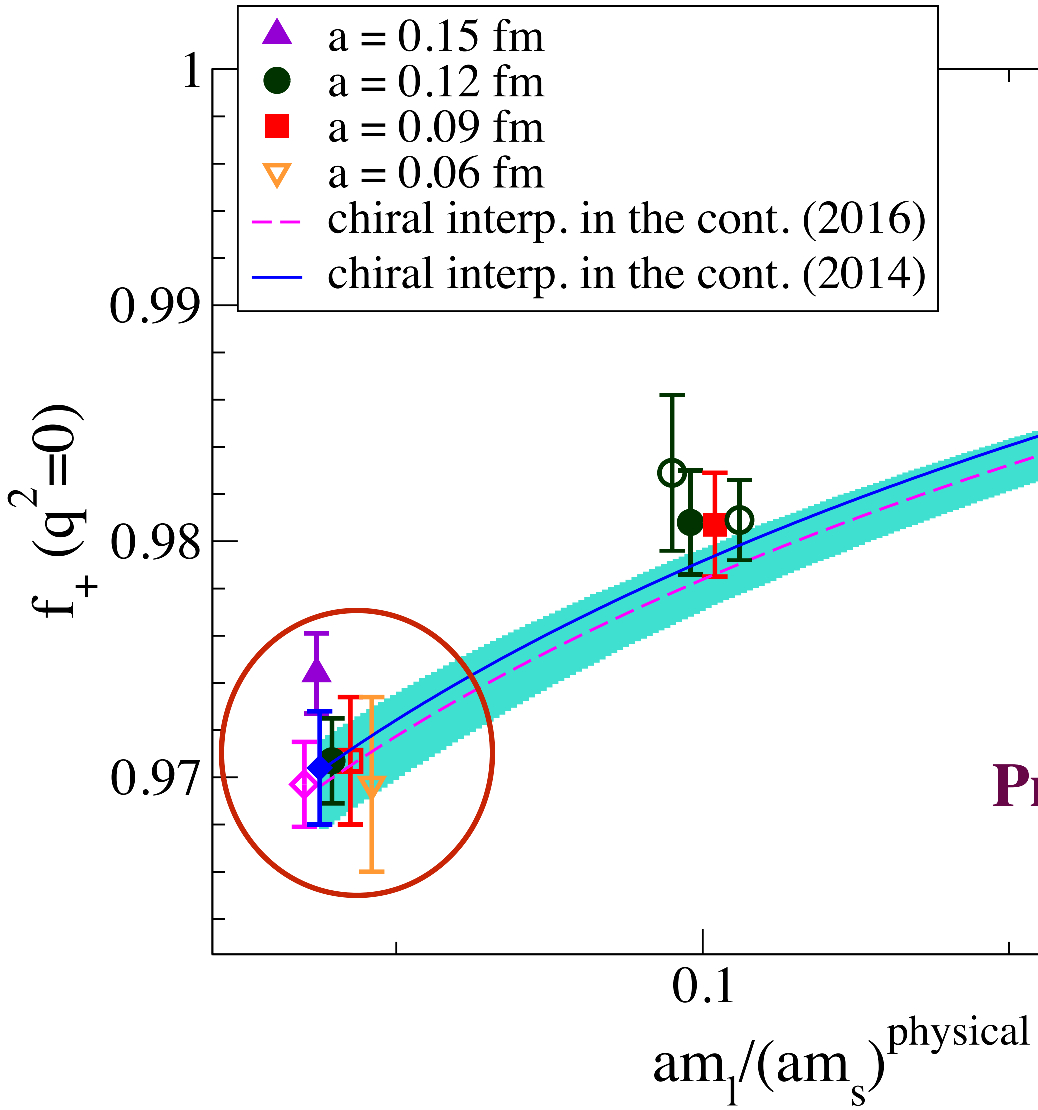


small errors due to

- ✦ **physical light quark masses**
- ✦ improved light-quark actions
- ✦ NPR or no renormalization

Kaon form factor update

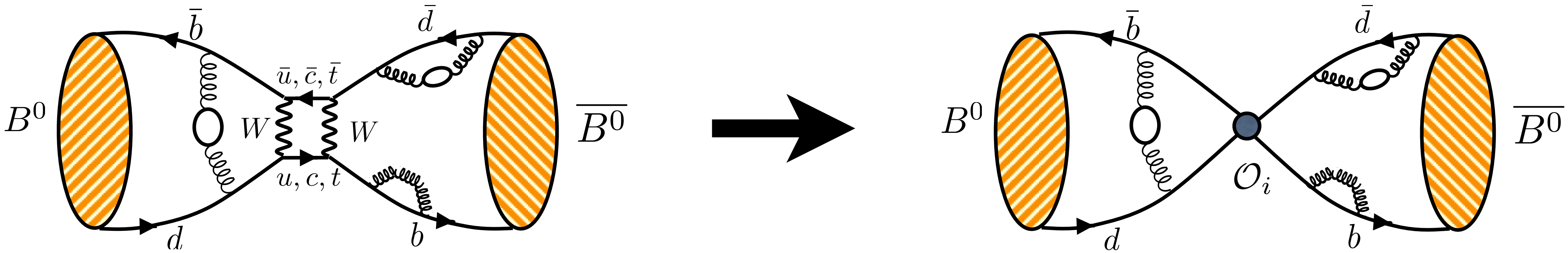
E. Gamiz @ Lattice 2016 [FNAL/MILC in preparation]



- ✦ two additional lattice spacings
- ✦ four ensembles at physical mass
- ✦ one-loop ChPT for FV with twisted boundary conditions [Bernard et al, arXiv:1702.03416, 2017 JHEP]
- ✦ expect ~0.2% total error

Neutral Meson Mixing

Standard Model



SM: $\Delta M_q = (\text{known}) \times |V_{tq}^* V_{tb}|^2 \times \langle \bar{B}_q^0 | \mathcal{O}_1 | B_q^0 \rangle$

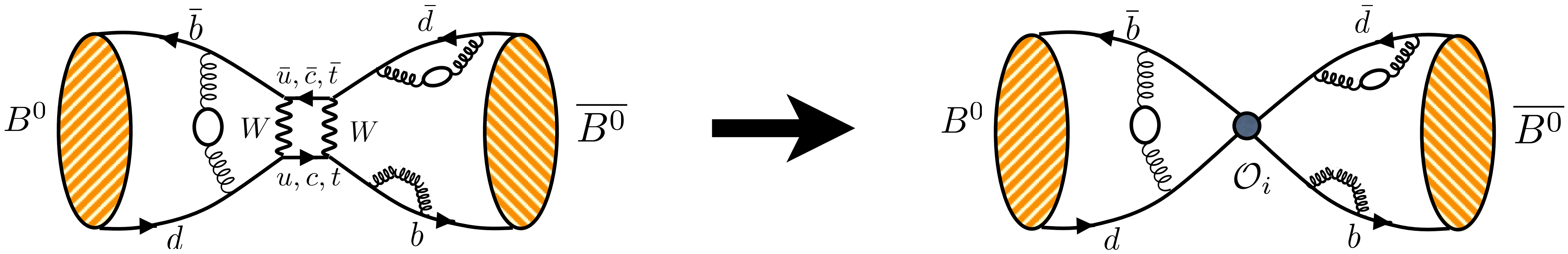
also:

$$\frac{\Delta M_s}{\Delta M_d} = \frac{m_{B_s}}{m_{B\,d}} \times \left| \frac{V_{ts}}{V_{td}} \right|^2 \times \xi^2 \quad \text{with} \quad \xi \equiv \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$$

$$\epsilon_K = (\text{known}) \times B_K \kappa_\epsilon \times |V_{cb}|^2 \times \bar{\eta} \times f(\bar{\rho}, \bar{\eta}, V_{cb}, \eta_i)$$

Neutral Meson Mixing

Standard Model



In general :

$$\mathcal{H}_{\text{eff}} = \sum_{i=1}^{5} c_i(\mu)\mathcal{O}_i(\mu)$$

SM:

$$\mathcal{O}_1 = (\bar{b}^\alpha \gamma_\mu L q^\alpha)\ (\bar{b}^\beta \gamma_\mu L q^\beta)$$

$$\mathcal{O}_2 = (\bar{b}^\alpha L q^\alpha)\ (\bar{b}^\beta L q^\beta)$$

$$\mathcal{O}_3 = (\bar{b}^\alpha L q^\beta)\ (\bar{b}^\beta L q^\alpha)$$

BSM:

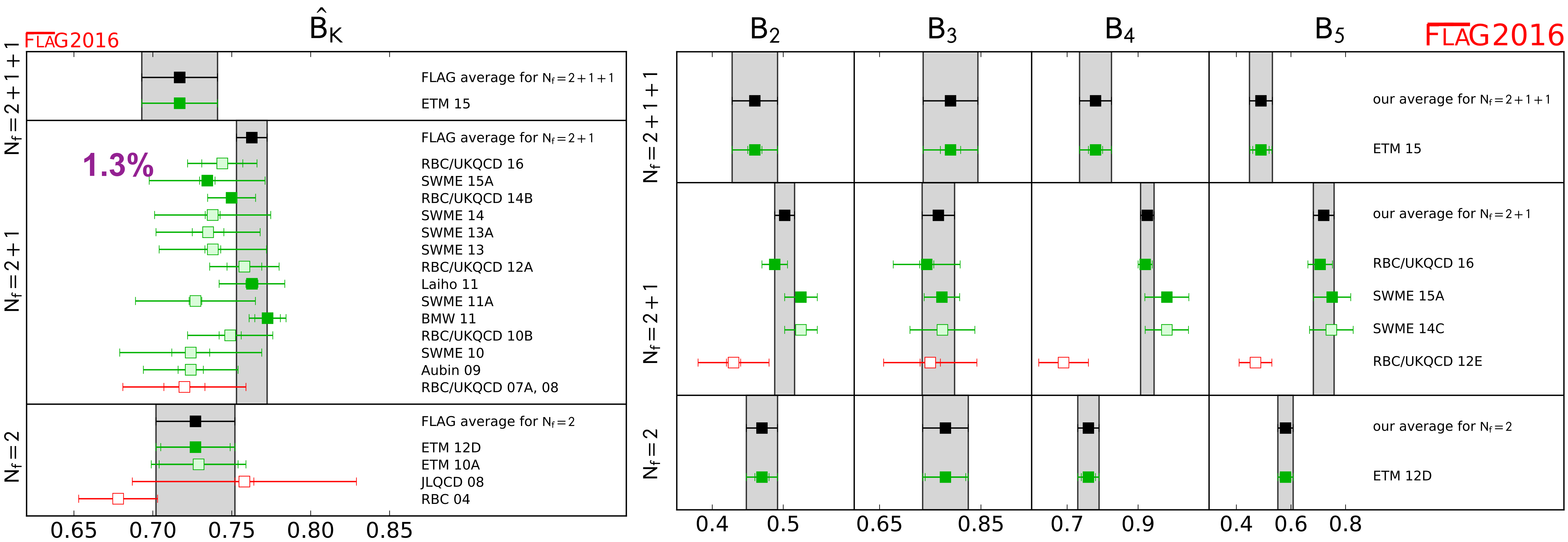
$$\mathcal{O}_4 = (\bar{b}^\alpha L q^\alpha)\ (\bar{b}^\beta R q^\beta)$$

$$\mathcal{O}_5 = (\bar{b}^\alpha L q^\beta)\ (\bar{b}^\beta R q^\alpha)$$

Recent and ongoing LQCD calculations of K, D, and B mixing quantities now include results for hadronic matrix elements of all five operators.

FLAG review of Kaon quantities

S. Aoki et al [FLAG-3 review, arXiv:1607.00299, EPJC 17, web update]



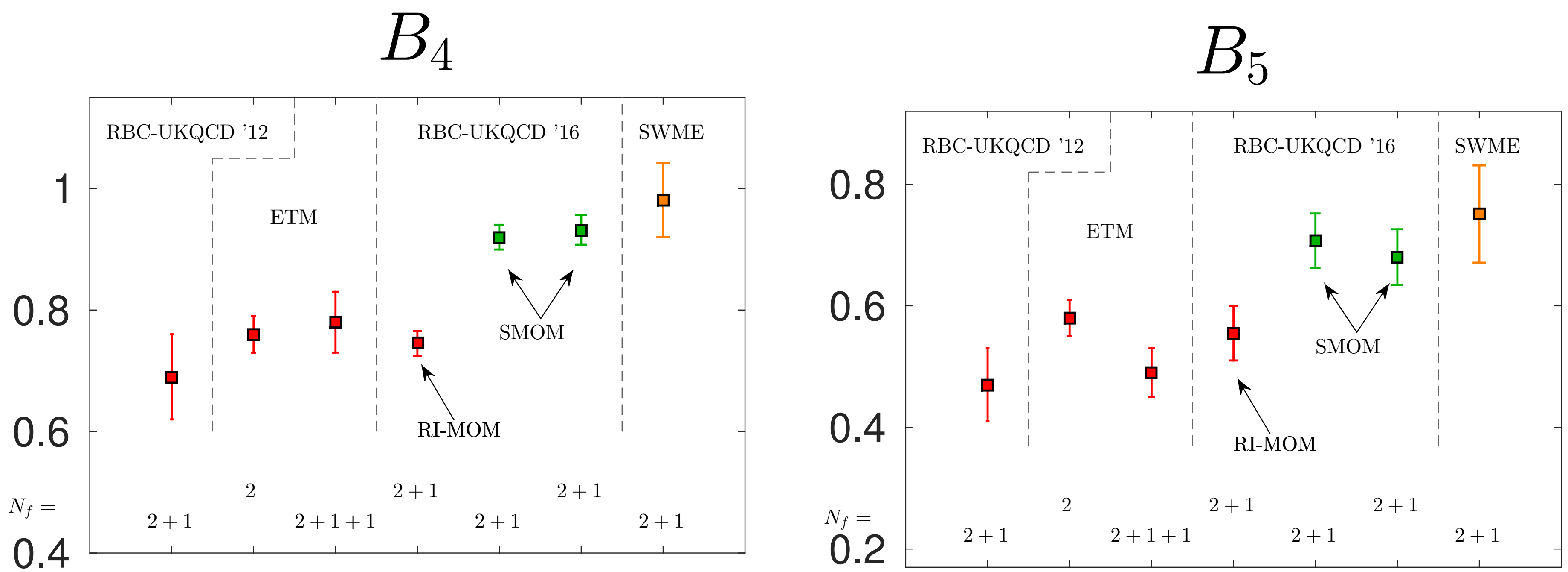
The apparent differences between results for $B_{2\text{-}5}$ with $N_f = 2+1+1$ vs $N_f = 2+1$ are due to the use of different intermediate renormalization schemes, in particular, RI/SMOM vs RI/MOM [N. Garron, R. Hudspith, and A. Lytle, arXiv:1609.03334, 2016 JHEP].

Kaon bag parameter update

Xu Feng @ Lattice 2017 (Kaon review):

Resolution of the discrepancy for B_4, B_5

$N_f = 2+1$ DWF, $a = 0.08$, 0.11 fm, $m_\pi = 300$ MeV [RBC-UKQCD, JHEP11(2016)001]



Plot, courtesy of N. Garron

- Use both RI/MOM and SMOM ⇒ the former is significantly smaller
- Use two RI/SMOM schemes, $(\not{q}, \not{q})$ and (γ_μ, γ_μ) ⇒ consistent results
- RI/(S)MOM result compatible with previous RI/(S)MOM calculation

Study suggests RI/MOM suffers from large IR artifacts ⇒ discrepancy

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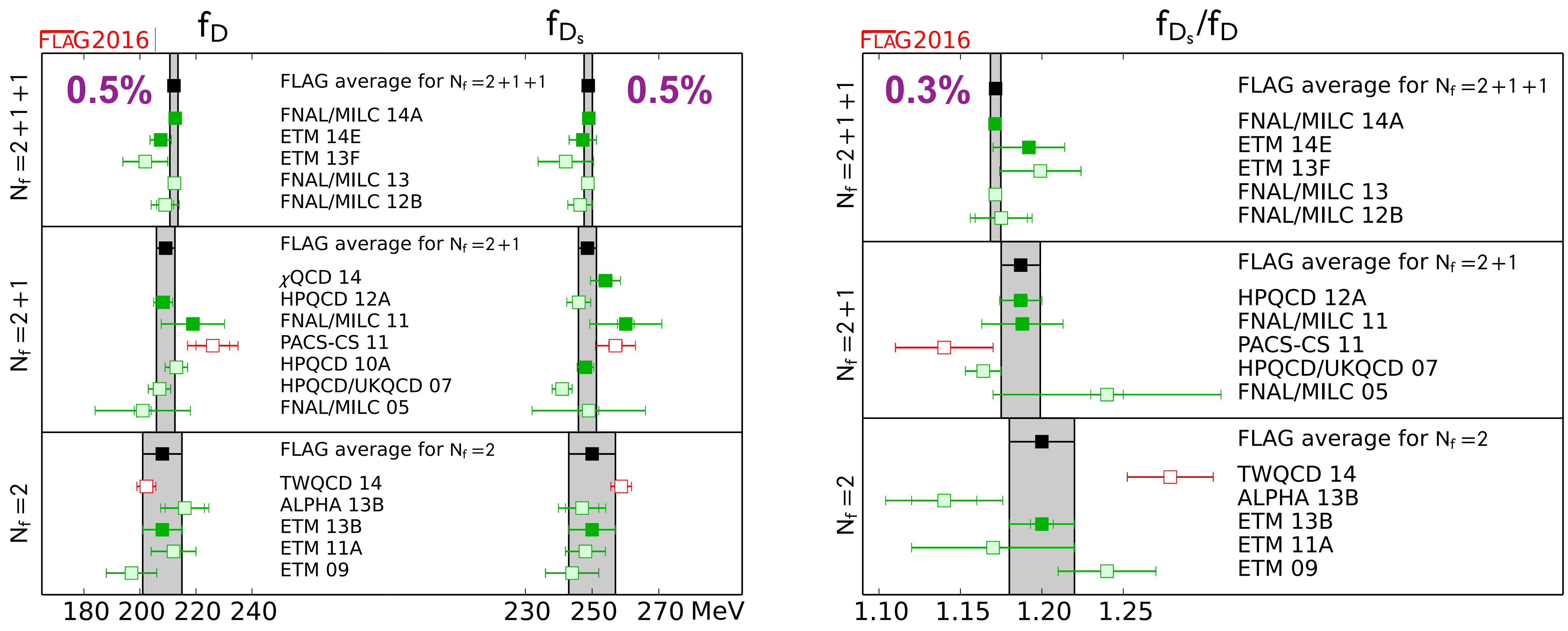
FLAG review of D-meson quantities

S. Aoki et al [FLAG-3 review, arXiv:1607.00299, EPJC 17, web update]

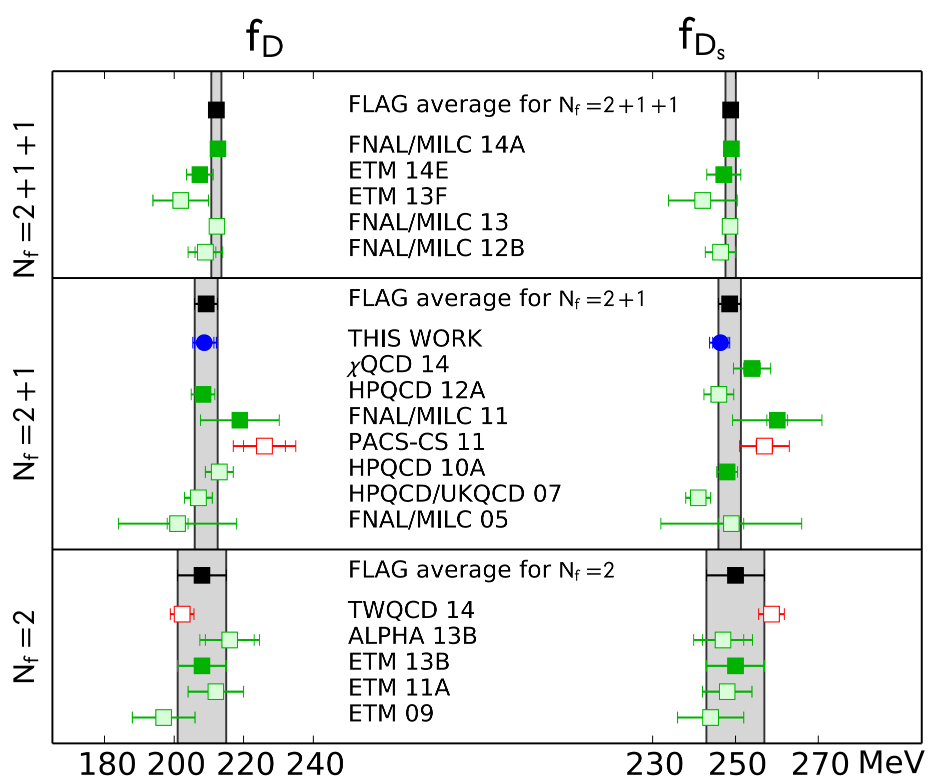


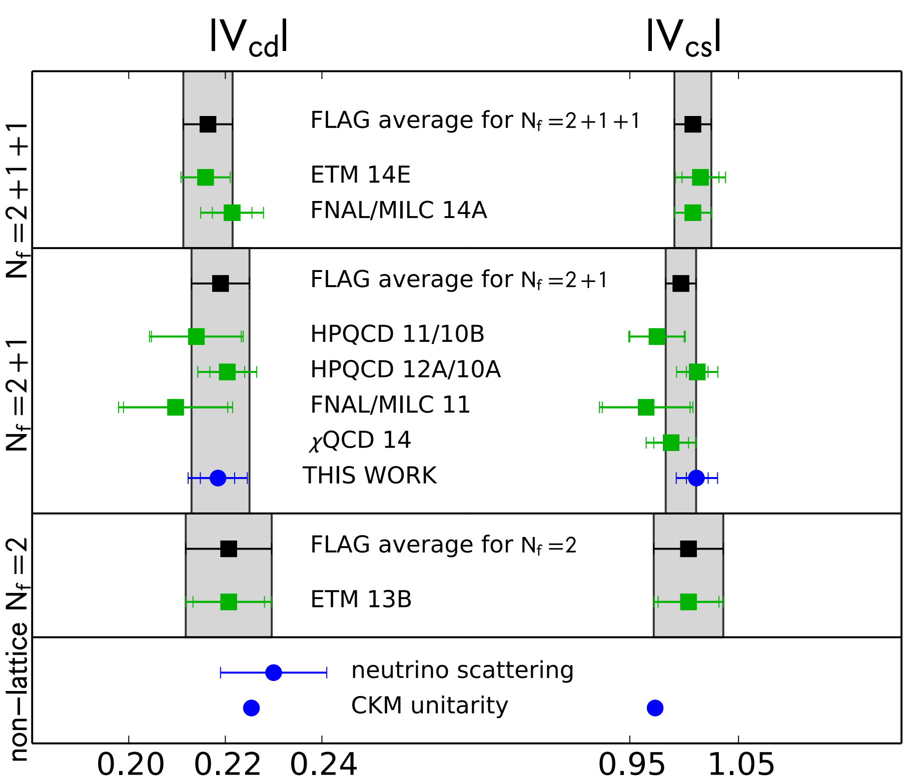
small errors due to

- physical mass ensembles
- improved actions (small discretization errors)
- small lattice spacings
- NPR or no renormalization

D-meson decay constant update

S. Aoki et al [FLAG-3 review, arXiv:1607.00299, EPJC 17, web update]





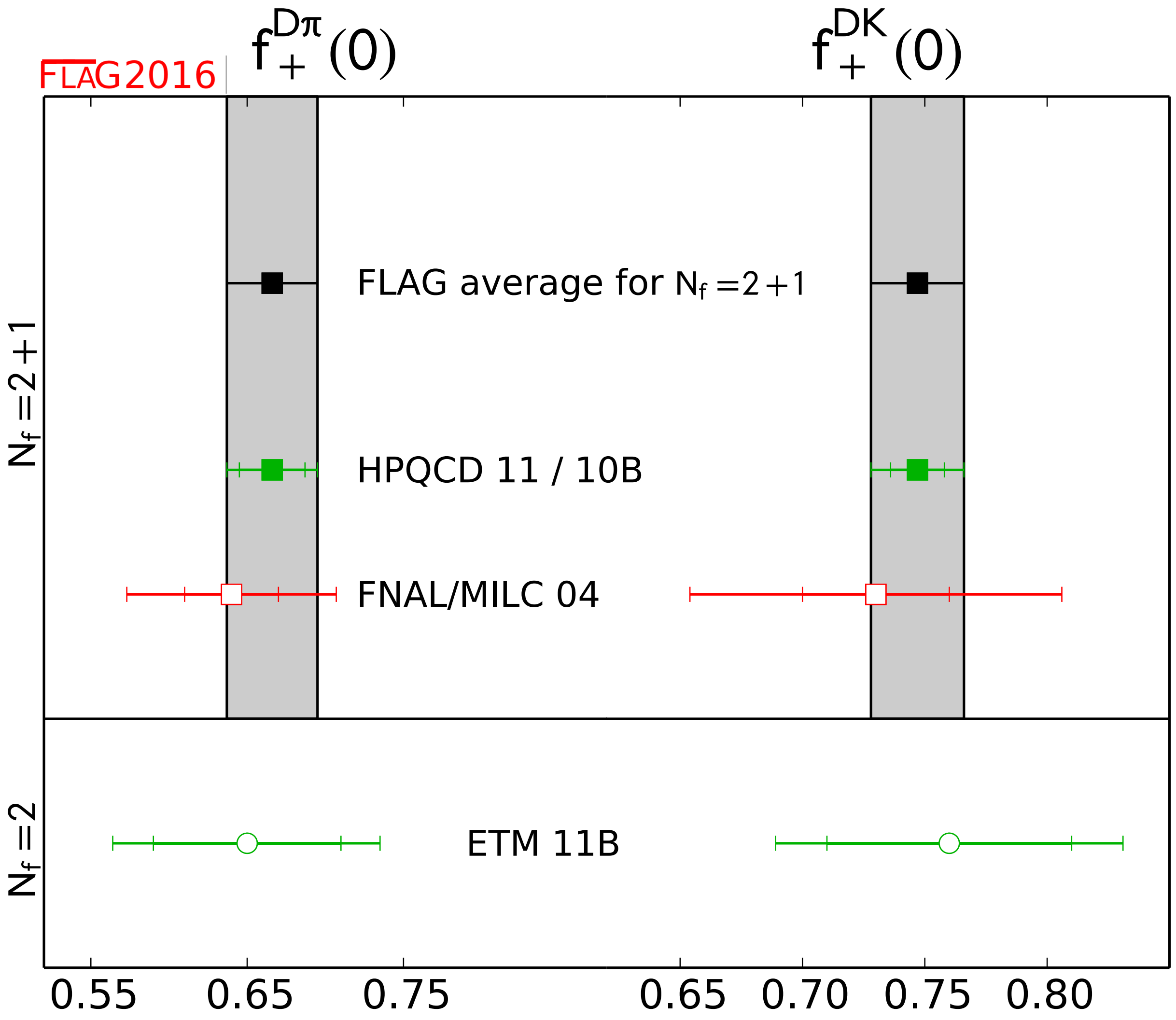
Plots inspired by FLAG III [arXiv:1607.00299]

- $N_f = 2 + 1$ Domain Wall Fermions
- **2 ensembles with physical pion masses**
- 3 Lattice spacings

- ongoing work by:
  CLS [Eckert @ Lattice 2017]
  FNAL/MILC
  …

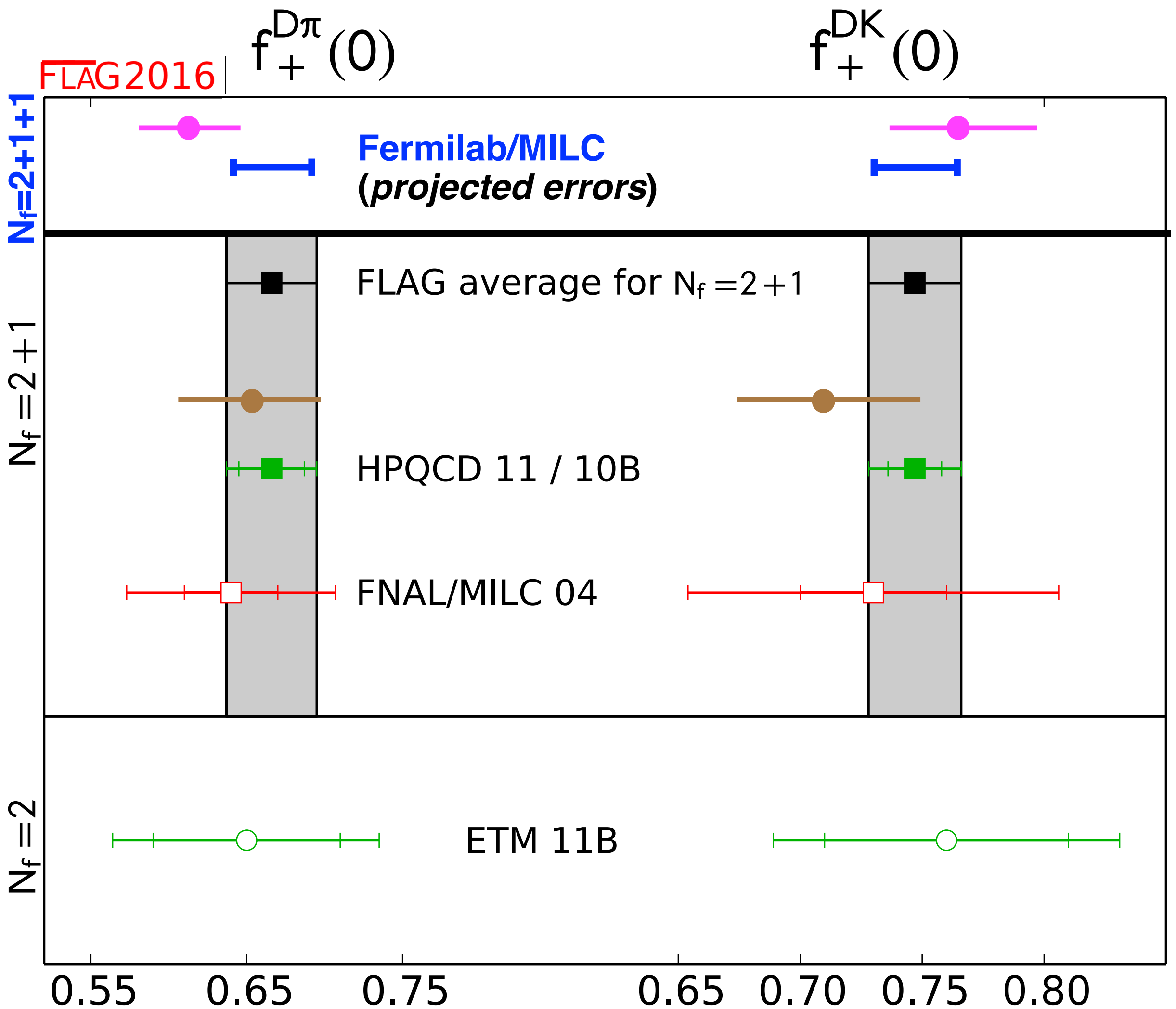
FLAG review of D-meson quantities

S. Aoki et al [FLAG-3 review, arXiv:1607.00299, EPJC 17, web update]



D semileptonic form factor update

adapted from S. Aoki et al [arXiv:1607.00299]



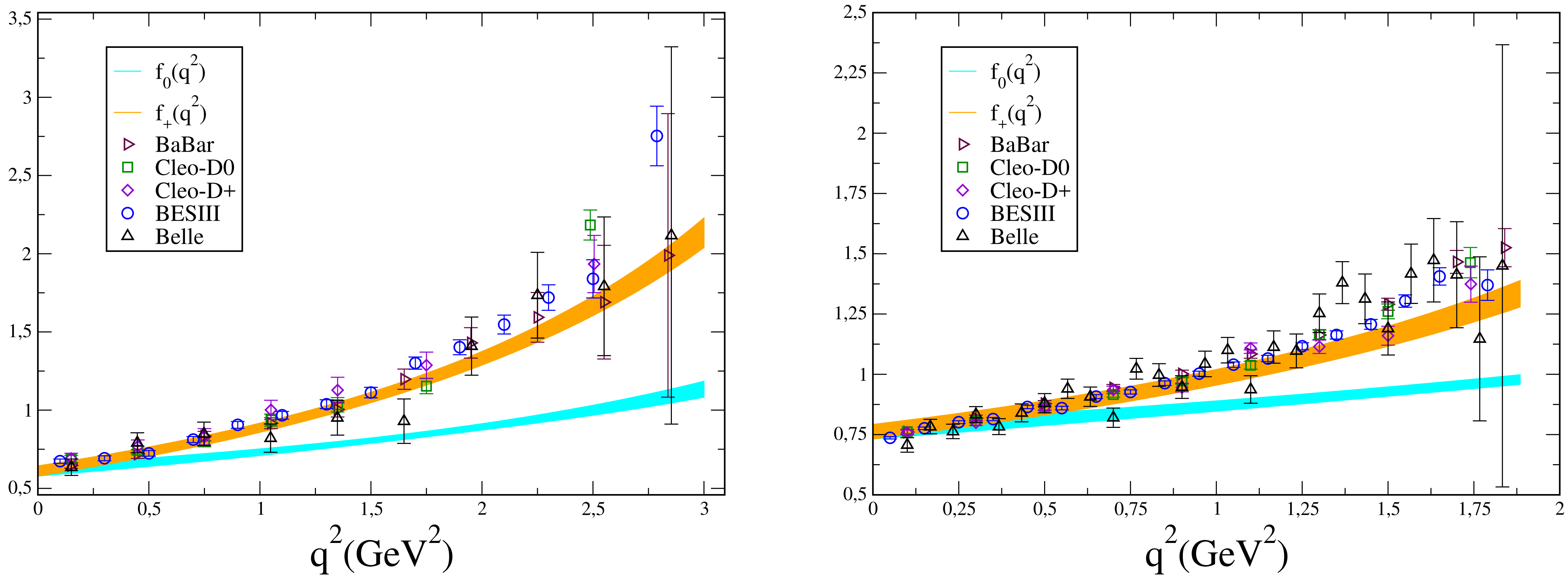
new results:

- ETM 2017 [arXiv:1706.03017]
  2+1+1 flavors of tmWilson
  calculate all form factors over whole q^2 range
  modified z-expansion
- JLQCD [T. Kaneko @ Lattice 2017]
  2+1 flavors of DW fermions
  form factors over whole q^2 range
  combined chiral-continuum extrapolaton
  systematic error analysis in progress
- FNAL/MILC [S. Gottlieb @ Lattice 2016]
  no central values (yet)
  2+1+1 flavors of HISQ
  physical mass ensembles
  calculate directly at zero q^2

☆ Note: First LQCD calculation of $\Lambda_c \to \Lambda \ell \nu$ form factors. Combine with BES-III Br measurement to determine $|V_{cs}|$ [Meinel, arXiv:1611.09696, 2017 PRL].

D semileptonic form factor update

ETM 2017 [V. Lubicz et al, arXiv:1706.03017]



- 2+1+1 flavors of tmWilson
- calculate f_+, f_0 over whole q^2 range
- modified z-expansion
- correct for hypercubic discretization effects

Three different values of the lattice spacing: 0.06 *fm* ÷ 0.09 *fm*

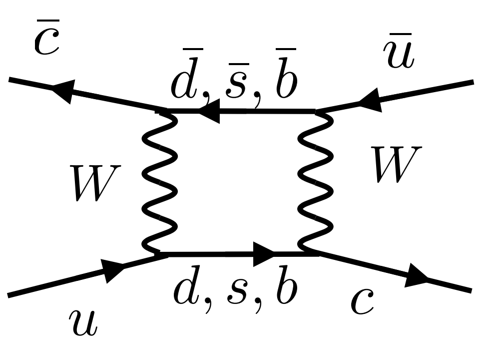
Different volumes: 2 *fm* ÷ 3 *fm*

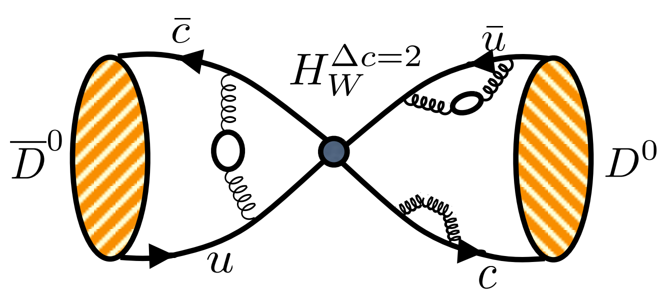
Pion masses in range 210 ÷ 440 MeV

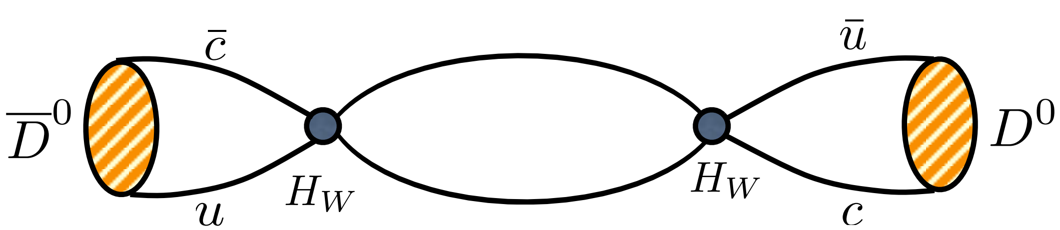
Neutral D-meson mixing introduction

Mixing between neutral D mesons yields off-diagonal elements in the time evolution matrices:

$$M_{12} - \frac{i}{2}\Gamma_{12} \;\propto\; \langle D^0|\mathcal{H}_W^{\Delta C=2}|\bar{D}^0\rangle \;+\; \sum_n \frac{\langle D^0|\mathcal{H}_W^{\Delta C=1}|n\rangle\langle n|\mathcal{H}_W^{\Delta C=1}|\bar{D}^0\rangle}{M_D - E_n + i\epsilon}$$







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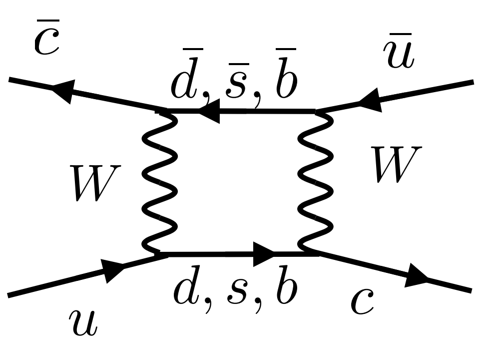
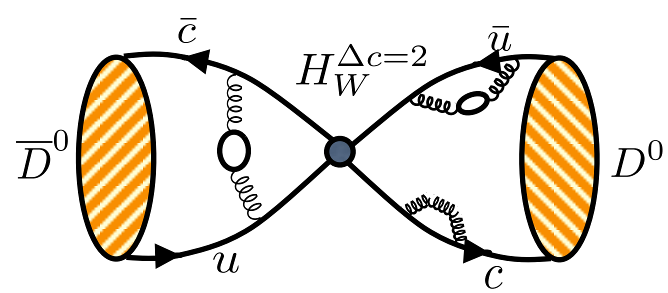
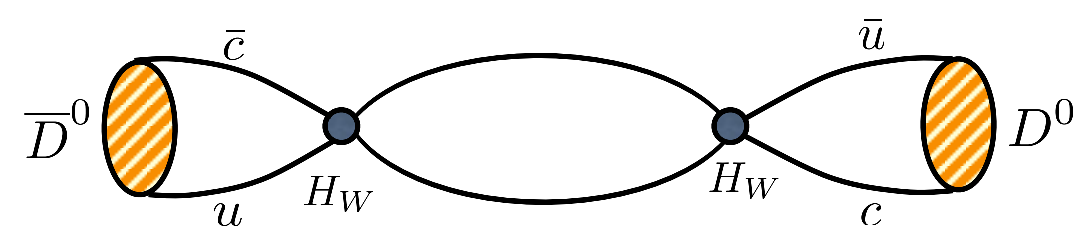






short distance

long distance

"Simple"

- can use the same methods as for B mixing
- BSM theories with heavy new particles can contribute here

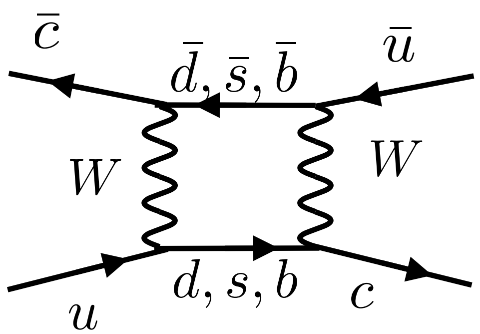
"Hard"

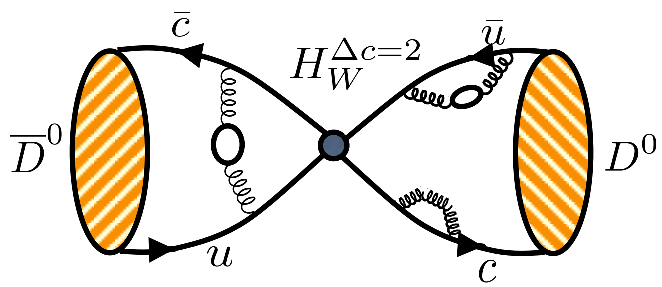
- large contribution to SM amplitude
- intermediate state includes multiple (>2) hadrons

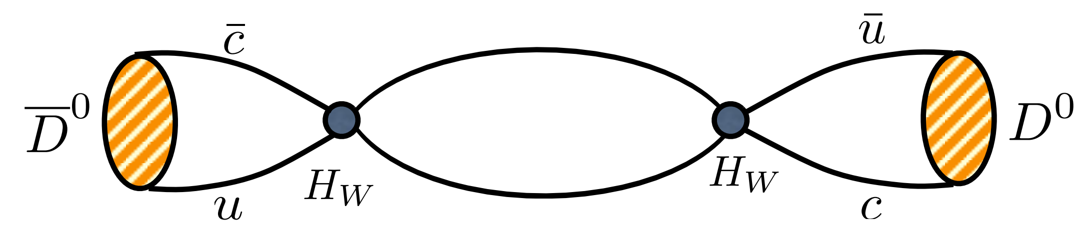
Neutral D-meson mixing introduction

Mixing between neutral D mesons yields off-diagonal elements in the time evolution matrices:

$$M_{12} - \frac{i}{2}\Gamma_{12} \;\propto\; \langle D^0|\mathcal{H}_W^{\Delta C=2}|\bar{D}^0\rangle \;+\; \sum_n \frac{\langle D^0|\mathcal{H}_W^{\Delta C=1}|n\rangle\langle n|\mathcal{H}_W^{\Delta C=1}|\bar{D}^0\rangle}{M_D - E_n + i\epsilon}$$







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Neutral D-meson mixing introduction

Relate M_{12} and Γ_{12} to $\Delta M \equiv M_1 - M_2$ and $\Delta\Gamma \equiv \Gamma_1 - \Gamma_2$.

Experimental averages [HFLAV 2016]:

$$x = \frac{\Delta M}{\Gamma} = 0.41^{+0.14\%}_{-0.15\%} \qquad y = \frac{\Delta\Gamma}{\Gamma} = 0.61 \pm 0.07\%$$

Fit to the data yields [HFLAV 2016]: $\phi_{12} \equiv \arg \frac{M_{12}}{\Gamma_{12}} = -0.17^{\text{o}} \pm 1.8^{\text{o}}$

SM predictions of x, y are affected by the long distance contributions.
Estimates from dispersion relations and HQE:

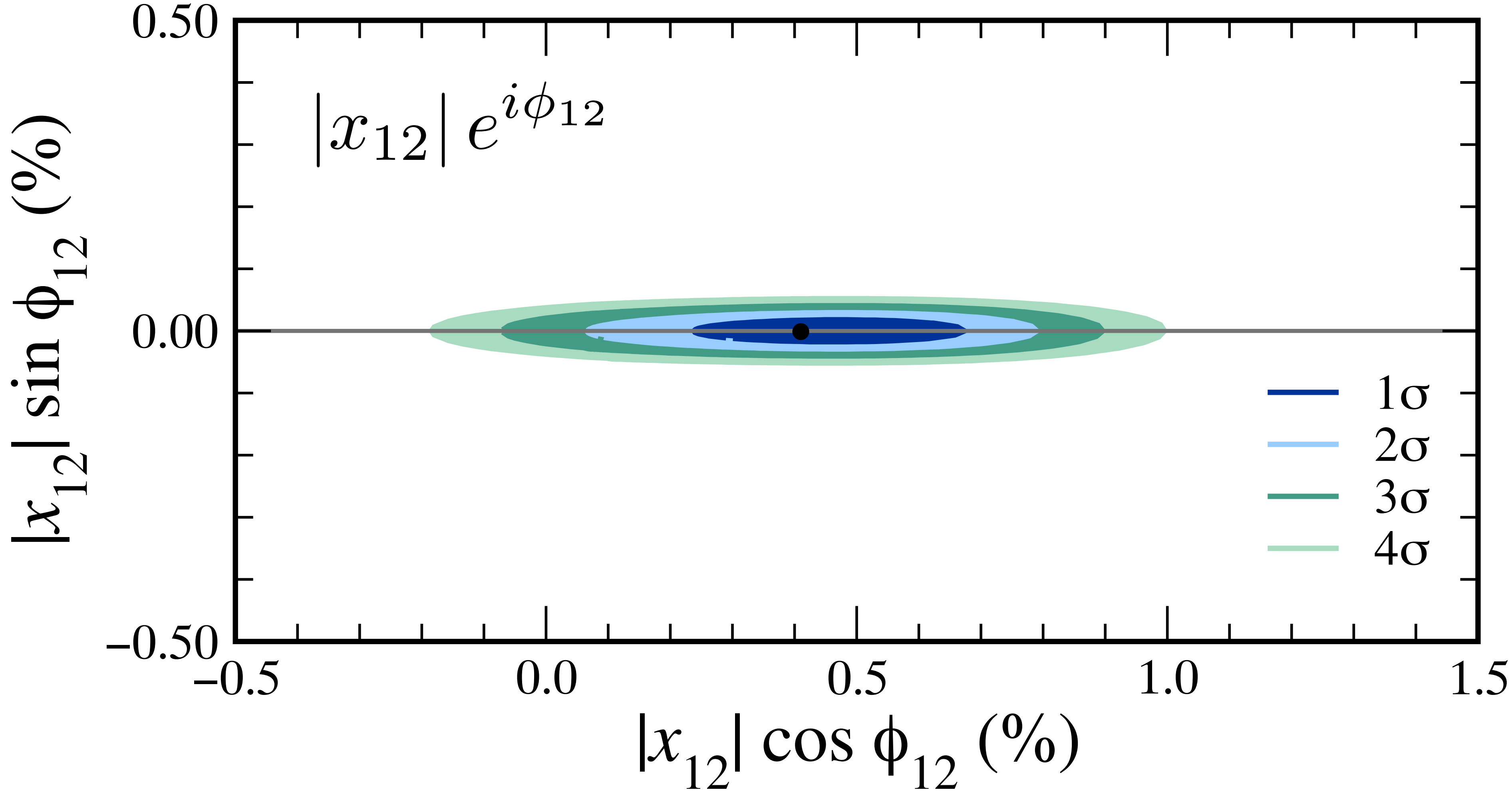
$$x \sim 10^{-3} - 10^{-2} \qquad y \sim 10^{-2}$$

But for ϕ_{12} the SM predicts: $\phi_{12} \sim 0$

➠ Use ϕ_{12} to constrain New Physics models.

Neutral D-meson mixing introduction

[HFLAV 2016, Alan Schwartz, private communication]



$$x_{12} \equiv 2\frac{M_{12}}{\Gamma}$$

If NP modifies only $\mathcal{H}^{\Delta C=2}$ then $M_{12}^{\rm NP} \sim \sum_{i=1}^{5} C_i^{\rm NP}(\mu) \langle \mathcal{O}_i \rangle(\mu)$

D mixing matrix elements in comparison

FNAL/MILC [A. Bazavov et al, arXiv:1706.04622]

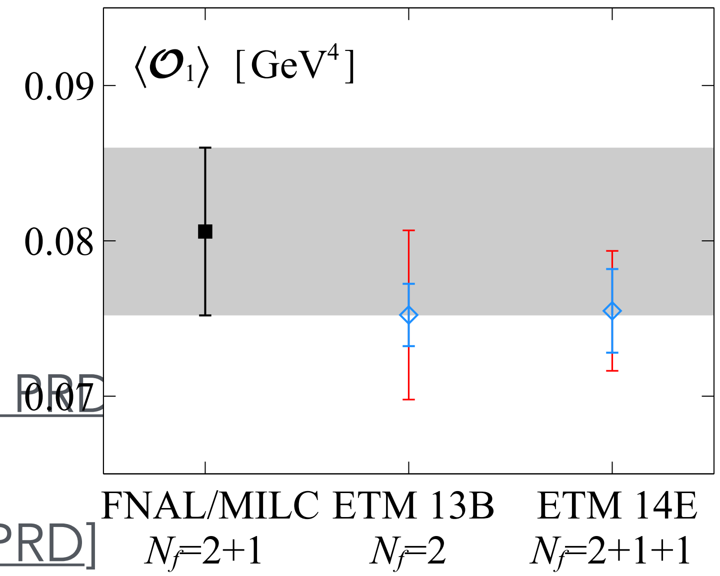
ETM:

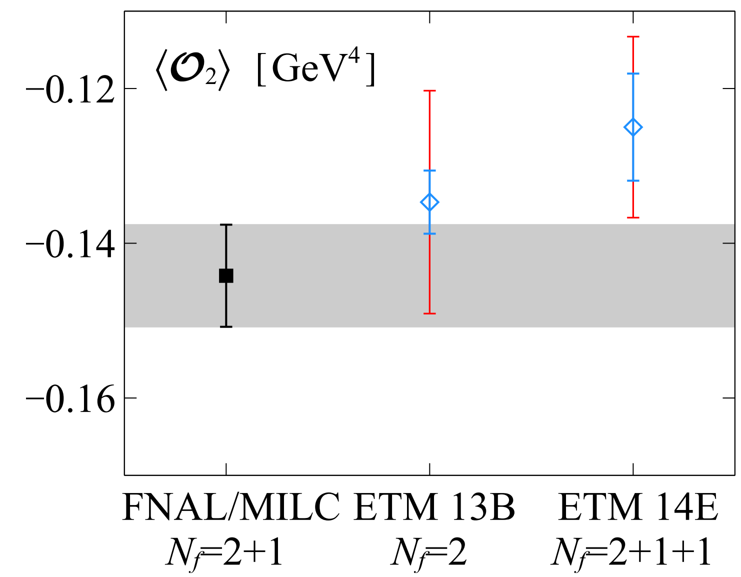
$n_f = 2+1+1$

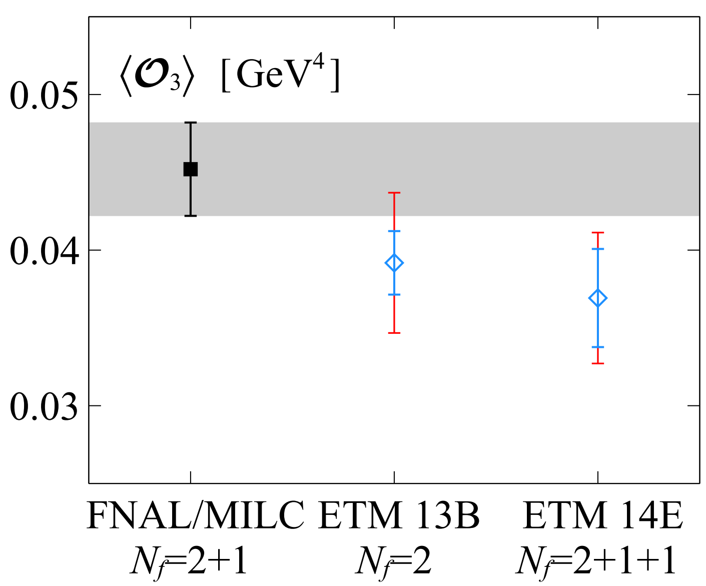
[arXiv:1505.06639, 2015 PRD]

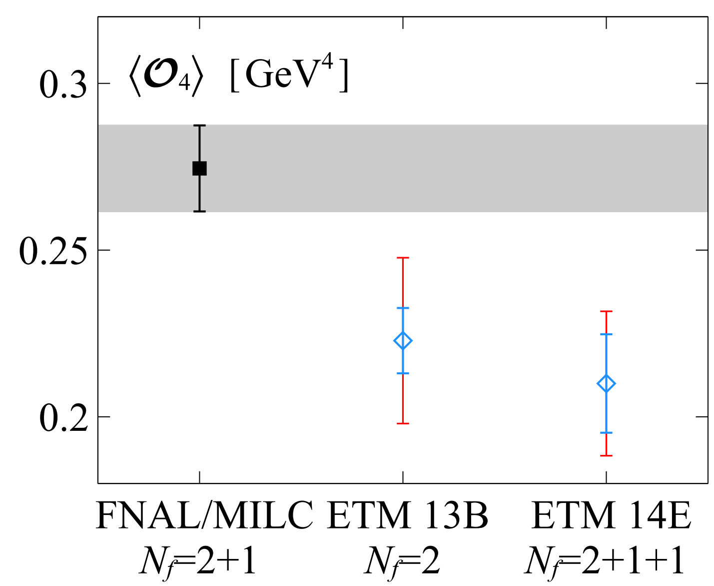
$n_f = 2$

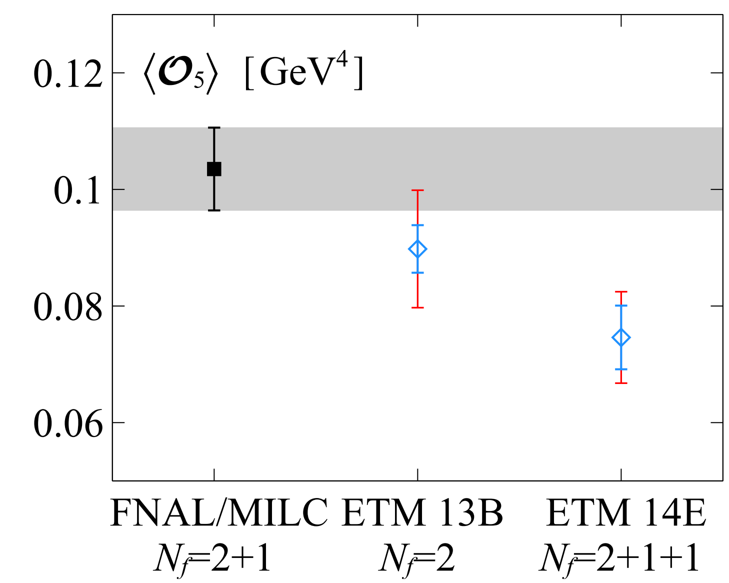
[arXiv:1403.7302, 2014 PRD]











- Conversion of ETM bag parameters to matrix elements requires as inputs the quark masses and D-meson decay constants.
- Here use m_q's and f_D's from ETM [2010 PRD, 2014 JHEP, 2014 NPB, 2015 PRD]

D mixing matrix elements in comparison

FNAL/MILC [A. Bazavov et al, arXiv:1706.04622]

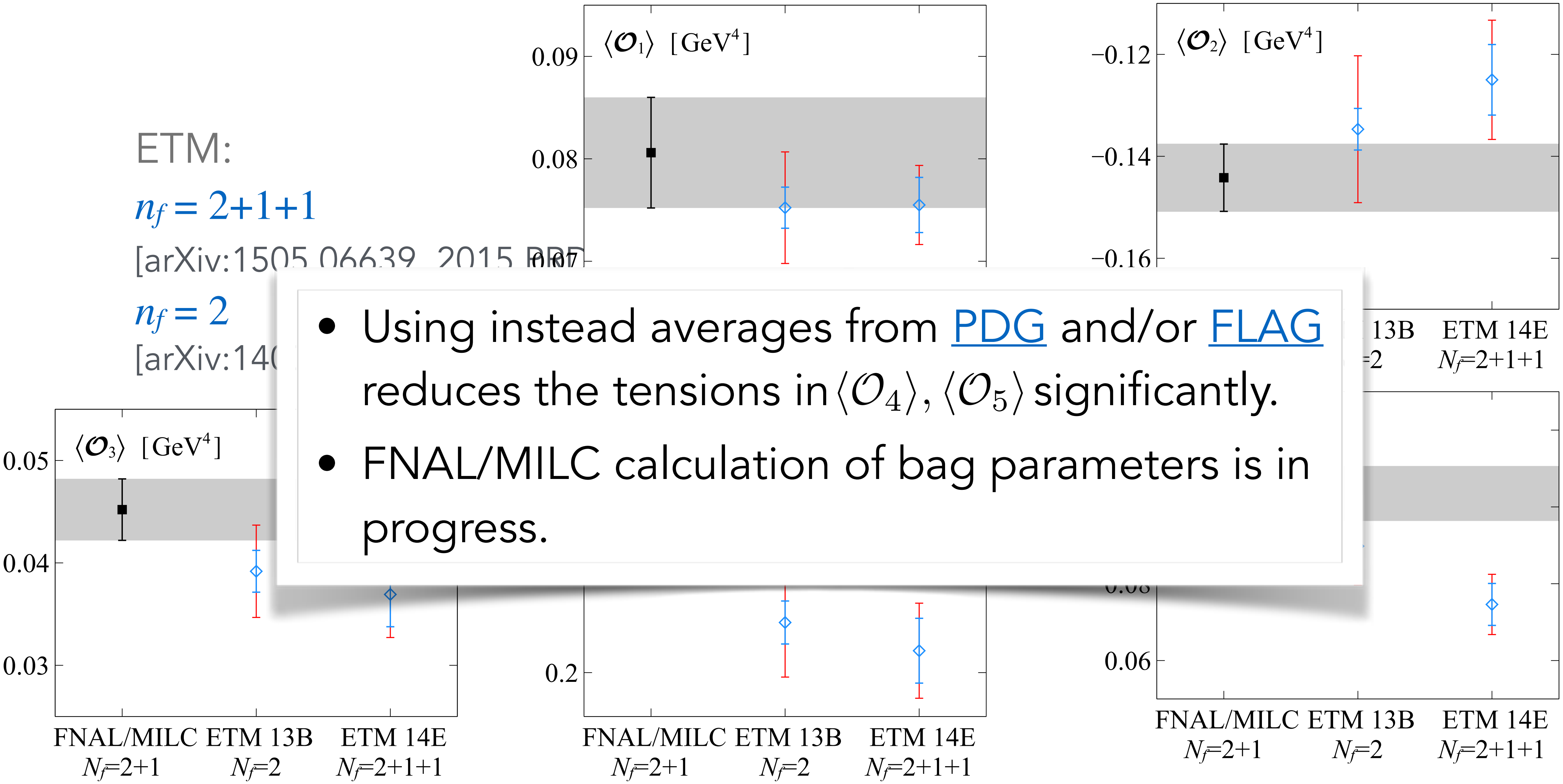
ETM:

$n_f = 2+1+1$

[arXiv:1505.06639, 2015 PRD

$n_f = 2$

[arXiv:14(



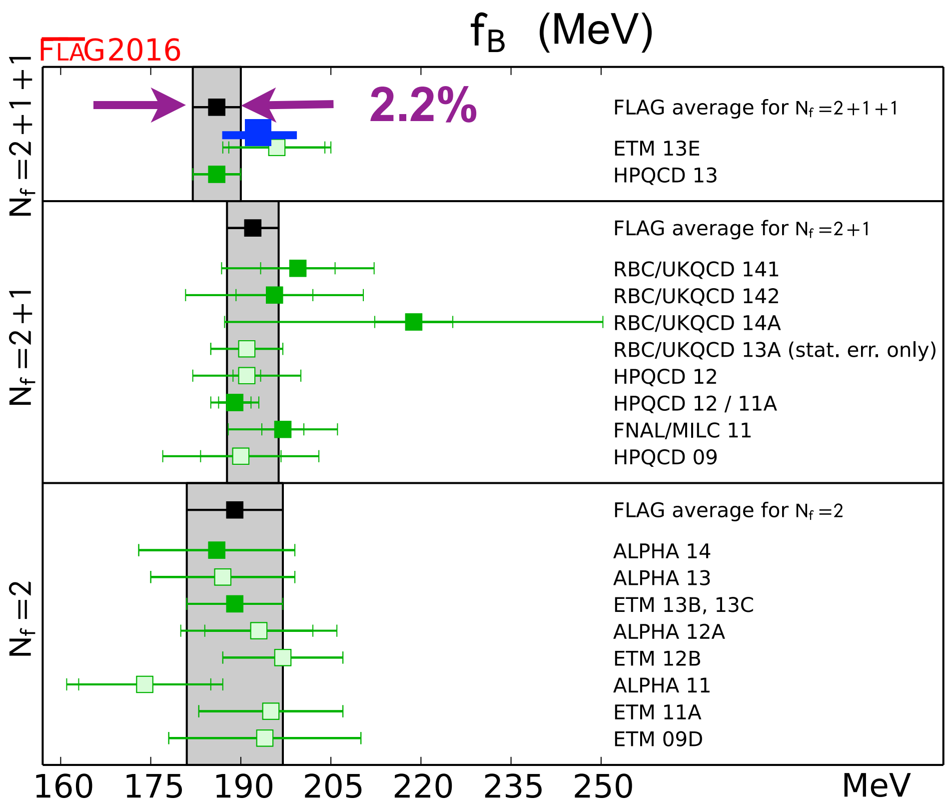
- Using instead averages from PDG and/or FLAG reduces the tensions in $\langle \mathcal{O}_4 \rangle, \langle \mathcal{O}_5 \rangle$ significantly.
- FNAL/MILC calculation of bag parameters is in progress.

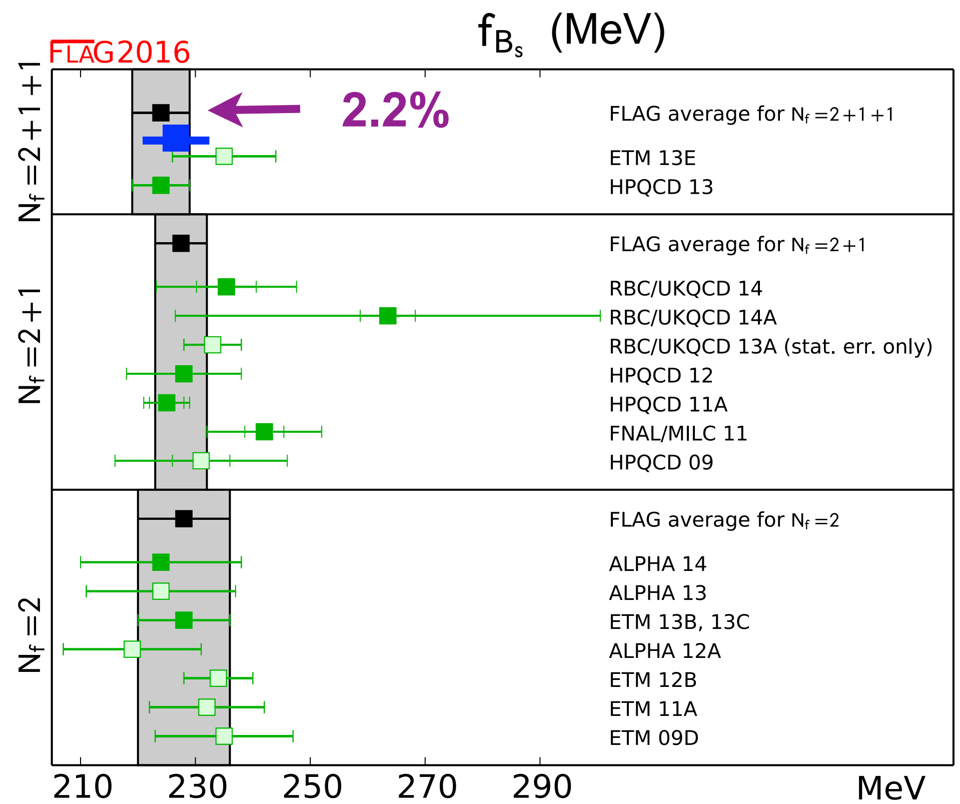
- Conversion of ETM bag parameters to matrix elements requires as inputs the quark masses and D-meson decay constants.
- Here use m_q's and f_D's from ETM [2010 PRD, 2014 JHEP, 2014 NPB, 2015 PRD]

Outline

- Motivation and Introduction
- Leptonic, Semileptonic Decays, Mixing
  - ⋆ Kaons
  - ⋆ D mesons
  - ⋆ B mesons
- Implications
  - ⋆ CKM determinations
- Summary

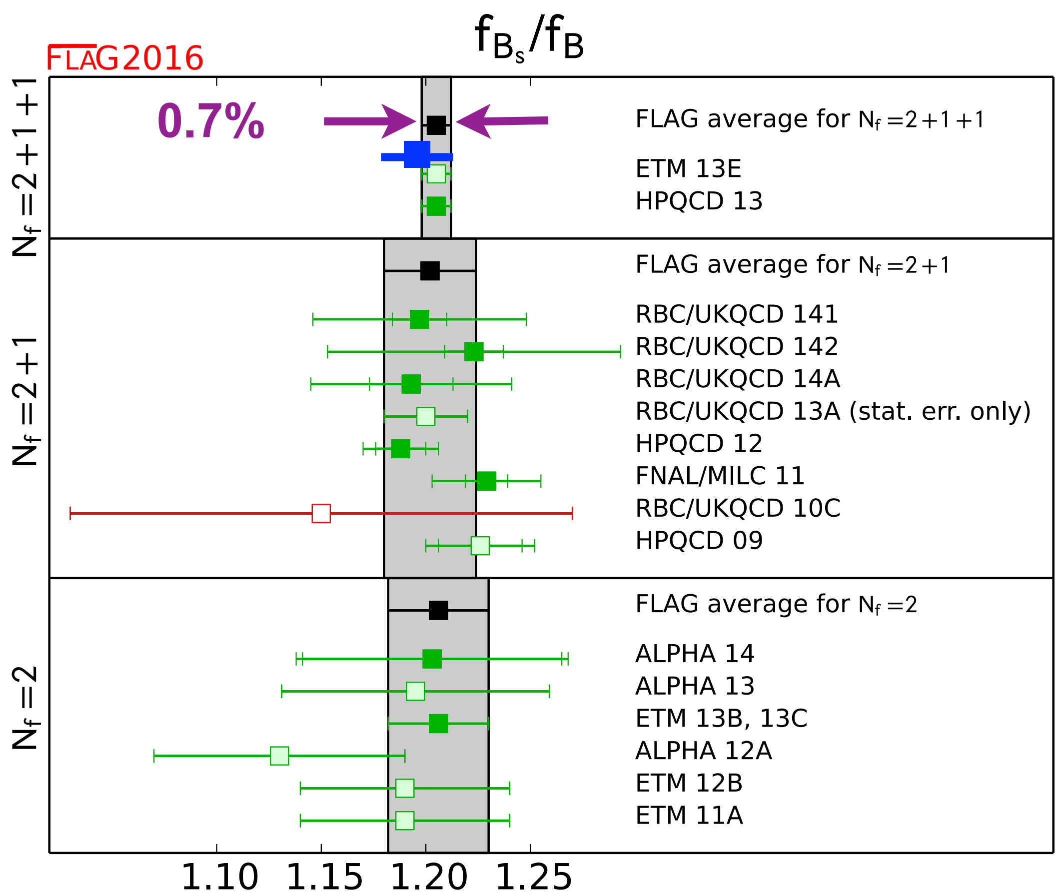
B decay constant summary





S. Aoki et al [FLAG-3 review, arXiv:1607.00299, EPJC 17, web update]

status
end 2015

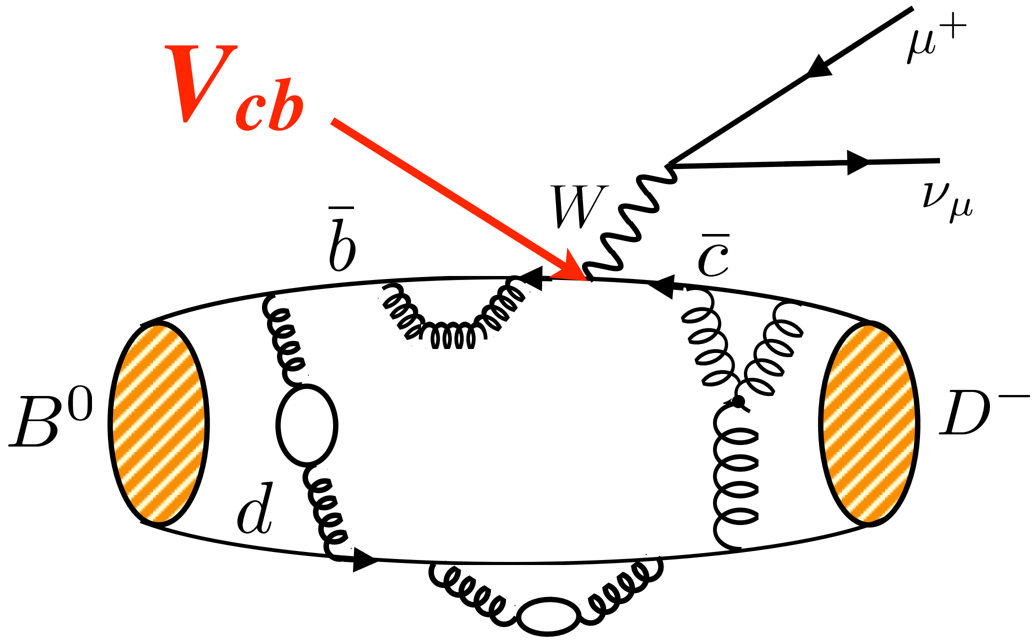


- new results:
  ETM [arXiv:1603.04306, 2016 PRD
  FNAL/MILC [arXiv:1712.09262],

- ongoing work by RBC/UKQCD, …

➠ errors on f_B, f_{Bs} to ≲ 1%

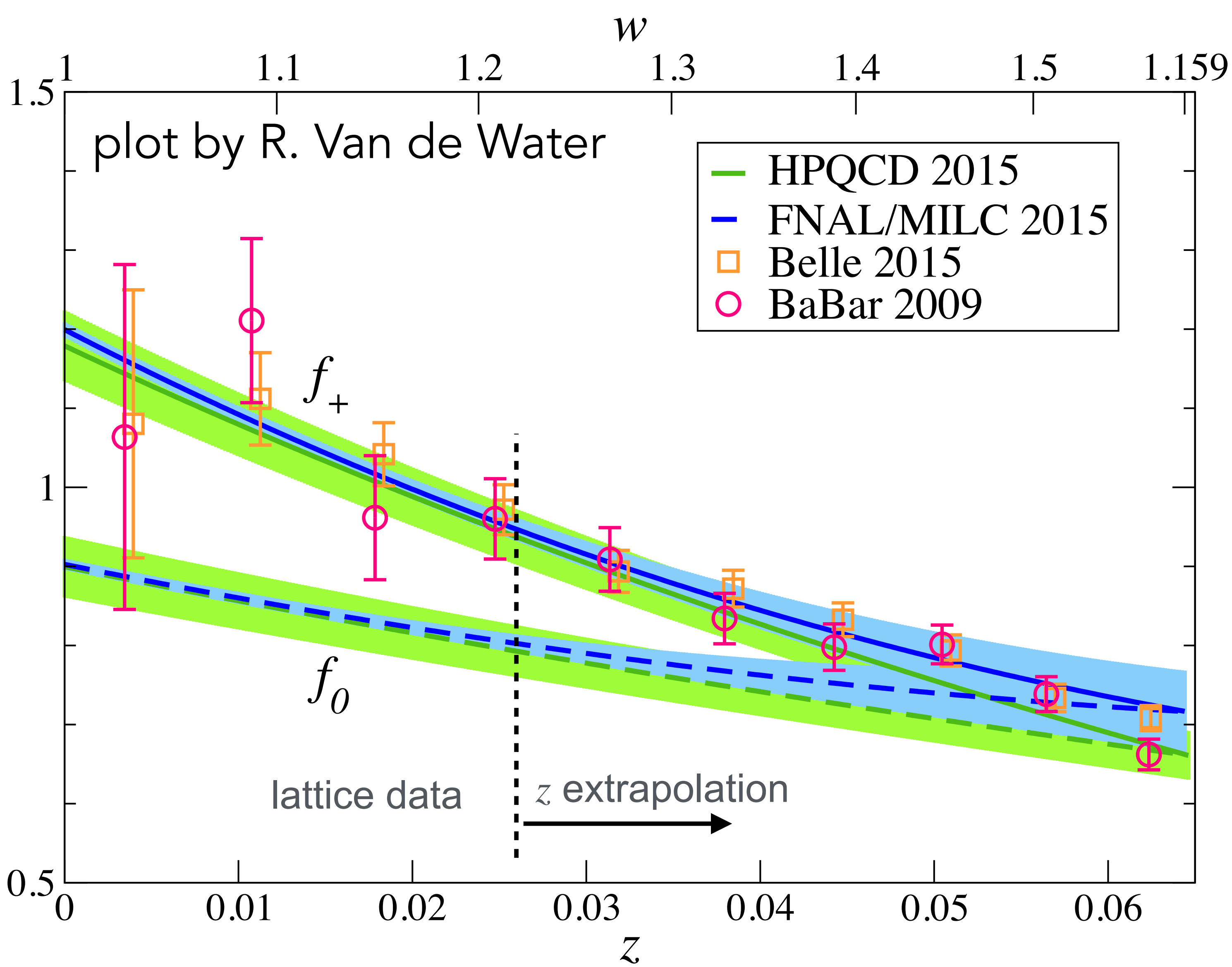
Form factors for $B \to D\,\ell\nu, \quad (\ell = e, \mu, \tau)$



$$\frac{d\Gamma(B \to D\,\mu\nu)}{dq^2} = (\text{known}) \times |V_{cb}|^2 \times f_+^2(q^2)$$

- ⋆ calculate the form factors in the low recoil energy (high q^2) range.
- ⋆ use z-expansion for model-independent parameterization of q^2 dependence.
- ⋆ calculate both form factors, $f_+(q^2), f_0(q^2)$.
- ⋆ for $f_+(q^2)$ compare shape between experiment and lattice.
- ⋆ Tensor form factor(s) are calculable in LQCD using the same methods.

Form factors for $B \to D\,\ell\nu, \quad (\ell = e, \mu, \tau)$



HPQCD [arXiv:1505.03925, PRD 2015, err]

FNAL/MILC [arXiv:1503.07237, PRD 2015]

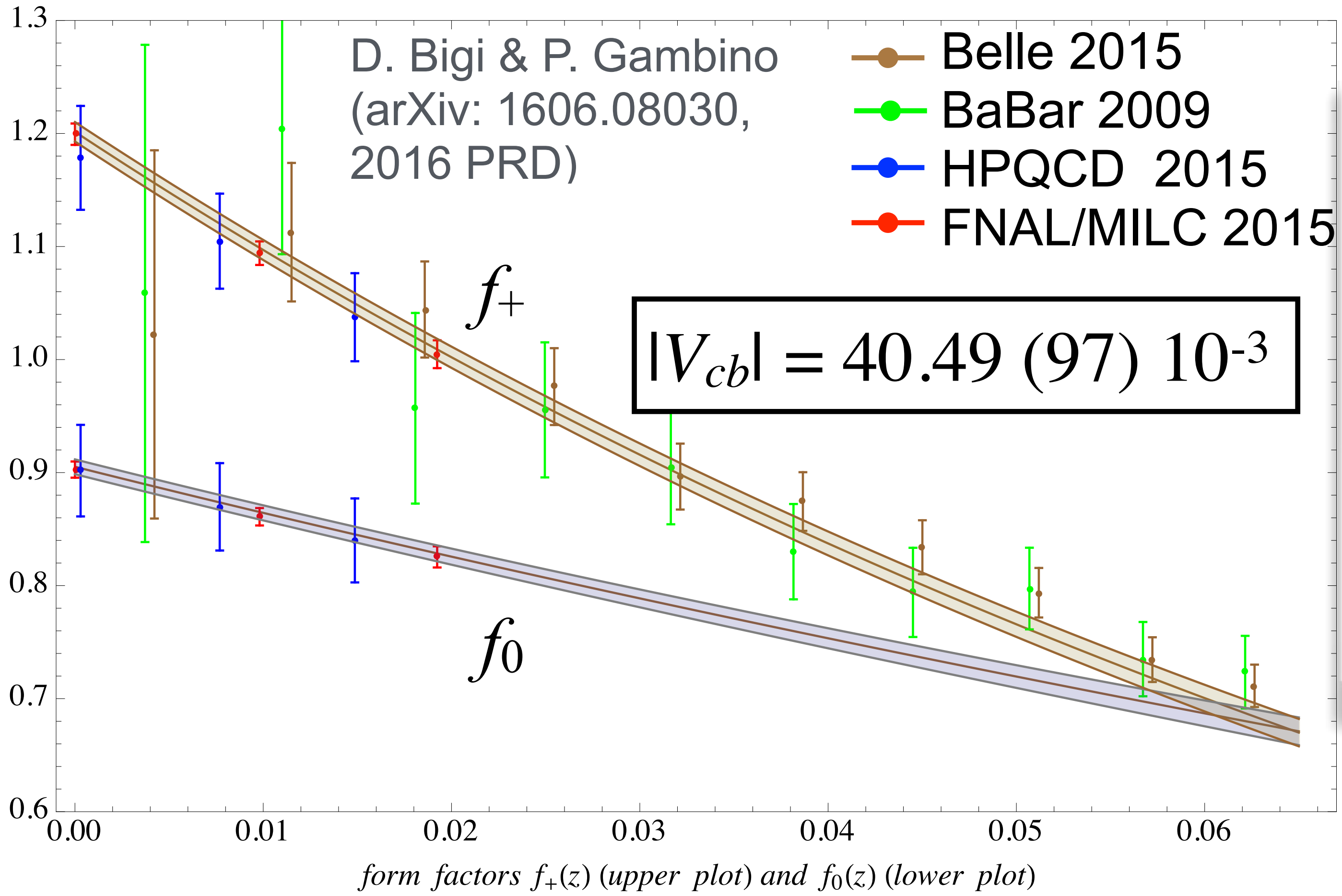
- Two LQCD calculations (FNAL/MILC, HPQCD)
- LQCD form factor uncertainties (~1.2%) smaller than experiment.

- LQCD form factors can be used to calculate the CKM free ratio:

$$R(D) \equiv \frac{\mathcal{B}(B \to D\tau\nu_\tau)}{\mathcal{B}(B \to D\ell\nu)}$$

$B \to D \ell \nu$ & $|V_{cb}|$

- combine LQCD form factors with experiment, using the BGL [Boyd, Grinstein, Lebed, hep-ph/9508211, 1996 NPB] parameterization:



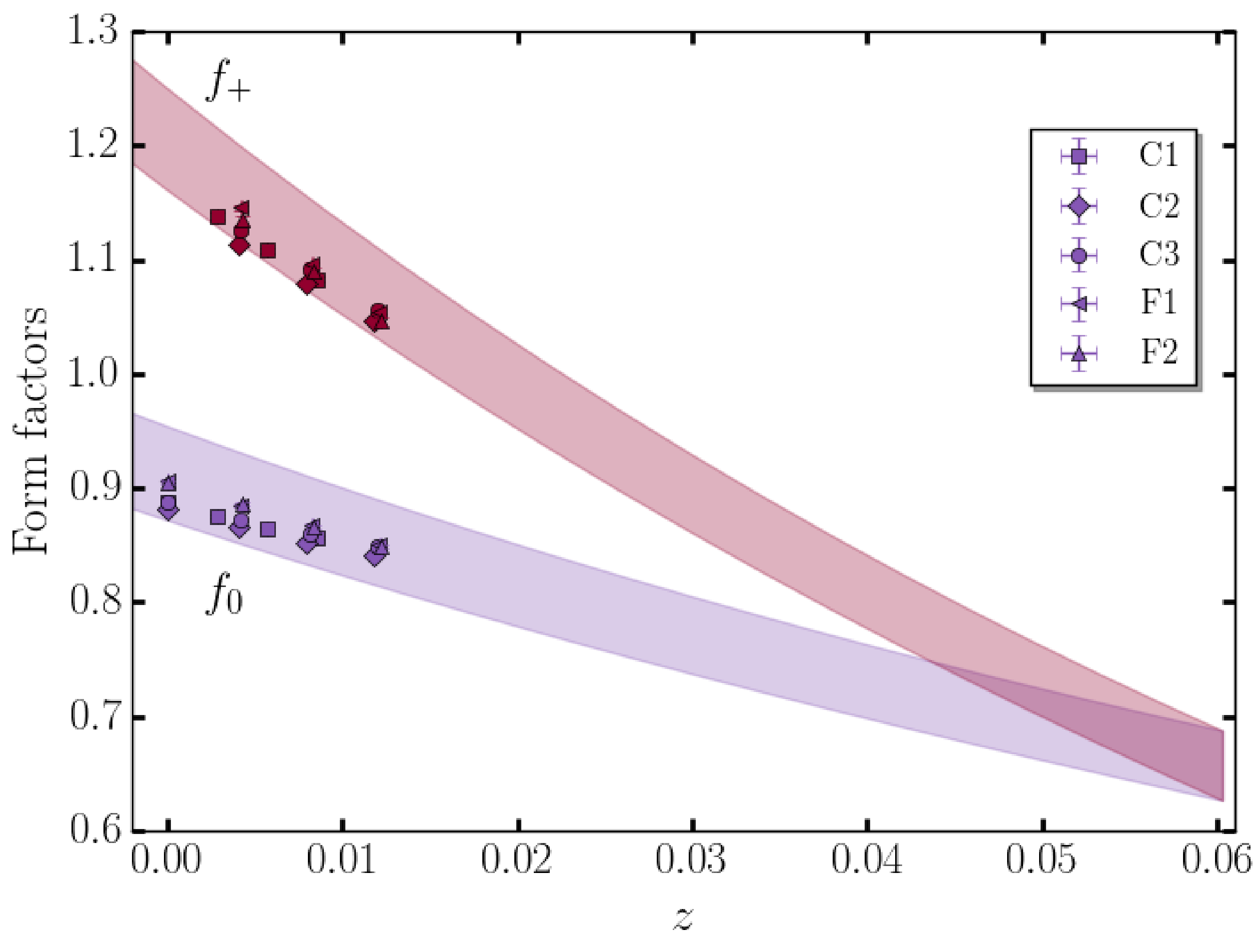
★ The form factors obtained from the combined exp/lattice fit are well determined over entire recoil range.

★ Can be used for an improved SM prediction of $R(D)$.

- FLAG-3 [S. Aoki et al, arXiv:1607.00299, EPJC 2017] performs a similar combined fit using the BCL [Bourrely, Caprini, Lellouch, arXiv:0807.2722, PRD 09] parameterization.
- Note: First LQCD calculations of semileptonic B_c decay form factors [HPQCD, A. Lytle @ Lattice 2017; Mathur @ Lattice 2017].

$B_s \to D_s$

☆ **new** LQCD form factors from HPQCD [Monahan et al, arXiv:1703.09728, 2017 PRD]



- ★ 5 MILC asqtad (2+1) ensembles two lattice spacings
- ★ NRQCD b and HISQ charm quarks
- ★ $O(a)$ improved current matched through $O(\alpha_s, \Lambda/m_b, \alpha_s a/m_b)$
- ★ $R(D_s) = 0.301\ (6)$
- ★ combine results with previous HPQCD calculation of $B \to D$ form factors to obtain ratios used for f_s/f_d.

- Consistent with previous results by FNAL/MILC [Bailey et al, arXiv:1202.6346, 2012 PRD] and ETMC [M. Atoui et al, arXiv:1310.5238, 2014 EPJC].
- ongoing work by: RBC/UKQCD [O. Witzel @ Lattice 2017], FNAL/MILC, …

form factor for $B \to D^* \ell \nu$ at zero recoil and V_{cb}

$$\frac{d\Gamma(B \to D^* \ell\nu)}{d\omega} = (\text{known}) \times |V_{cb}|^2 \times (\omega^2 - 1)^{1/2} |\mathcal{F}(\omega)|^2$$

HFLAV 2016: Use CLN* expression to extrapolate exp. data to ω=1:

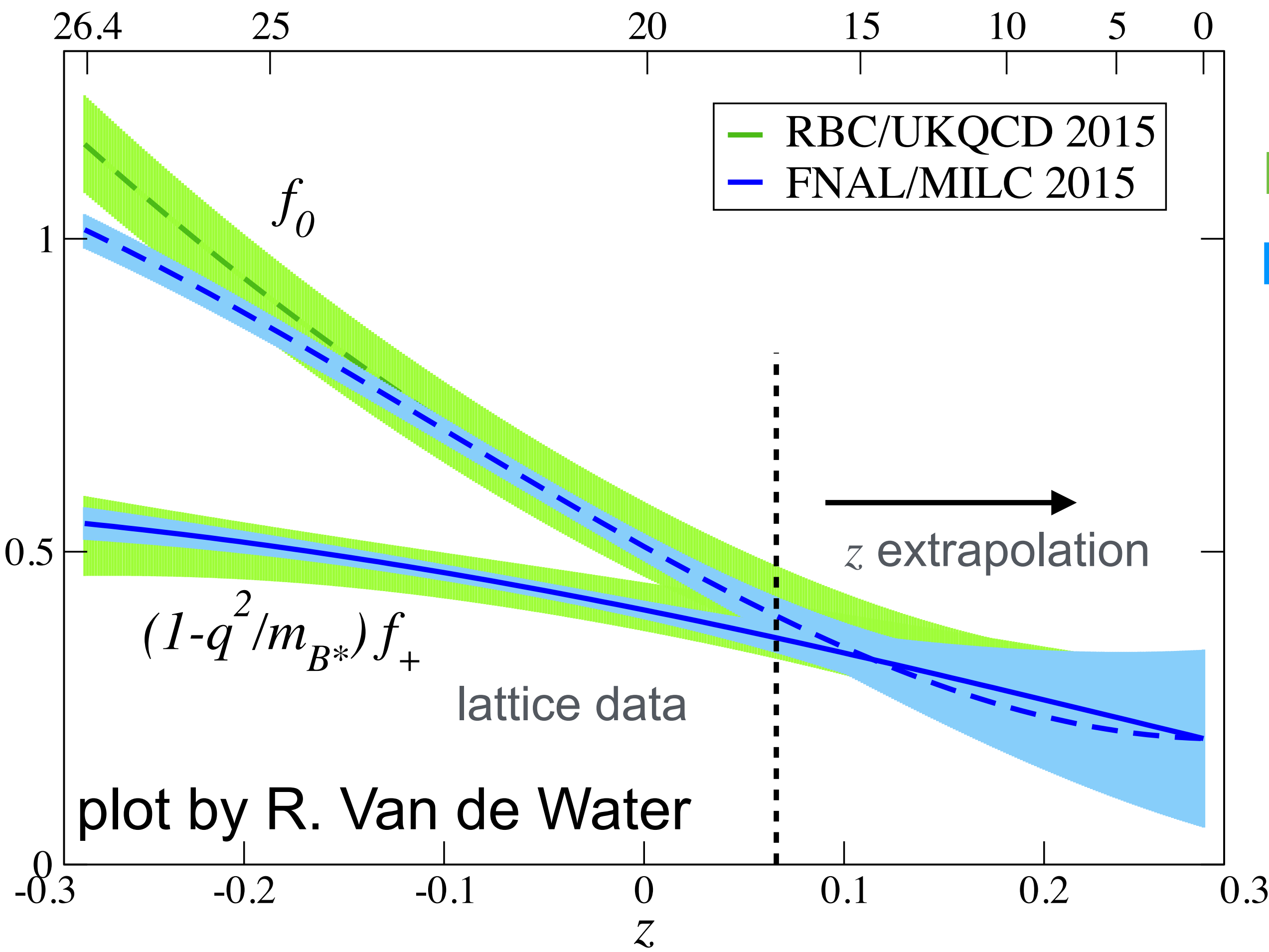
$$B \to D^* \ell\nu : \eta_{\rm EW} |V_{cb}| \mathcal{F}(1) = (35.61 \pm 0.11 \pm 0.41) \times 10^{-3}$$

combine with LQCD calculation of $\mathcal{F}(1)$:

❖ FNAL/MILC 2014 [J. Bailey et al, arXiv:1403.0635, 2014 PRD]: $\mathcal{F}(1) = 0.906(4)(12)$

- *CLN [Caprini, Lellouch, Neubert, hep-ph/9712417, NPB 98] is based on the model-independent z-expansion (just like BGL, BCL), but then add model-dependent assumptions about the parameters
  ➟ reduces the error from the extrapolation
- LQCD form factor data for $B \to D^*$ at nonzero recoil are not yet available.

form factors for $B \to \pi \ell \nu$ & V_{ub}

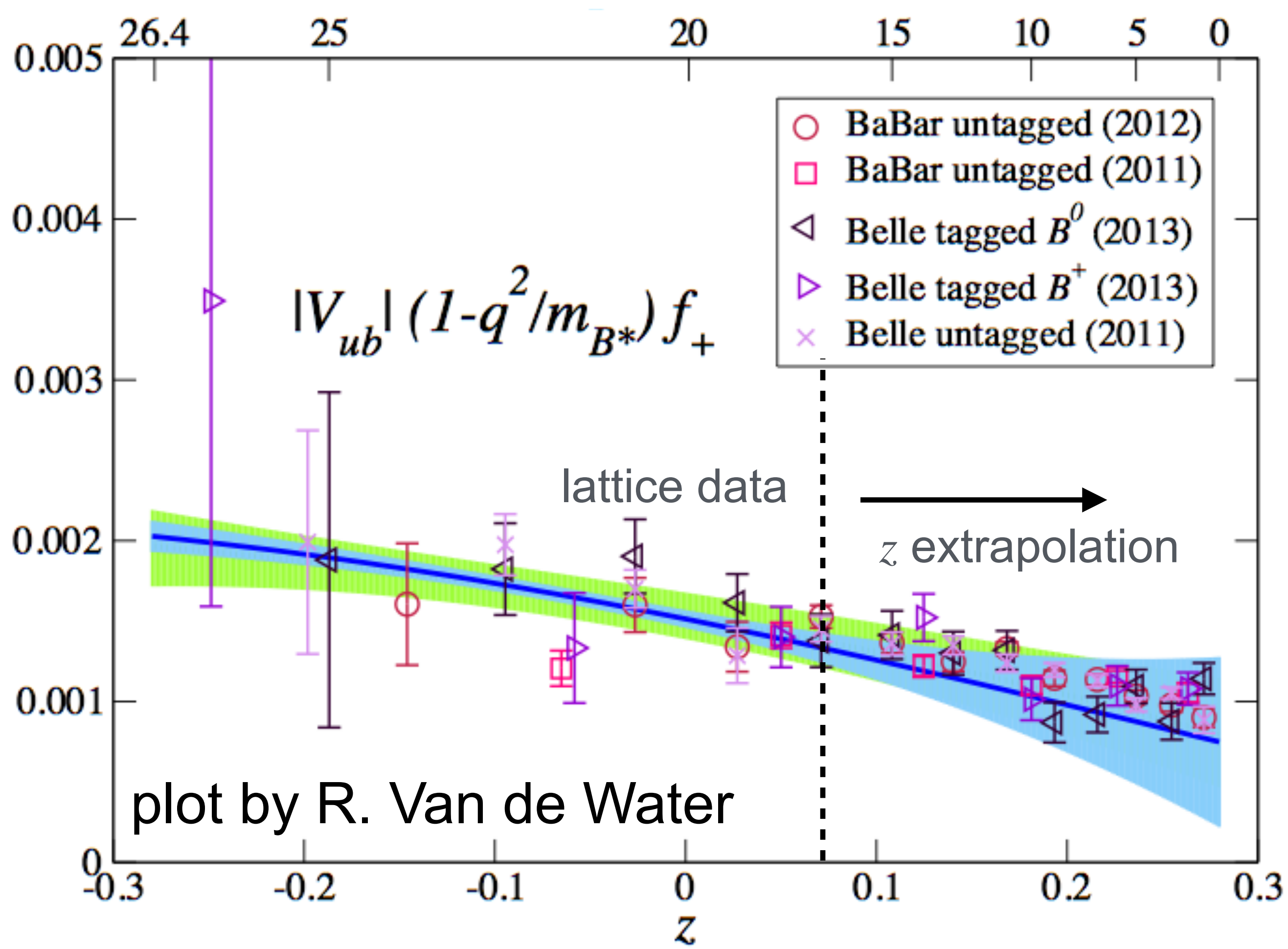


RBC/UKQCD [arXiv:1501.05373, PRD 2015]
FNAL/MILC [arXiv:1503.07839, PRD 2015]

- ☆ FNAL/MILC & RBC/UKQCD form factors are in good agreement.

- ☆ Two independent LQCD **predictions** for $B_s \to K\ell\nu$ form factors
  [HPQCD, arXiv:1406.2279, PRD 2014; RBC, arXiv:1501.05373, PRD 2015]
- ☆ ongoing work by:
  FNAL/MILC [S. Gottlieb, A. Kronfeld @ Lattice 2017]
  RBC [O. Witzel @ Lattice 2017]
  ALPHA [M. Koren @ Lattice 2017]

form factors for $B \to \pi \ell \nu$ & V_{ub}



RBC/UKQCD [arXiv:1501.05373, PRD 2015]

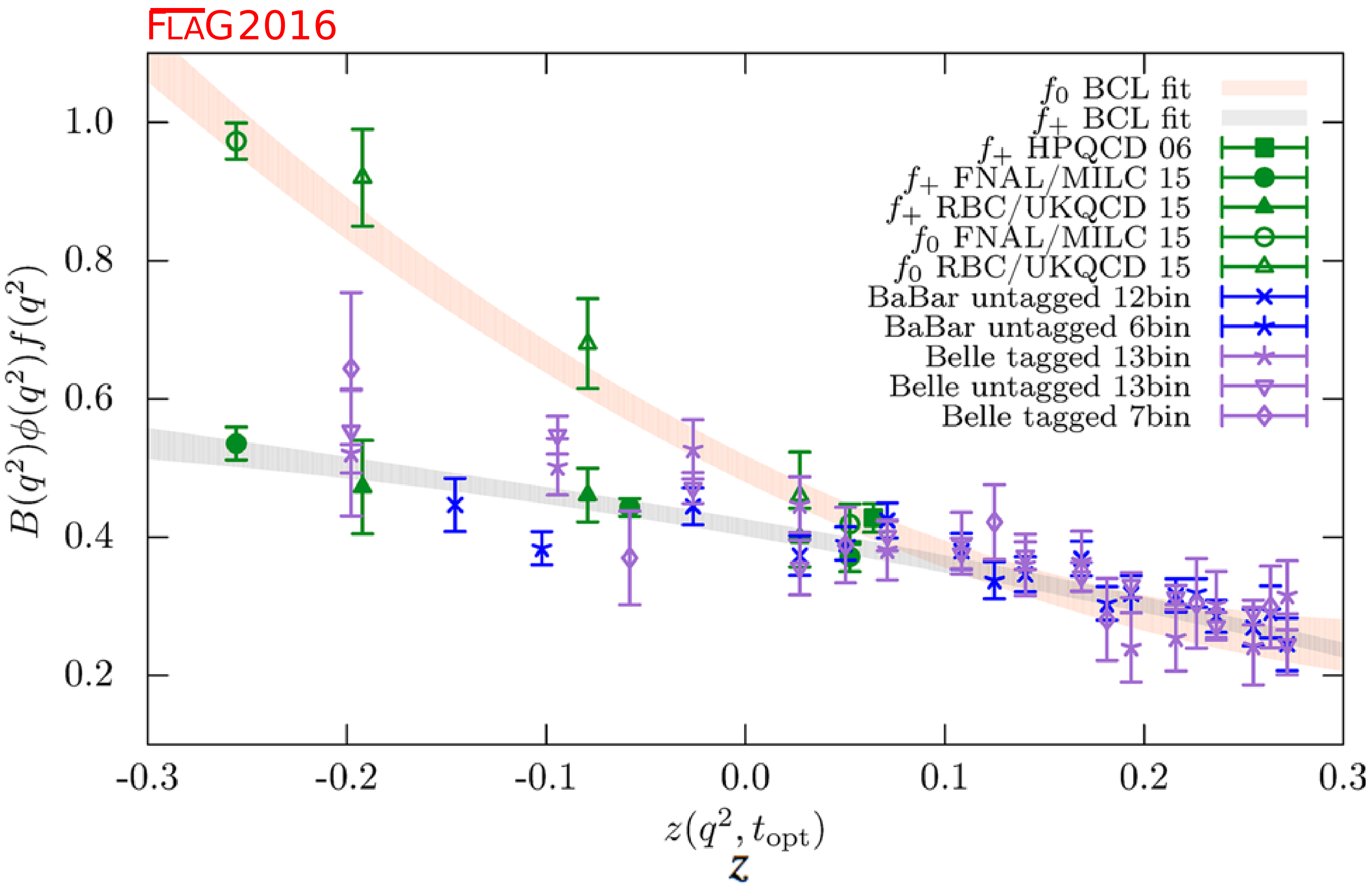
FNAL/MILC [arXiv:1503.07839, PRD 2015]

$|V_{ub}| = 3.72\ (16)\ 10^{-3}$

- shape of f_+ agrees with experiment and uncertainties are commensurate
- fit lattice form factors together with experimental data to determine $|V_{ub}|$ and obtain form factors (f_+, f_0) with improved precision…
- determination of $|V_{ub}/V_{cb}|$ from Λ_b decay with LHCb [arXiv:1503.01421, PRD 2015; arXiv:1504.01568, Nature 2015]:

$$R_{FF} = \frac{|V_{cb}|^2}{|V_{ub}|^2} \frac{\int_{15\mathrm{GeV}^2}^{q^2_{\max}} \frac{d\Gamma(\Lambda_b \to p \mu \nu)}{dq^2} dq^2}{\int_{7\mathrm{GeV}^2}^{q^2_{\max}} \frac{d\Gamma(\Lambda_b \to \Lambda_c \mu \nu)}{dq^2} dq^2} = 1.471 \pm 0.094 \pm 0.109$$

form factors for $B \to \pi \ell \nu$ & V_{ub}



S. Aoki et al [FLAG-3 review, arXiv:1607.00299, EPJC 17, web update]

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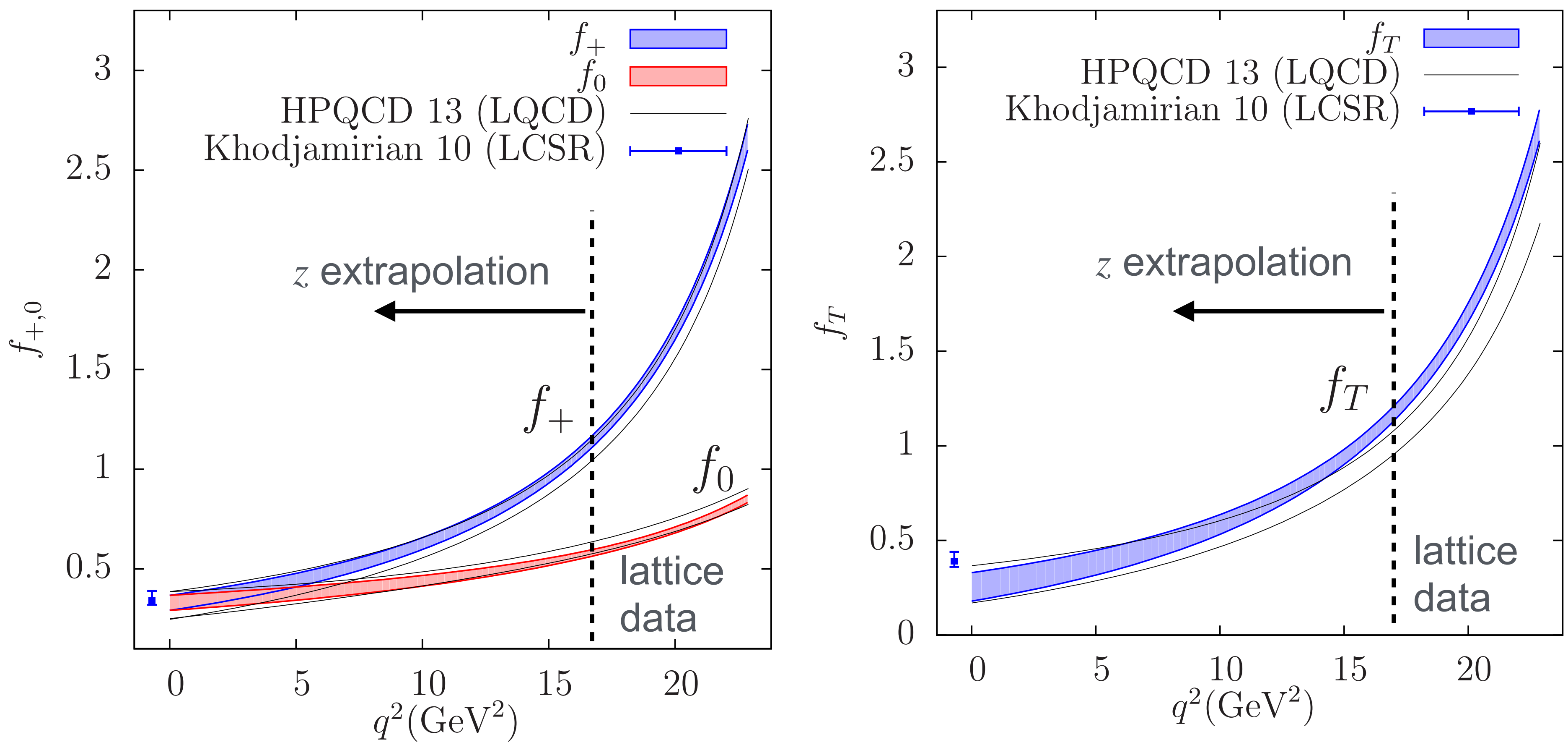
form factors for $B \to K\,\ell\ell$

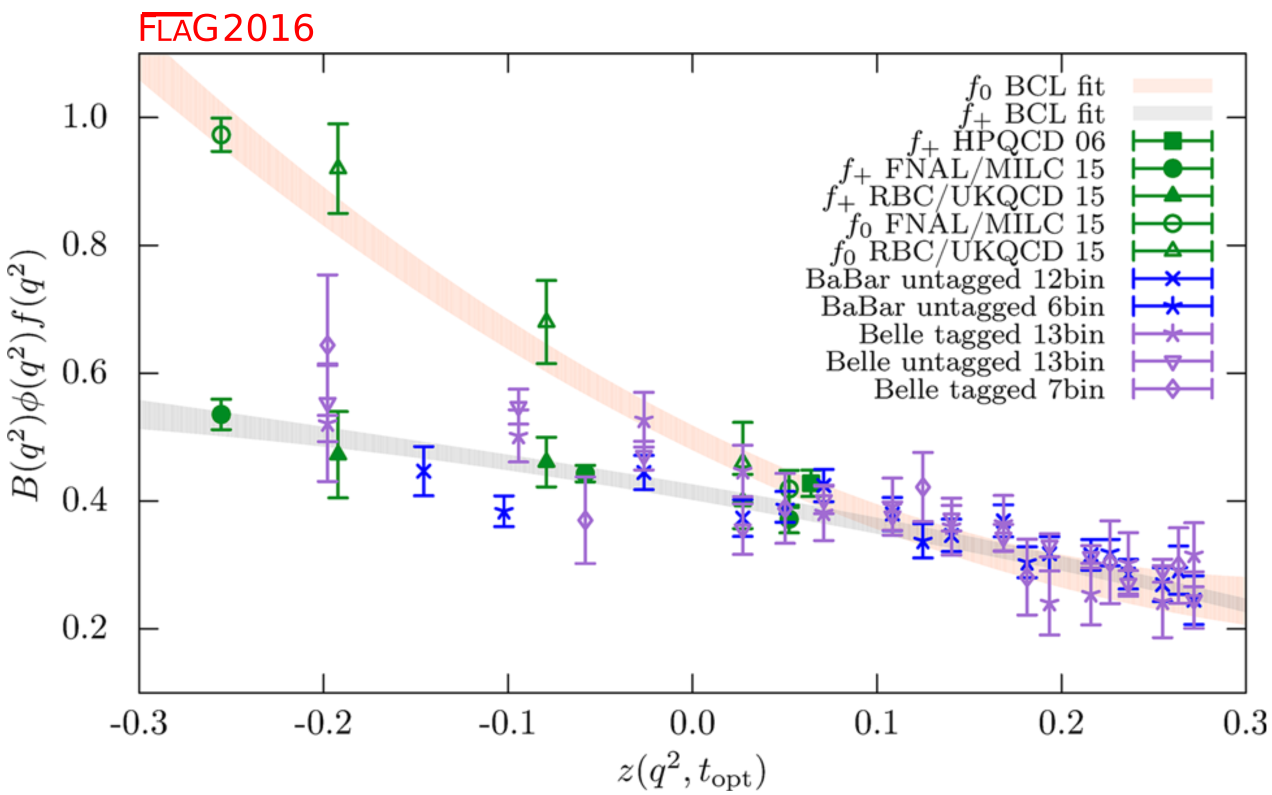


HPQCD [arXiv:1306.0434, 1306.2384, PRL 2013]

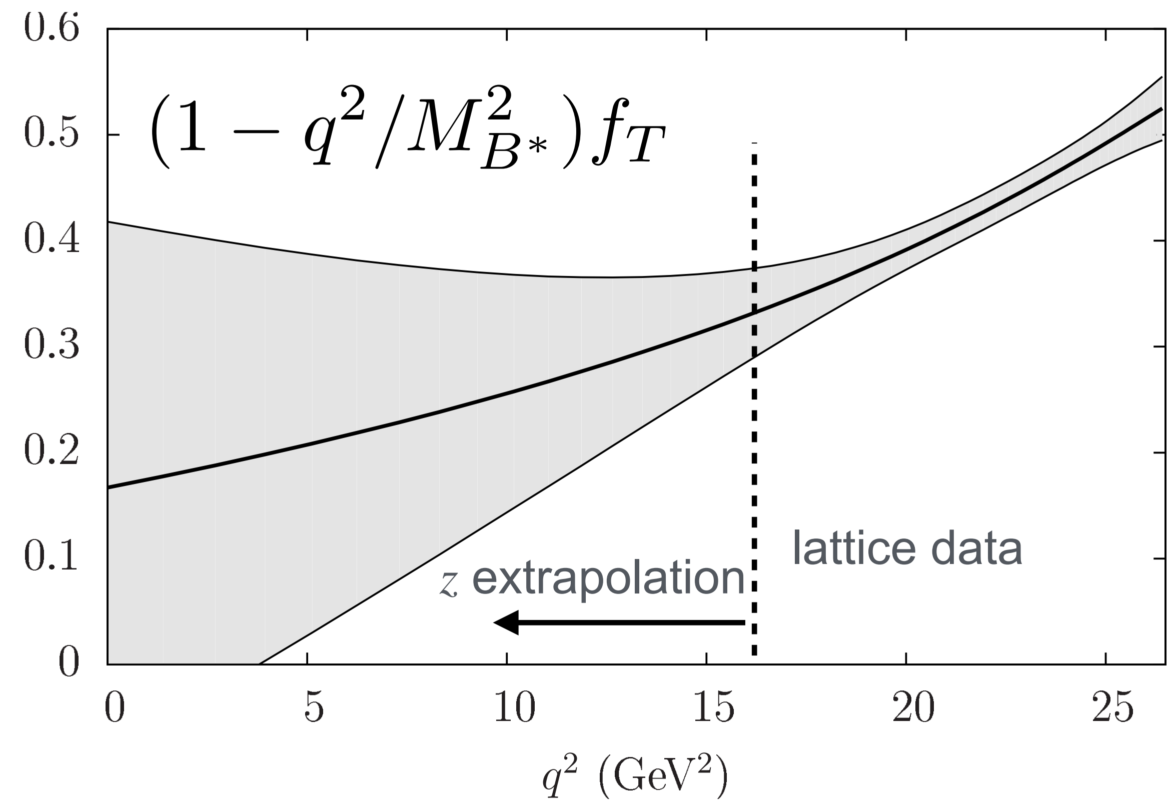
FNAL/MILC [arXiv:1509.06235, PRD 2016]

- Two LQCD calculations (on overlapping ensemble sets, different valence actions): HPQCD (NRQCD b + HISQ), FNAL/MILC (Fermilab b + asqtad)
- consistent results for all three form factors
- also consistent with LCSR [Khodjamarian et al, arXiv:1006.4945, JHEP 2010]
- Note: First LQCD calculation of $\Lambda_b \to \Lambda\,\ell^+\ell^-$ form factors (10 total) [Detmold & Meinel, arXiv:1602.01399, 2016 PRD].

form factors for $B \to \pi \ell\ell$



FNAL/MILC [arXiv:1507.01618, PRL 2015]

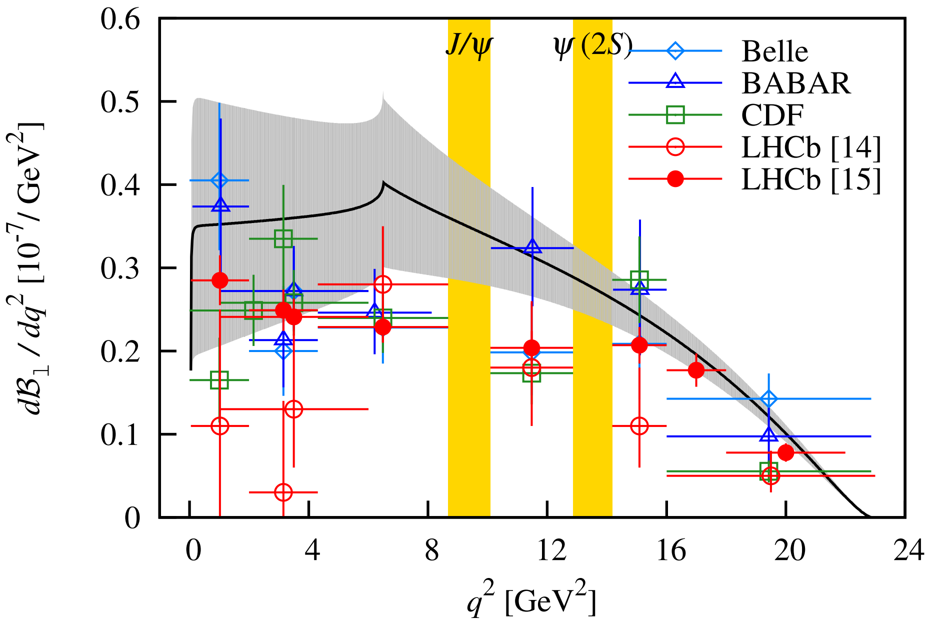


- LQCD calculation of f_T by FNAL/MILC
- Take f_+, f_0 from combined fit of lattice form factors + experimental data for $d\mathcal{B}(B \to \pi\ell\nu)/dq^2$

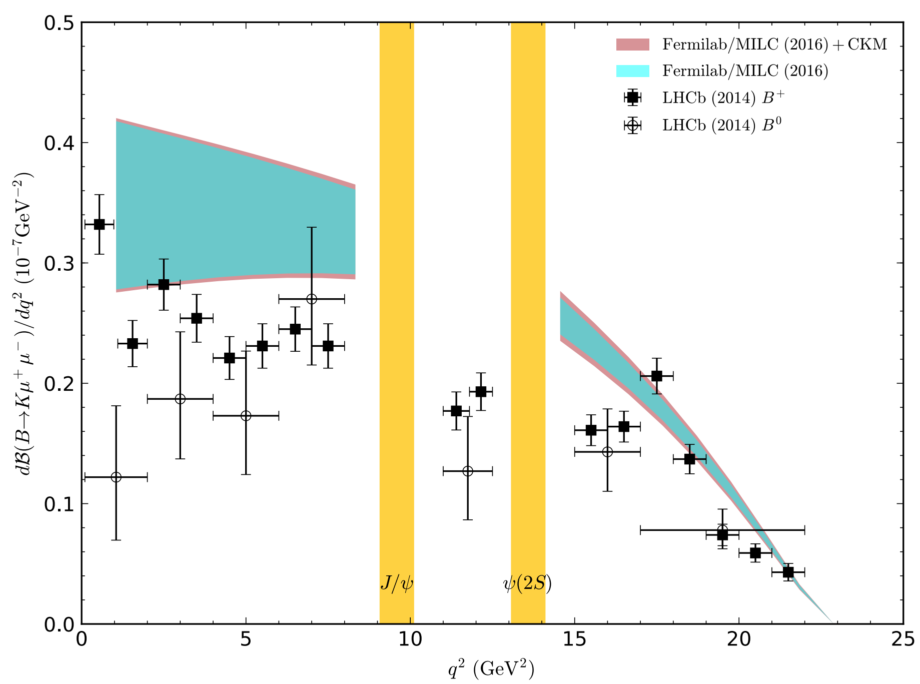
Phenomenology for $B \to K, \pi\, \ell^+ \ell^-$

$B \to K$ Experiment vs. Theory $B \to \pi$

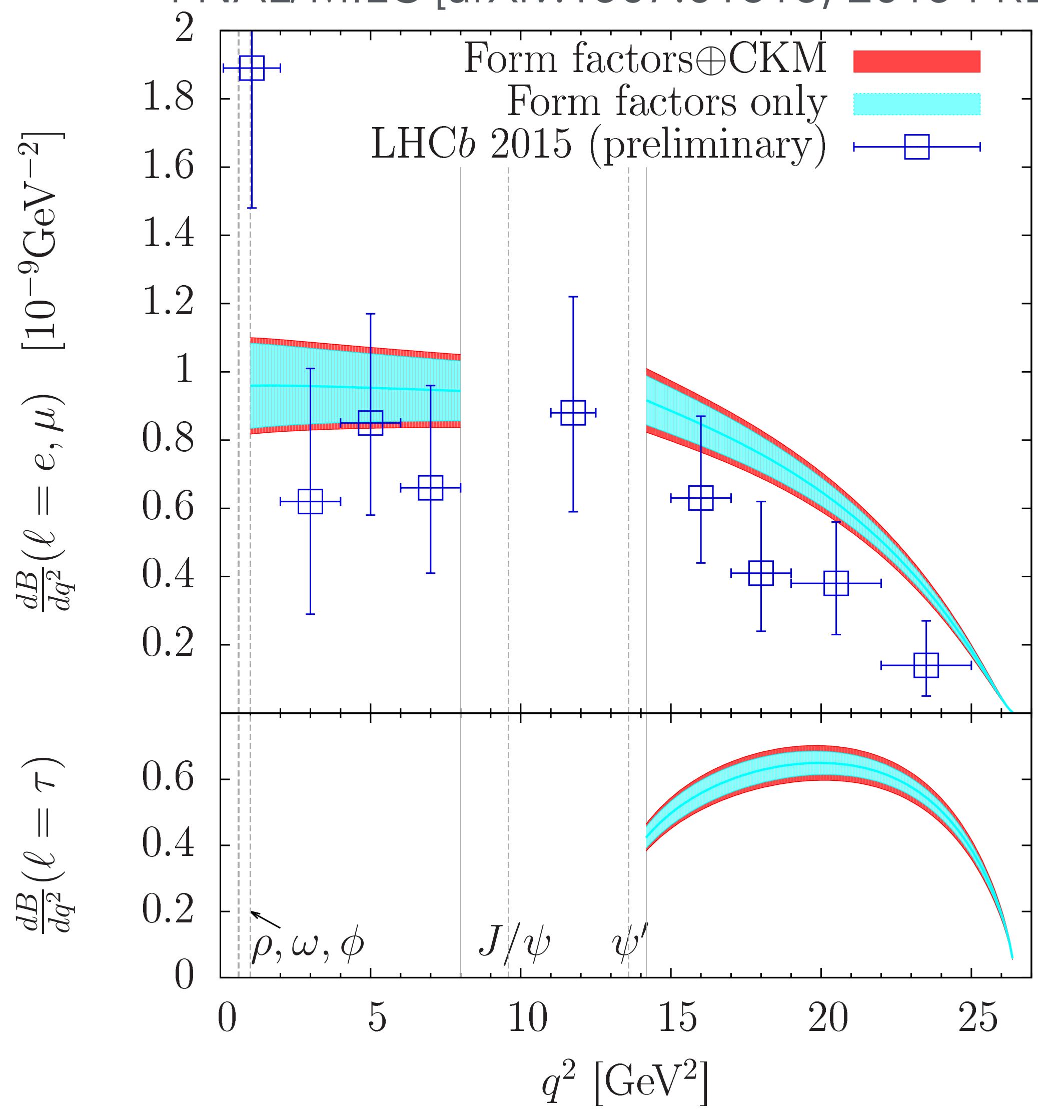
HPQCD [arXiv:1306.0434, 2013 PRL]



FNAL/MILC [arXiv:1509.06235, 2016 PRD



FNAL/MILC [arXiv:1507.01618, 2015 PRL]



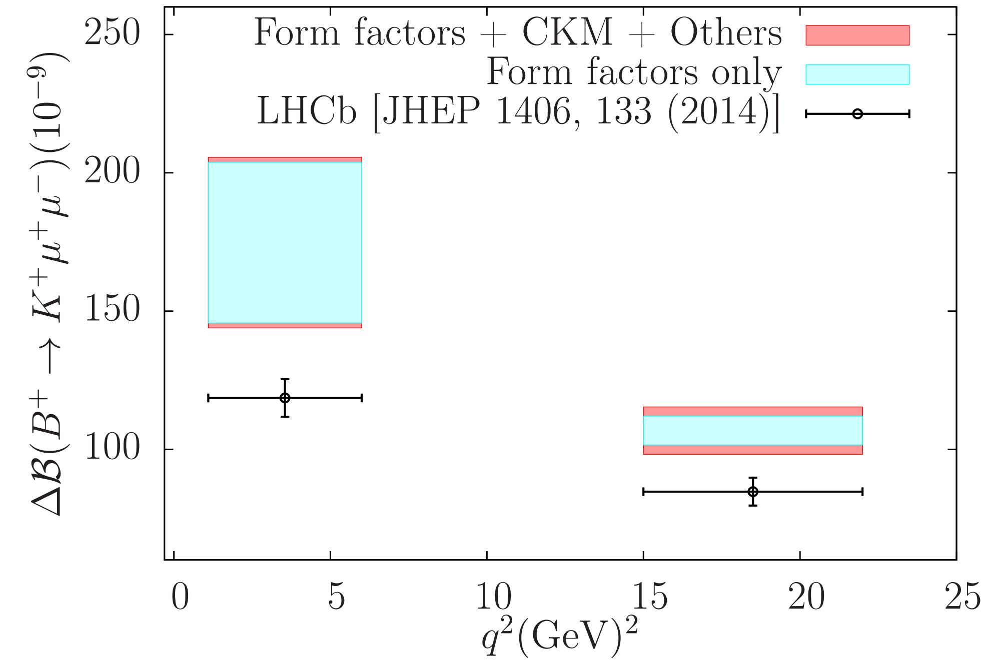
A. Kronfeld @ Lattice 2017
(omit older exp. data)

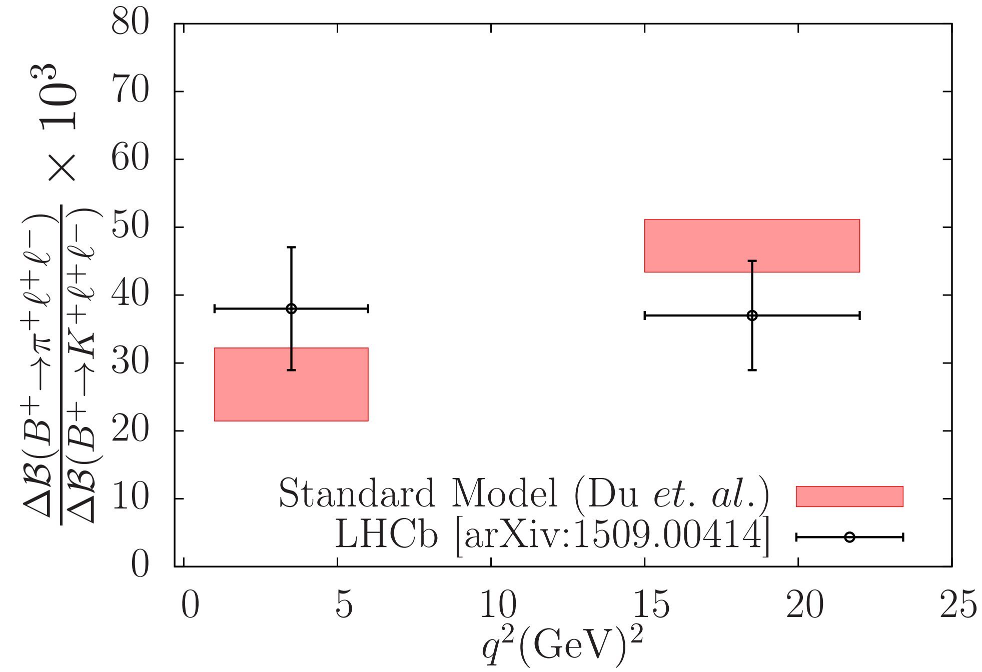
Phenomenology for $B \to K, \pi\, \ell^+ \ell^-$

Experiment vs. theory

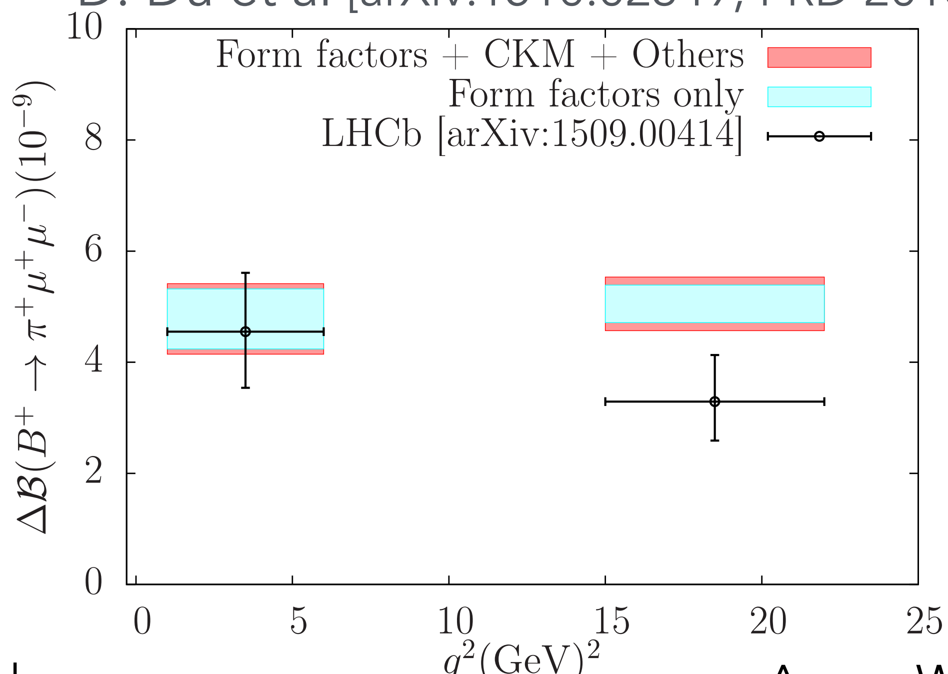
- LHCb data + FNAL/MILC form factors [arXiv:1509.00414, JHEP 2015;1403.8044, JHEP 2014]
- focus on large bins above and below charmonium resonances
- theory errors commensurate with experiment (but nonfactorizable contributions not under good control)
- yields $\sim$1-2σ tensions
- $\Rightarrow$ determine $|V_{td}/V_{ts}|$,$|V_{td}|$,$|V_{ts}|$ or constrain Wilson coefficients

D. Du et al [arXiv:1510.02349, PRD 2016]





D. Du et al [arXiv:1510.02349, PRD 2016]

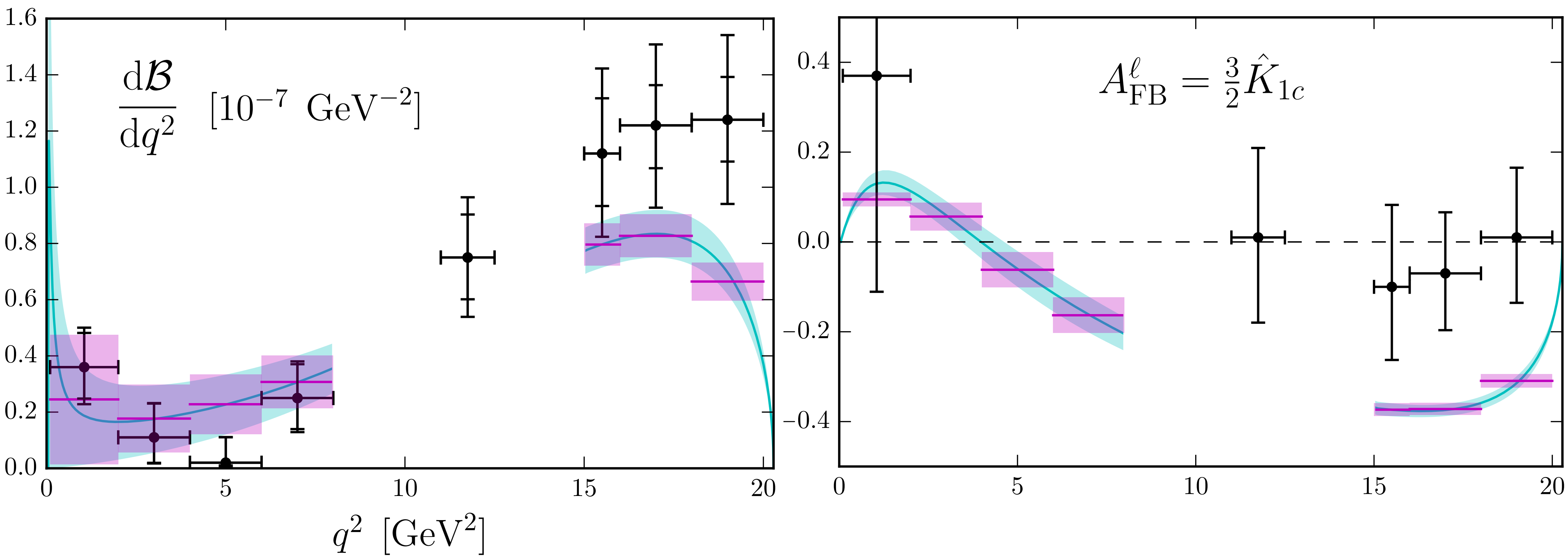


Phenomenology for $\Lambda_b \to \Lambda\, \ell^+ \ell^-$

Experiment vs. theory

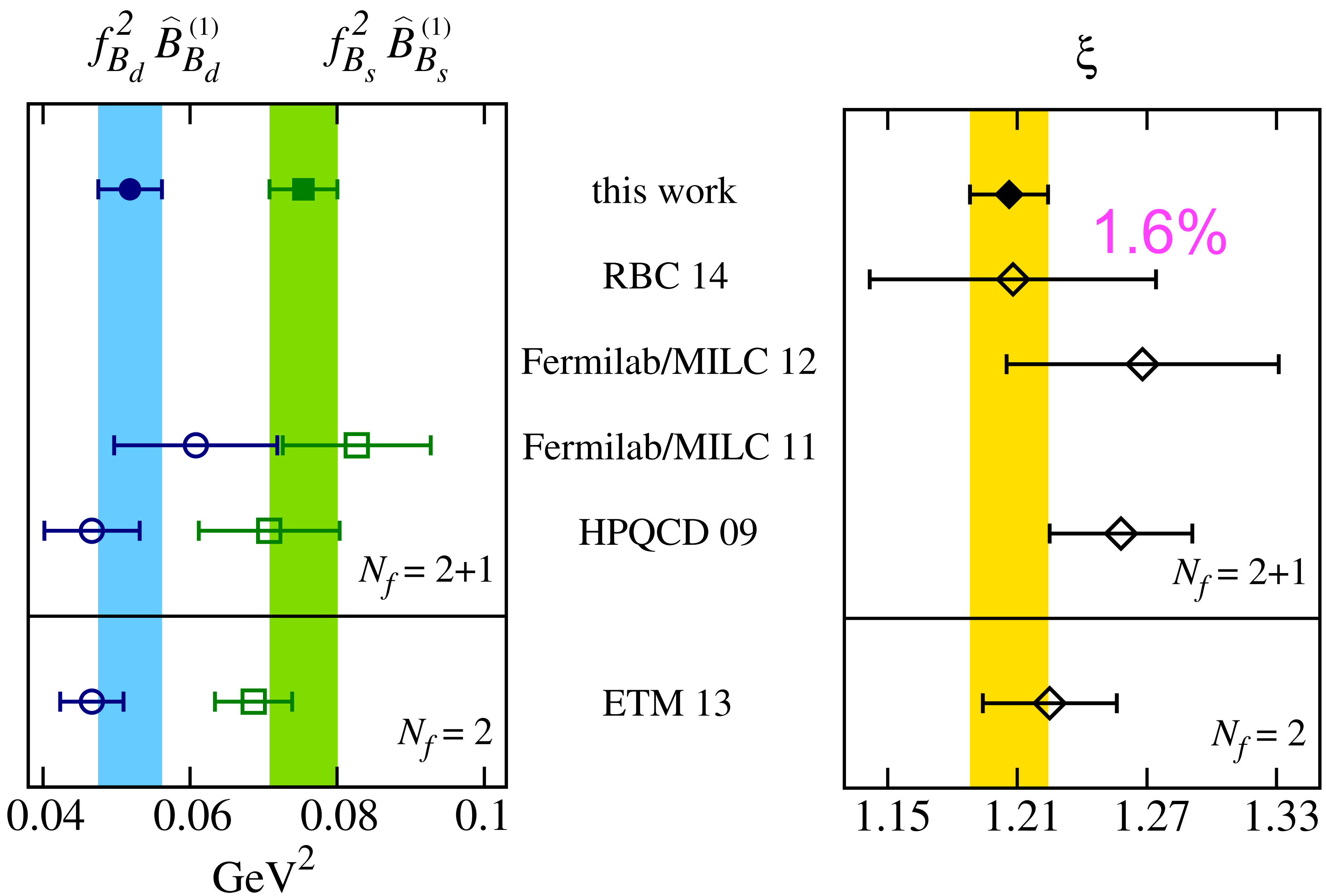
- LHCb data + Detmold & Meinel form factors
  [arXiv:1503.07138, JHEP 2015]
- focus on regions above and below charmonium resonances
- exp. data lie above SM theory ~1-3σ tensions

Detmold & Meinel [arXiv:1602.01399, PRD 2016]



B mixing results in comparison

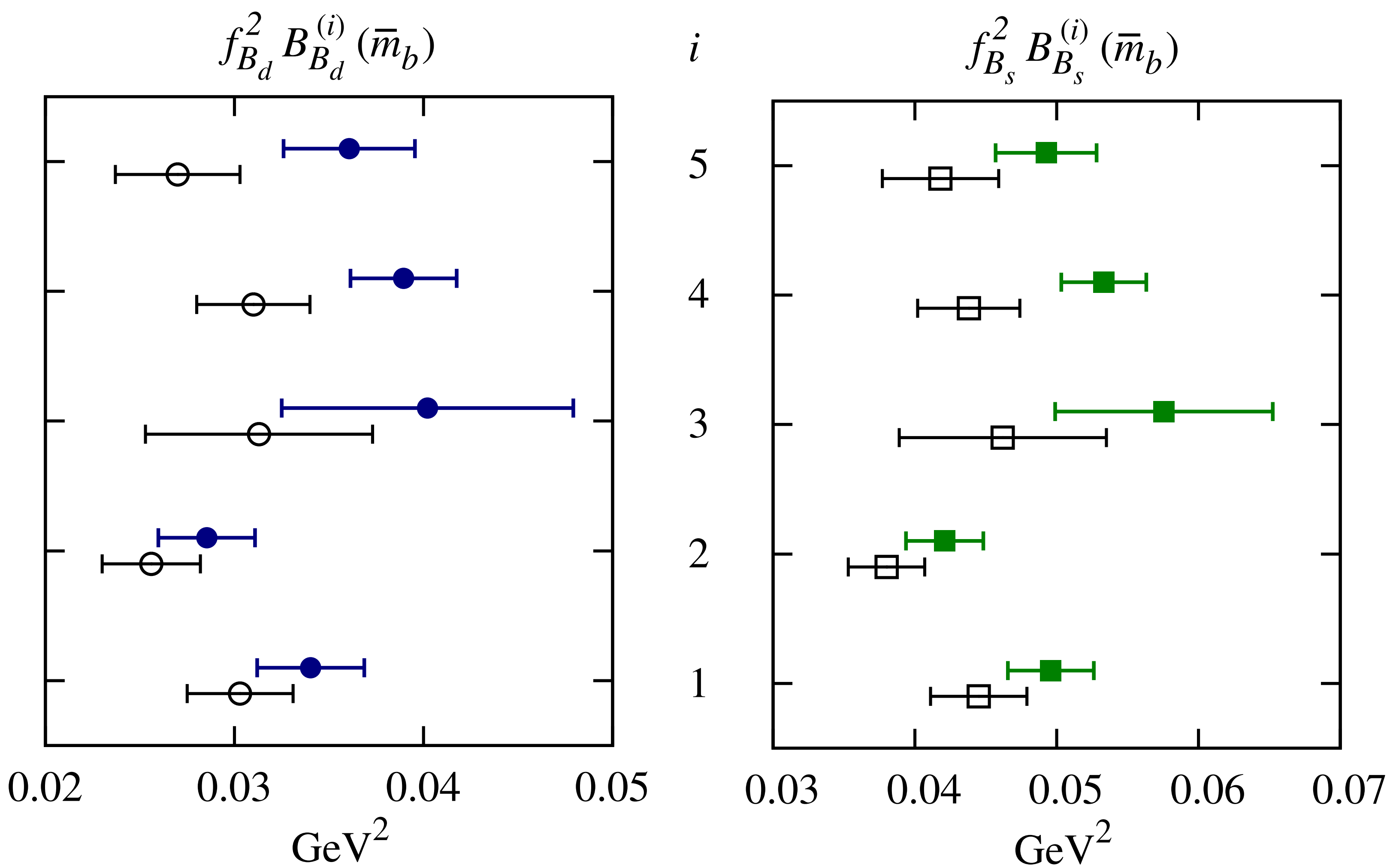
FNAL/MILC [arXiv:1602.03560, PRD 2016]



- Note: FLAG-3 is currently updating their averages for B mixing quantities to include the new FNAL/MILC results.
- ongoing LQCD calculations by HPQCD, ETM, RBC/UKQCD, …

B mixing results in comparison

ETM (n_f=2) [arXiv:1308.1851, JHEP 2014] vs. FNAL/MILC (n_f=3) [arXiv:1602.03560, PRD 2016]



⭐FNAL/MILC also provides the correlations between all 10 matrix elements.

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  - ★ D mesons
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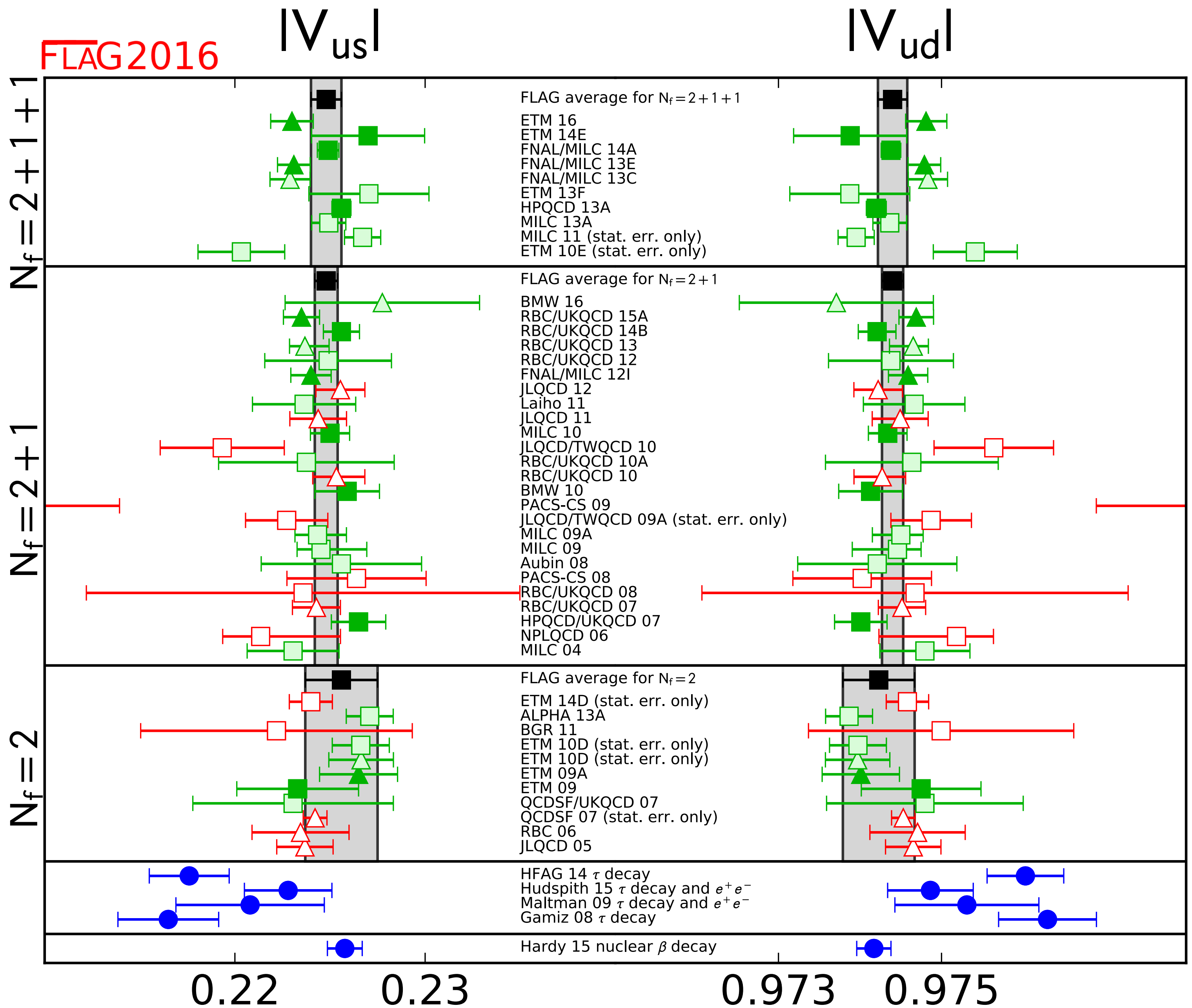
FLAG summary of $|V_{ud}|$ and $|V_{us}|$

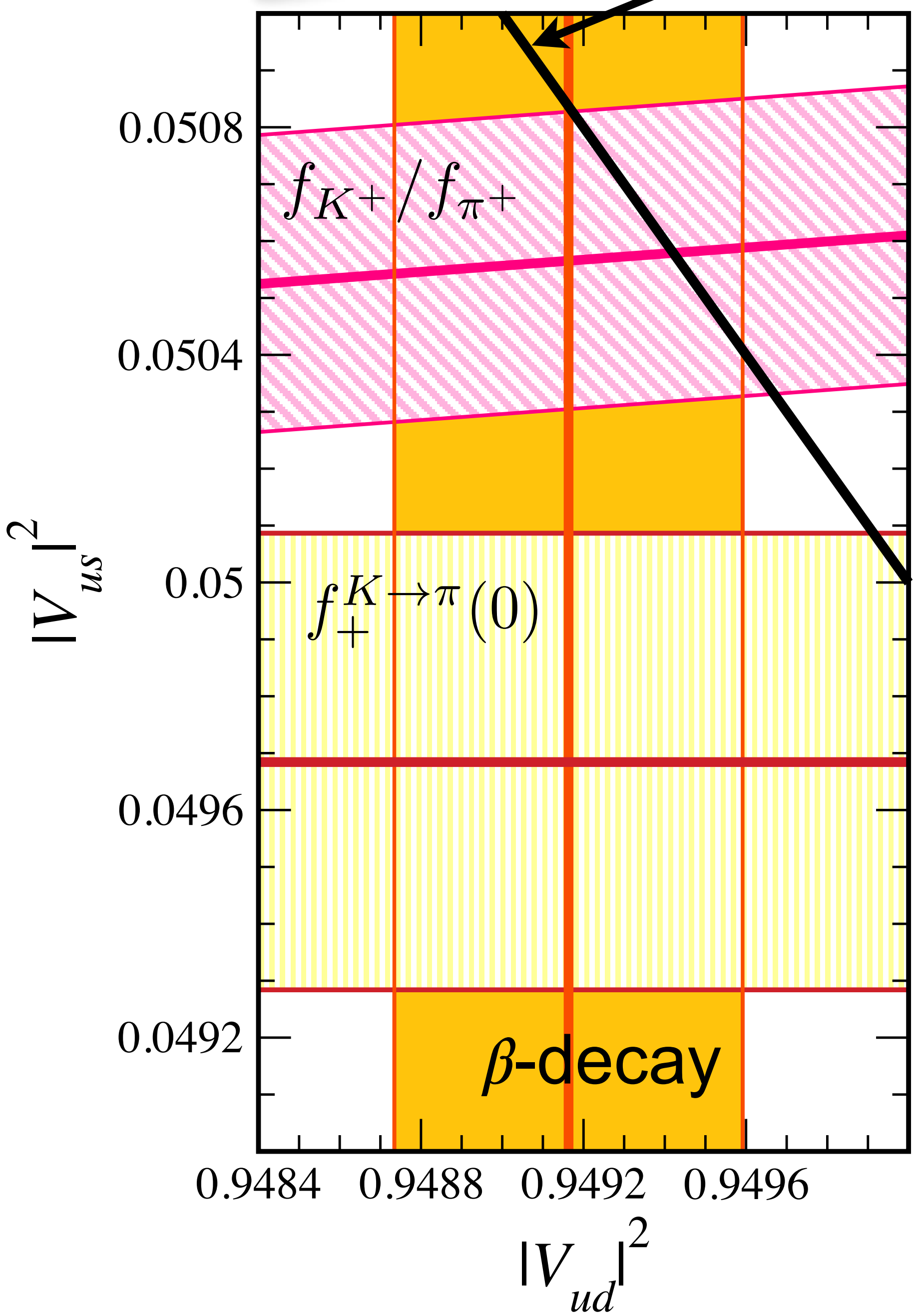
S. Aoki et al [FLAG-3 review, arXiv:1607.00299, EPJC 17, web update]



1^{st} row CKM unitarity test

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

$|V_{ub}| \approx 4 \times 10^{-3} \approx 0$



Xu Feng @ Lattice 2017:

$$|V_u|^2 \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

- Use $|V_{us}|$ for $K_{\ell 3}$ + $|V_{us}/V_{ud}|$ for $K_{\ell 2}/\pi_{\ell 2}$ as input

  $|V_u|^2 = 0.9798(82) \quad \Rightarrow \quad 2.5\sigma$ deviation from 1

Most precise value of $|V_{ud}| = 0.97417(21)$ is from superallowed nuclear β decay

- Use $|V_{us}|$ for $K_{\ell 3}$ + $|V_{ud}|$ for β decay

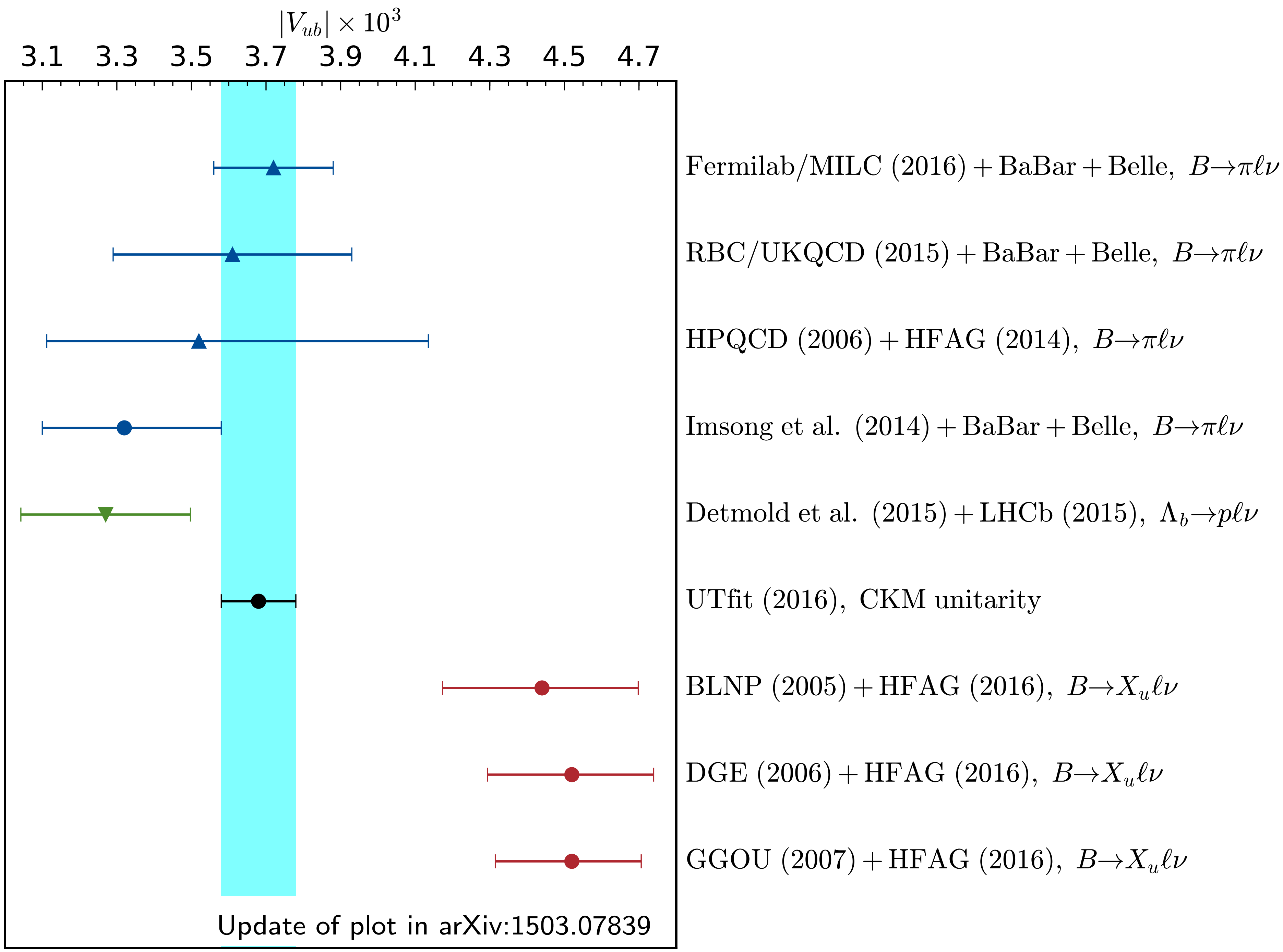
  $|V_u|^2 = 0.9988(5) \quad \Rightarrow$ sharpen the test, still 2.4σ deviation

- Use $|V_{us}/V_{ud}|$ for $K_{\ell 2}/\pi_{\ell 2}$ + $|V_{ud}|$ for β decay

  $|V_u|^2 = 0.9998(5) \quad \Rightarrow$ confirm CKM unitarity

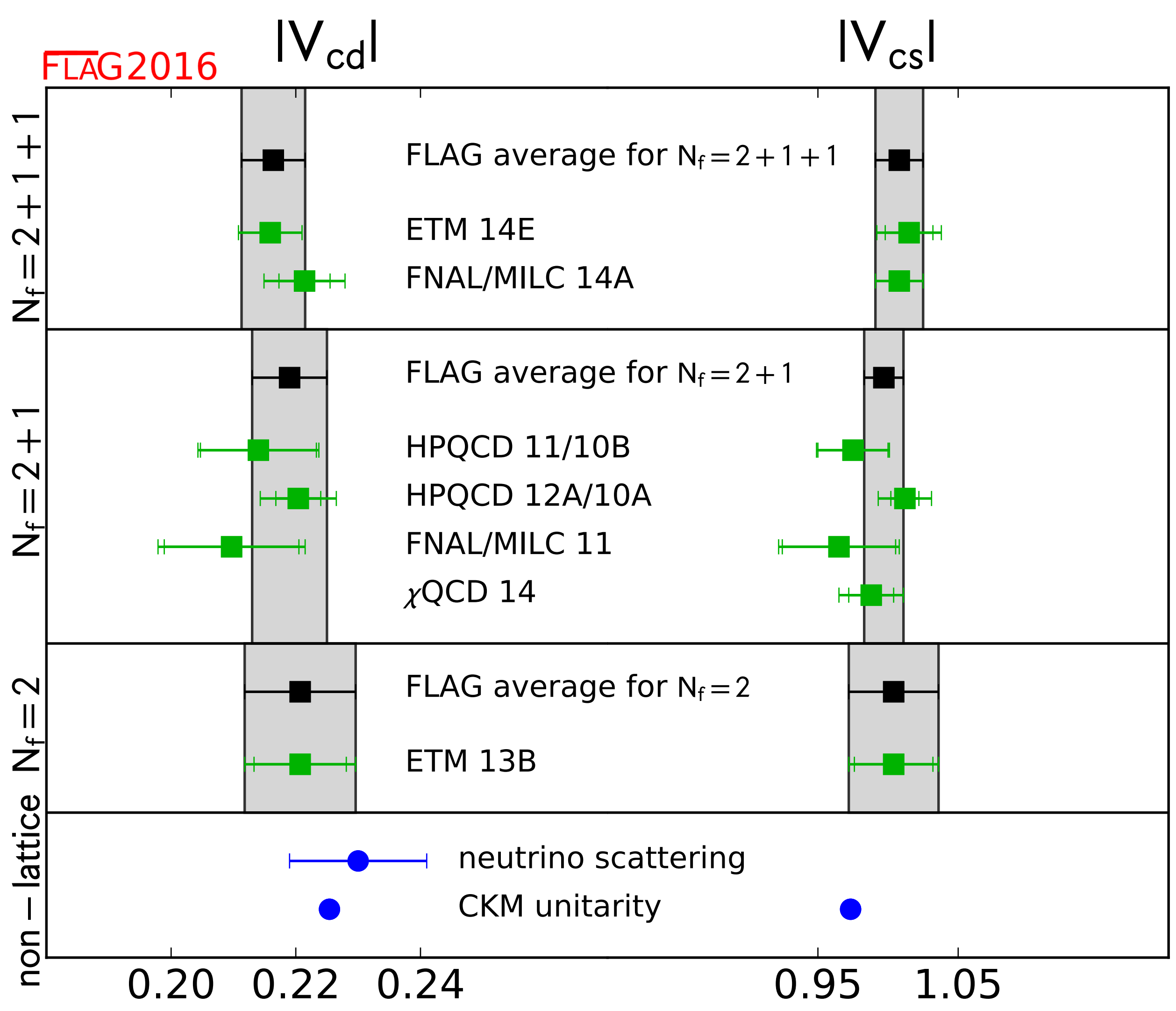
Exclusive $|V_{ub}|$

A. Kronfeld @ Lattice 2017:

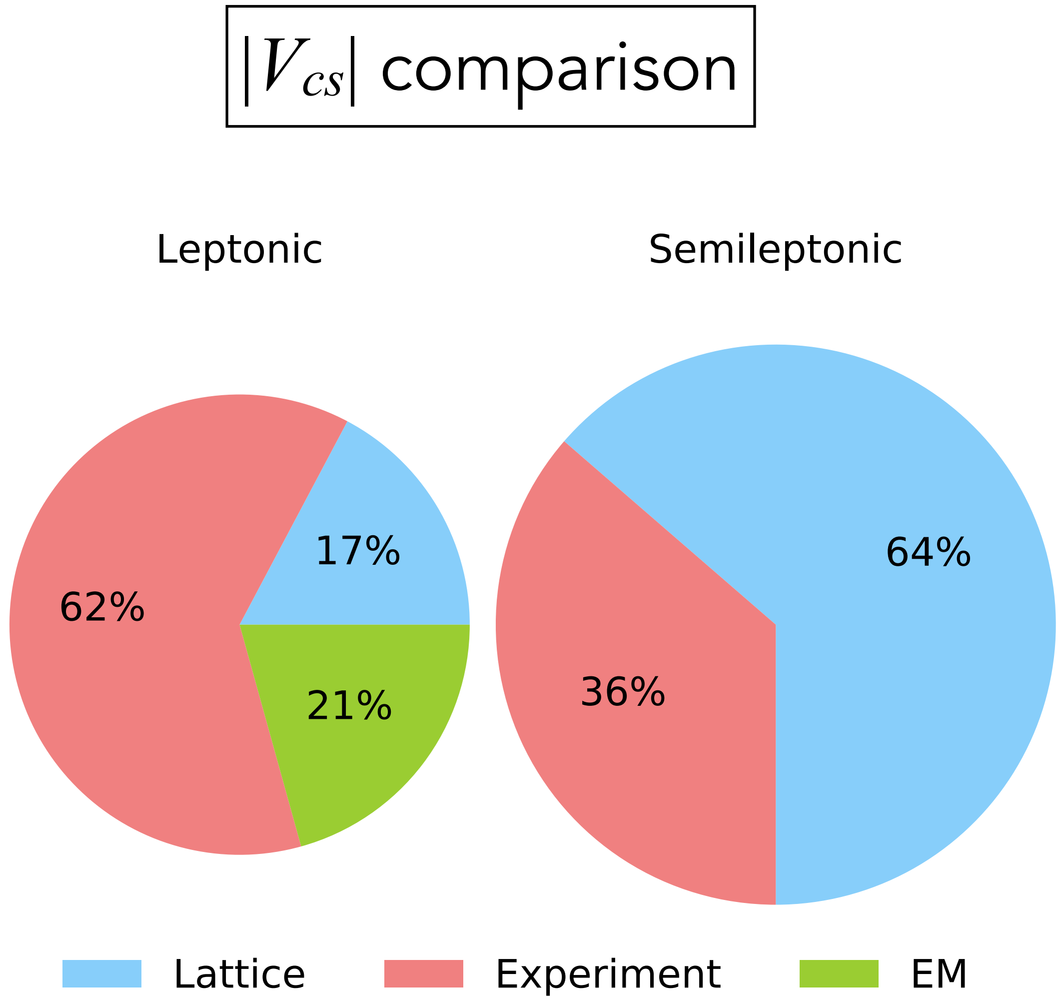


FLAG summary of $|V_{cd}|$ and $|V_{cs}|$

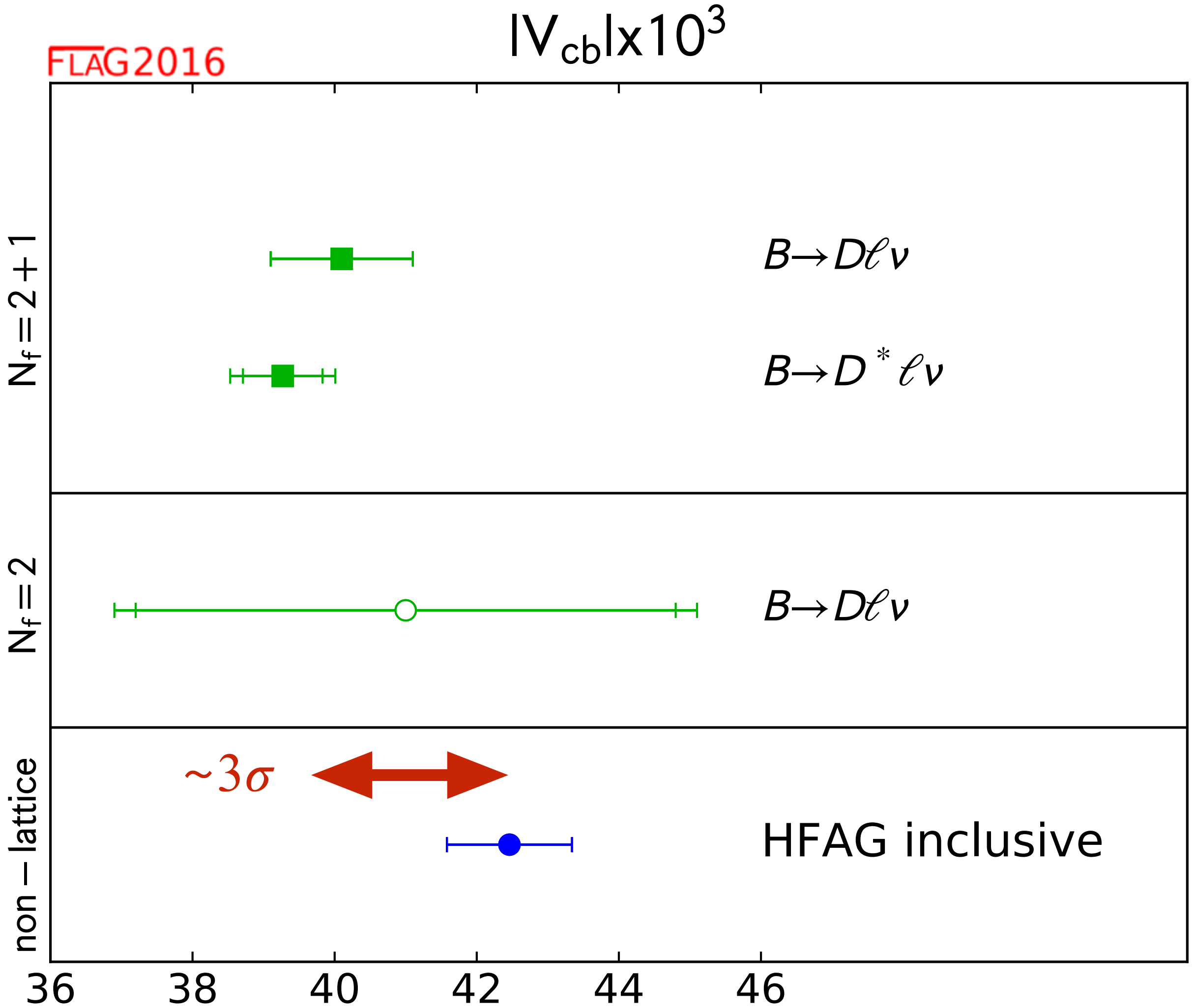
S. Aoki et al [FLAG-3 review, arXiv:1607.00299, EPJC 17, web update]



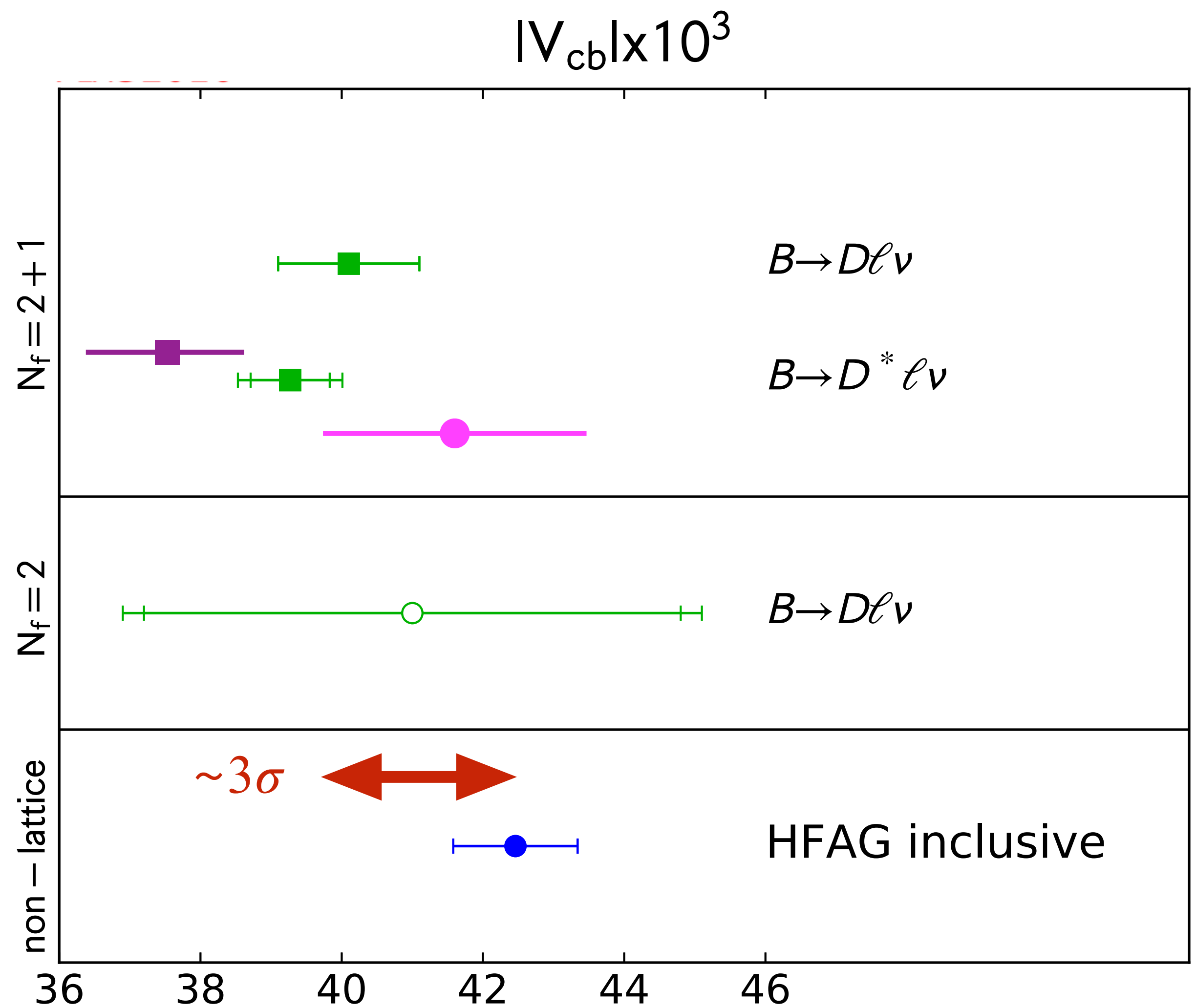
S. Gottlieb, T. Primer (FNAL/MILC) @ Lattice 2016:



Exclusive $|V_{cb}|$



Exclusive $|V_{cb}|$



New BELLE measurement of $B \to D^*$ decay with CLN extrapolation to w=1 and lattice $\mathcal{F}(1)$ [arXiv:1702.01521]

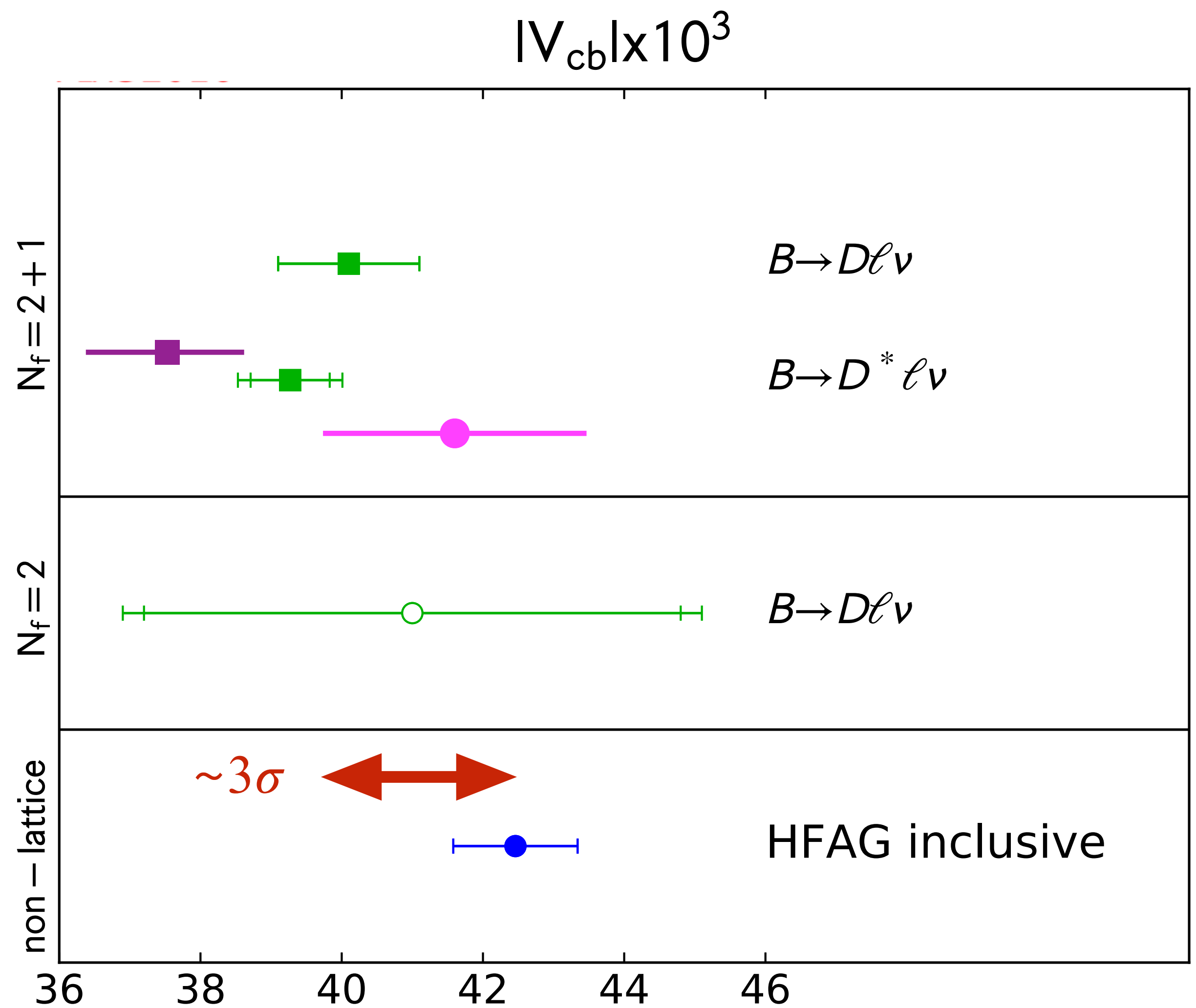
Two new theory analyses:

- Bigi, Gambino, Schacht [arXiv:1703.06124]
- Grinstein, Kobach [arXiv:1703.08170]

Both use new Belle data and BGL together with lattice $\mathcal{F}(1)$.

- Difference between the new CLN and BGL results may be due to assumptions in CLN, but BLPR [Bernlochner, Ligeti, Papucci, Robinson, arXiv:1703.05330, 2017 PRD] obtain values for $|V_{cb}|$ from BGL fits + HQET constraints for 1/m corrections that are similar to the $|V_{cb}|$ results from CLN fits.
- In addition, BLPR [Bernlochner et al, arXiv:1708.07134] find that the BGL (+lattice $\mathcal{F}(1)$) fits yield larger than expected 1/m corrections.

Exclusive $|V_{cb}|$



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- Grinstein, Kobach [arXiv:1703.08170]

Both use new Belle data and BGL together with lattice $\mathscr{F}(1)$.

- Difference between the new CLN and BGL results may be due to assumptions in ...]
  ob...
  tha...
- In ...
  $\mathscr{F}$(1)) fits yield larger than expected 1/m corrections.

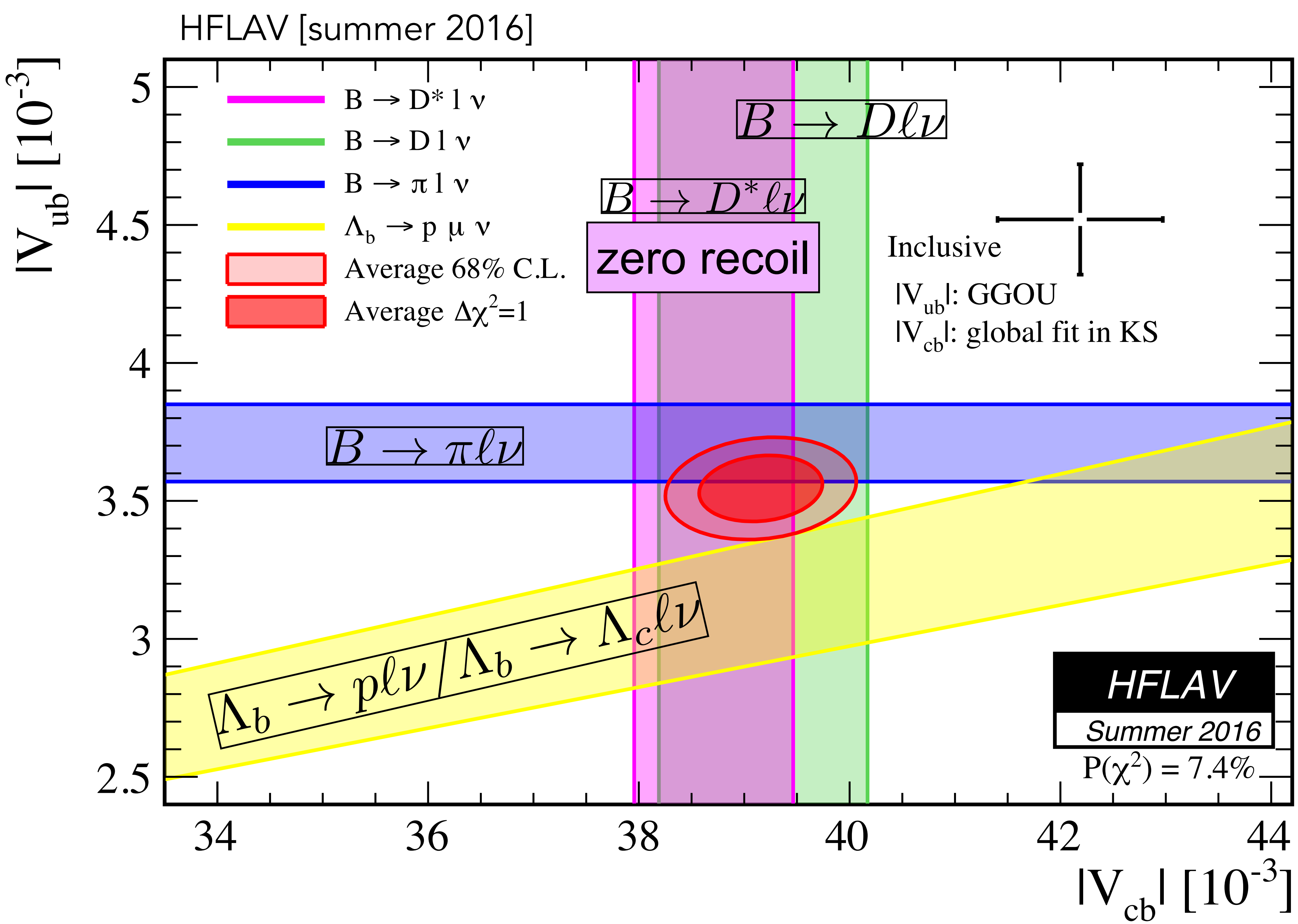
Need lattice form factor data for $B \to D^*$ at nonzero recoil:

combine with experimental data using BGL (same as for $B \to D$)

➡ improve precision on $|V_{cb}|$ and check exclusive/inclusive tension

.... in progress (FNAL/MILC, HPQCD, RBC, LANL/SNU)

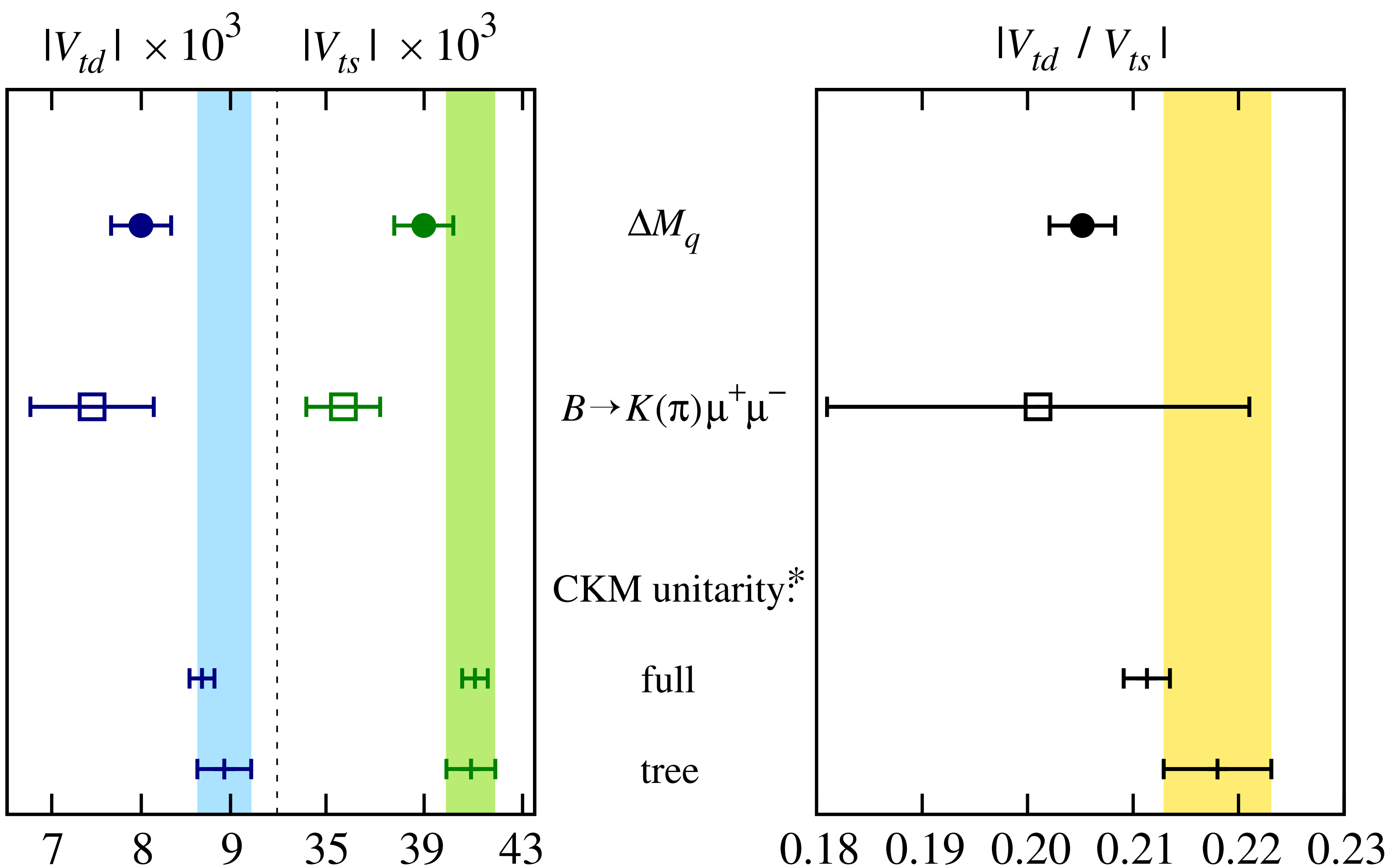
Exclusive vs. inclusive $|V_{cb}|$ and $|V_{ub}|$



~3σ tension between inclusive and exclusive $|V_{cb}|$ and $|V_{ub}|$

Implications for $|V_{td}|$ and $|V_{ts}|$

FNAL/MILC [arXiv:1602.03560, PRD 2016], plot by C. Bouchard

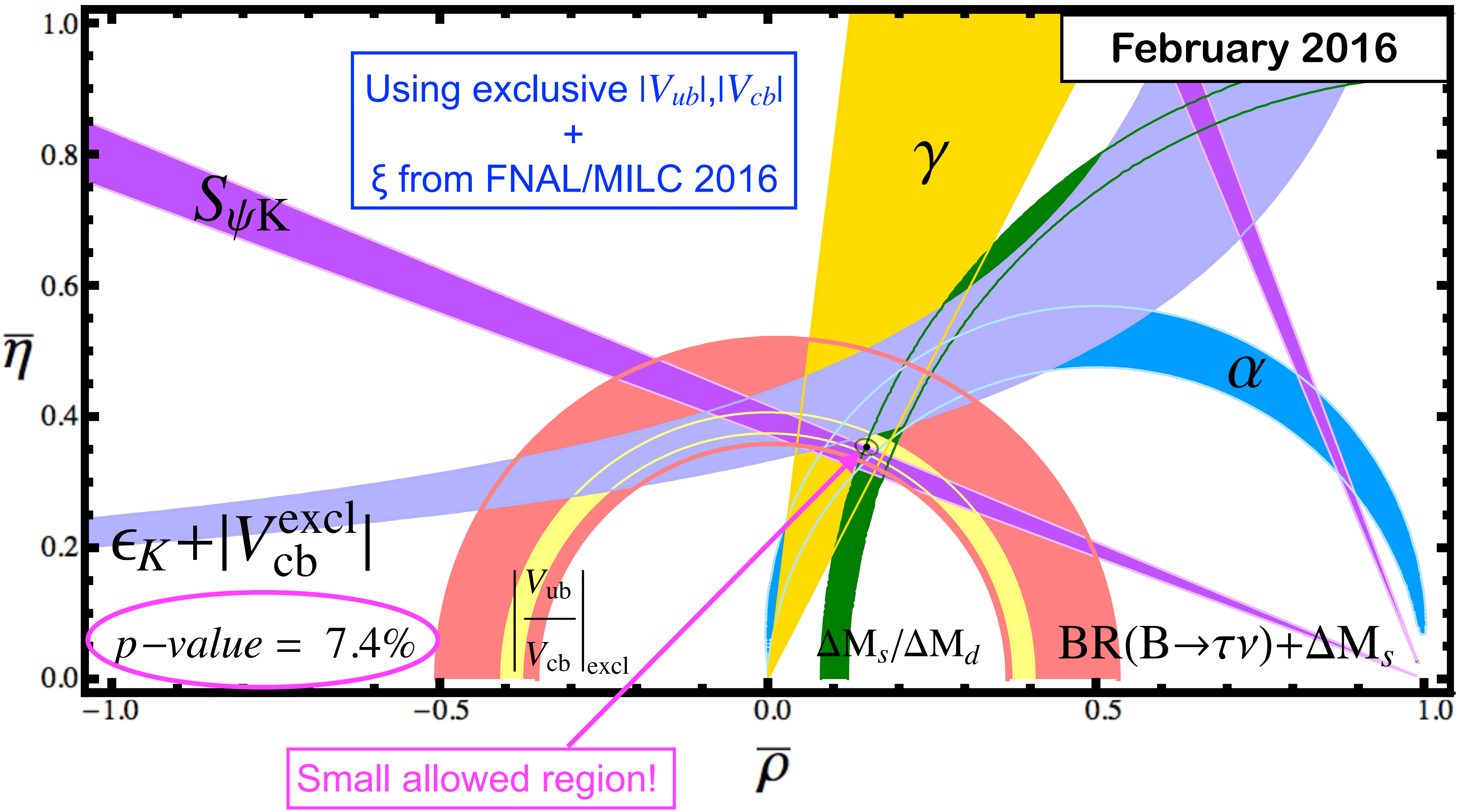


$\sim 2\sigma$ tensions between loop processes and CKM unitarity constraints from tree-level processes.

*from CKMfitter 2015 [hep-ph/0406186, http://ckmfitter.in2p3.fr]

UT analysis

Laiho, Lunghi, Van de Water [arXiv:0910.2928, arXiv:0910.2928, PRD 2010, E. Lunghi, priv. comm.]



Outline

- Motivation and Introduction
- Leptonic, Semileptonic Decays, Mixing
  - ⋆ Kaons
  - ⋆ D mesons
  - ⋆ B mesons
- Implications
  - ⋆ CKM determinations
- Summary

Summary

(inspired by A. Kronfeld)

$\langle \bar{B}_q^0 | \mathcal{O}_i^{\Delta B=2} | B_q^0 \rangle$

$\langle \bar{D}^0 | \mathcal{O}_i^{\Delta C=2} | D^0 \rangle$

$\hat{B}_K \quad \ldots$

$f_{+,0}^{B\to D}(q^2), \ldots$

$f_+^{K\to\pi} \quad f_{+,0,T}^{B\to\pi} \quad \ldots$

$f_{K^\pm} \quad f_{B_{(s)}} \quad \ldots$

Complexity

✓

LQCD flagship results

Summary

$\langle \bar{B}_q^0 | \mathcal{O}_i^{\Delta B=2} | B_q^0 \rangle$

$\langle \bar{D}^0 | \mathcal{O}_i^{\Delta C=2} | D^0 \rangle$

$\hat{B}_K \quad \ldots$

$f_{+,0}^{B\to D}(q^2), \ldots$

$f_+^{K\to\pi} \quad f_{+,0,T}^{B\to\pi} \quad \ldots$

$f_{K^\pm} \quad f_{B_{(s)}} \ldots$

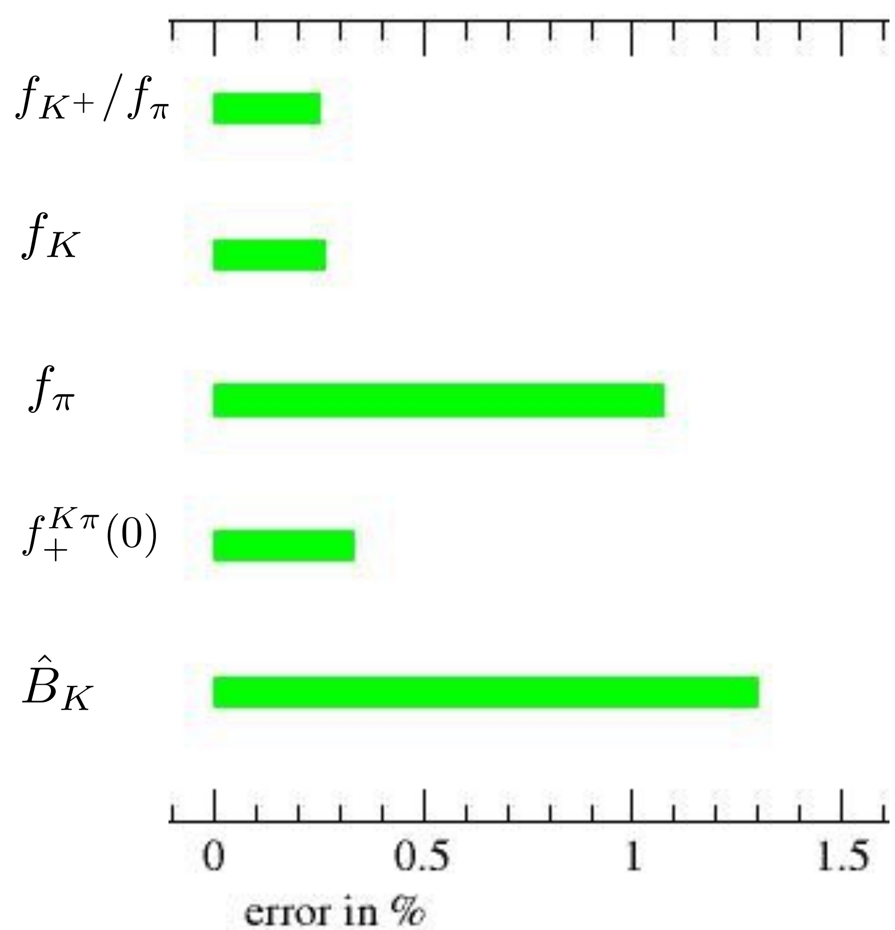
✓

LQCD flagship results

Kaon summary

For all quantities there are results that use **physical mass** ensembles

errors (in %) **FLAG-3 averages**



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Summary

$\langle \bar{B}_q^0 | \mathcal{O}_i^{\Delta B=2} | B_q^0 \rangle$

$\langle \bar{D}^0 | \mathcal{O}_i^{\Delta C=2} | D^0 \rangle$

$\hat{B}_K \quad \ldots$

$f_{+,0}^{B\to D}(q^2), \ldots$

$f_+^{K\to\pi} \quad f_{+,0,T}^{B\to\pi} \quad \ldots$

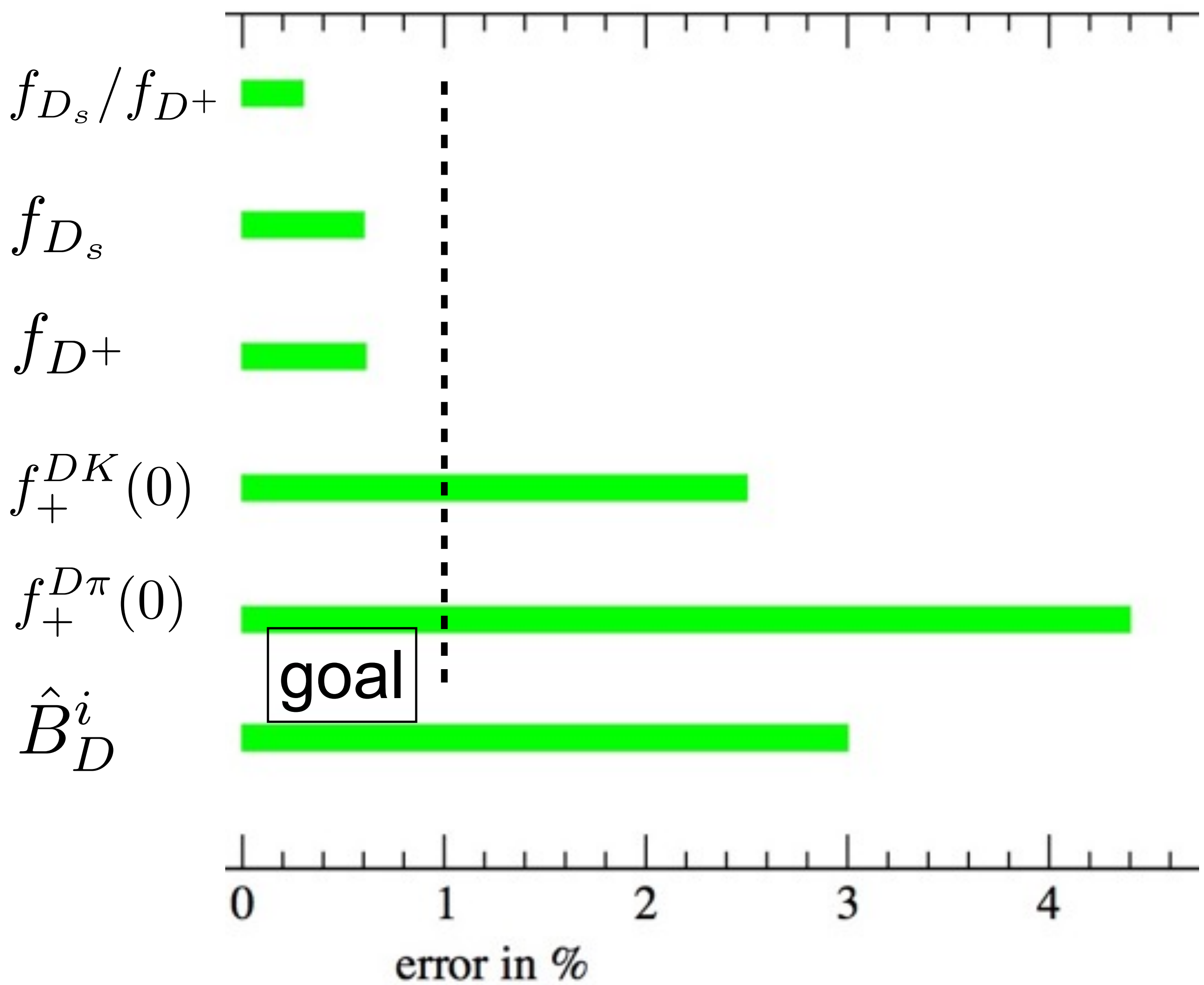
$f_{K^\pm} \quad f_{B_{(s)}} \quad \ldots$

✓

LQCD flagship results

D meson summary

errors (in %) comparison:



Summary

$\langle \bar{B}^0_q | \mathcal{O}_i^{\Delta B=2} | B^0_q \rangle$

$\langle \bar{D}^0 | \mathcal{O}_i^{\Delta C=2} | D^0 \rangle$

$\hat{B}_K \quad \ldots$

$f_{+,0}^{B\to D}(q^2), \ldots$

$f_+^{K\to\pi} \quad f_{+,0,T}^{B\to\pi} \quad \ldots$

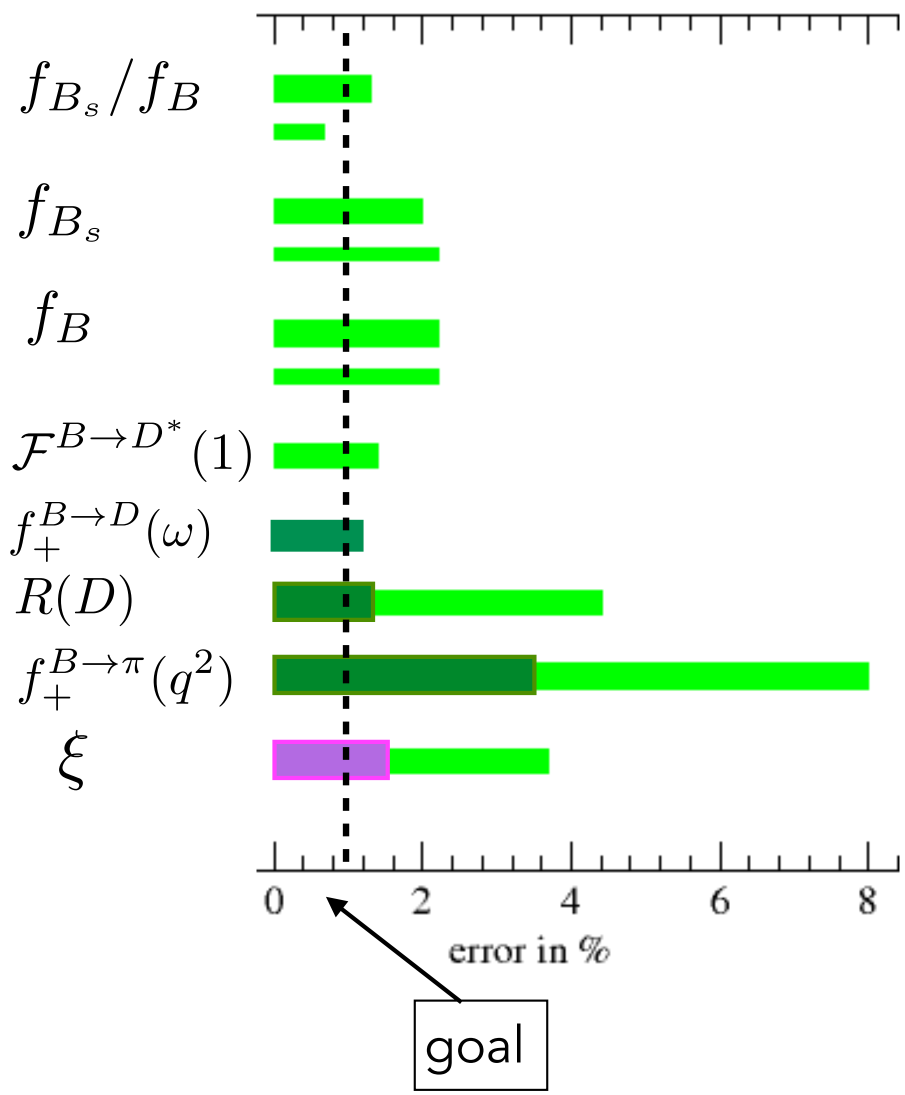
$f_{K^\pm} \quad f_{B_{(s)}} \ldots$

✓

LQCD flagship results

B-meson summary

errors (in %) FLAG-2/3 averages + new results



Several quantities where lattice errors are commensurate with experimental uncertainties

Summary

(inspired by A. Kronfeld)

$\langle \bar{B}_q^0 | \mathcal{O}_i^{\Delta B=2} | B_q^0 \rangle$

$\langle \bar{D}^0 | \mathcal{O}_i^{\Delta C=2} | D^0 \rangle$

$\hat{B}_K \quad \ldots$

$f_{+,0}^{B\to D}(q^2), \ldots$

$f_+^{K\to\pi} \quad f_{+,0,T}^{B\to\pi} \quad \ldots$

$f_{K^\pm} \quad f_{B_{(s)}} \quad \ldots \quad \langle \pi\pi_{(I=2)} | \mathcal{H}^{\Delta S=1} | K^0 \rangle$

$\Delta M_K,\ \epsilon_K$

$\langle \pi\pi_{(I=0)} | \mathcal{H}^{\Delta S=1} | K^0 \rangle$

$B \to K^* \ell\ell \to K\pi\, \ell\ell \ldots$

$K^+ \to \ell^+ \nu\, (\gamma) \quad \ldots$

$K^+ \to \pi^+ \ell^+ \ell^-$

$K^+ \to \pi^+ \nu\bar{\nu} \quad \ldots$

Complexity

LQCD flagship results

First complete LQCD results, large(ish) errors

First results, physical params, incomplete systematics

new methods, pilot projects, unphysical kinematics