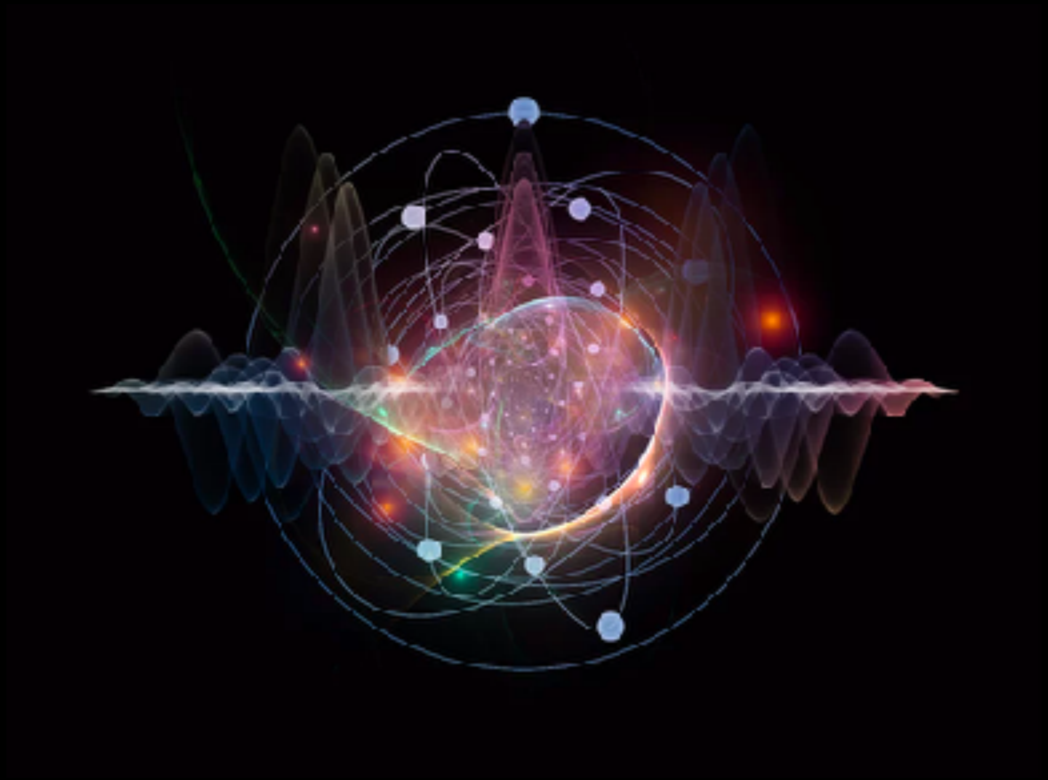
Unification from Scattering Amplitudes



Clifford Cheung
Caltech

Aspen Winter 2018

What defines a theory?

We typically define QFT from an action principle.

$$S = \int d^4x \, \mathcal{L}$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 + \ldots$$

We typically define QFT from an action principle.

locality in spacetime

$$S = \int d^4x \; \mathcal{L}$$

Poincare invariance

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 + \ldots$$

parameters not fixed by
symmetry are measured!

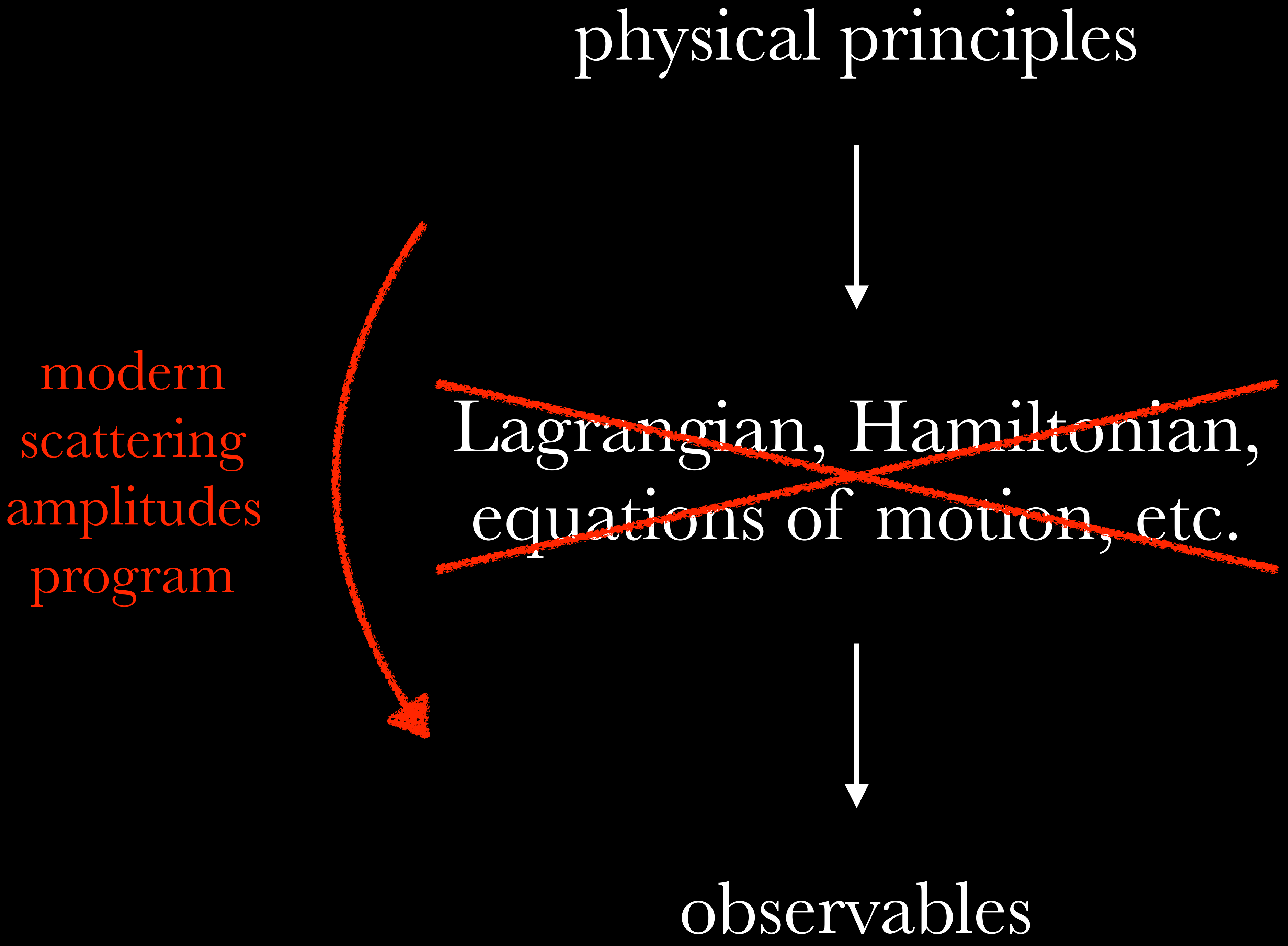
physical principles

↓

Lagrangian, Hamiltonian,
equations of motion, etc.

↓

observables



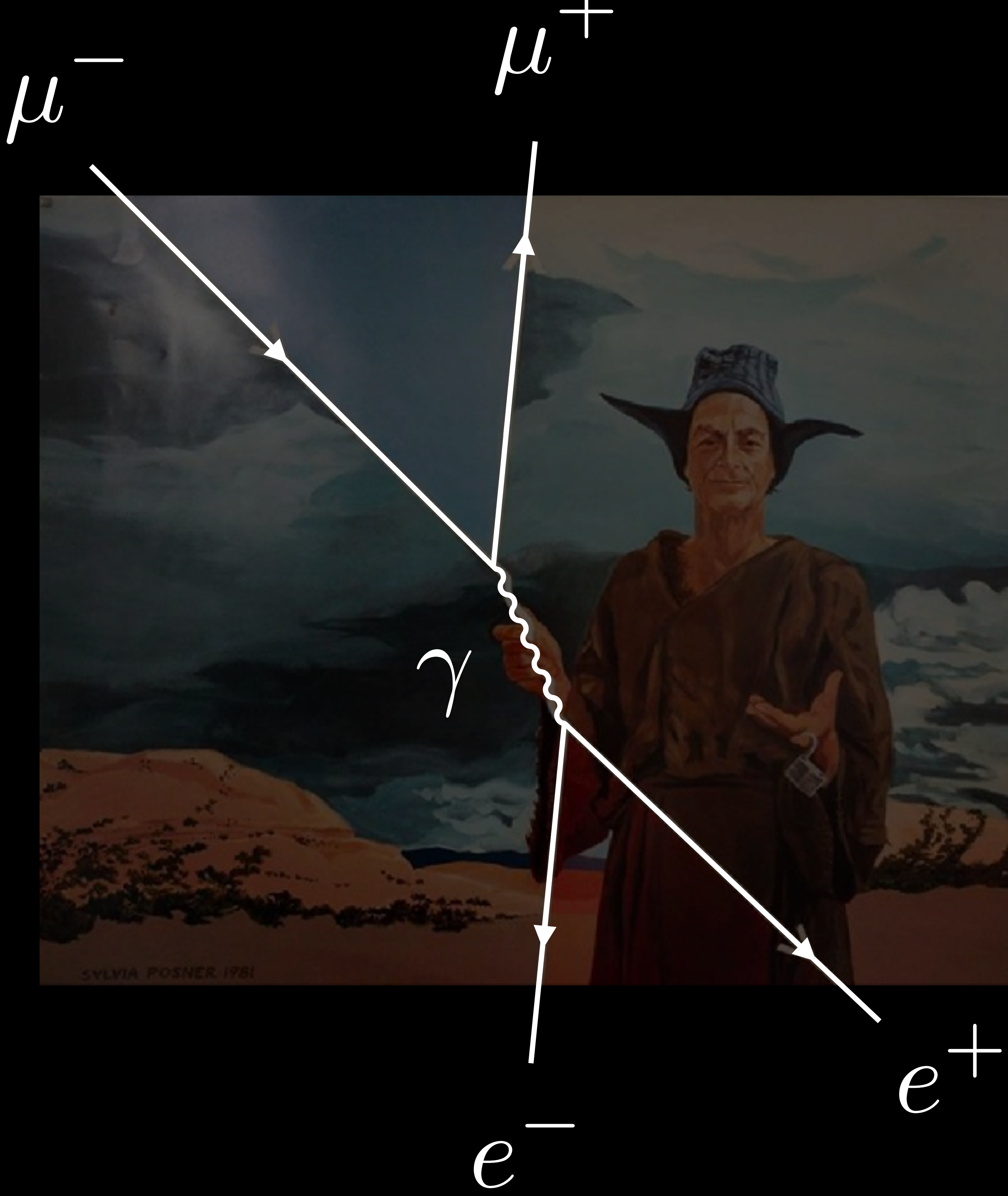
physical principles

~~Lagrangian, Hamiltonian, equations of motion, etc.~~

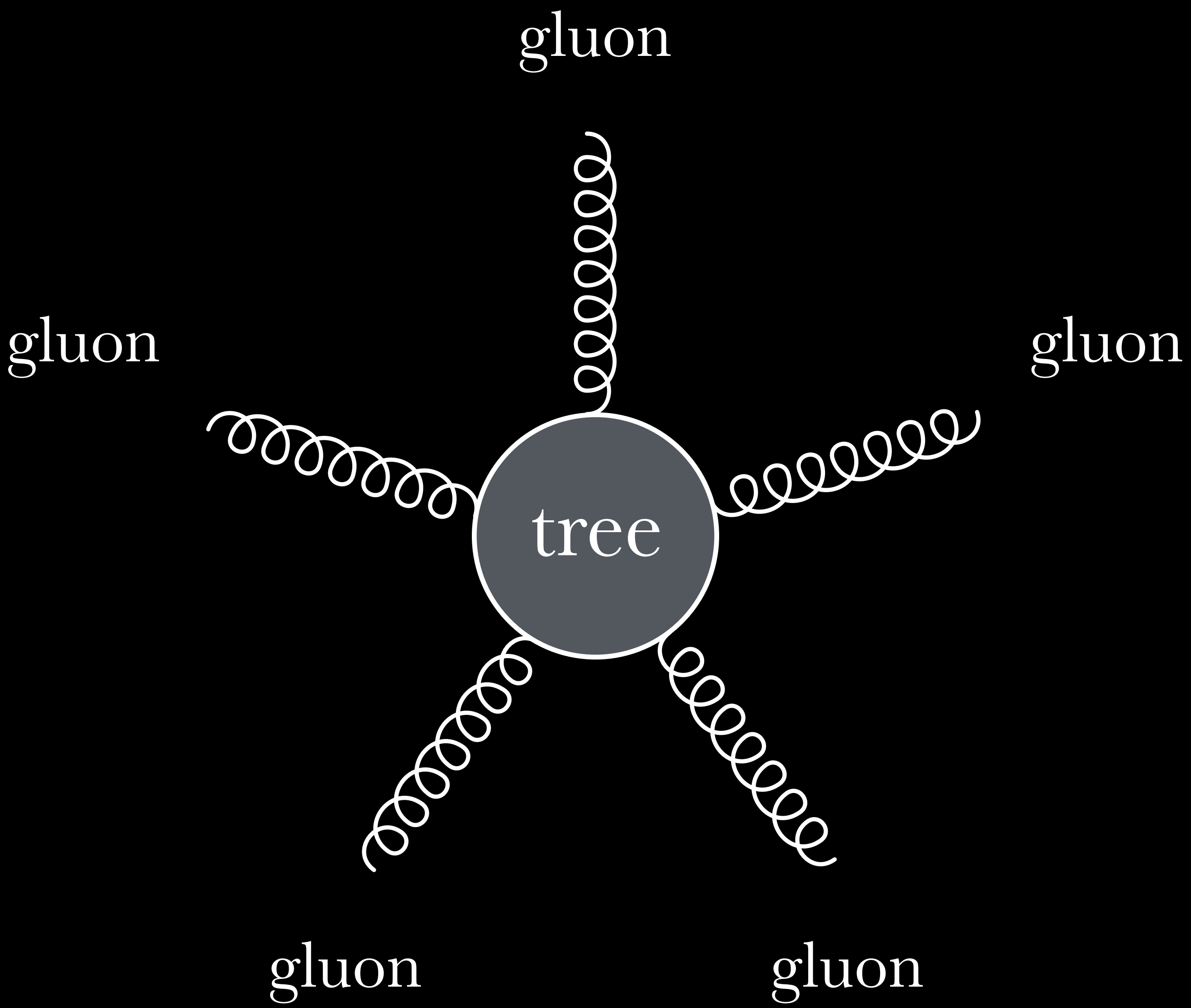
modern scattering amplitudes program

observables

Observables are computed with Feynman diagrams.


μ^-
μ^+
γ
e^-
e^+
SYLVIA POSNER 1981

But Feynman diagrams are *immensely* complicated.



5pt gluon amplitude

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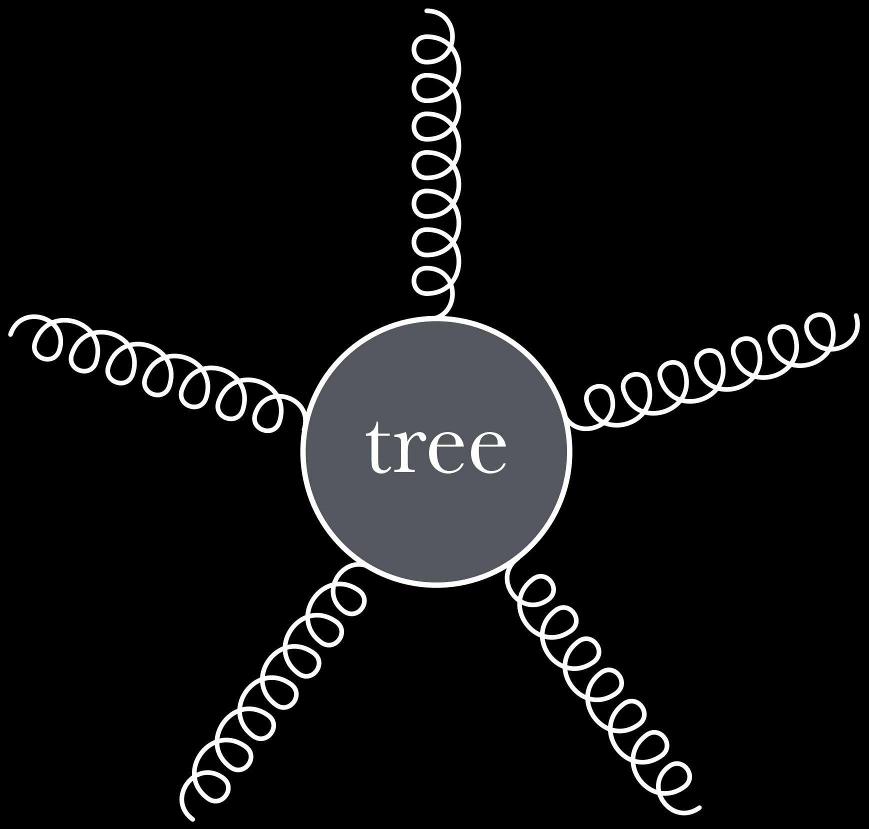

$$A(1^{h_1}2^{h_2}3^{h_3}4^{h_4}5^{h_5}) \quad =$$

$$\sim \frac{(e_1 \cdot e_5)(p_1 \cdot e_2)(p_2 \cdot e_3)(p_3 \cdot e_4)}{(p_1 + p_2)^2(p_4 + p_5)^2} + \dots$$

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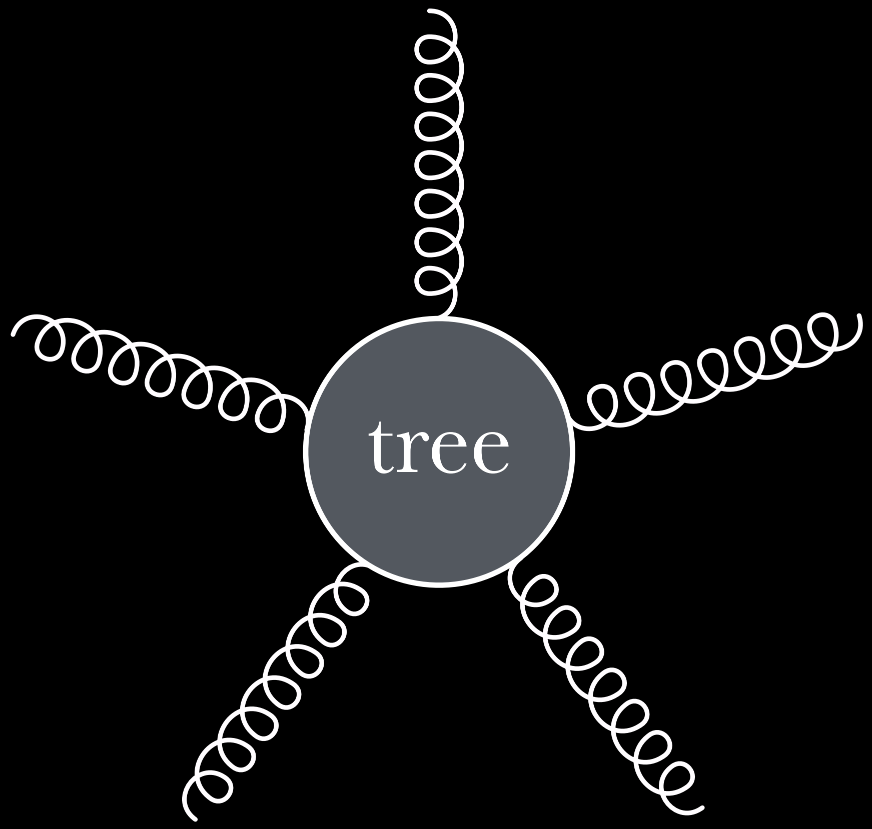
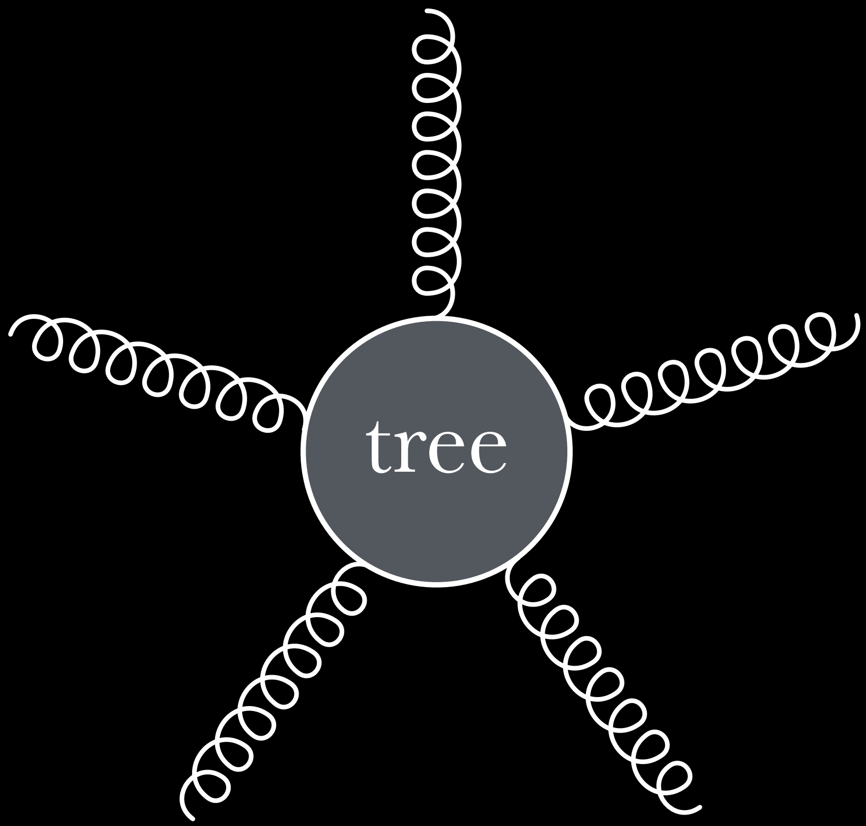


$$A(1^+ \, 2^+ \, 3^+ \, 4^+ \, 5^+ \,) \quad = \quad 0$$

But Feynman diagrams are *immensely* complicated.



$$A(1^+\,2^+\,3^+\,4^+\,5^+) = 0$$

$$A(1^-\,2^+\,3^+\,4^+\,5^+) = 0$$

Park, Taylor (1986)

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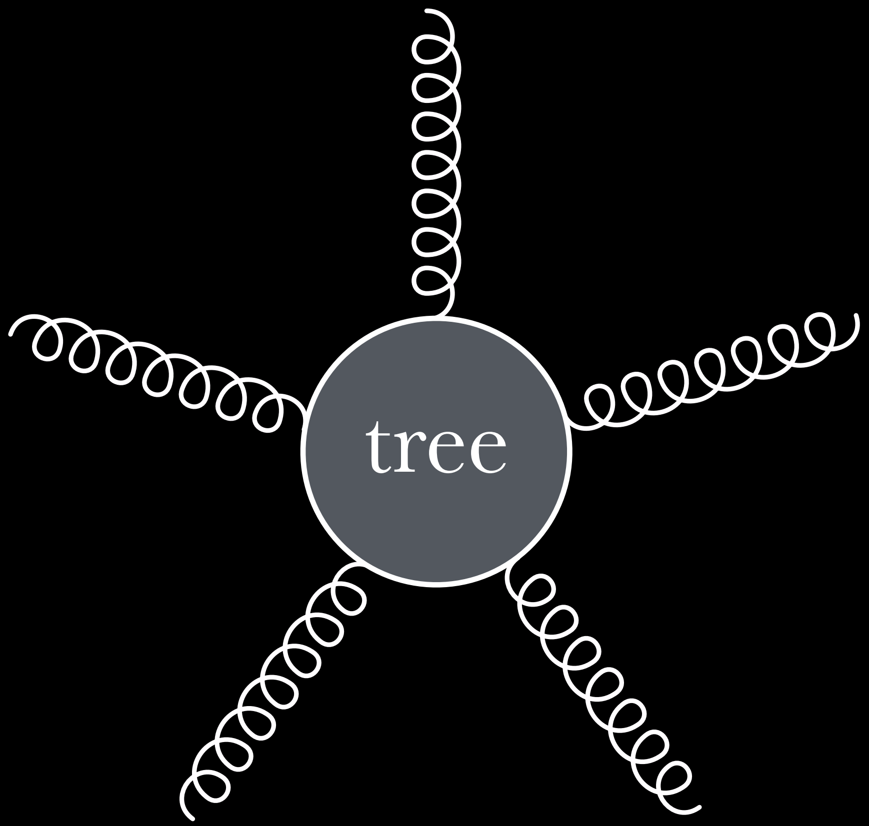

$$A(1^+\,2^+\,3^+\,4^+\,5^+\,) \quad = \quad 0$$

$$A(1^-\,2^+\,3^+\,4^+\,5^+\,) \quad = \quad 0$$

$$A(1^-\,2^-\,3^+\,4^+\,5^+\,) \quad = \quad \frac{\langle 12\rangle^3}{\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle}$$

Park, Taylor (1986)

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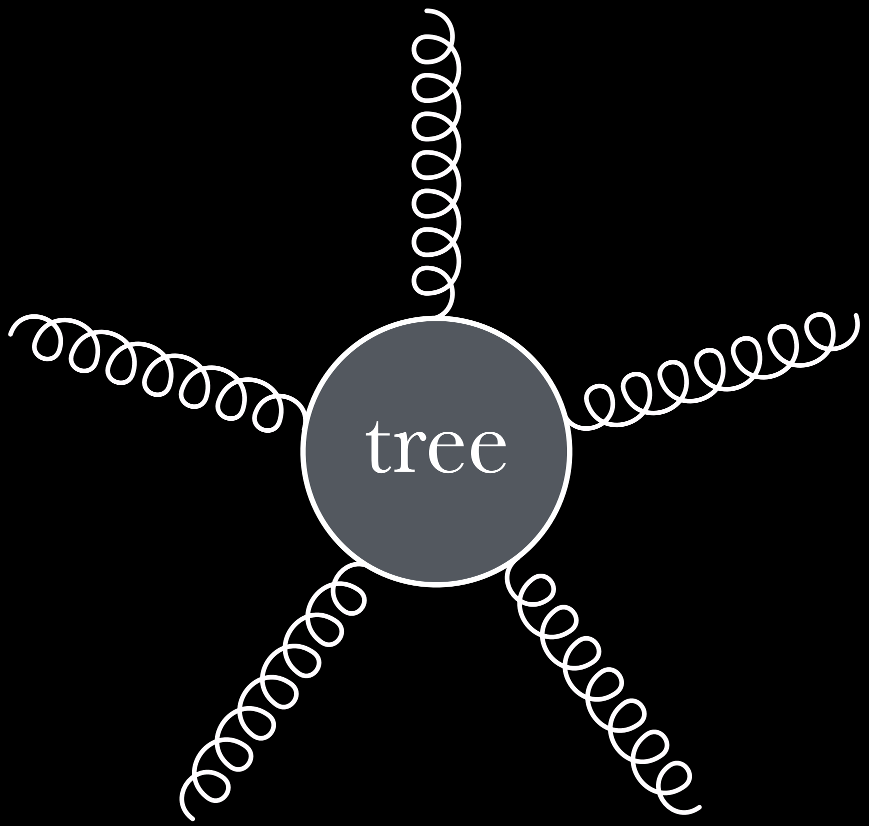

use the *right* variables

$$p_i^\mu = \sigma^\mu_{\alpha\dot{\alpha}} \lambda_i^\alpha \bar{\lambda}_i^{\dot{\alpha}}$$

$$\langle ij \rangle = \lambda_i^\alpha \lambda_j^\beta \epsilon_{\alpha\beta}$$

$$[ij] = \bar{\lambda}_i^{\dot{\alpha}} \bar{\lambda}_j^{\dot{\beta}} \epsilon_{\dot{\alpha}\dot{\beta}}$$

$$A(1^+\, 2^+\, 3^+\, 4^+\, 5^+\,) \quad = \quad 0$$

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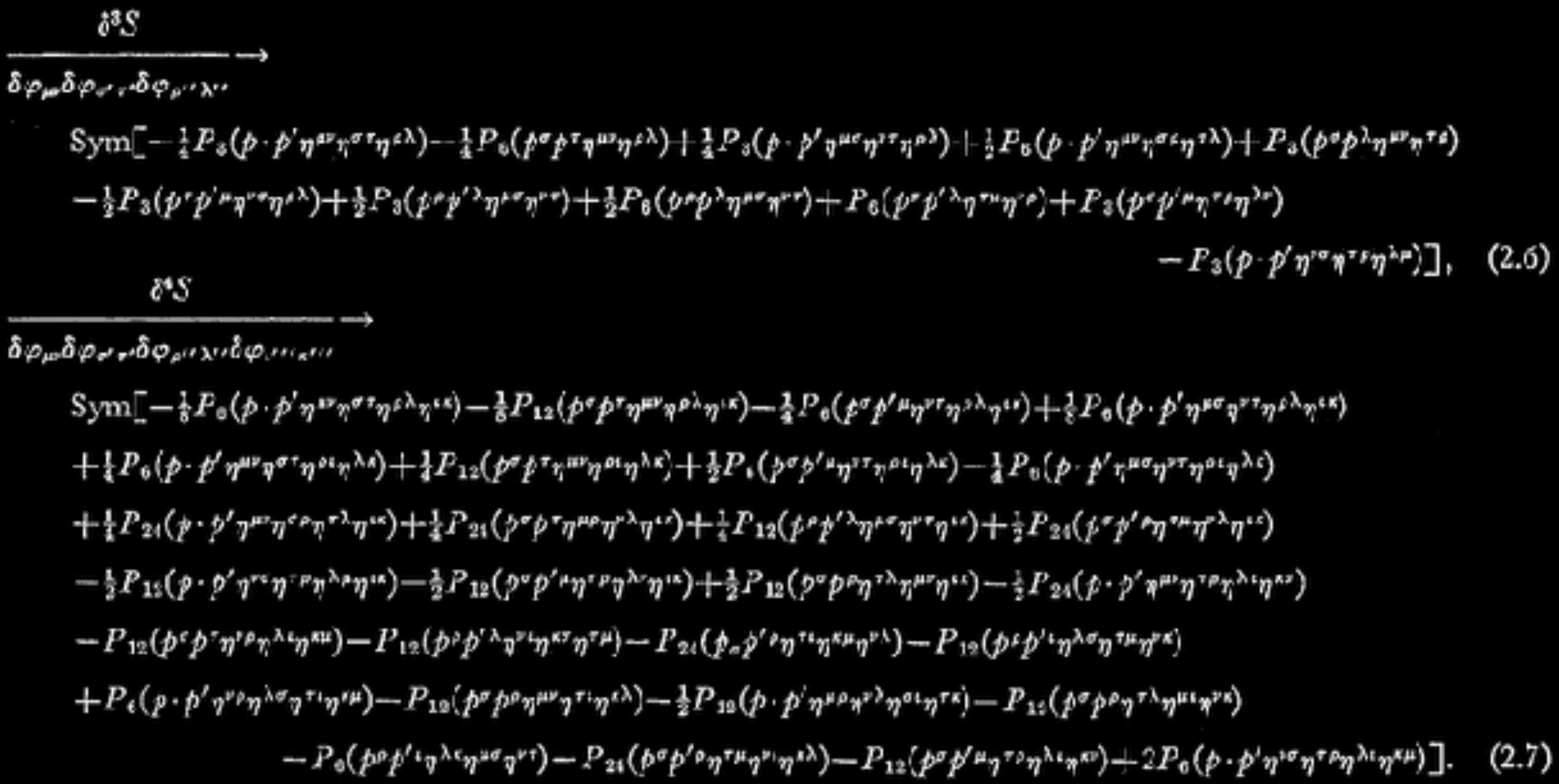
Park, Taylor (1986)

A simplification also happens for gravitons.



3pt graviton
Feynman vertex

4pt graviton
Feynman vertex

4pt graviton amplitude

$$M(1^- \; 2^- \; 3^+ \; 4^+ \,) \quad = \quad \frac{\langle 12\rangle^4 [34]^4}{stu}$$

DeWitt (1967)

Why the unnecessary complexity?

This complexity arises from gauge "symmetries" of the action which are in truth redundancies.

$$V \rightarrow V - \frac{\partial\theta}{\partial t}$$

$$\vec{A} \rightarrow \vec{A} + \vec{\nabla}\theta$$

High Voltage Differential!

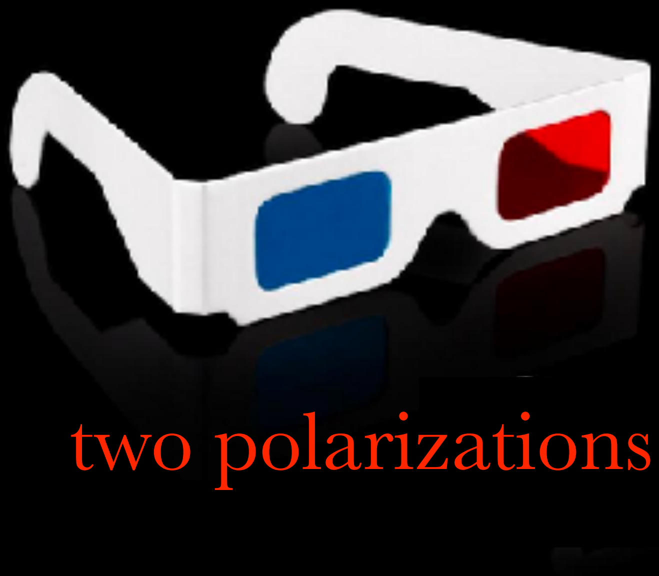
But the physically observable fields are invariant.

$$\vec{E} \rightarrow \vec{E} \qquad \vec{B} \rightarrow \vec{B}$$

To manifest locality and Lorentz invariance we introduce a four-vector field.

$$A_\mu = (V, \vec{A}) \qquad \longrightarrow$$

four degrees of freedom



two polarizations

We are *forced* to invent a gauge symmetry in order to remove the extra degrees of freedom.

A byproduct of gauge symmetry is complexity that worsens at higher orders in perturbation theory.

n-pt	4	5	6	7	8
Feynman diagrams	4	25	220	2485	34300
recursion relations	1	1	3	6	20

Even worse, some *non-symmetries* of the action are also redundancies of description.

"interacting" scalar theory

$$\mathcal{L} = \frac{(\partial\phi)^2}{2} \times \left(1 + \lambda_1 \phi + \frac{1}{2!}\lambda_2 \phi^2 + \cdots\right)$$

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"interacting" scalar theory

$$\mathcal{L} = \frac{(\partial\phi)^2}{2} \times \left(1 + \lambda_1\phi + \frac{1}{2!}\lambda_2\phi^2 + \cdots\right)$$

All scattering amplitudes vanish!

free scalar theory

$\phi \to g(\phi)$ sends $\mathcal{L} \to \frac{(\partial\phi)^2}{2}$

On-shell amplitudes are field basis invariant.

$$Z[J] \sim \int [\overset{\substack{\text{“integration variable”} \\ \downarrow}}{d\phi}]\, e^{iS[\phi]+i\int J\phi}$$

So many Lagrangians yield the same observables!

$$\begin{tikzcd}
\mathcal{L}_1 \arrow[drrr] & & \mathcal{L}_2 \arrow[dr] & & \cdots \arrow[dl] & & \mathcal{L}_\infty \arrow[dlll] \\
& & & A & & &
\end{tikzcd}$$

Bottom line: the action obscures important physics.

- absolute rigidity of certain QFTs

“only of all possible worlds…”

- hidden duality and unity of QFTs

“gravity as the mother of all theories…”

rigidity of QFTs

Nature conforms to at least two physical criteria:

- Poincare Invariance
- Locality

These principles *uniquely* define certain theories!

gluon = self-interacting massless spin 1
graviton = self-interacting massless spin 2

The 3pt gluon and graviton amplitudes are fixed.

gluon

$$A(1^-2^-3^+) = \frac{\langle 12\rangle^3}{\langle 13\rangle\langle 32\rangle}$$

$$A(1^+2^+3^-) = \frac{[12]^3}{[13][32]}$$

graviton

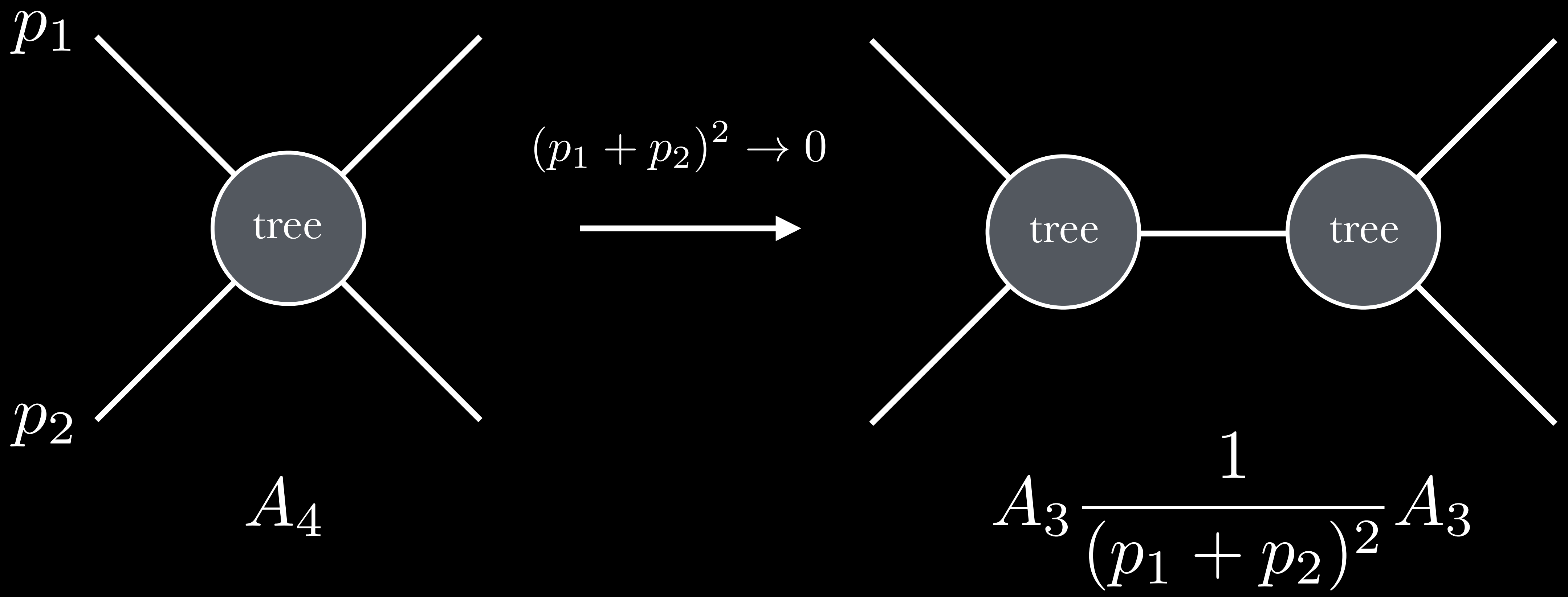
$$M(1^-2^-3^+) = \frac{\langle 12\rangle^6}{\langle 13\rangle^2\langle 32\rangle^2}$$

$$M(1^+2^+3^-) = \frac{[12]^6}{[13]^2[32]^2}$$

simplest example of "gluon^2 = graviton"

Benincasa, Cachazo (0705.4305)

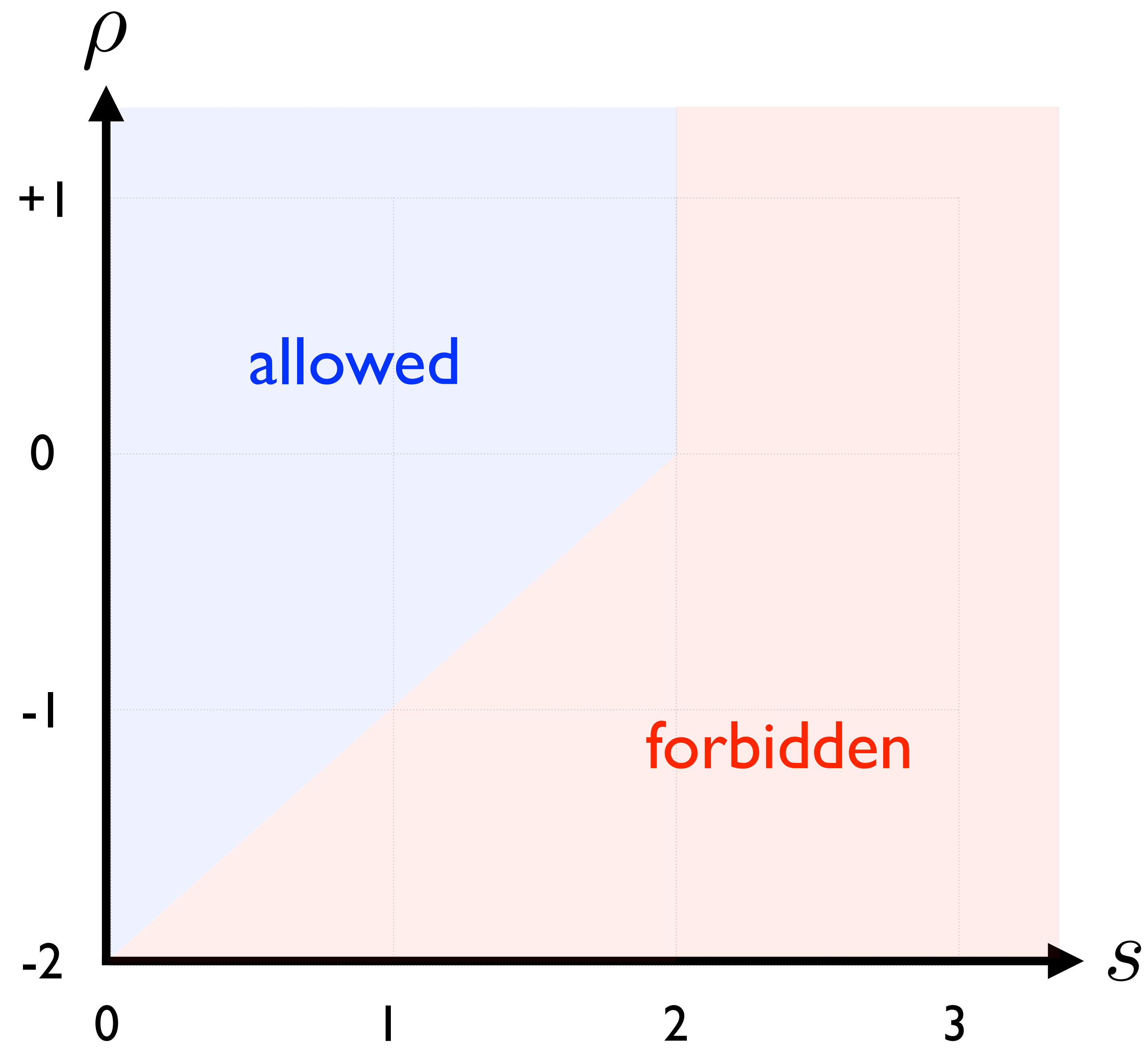
The 4pt amplitudes and up are fixed by locality.



Recursion relations automate this construction by relating higher point to lower point amplitudes.

Britto, Cachazo, Feng, Witten (hep-th/0412308, hep-th/0501052)

Let us draw a theory “map” for massless particles.



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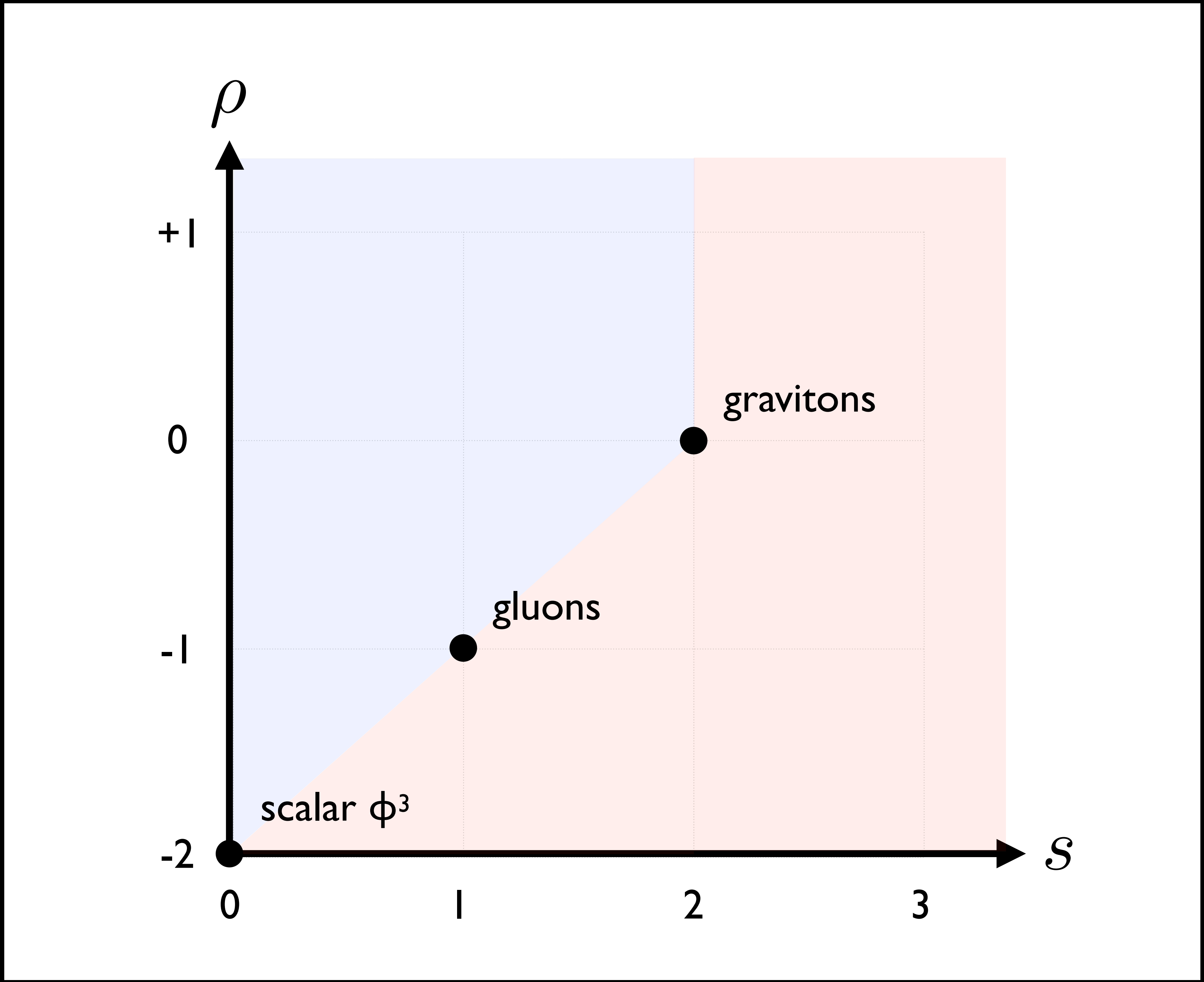


What property of the amplitude fixes an EFT?

- Poincare Invariance
- Locality
- ? ? ?

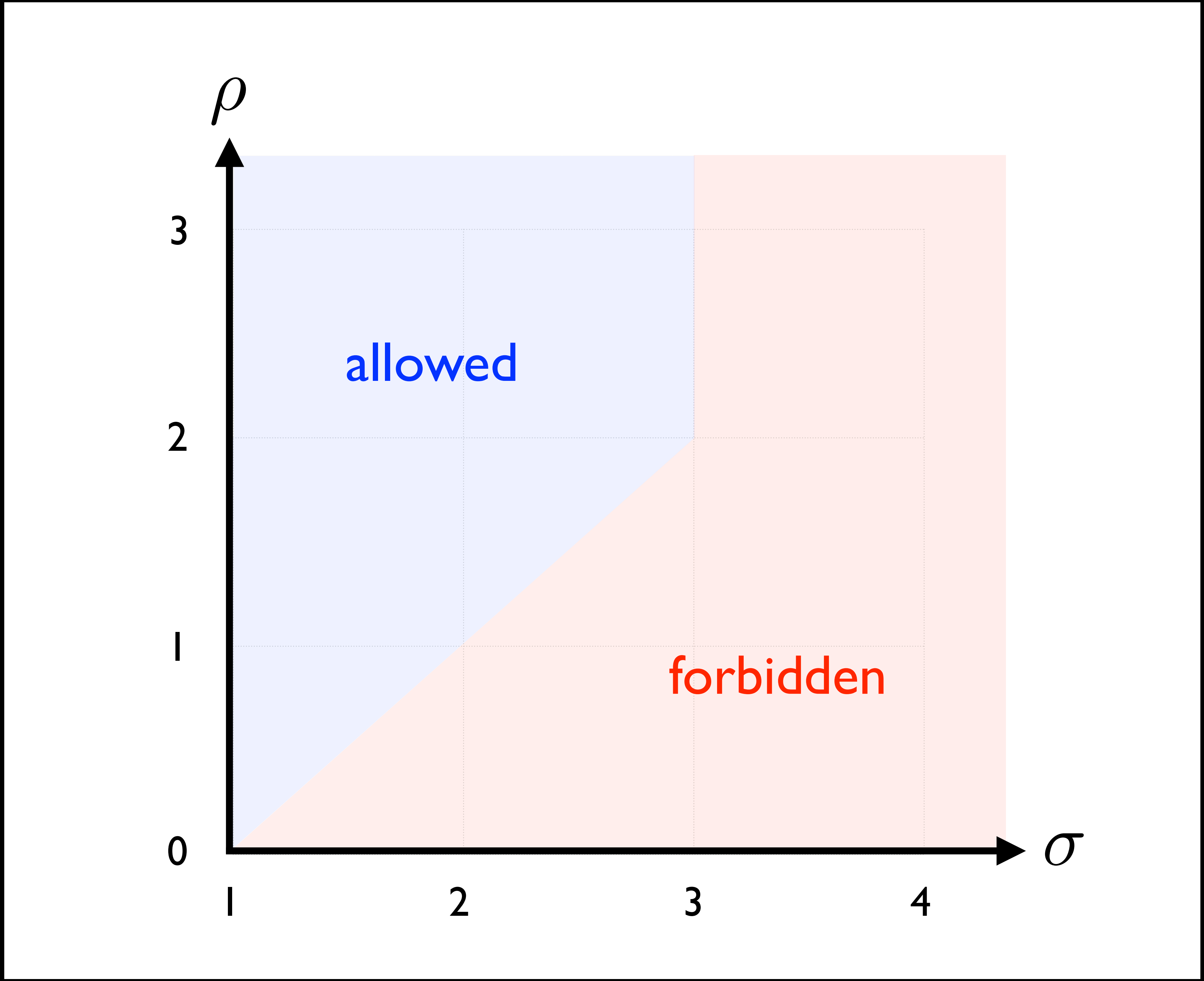
EFTs are typically built around symmetry breaking.
What we can do without an action?

What property of the amplitude fixes an EFT?

- Poincare Invariance
- Locality
- Infrared Structure

EFTs are typically built around symmetry breaking.
What we can do without an action?

Repeating this procedure yields a “map” of EFTs!



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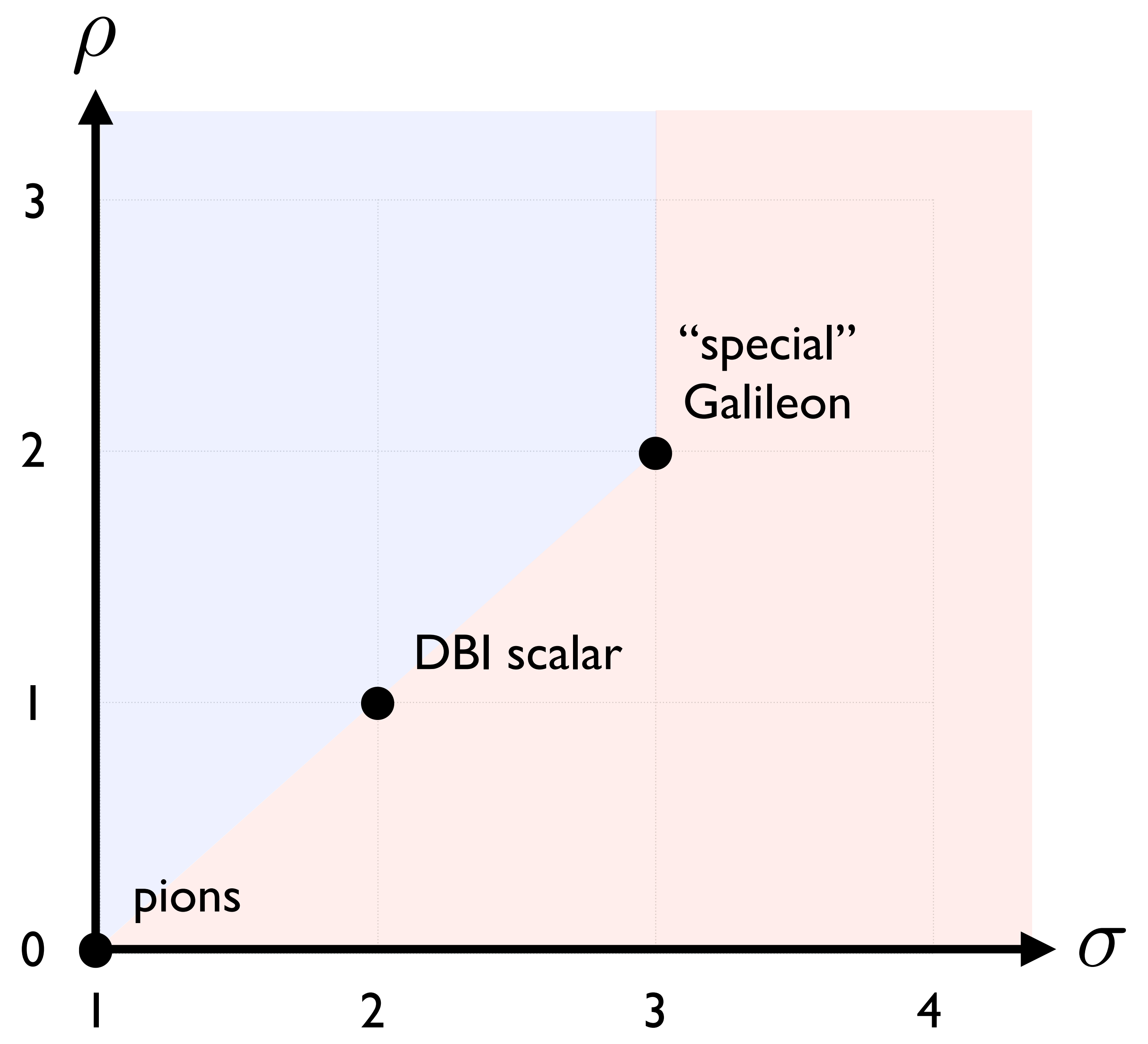
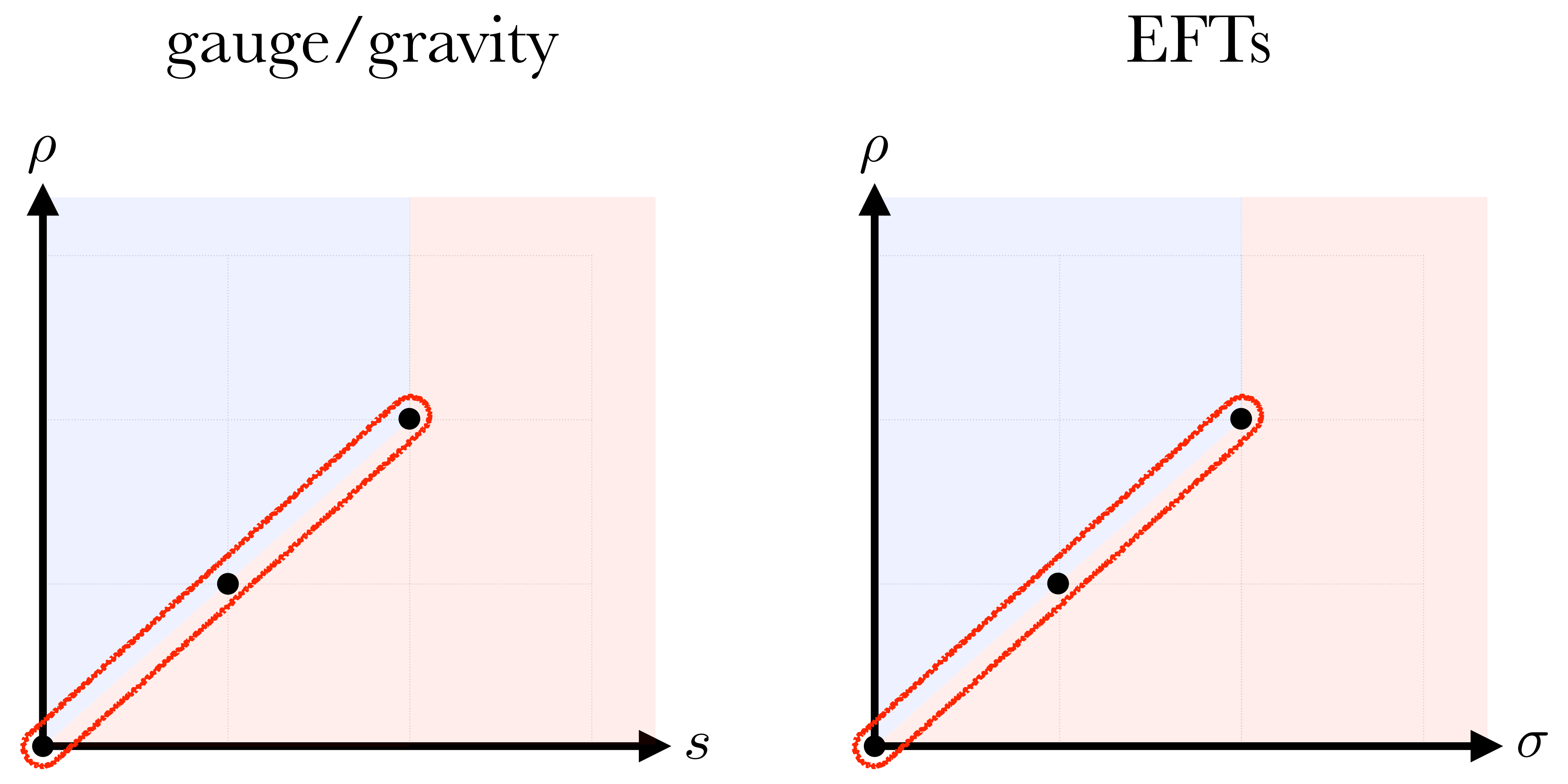


Tree-level amplitudes of a huge class of theories are constructible via on-shell recursion.

- Yang-Mills theory, gravity
- SUSY theories, standard model
- renormalizable theories in 4D
- EFTs for pions, DBI scalars, Galileons

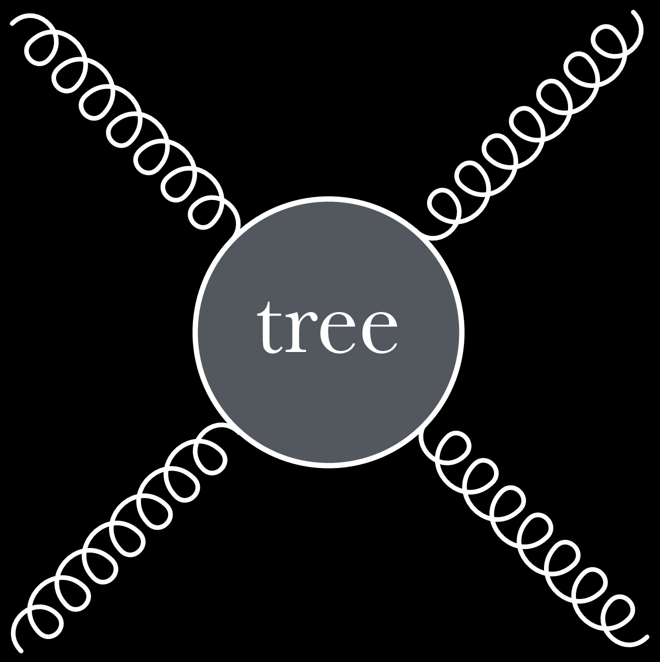
CC, Kampf, Novotny, Shen, Trnka (1509.03309)

The “edge” theories are *maximally constrained* and moreover *secretly connected* across maps!



duality of QFTs

Bern-Carrasco-Johansson (BCJ) showed that gluons have a duality between color and kinematics.



$$= \quad \frac{c_s n_s}{s} + \frac{c_t n_t}{t} + \frac{c_u n_u}{u}$$

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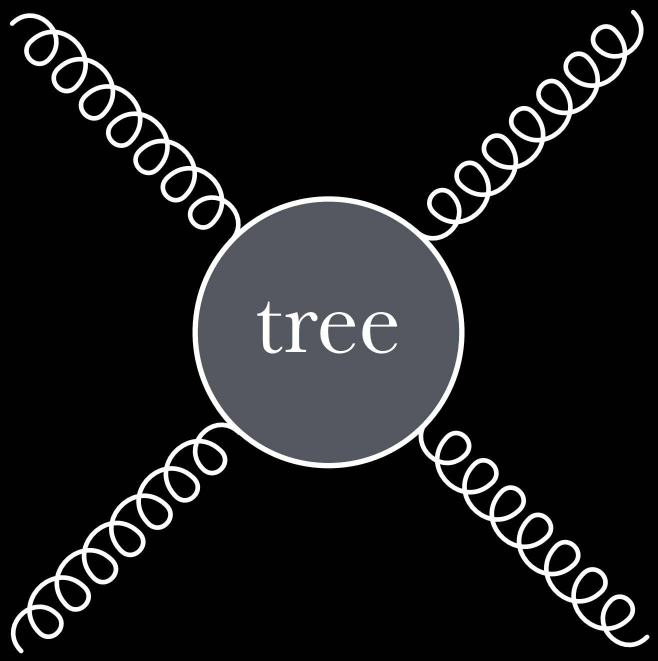

$$= \frac{c_s n_s}{s} + \frac{c_t n_t}{t} + \frac{c_u n_u}{u}$$

$f_{a_1 a_2 b} f_{a_3 a_4 b}$ $\quad$ $f_{a_2 a_3 b} f_{a_1 a_4 b}$ $\quad$ $f_{a_3 a_1 b} f_{a_2 a_4 b}$

$$c_s + c_t + c_u = 0$$

(automatic)

$\sim (e_1 \cdot e_2)(e_3 \cdot e_4)(p_2 \cdot p_3) + \ldots$

$$n_s + n_t + n_u = 0$$

(not automatic)

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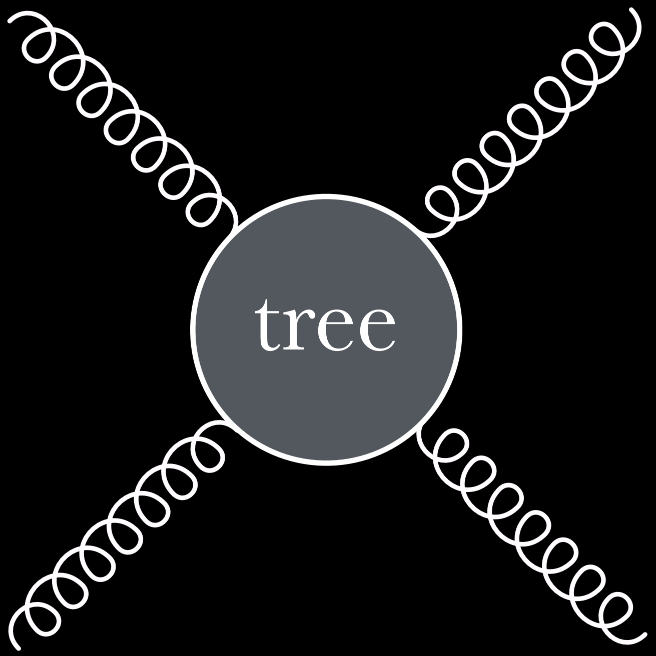

$$= \frac{c_s n_s}{s} + \frac{c_t n_t}{t} + \frac{c_u n_u}{u}$$

substitute color
for kinematics

$$\frac{n_s^2}{s} + \frac{n_t^2}{t} + \frac{n_u^2}{u}$$

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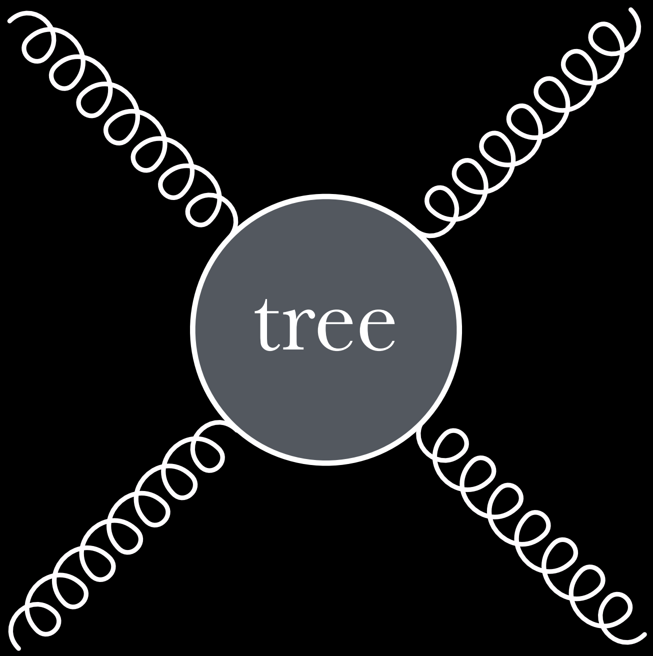
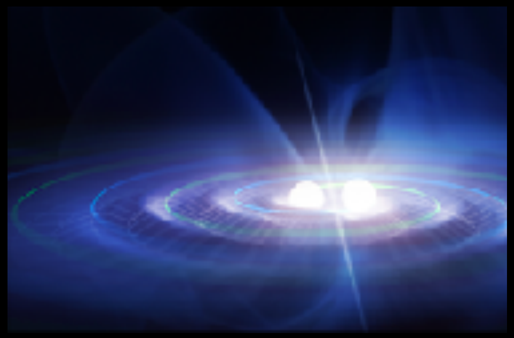


$$= \frac{c_s n_s}{s} + \frac{c_t n_t}{t} + \frac{c_u n_u}{u}$$

$\downarrow$ substitute color for kinematics $\downarrow$

this is the 4pt graviton scattering amplitude!!! $\Rrightarrow$ $$\frac{n_s^2}{s} + \frac{n_t^2}{t} + \frac{n_u^2}{u}$$

$($  $) \otimes ($  $) =$ 

gluon
amplitude

gluon
amplitude

graviton
amplitude

The “double copy” is proven at tree-level, verified at loop-level, and applicable to numerous QFTs!

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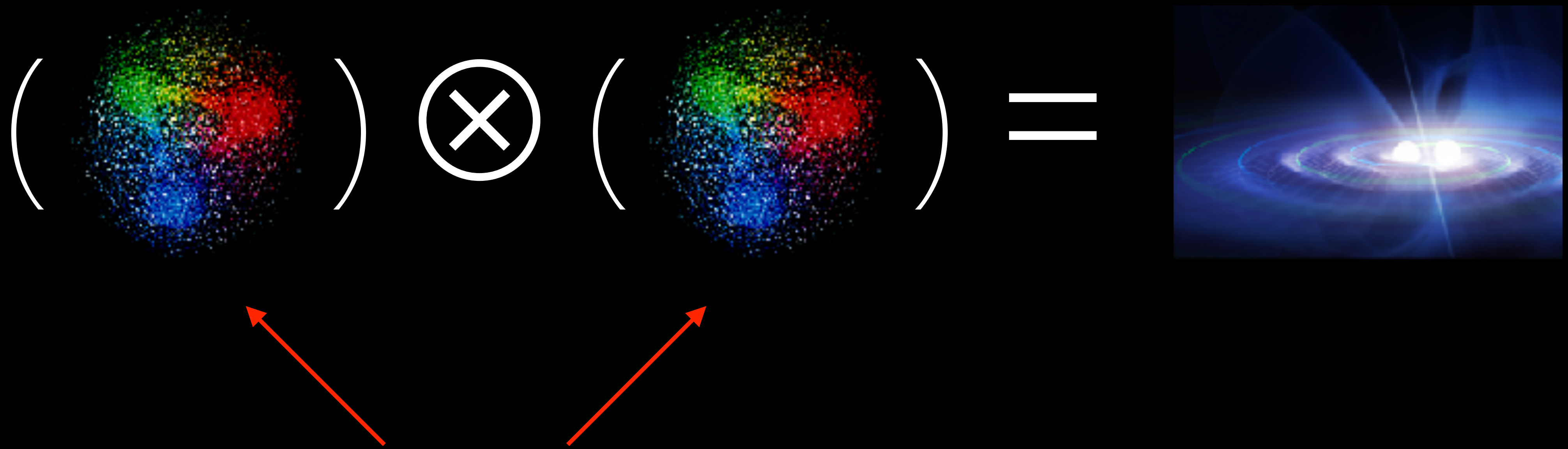
spin s_1 particle w/ multiplicity $\otimes$ spin s_2 particle w/ multiplicity $=$ spin $s_1 + s_2$ particle w/o multiplicity

gluon $\otimes$ gluon $=$ graviton

pion $\otimes$ pion $=$ special Galileon

gluon $\otimes$ pion $=$ Born-Infeld photon

This structure has remarkable implications for the underlying structure of gravitational theories.

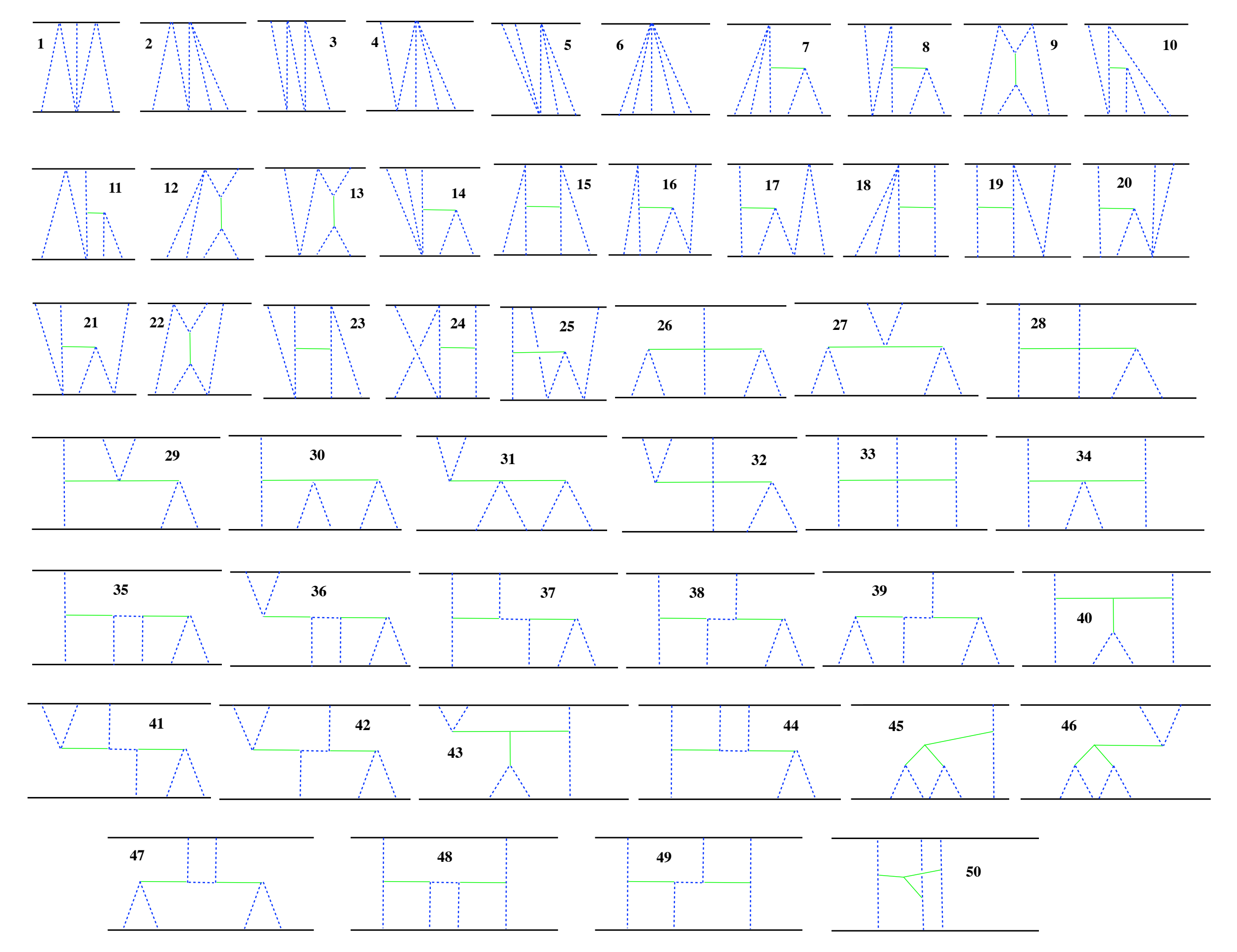


each gluon factor is
separately Lorentz invariant

Graviton amplitudes are *doubly* Lorentz invariant.
This can be manifested in gravity action!

Cheung, Remmen (1612.03927,1705.00626)

The double copy, together with modern methods, should also simplify post-Newtonian calculations.

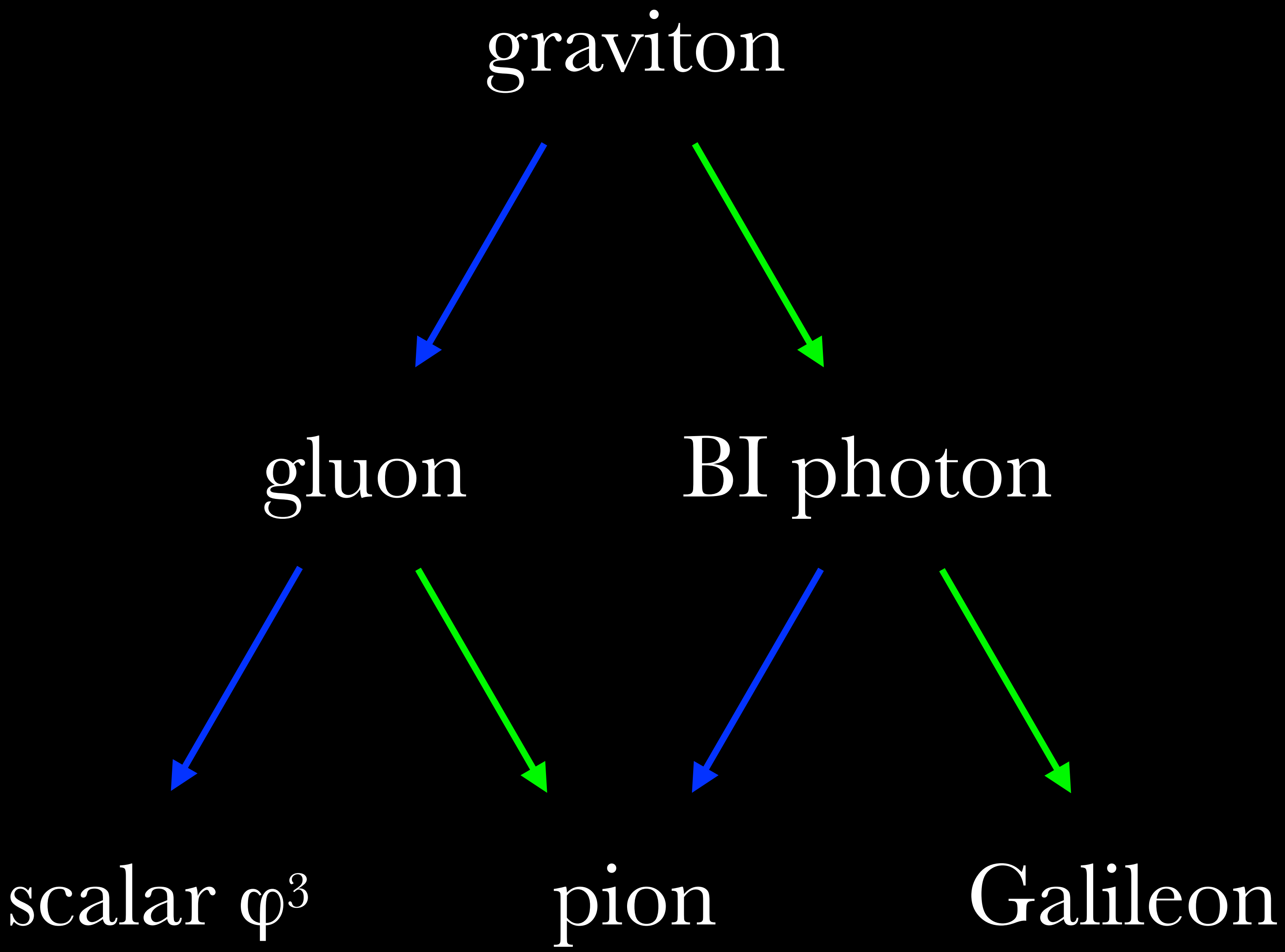


is there an easier way ?

Foffa, Mastrolia, Sturani, Sturm (1612.00482)

unity of QFTs

Amplitudes encode a hidden unity of theories!



graviton

gluon

BI photon

scalar φ^3

pion

Galileon

Treating the amplitude as an abstract function, we obtain relations from *simple* differential operators.

$$A = \text{function of } \{(p_i \cdot p_j), (p_i \cdot e_j), (e_i \cdot e_j)\}$$

$$\text{“}\longrightarrow\text{”} = \mathcal{T}_{ijk} = \frac{\partial}{\partial(p_i \cdot e_j)} - \frac{\partial}{\partial(p_k \cdot e_j)}$$

$$\text{“}\longrightarrow\text{”} = \mathcal{L}_i = \sum_j (p_i \cdot p_j) \frac{\partial}{\partial(p_j \cdot e_i)}$$

Cheung, Remmen, Shen, Wen (1705.05025,1709.04932)

Properties of gravity are inherited by descendants.

$$\begin{tikzcd}
\begin{pmatrix}\text{gluon}\\ \text{amplitude}\end{pmatrix} \arrow[d] & \otimes & \begin{pmatrix}\text{gluon}\\ \text{amplitude}\end{pmatrix} \arrow[d] & = & \begin{matrix}\text{graviton}\\ \text{amplitude}\end{matrix} \arrow[d] \\
\begin{pmatrix}\text{pion}\\ \text{amplitude}\end{pmatrix} & \otimes & \begin{pmatrix}\text{pion}\\ \text{amplitude}\end{pmatrix} & = & \begin{matrix}\text{Galileon}\\ \text{amplitude}\end{matrix}
\end{tikzcd}$$

Properties of gravity are inherited by descendants.

graviton general covariance

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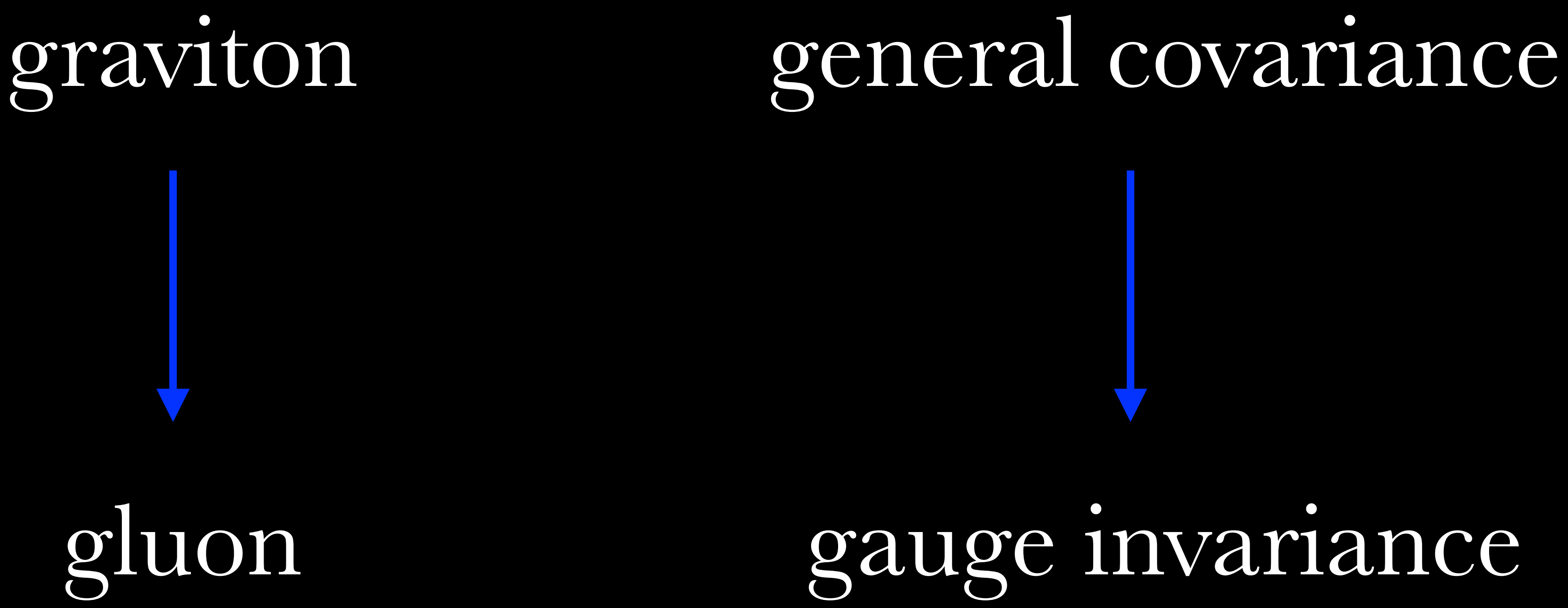


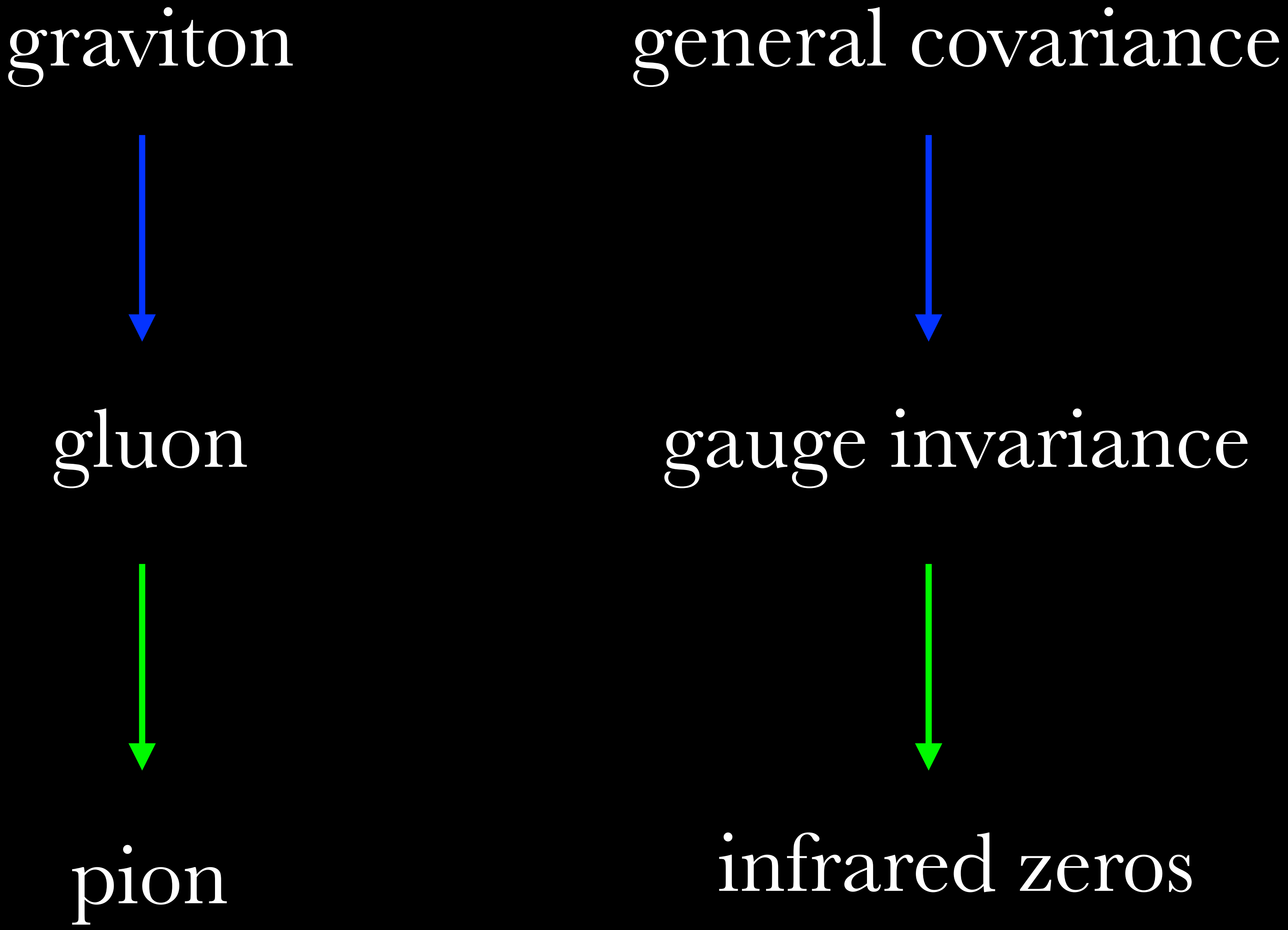
graviton

gluon

general covariance

gauge invariance

Properties of gravity are inherited by descendants.



graviton

gluon

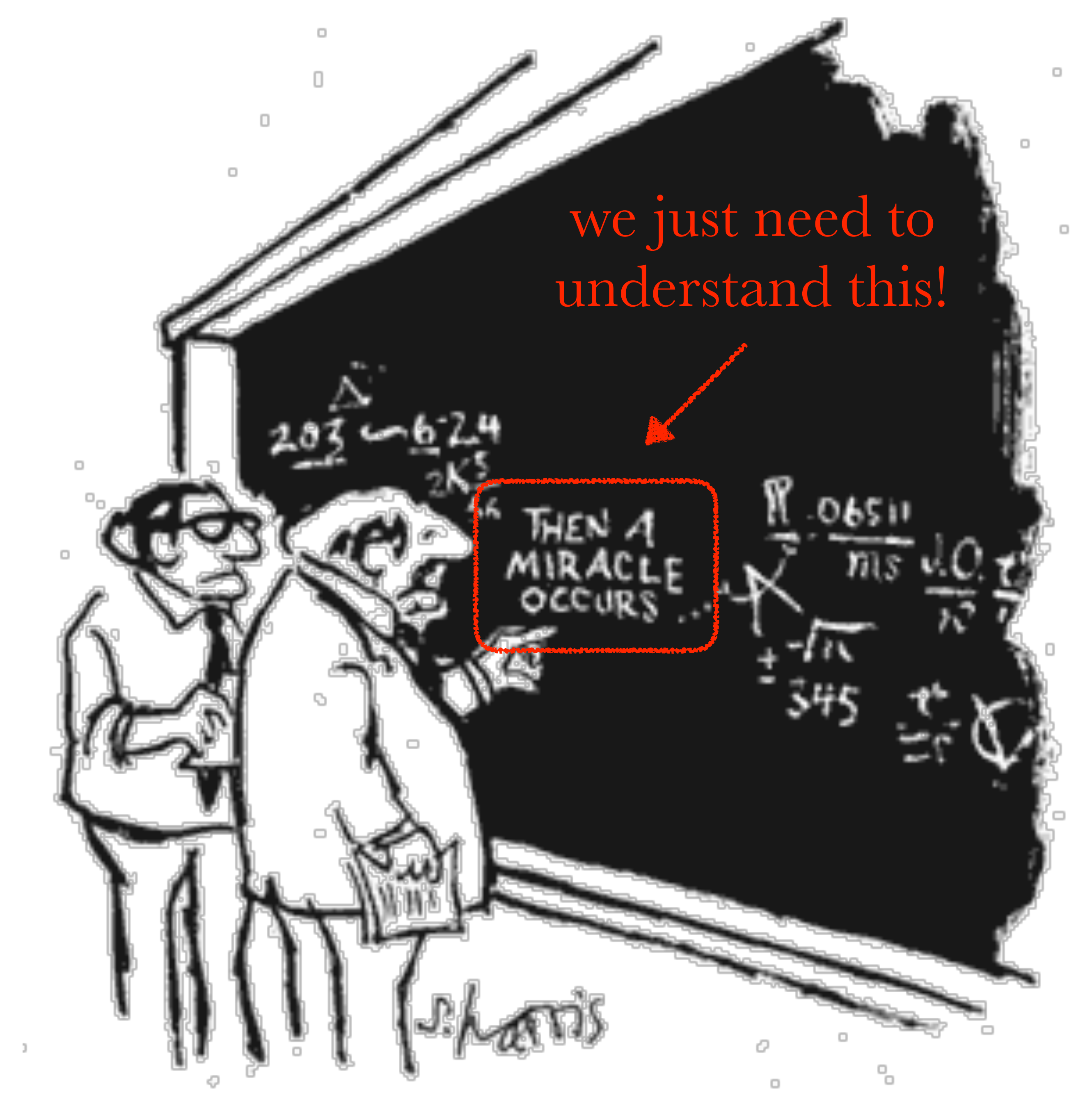
pion

general covariance

gauge invariance

infrared zeros

Despite immense progress, many question remain!



we just need to understand this!

stay tuned!