



# Reconciling lepton flavor violation and the muon anomalous magnetic moment

COFI workshop 2018, 05/22/2018, San Juan, PR

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Manfred Lindner, Moritz Platscher, Farinaldo S. Queiroz

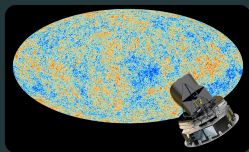
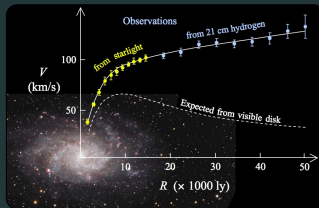
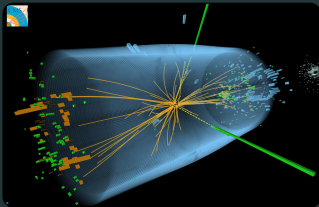
based on Phys. Rept. 731, (2018) 1-82 (arXiv:1610.06587 [hep-ph])



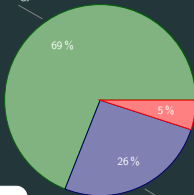
# Introduction

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# Introduction – New physics around the corner?



Dark Energy

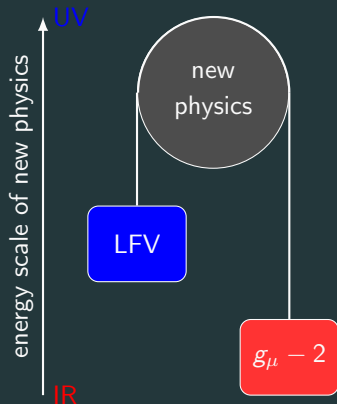


Baryonic Matter

Dark Matter



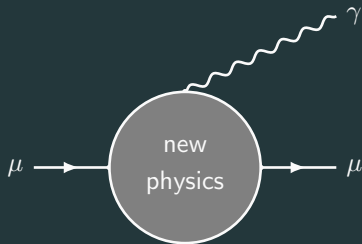
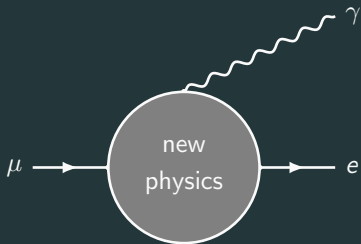
# Introduction – New physics around the corner?



- Non-observation of LFV:  
high NP scale
- $g_{\mu} - 2$  anomaly:  
rather low NP scale
- What to make of this?
  - $g_{\mu} - 2$  excess confirmed  
⇒ **should we see LFV?**
  - no  $g_{\mu} - 2$  excess  
⇒ **constraints on LFV!**

# Introduction – Summary

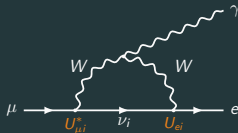
- We see no definite sign of new physics at colliders (UV experiments)
- But new physics is needed:  $\nu$  masses, DM, DE,  $B$  asymmetry, ...
- Maybe  $\Lambda \gg m_{\text{Higgs}}$ 
  - ⇒ Need a “telescope” to look at the distant physics!
  - ⇒ Lepton flavor violation (LFV) and
  - ⇒ Leptonic anomalous magnetic moments ( $g - 2$ ) are very sensitive
- Both are related:



# Introduction – Standard Model

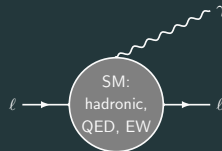
$\mu \rightarrow e\gamma$ :

- forbidden in SM with  $m_\nu = 0$ ,
- $m_\nu \neq 0$  implies **LFV** by neutrino flavor conversion
- **charged LFV**:  
 $BR(\mu \rightarrow e\gamma) \sim 10^{-55}$  due to tiny neutrino mass



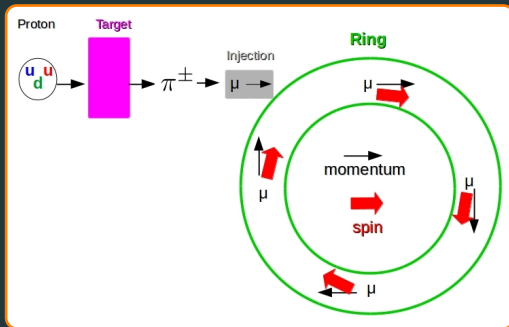
$(g - 2) \propto m_{e/\mu/\tau}^2$ :

- electron: very precise measurement of fine structure constant  $\alpha_{em}$
- muon:  $3.3\sigma$  discrepancy between SM prediction and measurement
- tau: very short life-time, but most sensitive to NP



# Introduction – current status

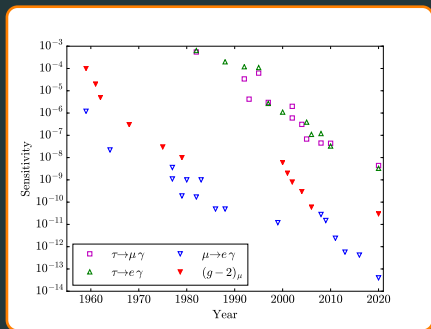
measurement principle:



spin-precession frequency:

$$\vec{\omega}_{a_\mu} = \frac{e}{m_\mu} \left[ (g_\mu - 2) \vec{B} - \underbrace{\left( a_\mu - \frac{1}{\gamma^2 - 1} \right)}_{=0 \text{ for "magic" } \gamma=29.3} \vec{v} \times \vec{E} \right],$$

# Introduction – current status



- $\mu \rightarrow e \gamma$ : reached unprecedented precision: [MEG]  
 $BR(\mu \rightarrow e \gamma) \leq 4 \cdot 10^{-13}$
- $(g_\mu - 2)$ : 3.3  $\sigma$  excess over SM prediction:  
 $\Delta a_\mu = 288 \cdot 10^{-11}$  [BNL E821]

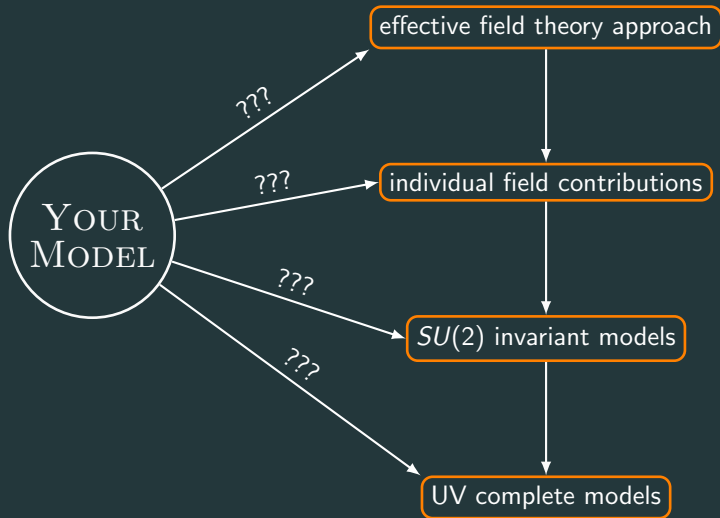
Maybe we are at the verge of seeing new physics in  $\mathcal{O}(1 \dots 10)$  years!



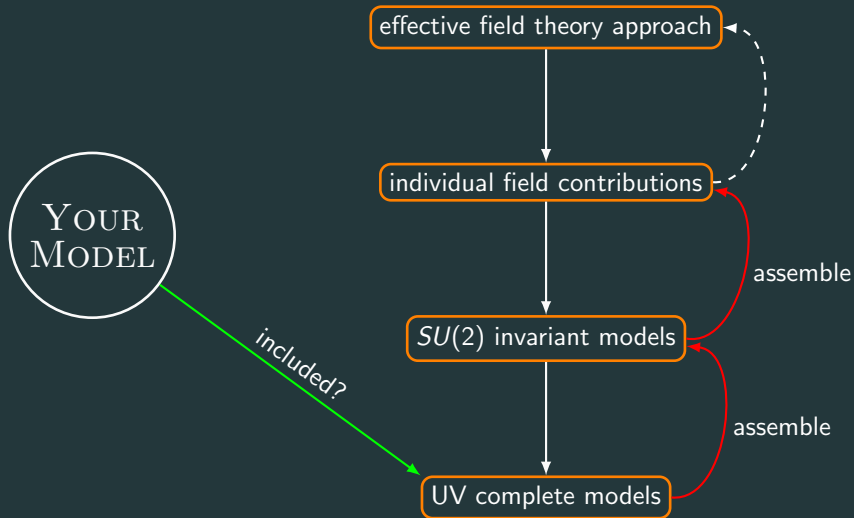
## Our work

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# How to use our paper



# How to use our paper



## The general approach – EFTs

Start from an EFT point of view (NP  $\Rightarrow d = 6$  operators):

$$\mathcal{L}_{\text{eff}} = \frac{\mu_{ij}^M}{2} \bar{l}_i \sigma^{\mu\nu} l_j F_{\mu\nu} + \frac{\mu_{ij}^E}{2} \bar{l}_i i\gamma^5 \sigma^{\mu\nu} l_j F_{\mu\nu} + \text{off-shell contributions}$$

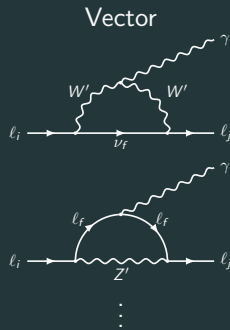
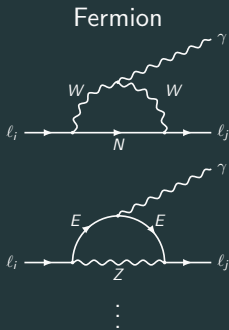
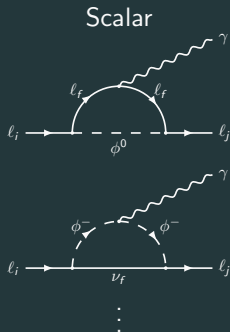
Consider form factors  $\mu_{ij}^{M/E} \equiv e m_i A_{ij}^{M/E} / 2$ :

$$\Rightarrow \Delta a_{l_i} \equiv (g - 2)/2 - (g - 2)_{\text{SM}}/2 = A_{ii}^M m_i^2 \text{ (no sum)}$$

$$\Rightarrow BR(l_i \rightarrow l_j \gamma) = \frac{3(4\pi)^3 \alpha_{\text{em}}}{4G_F^2} \left( |A_{ji}^M|^2 + |A_{ji}^E|^2 \right) BR(l_i \rightarrow l_j \nu_i \bar{\nu}_j)$$

# The general approach – simplified models

Calculate contributions to  $A^{E/M}$  from **one** field of spin  $s = 0, 1/2, 1$  and electric charge  $Q = 0, 1, 2$  coupling to SM leptons:



# The general approach – $SU(2)_L$ invariant models

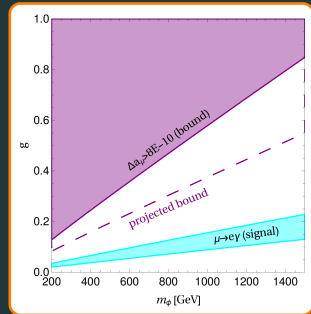
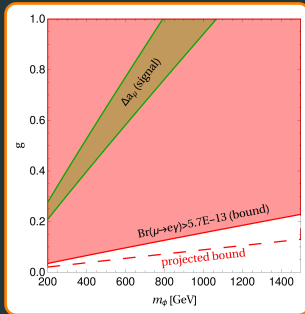
An example: scalar doublet  $\phi = (\phi^+, \phi^0)^T$ ,

$$\mathcal{L}_{\text{int}} = g_{ij} \overline{e_R^i} \phi^\dagger \cdot L^j + \text{h.c.}$$

$$g_{ij} = g \begin{pmatrix} 1 & 10^{-3} & 10^{-6} \\ 10^{-3} & 1 & 10^{-3} \\ 10^{-6} & 10^{-3} & 1 \end{pmatrix}$$

$$\mathcal{A}(l_i \rightarrow l_j \gamma) =$$

The first diagram shows an incoming lepton  $l_i$  and an outgoing lepton  $l_j$  connected by a fermion line. A scalar  $\phi^0$  is exchanged in the loop, with a fermion  $l_f$  running around it. A photon  $\gamma$  is emitted from the fermion line. The second diagram is similar, but the scalar is  $\phi^-$  and the fermion in the loop is  $\nu_f$ .



# The general approach – $SU(2)_L$ invariant models

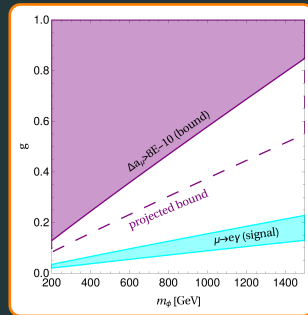
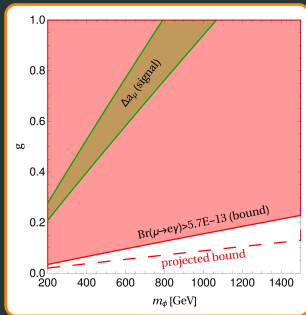
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$$\mathcal{A}(l_i \rightarrow l_j \gamma) = \text{[Diagram 1]} + \text{[Diagram 2]}$$

The diagrams show two Feynman diagrams for the process  $l_i \rightarrow l_j \gamma$ . The first diagram (left) shows a loop with a scalar  $\phi^0$  and a fermion  $l_f$ . The second diagram (right) shows a loop with a scalar  $\phi^-$  and a neutrino  $\nu_f$ .



# The general approach – $SU(2)_L$ invariant models

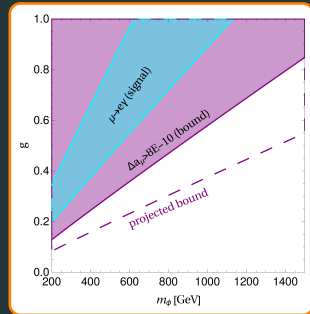
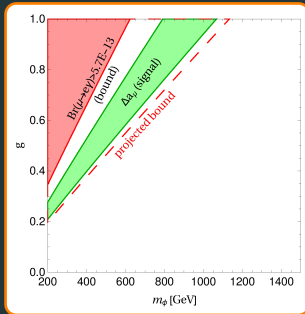
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$$g_{ij} = g \begin{pmatrix} 1 & 10^{-5} & 10^{-8} \\ 10^{-5} & 1 & 10^{-5} \\ 10^{-8} & 10^{-5} & 1 \end{pmatrix}$$

$$\mathcal{A}(l_i \rightarrow l_j \gamma) =$$

The first diagram shows an incoming lepton  $l_i$  and an outgoing lepton  $l_j$  connected by a dashed line representing a scalar  $\phi^0$ . A fermion  $l_f$  loop is attached to this line, with a wavy line representing a photon  $\gamma$  emitted from the loop. The second diagram is similar, but the scalar is  $\phi^-$  and the fermion is a neutrino  $\nu_f$ .





- Minimal Supersymmetric Standard Model (MSSM)
- Left-Right symmetric model
- Two-Higgs-doublet models
- Scotogenic model (radiative seesaw)
- Zee-Babu model
- $B - L$  gauge symmetry (also with inverse seesaw)
- $SU(3) \times SU(3) \times U(1)$  model
- $L_\mu - L_\tau$  gauge symmetry
- Dark Photon
- Seesaws

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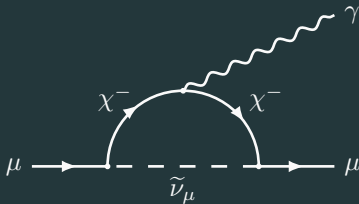
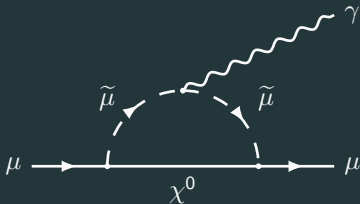
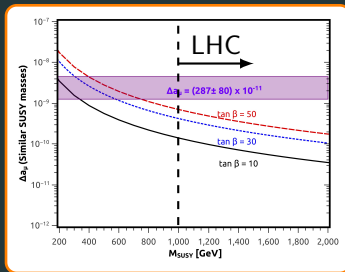
# MSSM - ( $g_\mu - 2$ )

SUSY: connects bosons & fermions  $\Rightarrow$  'doubling' of SM field content + 2<sup>nd</sup> Higgs

- Many contributions ( $\chi^0, \chi^\pm, \tilde{\mu}, \tilde{\nu}_\mu$ )
- Large viable parameter space, e.g.  $\tan \beta = \frac{\langle H_2 \rangle}{\langle H_1 \rangle}$

$\Rightarrow$  Make simplifying assumptions

similar SUSY masses  $\rightarrow$



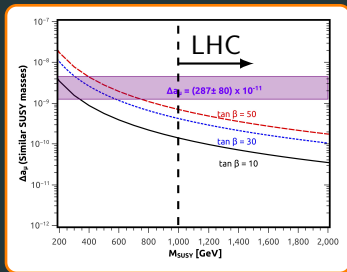
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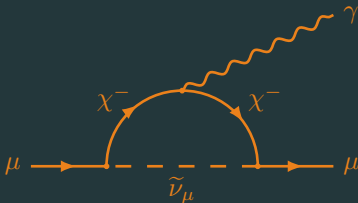
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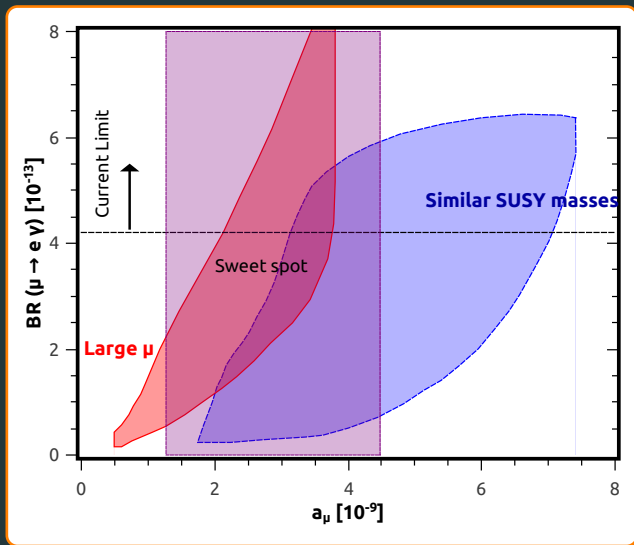


$$\begin{aligned} \Delta a_\mu^{\text{SUSY}} &\simeq \Delta a_\mu^{\chi^\pm} \\ &\simeq 10^{-9} \tan \beta \left( \frac{100 \text{ GeV}}{M_{\text{SUSY}}} \right)^2 \end{aligned}$$



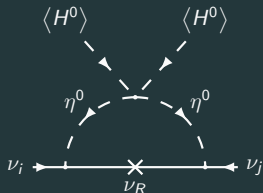
# MSSM – general discussion

parameter study:



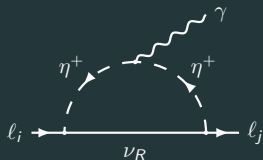
# Radiative seesaw model [Ma, 2006]

- 1-Loop-level  $\nu$ -mass generation
- 2<sup>nd</sup> inert Higgs doublet & RH  $\nu_R$
- $\nu$ -masses via DM interactions  
→ *scotogenic*
- $\mathcal{L}_{\text{Yuk}} = -y_\nu^{ij} \overline{\nu_{Ri}} \hat{\eta}^\dagger \cdot L_j + \text{h.c.}$



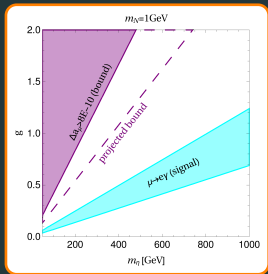
$$\Delta a_\mu < 0!$$

$$A_{\mu e}^{M/E} = \sum_i \frac{(y_\nu^\dagger y_\nu)_{\mu e}}{2(4\pi)^2} \frac{F\left(\frac{m_{\nu_R}}{m_{\eta^+}}\right)}{m_{\eta^+}^2}$$

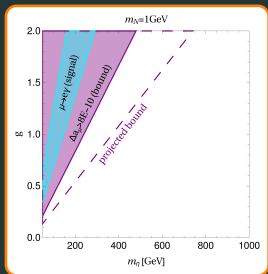
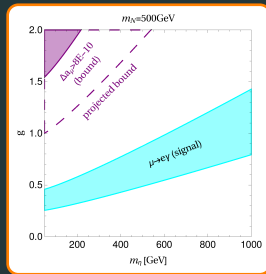


This model gives no viable explanation for the  $(g_\mu - 2)$  excess!

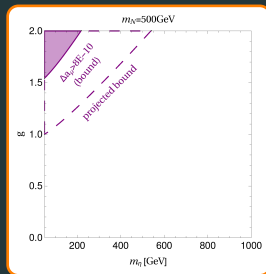
# Radiative seesaw – results



“mild hierarchy”

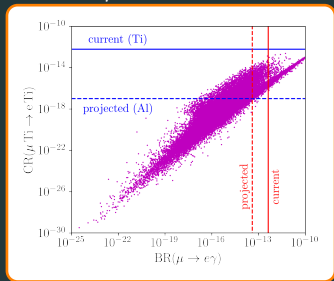


“strong hierarchy”



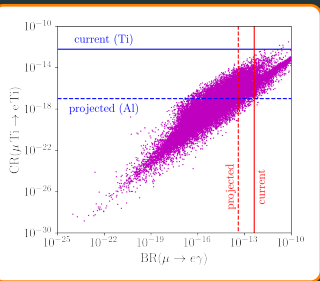
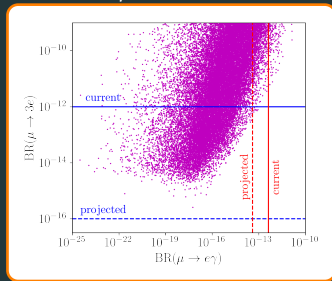
# Radiative seesaw – other LFV observables [Vicente, Yaguna, 2014]

$$\mu N \rightarrow e N$$

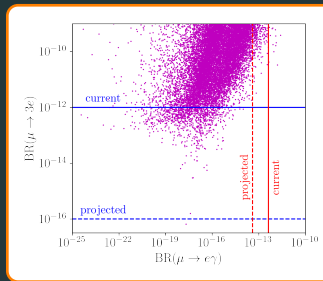


normal  
ordering

$$\mu \rightarrow 3e$$



inverted  
ordering





# Conclusions

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# What else is out there? Some recent ideas

A fair account:

## An extended gauge mediation for muon $(g - 2)$ explanation

Gautam Bhattacharyya, Tsutomu T. Yanagida, Norimi Yokozaki

(Submitted on 4 May 2018)

It is increasingly becoming difficult, within a broad class of supersymmetric models, to satisfactorily explain the discrepancy between the measured  $(g - 2)_\mu$  and its standard model prediction, and at the same time satisfy all the other constraints. In this paper we propose a new scheme of gauge mediation by introducing new soft supersymmetry breaking mass parameters for the Higgs sector in a minimal setup containing only a pair of  $(5 + \bar{5})$  messenger fields of  $SU(5)$ . This enables us to explain the  $(g - 2)_\mu$  discrepancy while avoiding all the existing constraints. We also provide possible dynamical origin of the new soft mass parameters. The wino and higgsino weighing below 500 GeV constitute the smoking gun signal at the (high luminosity) LHC.

- Use gauge structure to evade LFV
- However, light sleptons and gauginos strongly constrained by LHC
- extended soft SUSY breaking required

# What else is out there? Some recent ideas

## Post-Newtonian effects of Dirac particle in curved spacetime – III : the muon $g-2$ in the Earth's gravity

Takahiro Morishima, Toshifumi Futamase, Hirohiko M. Shimizu

(Submitted on 30 Jan 2018 (v1), last revised 11 Apr 2018 (this version, v2))

The general relativistic effects to the anomalous magnetic moment of muons moving in the Earth's gravitational field have been examined. The Dirac equation generalized to include the general relativity suggests the magnetic moment of fermions measured on the ground level is influenced by the Earth's gravitational field as  $\mu_m^{\text{eff}} \simeq (1 + 3\phi/c^2) \mu_m$ , where  $\mu_m$  is the magnetic moment in the flat spacetime and  $\phi = -GM/r$  is the Earth's gravitational potential. It implies that the muon anomalous magnetic moment measured on the Earth  $a_\mu \equiv g_\mu/2 - 1$  contains the gravitational correction of  $|a_\mu| \simeq 2.1 \times 10^{-9}$  in addition to the quantum radiative corrections. The gravitationally induced anomaly may affect the comparison between the theoretical and experimental values recently reported:  $a_{\mu(\text{EXP})} - a_{\mu(\text{SM})} = 28.8 (8.0) \times 10^{-10} (3.6 \sigma)$ . In this paper, the comparison between the theory and the experiment is examined by considering the influence of the spacetime curvature to the measurement on the muon  $g_\mu - 2$  experiment using the storage ring on the basis of the general relativity up to the post-Newtonian order of  $O(1/c^2)$ .

- Could gravity induce the shift in  $(g - 2)_\mu$  measurements?
- No flavor structure!
- However, . . .

# What else is out there? Some recent ideas

## Post-Newtonian particle physics in curved spacetime

[Matt Visser](#) (Victoria University of Wellington)

(Submitted on 2 Feb 2018)

In three very recent papers, (an initial paper by Morishima and Futamase, and two subsequent papers by Morishima, Futamase, and Shimizu), it has been argued that the observed experimental anomaly in the anomalous magnetic moment of the muon might be explained using general relativity. It is my melancholy duty to report that these articles are fundamentally flawed in that they fail to correctly implement the Einstein equivalence principle of general relativity. Insofar as one accepts the underlying logic behind these calculations (and so rejects general relativity) the claimed effect due to the Earth's gravity will be swamped by the effect due to Sun (by a factor of fifteen), and by the effect due to the Galaxy (by a factor of two thousand). In contrast, insofar as one accepts general relativity, then the claimed effect will be suppressed by an extra factor of  $[(\text{size of laboratory})/(\text{radius of Earth})]^2$ . Either way, the claimed effect is not compatible with explaining the observed experimental anomaly in the anomalous magnetic moment of the muon.

- The absolute potential is not physical due to the equivalence principle
- $\phi \sim \frac{M}{R} \Rightarrow \text{Sun} > \times 10, \text{ galaxy} > \times 2000$
- Finally, it was noted that . . .

## Can effective muon $g-2$ depend on the gravitational potential?

[H. Nikolic](#)

(Submitted on 12 Feb 2018)

Contrary to the claim in a series of recent papers, we show that it cannot. A source of the error in those papers is misinterpretation of coordinate time as a physical time.

# Summary

- LFV decays and  $(g - 2)$  are closely related
- Use one to constrain the other and reconcile potential signals with constraints
- Catalog of contributions to both processes ranging from simplified models,  $SU(2)_L$  invariant models to UV complete models
- Some recent ideas as to how evade LFV, while keeping  $(g - 2)_\mu$  sufficiently large
- Increasing tension between  $(g - 2)_\mu$  and other constraints (LHC)

# Summary Table

Model	$\Delta a_\mu > 0?$	LFV okay?
<b>MSSM</b>	✓ $M_{\text{SUSY}}^2 \sim \tan \beta (100 \text{ GeV})^2$	✓ (LHC: $M_{\text{SUSY}} \gtrsim 1 \text{ TeV}$ )
LR-symmetric	✓ $M_{W_R} \sim 100 \text{ GeV} \times g_R/g_L$	✓ $M_{W_R} \sim 5 \text{ TeV} \times g_R/g_L$
2HDM (type III)	✓	✓
<b>radiative Seesaw</b>	✗	✓
Zee-Babu	✗	✓ $m_{h,k} \gtrsim 1 \text{ TeV} \cdot \frac{ y }{0.1}$
gauged $B - L$	✓ (ruled out by LHC)	✓ (gauge)
with inverse seesaw	✓ (ruled out by LHC)	$ \sum_i U_{\mu\nu_R^i} U_{e\nu_R^i}  < 10^{-5}$
$SU(3)_c \times SU(3)_L \times U(1)_X$	✓ $M_{Z'} \sim 800 \text{ GeV}$	✓ $M_{Z'} \gtrsim 3 \text{ TeV}$
$L_\mu - L_\tau$	✓ $M_{Z'} \sim 600 \text{ GeV} \cdot g'$	$M_{Z'} \gtrsim 750 \text{ GeV} \cdot g'$
	✓ $M_{Z'} \lesssim 100 \text{ MeV}$ & $g' \sim 10^{-3}$	( $\nu$ trident) ✓ $M_{Z'} \lesssim 100 \text{ MeV} \cdot g'$
Dark photon	✓ (ruled out by kin. mix.)	✓ (no LFV)
seesaw type I	✓	✓
seesaw type II	✗	✓ $m_{\Delta^{++}} \gtrsim 500 \text{ GeV}$ , but LHC: $\gtrsim 2 \text{ TeV}$
seesaw type III	✓ (?)	✓ (LHC: $m_\Sigma \gtrsim 1 \text{ TeV}$ )

✓ = okay, ✓ = "okay, but. . ." , ✗ = not viable

# Thank You!

*How much money would you bet?*

**Backup slides**

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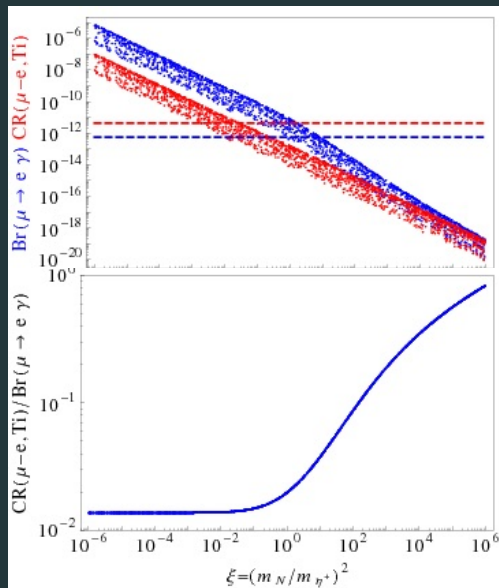
# The full photonic amplitude

Both on- and off-shell contributions:

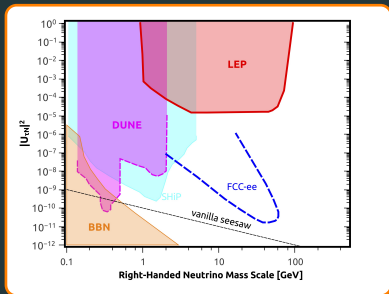
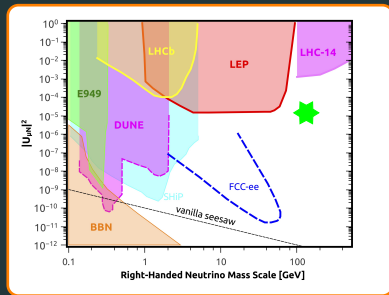
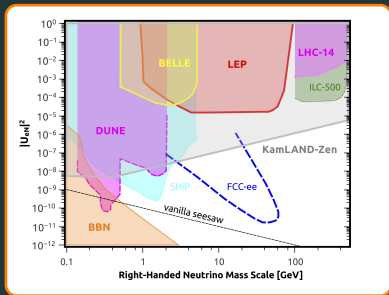
$$\mathcal{A}^{\text{photonic}} = -eA_{\mu}^*(q)\bar{u}_{\ell_j}(p_j) \left[ \overbrace{\left( f_{E0}^{ji}(q^2) + \gamma_5 f_{M0}^{ji}(q^2) \right)}^{\text{off-shell}} \left( \gamma^{\mu} - \frac{\not{q}q^{\mu}}{q^2} \right) + \right. \\ \left. + \underbrace{\left( f_{M1}^{ji}(q^2) + \gamma_5 f_{E1}^{ji}(q^2) \right)}_{\text{on-shell}} \frac{i\sigma^{\mu\nu}q_{\nu}}{m_i} \right] u_{\ell_i}(p_i).$$

- on-shell:  $\mu \rightarrow e\gamma$ ,  $g - 2$
- off-shell:  $\mu N \rightarrow eN$ ,  $\mu \rightarrow 3e$ , etc.

# Radiative seesaw – results from [Toma, Vicente, 2013]



# Type I seesaw bounds



★ = parameter point in agreement with all constraints and  $\Delta a_\mu = 288 \cdot 10^{-11}$